A Principal-Agent Model of Bidding Firms in Multi-Unit Auctions

Completed Research Paper

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Abstract

Principal-agent relationships in bidding firms are widespread in high-stakes auctions. Often only the agent has information about the value of the objects being sold. The board wants to maximize the profit, but the management wants to win the package with the highest value. In environments in which it is efficient for firms to coordinate on jointly winning packages, we show that the principals would coordinate, while the agents would not. We analyze environments with decreasing levels of information that the principal has about the valuations. Depending on the auction format it can be impossible to set budget constraints that align the agents' strategies in equilibrium. The analysis helps explain price wars in high-stakes auctions.

Keywords: Auction, Electronic Marketplace, Principal-Agent Model

Introduction

The IS literature has always been concerned with models about bidder behavior on markets. Many IS contributions have analyzed behavior in online or multi-item auctions that deviate from traditional game-theoretical assumptions (Adomavicius and Gupta 2005; Bapna et al. 2000; Bapna et al. 2003). The design of markets and the development of adequate market models has become a significant IS research stream contributing to the interdisciplinary field of market design (Roth 2008), a field that was honored with the Nobel Memorial Prize in Economic Sciences in 2012. In this paper, we suggest to modify a fundamental assumption in traditional models of auctions, that of a single payoff-maximizing bidder. This assumption has been central for the entire literature on market design in IS and beyond.

Payoff maximization with quasilinear utility functions is a standard assumption in auction theory that allows for dominant strategy incentive compatibility via the well-known Vickrey-Clarke-Groves mechanism. Unfortunately, the assumption of a quasilinear utility function is often violated in the field. In particular, budget constraints have received significant attention in recent years (Brusco and Lopomo 2009; Burkett 2015; Che and Gale 2000; Dobzinski et al. 2012; Engelbrecht-Wiggans 1987; Malakhov and Vohra 2008; Shapiro et al. 2013). Typically, the literature assumes exogenous budget constraints often motivated as the result of imperfect capital markets and agency problems (Che and Gale 1998). Indeed, principal-agent relationships are widespread in bidding firms participating in high-stakes auctions. Engelbrecht-Wiggans (1987) discusses such relationships with regard to the auctions of mineral leases, defense systems, and construction contracts. It is also well-known that such relationships between the supervisory board and the bidding team or management of a telecom play an important role in spectrum auctions (Chakravorti et al. 1995; Shapiro et al. 2013).

We model a related but complementary environment with multi-unit auctions and focus on hidden information about the valuations in symmetric principal-agent pairings. In this model, agents have precise information about the valuations of the goods, but principals do not. Unlike Burkett (2015), the types of manipulation for agents in multi-object auctions are quite different to single-dimensional environments which leads to different types of inefficiency. We show that the information asymmetry and the different preferences result in an agency dilemma that is difficult to resolve.

Information Asymmetries in the Bidding Firm

Let us first motivate the information asymmetries in principal-agent relationships of bidding firms by looking at the relationship between the supervisory board (principal) and the management (agent) of a telecom in spectrum auctions. Similar relationships arise between the management of a multinational telecom and the management of a national subsidiary bidding in the auction. The payments made by telecommunication firms in spectrum auctions are often billions of dollars, and thus the management cannot cover the cost of the auction. The agent in these environments has limited liability and the principal has to pay in the auction.

In spectrum auctions, firms have preferences over different packages of spectrum licenses. Each of these packages can be assigned a business case with a net present value. The management knows the market best, it knows the technology, the competition, and the end consumer market, and so they can compute business cases that allow for a good estimate of the net present value of each package. The board of directors does not have this information, and the management has no incentive to reveal it. Principals cannot force agents to be truthful and often only have analyst estimates that typically have an enormous variance. Bulow et al. (2009) writes that “Prior to the AWS auction, analyst estimates of auction revenue ranged from $7 to $15 billion. For the recent 700 MHz auction, they varied over an enormous range – from $10 to $30 billion.” This is by no means an exception and estimates of investment banks and other external observers can be quite different from the actual revenue of the auction.[1] Sometimes external consultants are hired to provide the principal with more information about the value of the licenses.

In our model, the agent is not a shareholder of the company and does not internalize the price in his utility function. The shares for the management would need to be significant to set profit-maximizing incentives in our scenario, thus making the agent a shareholder is not a viable option. He wants to win packages with high values because he indirectly benefits from the respective large business cases. For example if the management acquires more valuable spectrum at a lower profit, it would be in a better strategic position in the local market. Furthermore, the value of the local subsidiary within the multinational increases and gives the local management more weight. Empire-building motives are a widespread reason for such behavior on the part of agents in the principal-agent literature (Jensen 1986). Spectrum auctions are only one example of high-stakes auctions in which such empire-building motives matter. Engelbrecht-Wiggans (1987) discusses other auction applications with agency dilemmas and writes “in bidding for mineral leases, a firm may wish to maximize expected profits while its bidder feels it should maximize the firm's proven reserves." Actually, payoff maximization is hard to defend for an agent in such high-stakes auctions in which agents typically try to “win within budget.” Contrary, value maximization is a good approximation of agent motives.

The principal would like to maximize profit, but he typically does not have enough information about the valuations and the competitors’ preferences to bid on his own. In spectrum auctions, the principal will not learn the true valuations of the licenses, as the future profits of the firm depend on many other decisions, so that it is also not possible for the principal to understand the impact of winning a specific package in an auction ex post. This has also been observed in other markets (Engelbrecht-Wiggans 1987).

There are a number of parallels to the literature on optimal delegation in which an uninformed principal delegates decision rights to an informed but biased agent. Holmström (1977) already showed that the optimal mechanism for the principal when utility is not transferable consists of choosing a subset of actions, from among which the agent is allowed to pick the most desired one. In our principal-agent relationship without asymmetric information in Section “The Limits of Budget Constraints”, we focus on *delegation sets*, in particular, budget constraints as a second-best solution to overcome the agency problem and to implement the principal’s equilibrium strategy. Alonso and Matouschek (2008) write that “capital budgeting rules typically put limits on the size of the investments that managers can decide on,

[1] Prior to the German spectrum auction in 2010, most analysts expected low revenue (Berenberg Bank estimated €1.67 bn., the LBBW bank estimated €2.1 bn.). The actual revenue from the auction was €5 bn.

but do not link their pay directly to the investments they recommend." This is similar in spectrum auctions.

Contributions

We develop a principal-agent model of a firm participating in a multi-unit auction in which the agent has hidden information about the valuations of the goods. The principal tries to screen the agent optimally with budget constraints to overcome the adverse selection problem arising in this information environment.[2] We focus on package auctions in our analysis due to their relevance in spectrum auctions nowadays and because they allow bidders to express general preferences without presenting an exposure problem. For example, France (2011)[3] and Norway (2013)[4] used a first-price sealed-bid package auction, whereas Romania (2012) used an ascending combinatorial clock auction. The ascending package auction model can also be seen as an abstraction of the two-stage combinatorial clock auction that has been utilized in many countries worldwide (Cramton 2013). Combinatorial auctions have also been used in other domains, such as the procurement of bus services for bus routes in London (Cantillon and Pesendorfer 2006), of school meals in Chile (Olivares et al. 2012), and of raw materials at Mars, Inc. (Bichler et al. 2006).

Interesting strategic problems of principal-agent relationships within the bidding firms arise in multi-unit or more generally multi-object auctions. At this point, there is neither a Bayesian Nash equilibrium analysis of wide-spread multi-unit auction formats nor of multi-unit package auction formats when bidders have general valuations. In our environment, however, we can derive Bayesian Nash equilibria for quasi-linear principals in the package auction which also makes a contribution to the literature on package auctions.

First, in the next section we describe the environment formally as the principal-agent $m \times n$ package auction model in which n bidders compete for m units of a homogeneous good.

Next, in Section "The Agency Dilemma" we analyze the different auction formats as Bayesian games in order to analyze the agency problems that arise. The pivotal example is a 2×2 package auction with two units and two bidders in which it is ex-ante common knowledge that the solution with both bidders winning one unit each is efficient. In many auction markets, bidders actually have a good understanding about the type of efficiency in a market, and we refer to this environment as *dual-winner efficiency* (*DWE*). We emphasize this environment in the article over other efficiency environments not only because it is practically relevant but because it highlights the pivotal strategic problem in the principal-agent 2×2 package auction model: bidding firms need to coordinate in the efficient solution, but agents do not, leading to an agency dilemma and inefficiency. It is also straightforward to find similar strategic situations in larger markets in which a bidder could win a larger, more valuable package by not bidding on a smaller package. Such problems only arise in multi-object auctions, but not in single-object markets.

To demonstrate the agency bias in the symmetric information environment, we derive equilibrium bidding strategies of value-maximizing agents, and show that the agent does not bid on a single-unit package in the unique ex-post Nash equilibrium. This equilibrium bidding strategy is independent of the package auction format. The result is surprising because in equilibrium there is a risk that the agent will win nothing. It is also independent of the efficiency in the market and the risk attitude of the agent.

Then we analyze the equilibrium bidding strategies of quasilinear principals in case they had exact information about the valuations of the firm and their prior distributions. Given ex-ante common knowledge about the *DWE* environment of the bidders, we can derive equilibrium bidding strategies. We show that coordination on the dual-winner outcome actually constitutes a Bayesian Nash equilibrium for a risk-neutral principal in a first-price sealed-bid package auction, and we characterize environments in

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[2] Che and Gale (1998) identify moral hazard as a reason for the use of budget constraints to incentivize purchasing departments in organizations. Our results also hold in a hidden action environment in which the principal sets a budget constraint as a function of the valuations, but cannot observe bids.

[3] http://www.arcep.fr/fileadmin/reprise/dossiers/4G/proj-dec-appel-800mhz-160511.pdf

[4] http://eng.nkom.no/technical/frequency-auctions/auctions/planned-completed-auctions/_attachment/9106

which the *dual-winner equilibrium* is payoff-dominant for the principal compared to a *single-winner equilibrium*. A similar result, an ex-post equilibrium to coordinate, is provided for the ascending package auction. As a consequence, we have adverse selection in our principal agent model.

In Section "The Limits of Budget Constraints" we analyze the possibilities for the principal to implement his equilibrium strategy with a delegation set, i.e., budgets constraining the agent. One might think that whether the principal can implement his strategy with the agent or not crucially depends on the level of information he has about the valuations. If the principal knew the valuations ex interim, one might think that he can always dictate the agent his equilibrium bids in order to coordinate with other firms on a dual-winner outcome. However, we show that even without hidden information there cannot always be a budget constraint that sets incentives for the agents to bid their budget truthfully and at the same time is in equilibrium for the principal in the first-price sealed-bid auction. In the ascending package auction this is easier: the principals can set the budget for the package of two units to zero and the budget constraint for the single-unit package equal to the corresponding valuation. However, the principal would need to know at least that there is DWE in the market which is already a strong assumption. Still, this provides further support for the value of ascending auctions even in an independent private values model. Most IS literature on package auctions has focused on ascending auctions and the information feedback provided in these auctions (Adomavicius and Gupta 2005; Bichler et al. 2010; Scheffel et al. 2011).

Finally, we present some extensions of our model to general second-price sealed-bid package auctions mechanism and show that they are outcome equivalent to the ascending package auction in our benchmark model of two bidders and two units. Moreover, we demonstrate how the main findings of our model help to analyze larger $m \times n$ markets. In the concluding section we provide examples of how the model can explain developments in the field.

The Model

In our model we consider n ex-ante symmetric firms $i, j \in I = \{1, \dots, n\}$ competing in a multi-object package auction for m units $l \in L = \{1, \dots, m\}$ of a homogeneous good. We denote this setting as $m \times n$ *principal-agent package auction model*. Let us first outline the auctions, before discussing the payoff environment.

The Auctions

We focus on different multi-unit package auction formats which allow bidders to submit multiple all-or-nothing package bids. In multi-unit package auctions, each bidder $i \leq n$ submits an all-or-nothing bid for every package. Here, each package is identified by the number of units, $l \leq m$, it contains. We assume an XOR bid language because it is the most general bid language allowing the expression of complements and substitutes (Nisan 2006), and it is regularly used in spectrum auctions. After the bidding has ended, the risk-neutral auctioneer selects the revenue-maximizing combination of package bids. Due to the XOR bidding language, every bidder can win one package at most. The XOR language is the most expressive language and it avoids exposure problems for the bidder. For example, a bidder might bid on one and a package of two items, but not want to win three items. We assume that the auctioneer has zero reservation prices for the units.

In the first-price sealed-bid (FPSB) package auction, all n bidders simultaneously submit their package bids to the auctioneer without knowing the opponents' bids. The ascending auction is modeled as iBundle (Parkes and Ungar 2000) which is efficient if the bidders reveal their demand correspondence in each round, i.e., they bid straightforward. In this auction, bidders observe discriminatory and non-linear prices for each package after each round. In each round, every bidder can place bids (reveal demand) on one or more packages. After the round, the auctioneer determines the winning allocation. The auction ends if there are no new bids. For all bundle bids from bidders not receiving a bundle, the ask price is the last bid price plus a minimum bid increment. If every bidder bids on his payoff-maximizing packages in each round, then this auction will end in a competitive equilibrium (Bikhchandani and Ostroy 2002).

We focus on FPSB and ascending package auctions in the paper, however, it is straightforward to analyze the VCG mechanism with our model as is done in the Section "Extensions". The VCG mechanism is the extension of the well-known second-price sealed-bid (Vickrey) auction for the sale of a single bundle to any combinatorial auction setting. For our benchmark model we demonstrate equivalence for the optimal delegation mechanism between the VCG mechanism and the ascending package auction.

The Payoff Environment

Each agent i learns ex-interim package values $v_i(l) \in V \subseteq \mathbb{R} \; \forall \, l \in L$ and $v_i(l)$ is the net present value of firm i for the bundle of l units. [5] These values are unknown to the principal. The vector $v_i = [v_i(1), \dots, v_i(m)] \in \mathbb{R}^m$ contains the value draws for all m packages of any agent i. Let us define the valuation vectors of all agents other than i as v_{-i}. We assume value draws that are strictly increasing in the number of units for any firm i: $v_i(l) < v_i(l')$ with $l < l' \; \forall \, l, l' \in L$. The values are normalized with $v_i(0) = 0 \; \forall \, i \in I$.

Given the standard symmetry assumption, each firm i's vector of valuation draws v_i is a priori distributed according to a strictly monotonically increasing joint cumulative distribution function $\mathrm{F}(v_i)\colon \mathbb{R}^m \to \mathbb{R}$ with corresponding marginal distribution function for value $v_i(l)$ of the form $F_l\big(v_i(l)\big)\colon \mathbb{R} \to \mathbb{R}$. The marginal distribution functions $F_l\big(v_i(l)\big)$ and $F_{l'}\big(v_i(l')\big)$, with $l' = l + 1 \; \forall \, l, l' \in L$, have support of $[\underline{v}(l), \overline{v}(l)]$ with $\underline{v}(l) < \overline{v}(l)$ and $[\underline{v}(l'), \overline{v}(l')]$ with $\underline{v}(l') < \overline{v}(l')$, respectively. The relationship between the different supports is $\overline{v}(l) < \overline{v}(l')$ and $\underline{v}(l) < \underline{v}(l')$.

In every firm $i \in I$, the principal provides the agent with monetary budget constraint(s) (i.e., budget(s)) and the agent places bids in the auction on behalf of the principal. The values of the packages are private information to each agent, whereas the corresponding probability distributions are ex ante common knowledge among agents participating in the auction. Let agent i's bid on a bundle of l units be denoted by $\beta_i(l) \in \mathbb{R} \; \forall \, l \in L$ which results in a vector of package bids on all m units of $\beta_i = [\beta_i(1), \dots, \beta_i(m)] \in \mathbb{R}^{\mathrm{m}}$. We define the bid vectors of all agents other than i as β_{-i}. In our *principal-agent package auction model* illustrated in **Figure 1**, the risk-neutral principal wants to maximize expected profit in which his profit of winning any package of l units is given by $\pi_i(l) = v_i(l) - \beta_i(l)$.

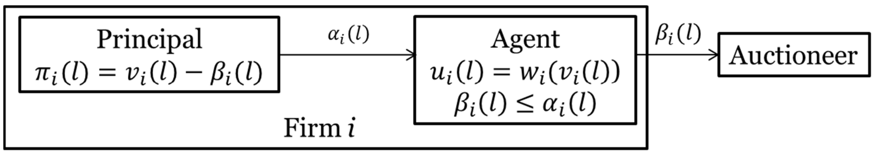

Figure 1. Illustration of the principal-agent model of bidding firms

The principal determines a budget constraint for the package of l units of $\alpha_i(l)$, and we refer to the vector of all package-dependent budget constraints for any firm i as $\alpha_i = [\alpha_i(1), \dots, \alpha_i(m)] \in \mathbb{R}^{\mathrm{m}}$.

The agent's gross utility includes his value-maximizing motives and is modeled by a Bernoulli utility function of winning a package of l units $u_i(l) = w\big(v_i(l)\big)$ with $w\colon \mathbb{R} \to \mathbb{R}$ being strictly increasing and continuous in package value $v_i(l)$. The agent independently decides on the amount of money bid on each package. As long as the payment for a bundle of l units is weakly lower than his respective monetary budget constraint of $\alpha_i(l)$, the agent obtains a utility of $w_i\big(v_i(l)\big)$. Any bid $\beta_i(l) > \alpha_i(l)$ is an unacceptable action for him, as he will be fired, for example if payments exceed the budget constraint.

The overall market with principals, agents, and the auctioneer is modeled as a sequential game of incomplete information. From firm i's point of view, its opponent's values and budget constraints correspond to unknown random variables, i.e., their types. Furthermore, this game consists of two subsequent stages. First, nature determines every agent i's vector of package valuation draws v_i. In *stage 1*, each principal decides on a vector of budget constraints α_i with which to provide his agent. In *stage 2*,

[5] In reality, agents only have estimates of the net present values too, but still, these are more precise estimates of his license values. In our model, we assume that the agent has exact information about the firm's private values ex interim, as is customary in most spectrum auction models.

agents compete against each other and decide on a vector of bids β_i in the auction. The risk-neutral auctioneer selects the revenue maximizing set of packages. We apply backward induction starting at the second stage and then continue with the first stage to examine Perfect Bayesian equilibria of this game.

In our analysis we focus on the *principal-agent* 2×2 *package auction model* with two bidders and two units. This simple market helps to highlight the differences in strategies of principals and agents, and the potential contracts necessary to overcome the adverse selection problem that arises.

The Agency Dilemma

We first analyze the agents' equilibrium strategies in stage 2 of the *principal-agent* 2×2 *package auction model*, before we derive equilibria of the profit-maximizing principals. The analysis illustrates the adverse selection problem arising between principal and agent.

Equilibrium Bidding Strategies of the Agent

In our analysis, we assume the agents do not bid beyond their valuation. This "no overbidding" assumption helps us understand the strategic bias of the agent and it is frequently made in the literature (Bhawalkar and Roughgarden 2011; Caragiannis et al. 2011; Leme and Tardos 2010; Lucier and Borodin 2010). It is a reasonable assumption because although the agent (management) does not internalize prices in its utility it is very likely to care about the firm not making losses and going bankrupt.[6] Alternatively, one can assume budget constraints that are increasing functions of the package valuations.[7] There will always be some budget constraint such that the agent cannot bid infinity, and the following analysis only serves to understand the bias of the agent and how his equilibrium bidding strategy differs from that of the principal. Understanding this bias is important for Section „The Limits of Budget Constraints" in which we aim to set optimal budget constraints. We first derive two useful lemmata for $2 \times n$ package auctions with two units and $n > 1$ bidders.

Lemma 1: *It is a weakly dominant action for any agent i to submit a bid in the amount of his valuation on the two-unit package in the second stage of the principal-agent $2 \times n$ FPSB package auction model: $\beta_i(2) = v_i(2)$.*

Proof: For opponents' fixed bids β_{-i}, suppose agent i bids strictly less than his valuation on the bundle of 2 units. Increasing his bid to $\beta_i(2) = v_i(2)$ does not reduce the agent's payoff when winning but strictly raises the probability of winning the respective bundle. This comes at the cost of proportionately lowering the chances to win the one-unit package. However, as valuations are strictly increasing in the number of units, $v_i(2) > v_i(1)$, and the utility function $w(\cdot)$ is strictly increasing in the package value $v_i(l)$ the smaller package offers less utility than the larger package. Submitting a bid in excess of his valuation on the two-unit package cannot be optimal for any agent, as it is an unacceptable action. **QED.**

Lemma 1 can be generalized in the following way: Any agent faces the weakly dominant action of submitting a bid in the amount of his valuation on the largest package in the second stage of the *principal-agent* $m \times n$ *FPSB package auction model*. Moreover, it is a weakly dominant strategy for an agent to bid his entire valuation if all units of the $m \times n$ *FPSB auction* are sold as a single package.

Lemma 2: *An agent i's best response on the one-unit package must either be zero or equal to his valuation in the second stage of the principal-agent $2 \times n$ FPSB package auction model: $\beta_i(1) \in \{0, v_i(1)\}$.*

Proof: Any single-unit bid of agent i, $\beta_i(1)$, from the range of $(0, v_i(1)]$ does not affect payoff when winning the one-unit bundle. He receives a utility of $w(v_i(1))$. For his opponents' fixed bids β_{-i}, a bid of

[6] One could also assume the principal to be informed about losses ex-post and having to fire the agent in this case.

[7] In this symmetric information environment the agents are restricted by symmetric budgets of $\alpha_i(l) = \alpha(v_i(l))$ in which $\alpha: \mathbb{R} \to \mathbb{R}$ is a strictly increasing and continuous function in the package value for l units, $v_i(l)$. Furthermore, we define $\alpha(0) = 0$. Note that the "no overbidding" assumption simply corresponds to a special case of these budget constraints with $\alpha_i(l) = v_i(l)$. The budget constraint is still strictly increasing and continuous in the package valuation.

$\beta_i(1) = v_i(1)$ (0), however, maximizes (minimizes) the probability of coordinating on the small bundle. It thereby minimizes (maximizes) the chances of winning the two-unit package. Any bid from within the range of $(0, v_i(1))$ neither maximizes nor minimizes the probability of obtaining the one-unit package and therefore cannot be optimal. Again, any bid greater than the valuation is an unacceptable action. **QED.**

Lemma 2 can be generalized as follows.: For an agent i's best response on a package of l units, with $l < m$, it must be true that $\beta_i(l) \in \{0, v_i(l)\}$ in the second stage of the *principal-agent* $m \times n$ *FPSB package auction*. As in the $2 \times n$ setting, any bid from the range of $(0, v_i(1))$ cannot be a best response as it neither maximizes nor minimizes the probability of winning the respective package and all bids from the range of $(0, v_i(1)]$ do not affect payoff anyway.

Note that **Lemma 1** and **Lemma 2** also hold for the second stage of the *principal-agent* $2 \times n$ *ascending package auction model* in a slightly modified way. To use identical notation for both auction formats, let us denote the highest package price for l units that bidder i is willing to accept as $\beta_i(l)$. Then, it is a weakly dominant action for any agent i to remain active for the package of two units until its price reaches his valuation: $\beta_{\text{i}}(2) = v_i(2)$. Furthermore, any agent i's optimal action for the single-unit package is either to not bid on the package at all, or to remain active until he is winning or its price reaches his corresponding valuation: $\beta_i(1) = v_i(1)$.

FPSB Package Auction

With these findings, we can now derive the agents' equilibrium strategies in both formats of the second stage of the *principal-agent* 2×2 *package auction model*. The analysis of this market is tractable, and the same strategic problems can be found in larger markets with more bidders. Our first observation for the FPSB package auction is that agents would not coordinate in equilibrium.

Theorem 1: *It is the unique symmetric ex-post equilibrium for any agent i to submit a vector of bids* $\beta_i = [0, v_i(2)]$ *in the second stage of the principal-agent* 2×2 *FPSB package auction model for any vector of package values* v_i.

Proof: According to **Lemma 1** and **Lemma 2** it must be true for an agent i that $\beta_i(2) = v_i(2)$ and $\beta_i(1) \in \{0, v_i(1)\}$. The same must hold for opponent j correspondingly. WLOG let us first assume the sum of both agents' single-unit valuations always strictly exceeds each agent's valuation for the package of two units. We are now able to construct the following payoff matrix for agent i in **Table 1**, to determine his optimal choice of $\beta_i(1)$.

Payoff matrix for agent *i*		Agent j	
		$\beta_j(1) = v_j(1)$	$\beta_j(1) = 0$
Agent i	$\beta_i(1) = v_i(1)$	$w(v_i(1))$	$w(v_i(2)) \cdot F_2(v_i(2))$
	$\beta_i(1) = 0$	$w(v_i(2)) \cdot F_2(v_i(2))$	$w(v_i(2)) \cdot F_2(v_i(2))$

Table 1. Payoff matrix for agent i in the 2×2 *FPSB package auction*

Suppose both agents submit bids in the amount of their valuations for one unit. Each agent wins a single unit, and agent i obtains a utility of $w(v_i(1))$ with certainty. By assumption, the sum of both agents' single-unit package valuations strictly exceeds each agent's valuation for two units. Now consider all other bid combinations in which at least one agent does not bid on the single-unit package. Here, the sum of both agents' single-unit bids cannot exceed each agent's bid on the bundle of two units. Thus, only the agents' bids on two units compete against each other. Agent i receives a utility of $w(v_i(2))$ only if his valuation exceeds opponent j's valuation: $v_i(2) > v_j(2)$. This occurs with probability $F_2(v_i(2))$.

Assume opponent j randomizes for his bid on the single unit between the elements of $\{0, v_j(1)\}$ with some fixed probabilities. Using **Table 1**, agent i's expected utility of bidding his valuation on one unit exceeds the expected utility of not bidding on one unit if inequality (I) holds:

$$w\big(v_i(1)\big) \geq w\big(v_i(2)\big) \cdot F_2\big(v_i(2)\big) \qquad \textbf{(I)}$$

Keep in mind that the probabilities cancel out. Note that if $v_i(2) = \overline{v}(2)$, inequality **(I)** reverses to $w(v_i(1)) < w(\overline{v}(2))\ \forall\ v_i(1) \in [\underline{v}(1), \overline{v}(1)]$ because $F_2(\overline{v}(2)) = 1$ and $\overline{v}(1) < \overline{v}(2)$. Moreover, the product of $w(\cdot)$ and $F_2(\cdot)$ is monotonically increasing in the package value by assumption. Thus, inequality **(I)** cannot hold for any two-unit package value of $v_i(2) \in (\dot{v}(2), \overline{v}(2)]$ for which $\dot{v}(2)$ is defined by $w(\overline{v}(1)) = w(\dot{v}(2)) \cdot F_2(\dot{v}(2))$. Any agent i with value for two units of $v_i(2) > \dot{v}(2)$ never bids on one unit.

Similarly, opponent j can only bid his valuation on one unit if he has a valuation of $v_j(2) \leq \dot{v}(2)$. If this condition is not given, then he will not coordinate and inequality **(I)** cannot hold. Adjusting agent i's beliefs in **Table 1** accordingly, his expected utility of bidding his valuation exceeds the expected utility of not bidding on one unit if inequality **(II)** is true in which the probability of agent i winning is now conditional on opponent j having a value of $v_j(2) \leq \dot{v}(2)$:

$$w(v_i(1)) \geq w(v_i(2)) \cdot F_2(v_i(2) | v_j(2) \leq \dot{v}(2)) \qquad \textbf{(II)}$$

Assume agent i to have the threshold package value for two units of $v_i(2) = \dot{v}(2)$, as defined above, and some corresponding valuation for one unit of $\dot{v}(1)$. Then, inequality **(II)** becomes $w(\dot{v}(1)) \geq w(\dot{v}(2))$ because $F_2(\dot{v}(2) | v_j(2) \leq \dot{v}(2)) = 1$. This again contradicts the assumption of strictly monotonically increasing net present values in the number of units as $\dot{v}(1) < \dot{v}(2)$. As this reasoning is true for any value $\dot{v}(2) \in [\underline{v}(2), \overline{v}(2)]$, full shading on one unit is optimal for any vector of valuations v_i.

Dropping the assumption that the sum of both agents' single-unit package valuations strictly exceeds each agent's valuation for two units would result in expected utility weakly smaller than $w\big(v_i(1)\big)$ for agent i if both agents submit their single-unit valuations truthfully. As it is not optimal to bid on one unit in the case of receiving a utility of $w\big(v_i(1)\big)$ with certainty, it cannot be optimal if receiving an expected utility weakly smaller than $w\big(v_i(1)\big)$. Note that in this equilibrium no agent i would want to deviate from his equilibrium strategy $\beta_i = [0, v_i(2)]$, even if he knew the opponent's type. Therefore, the proposed equilibrium strategy constitutes the unique ex-post equilibrium in the second stage of the *principal-agent* 2×2 *FPSB package auction model*. **QED.**

Intuitively, any agent i's opponent j would only be willing to coordinate on one unit if his valuation for two units was low. In this case, however, it would be a best response for bidder i not to coordinate, but try to win the package of both units independent of both agents' actual values. We now show that agents do not bid on a single unit in the second stage of the *principal-agent* 2×2 *ascending package auction model*, either.

Ascending Package Auction

Let us first introduce an adapted definition of *straightforward bidding* for agents in the second stage of the *principal-agent* 2×2 *ascending package auction model.*

Definition 1: A *straightforward bidding* strategy for agents in the second stage of the *principal-agent* $m \times n$ *ascending package auction model* is defined as follows: an agent begins to bid on the most valuable package and remains active until he is overbid. As long as he is winning, he does not bid for a smaller package. If he is overbid, he starts to bid for the next smaller package and again remains active until he is overbid. This process continues iteratively for all next smaller packages.

We now employ this definition to proof the following **Theorem 2**.

Theorem 2: *In the second stage of the principal-agent* 2×2 *ascending package auction model, straightforward bidding constitutes an ex-post equilibrium. In this equilibrium the agent with the highest valuation for two units does not get active on one unit.*

Proof: Lemma 1 implies that both agents start bidding on the two-unit package immediately and remain active until their valuations for two units are reached. According to **Lemma 2**, each bidder has to decide whether to get active on one unit or not. WLOG, assume agent i is the last remaining active bidder on the package of two units and suppose he decides to bid for the single unit. If opponent j gets active on this unit as well the sum of both agents' single-unit prices might eventually exceed bidder i's valuation for two units. This cannot happen if agent i does not bid on one unit and therefore it is a weakly dominant action for bidder i not to get active on the single unit. Remember, agent i strictly prefers two units to one unit. Given this behavior, opponent j is in fact indifferent between becoming active and not bidding on the single-unit package, as he cannot win anyway.

Even knowing the opponent's type, no agent can benefit by deviating from his equilibrium strategy. Thus, straightforward bidding constitutes an ex-post equilibrium in the second stage of the *principal-agent* 2×2 *ascending package auction model*. **QED.**

Similar to **Theorem 1** of the FPSB package auction, **Theorem 2** describes an ex-post equilibrium that is robust against risk aversion[8]. In both auction formats, agents never coordinate on winning one unit each. Moreover, this result is completely independent of the efficiency environment. The analysis shows that in the second stage of the *principal-agent* 2×2 *package auction model*, agents will bid their valuation on the large package and nothing on the smaller package in equilibrium. Let us now focus on the principals' bidding behavior by abstracting from the agents.

Equilibrium Bidding Strategies of the Principal

In this subsection, we analyze how a quasilinear principal would bid in equilibrium if he had perfect knowledge about his firm's valuations and prior type distributions. This will provide a baseline to compare the strategies of the agent against. Unfortunately, there is no closed-form equilibrium bid function for package auctions. However, we derive conditions under which profit-maximizing principals prefer to coordinate on winning one unit each (dual-winner outcome) in the 2×2 *package auction*, assuming bidders have public information about *DWE*. With *DWE*, it is always efficient to have the dual-winner outcome: $v_i(1) + v_j(1) - max\{v_i(2), v_j(2)\} > 0$ for all $v_i(1), v_j(1) \in [\underline{v}(1), \overline{v}(1)]$ and $v_i(2), v_j(2) \in [\underline{v}(2), \overline{v}(2)]$. The inequality $\overline{v}(2) < 2 \cdot \underline{v}(1)$ ensures *DWE*, independent from the realization of valuations. This environment is strategically interesting because bidders need to coordinate on the efficient solution. With this assumption, we can derive Bayesian Nash equilibrium strategies similar to earlier work on split-award auctions (Anton and Yao 1992).

Theorem 3 and **Theorem 6** show that a dual-winner outcome can be supported as equilibrium for all possible valuations $v_i(1), v_j(1) \in [\underline{v}(1), \overline{v}(1)]$ and $v_i(2), v_j(2) \in [\underline{v}(2), \overline{v}(2)]$ in the case of *DWE* in the FPSB and ascending package auction, respectively. **Theorem 4** and **Theorem 7** derive conditions under which the single-winner outcome constitutes an equilibrium for all possible valuations $v_i(1), v_j(1) \in [\underline{v}(1), \overline{v}(1)]$ and $v_i(2), v_j(2) \in [\underline{v}(2), \overline{v}(2)]$ in *DWE*. We refer to these two types of equilibria as *dual-winner equilibrium* and *single-winner equilibrium*, respectively. Finally, **Theorem 5** and **Theorem 8** establish equilibrium selection criteria for both auction formats in terms of payoff dominance.

FPSB Package Auction

We assume quasilinear bidders (the principals) to be symmetric, and bidder i's optimal bid on a bundle of l units $\beta_i(l)$ is a strictly increasing and continuous function in his valuation for this package $v_i(l)$: $\beta_i(l) = \beta(v_i(l))$.

Theorem 3: *Any bidder i's vector of bids* $\beta_i = [\beta(1), \beta_i(2)]$ *is a symmetric dual-winner equilibrium in a* 2×2 *FPSB package auction if the following conditions hold:*

(1.) $\overline{v}(2) < 2 \cdot \underline{v}(1)$

(2.) $\beta_i(1)$ *is constant over* $v_i(1) \in [\underline{v}(1), \overline{v}(1)]$ *and denoted by* $\beta_i(1) = \beta(1)$ *for all* $i \in I$

(3.) $\beta(1) \in [\overline{v}(2) - \underline{v}(1), \underline{v}(1)]$

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[8] We refer to risk aversion as is implied by a concave utility function $w(\cdot)$ over possible outcomes of the auction (lottery) $\{0, v_i(1), v_i(2)\}$ for some agent i. Regarding **Theorem 1** and **Theorem 2** one could expect this risk averse agent to prefer the certain dual-winner outcome with utility of $w(v_i(1))$ over the lottery of winning the single-winner outcome with utility of $w(v_i(2)) \cdot F_2(v_i(2))$ and not winning at all.

(4.) $\beta(\overline{v}(2)) = 2 \cdot \beta(1)$

(5.) $G(v_i(2), \beta(1)) \leq \beta(v_i(2))$ *for all* $v_i(2) \in [\underline{v}(2), \overline{v}(2)]$ *and all* $i \in I$

The proofs for **Theorems 3**, **4**, and **5** have several pages and needed to be skipped to fit the page restrictions of the conference proceedings. These proofs draw on techniques developed by Anton and Yao (1992), but are independent of a publicly known efficiency parameter. First, condition **(1.)** ensures *DWE*. Second, according to condition **(2.)** both bidders pool at a constant single-unit bid of $\beta(1)$ out of its range from condition **(3.)**. Third, condition **(4.)** ensures that the auctioneer always selects the dual-winner outcome in equilibrium. Finally, note that condition **(5.)** restrains any bidder i's equilibrium bidding function for two units. It is not allowed to fall below the lower bound of $G(v_i(2), \beta(1))$ in order to support the pooling bid for one unit. The lower bound is defined as follows:

$$G\big(v_i(2), \beta(1)\big) \equiv \beta(1) + \frac{\beta(1) - \underline{v}(1) \cdot \big(1 - F_2(v_i(2)\big)}{F_2\big(v_i(2)\big)}$$

This restriction ensures that winning the double-unit package is less profitable in expectation than obtaining a single unit in equilibrium. For our analysis we focus on the lowest pooling bid for one unit of $\beta(1) = \overline{v}(2) - \underline{v}(1)$. This maximizes the utility of both bidders and therefore serves as a natural focal point for implicit coordination in the *dual-winner equilibrium*. Let us next characterize the *single-winner equilibrium*.

Theorem 4: *Any bidder i's vector of bids* $\beta_i = [\beta_i(1), \beta_i(2)]$ *is a symmetric single-winner equilibrium in the* 2×2 *FPSB package auction if the following conditions hold:*

(1.) $\beta_i(2) = v_i(2) - F_2(v_i(2))^{-1} \cdot \int_{\underline{v}(2)}^{v_i(2)} F_2\left(v_j(2)\right) \cdot dv_j(2)$

(2.) $\beta_i(1) \in [0, \underline{v}(2) - \overline{v}(1))$

The equilibrium bid on the double-unit package in the *single-winner equilibrium* of the 2×2 *FPSB package auction* from condition **(1.)** corresponds to the equilibrium strategy of the well-known standard FPSB auction in which two units are sold as the sole package to two bidders. Condition **(2.)** ensures any bidder can enforce the *single-winner equilibrium* by bidding low enough on one unit to make the dual-winner outcome unprofitable for the opponent.

Let us now examine when principals prefer to coordinate on the *dual-winner equilibrium* rather than select the *single-winner equilibrium*.

Theorem 5: *Any principal i prefers the dual-winner equilibrium to the single-winner equilibrium in the* 2×2 *FPSB package auction if* $2 \cdot \beta(1) < E\left(v_j(2)\right)$ *is true.*

Intuitively, the expected double-unit valuation exceeds twice the pooling bid if the probability for high double-unit value draws is high enough. If high-value draws for the package of two units are very likely, however, any bidder prefers the *dual-winner equilibrium* to the *single-winner equilibrium* because he is likely to lose in the latter equilibrium. Let us now turn to the analysis of the principals' optimal bidding behavior in the 2×2 *ascending package auction*.

Ascending Package Auction

We start with the characterization of the *dual-winner equilibrium*.

Theorem 6: *In a symmetric ex-post dual-winner equilibrium of the* 2×2 *ascending package auction, both bidders get active on the single-unit package until the price reaches their valuations and do not bid on the package of two units.*

Proof: If both bidders follow the proposed equilibrium strategy there is no over-demand, and the auction immediately stops at a price of zero for the package of one unit. Any bidder i receives equilibrium profit of $v_i(1)$ with certainty. In the remaining proof, we assume opponent j follows this proposed equilibrium strategy.

First, suppose principal i tries to win one unit, but does not immediately drop out from bidding on the double-unit package. Now the sum of both players' prices for one unit has to exceed bidder i's price for two units. This cannot be optimal because any player i has to pay a positive price for one unit. His profit is decreased strictly below equilibrium profit of $v_i(1)$.

Second, assume principal i tries to win two units and suppose he does not bid on the single unit. If player i wins two units at a price of $v_j(1)$, he obtains a profit of $v_i(2) - v_j(1)$. Under DWE the equilibrium profit of $v_i(1)$ strictly exceeds $v_i(2) - v_j(1)$. Now, let bidder i also bid on the package of one unit. This cannot possibly raise i's profit on two units compared to not bidding on one unit. Finally, note that the proposed equilibrium strategy is independent of any bidder's package valuations and therefore constitutes a symmetric ex-post equilibrium. **QED.**

There is also a *single-winner equilibrium* in the 2×2 *ascending package auction*.

Theorem 7: *In a symmetric ex-post single-winner equilibrium of the* 2×2 *ascending package auction, any bidder i remains active on the single-unit package as long as its price is strictly below* $\underline{v}(2) - \overline{v}(1)$ *and continues to bid on the package of two units until the respective price reaches his valuation of* $v_i(2)$.

Proof: Suppose opponent j follows the proposed equilibrium strategy from **Theorem 7**. Bidder i has no chance to profitably enforce the dual-winner outcome because opponent j does in fact use his "veto" bid, given $\underline{v}(2) > \overline{v}(1)$ is true. The reasoning is analogue to the proof of **Theorem 4** and therefore omitted. Bidder i is indifferent between submitting any single-unit bid of $b_i(1) \in [0, v_i(1)]$ as long as he continues to be active on the large bundle until the price reaches his corresponding value of $v_i(2)$. He obtains an expected profit of $F_2\big(v_i(2)\big) \cdot \big(v_i(2) - v_j(2)\big)$. Bidder i has the highest double-unit package value with probability of $F_2\big(v_i(2)\big)$. In this case, he wins two units at a price of the second highest value, $v_j(2)$, and receives a profit of $v_i(2) - v_j(2)$. However, if bidder i decides to drop out on two units before the price reaches his value of $v_i(2)$, he strictly lowers his probability of winning. This strictly decreases his expected profit and cannot be optimal.

In the *single-winner equilibrium*, by symmetry, both bidders quit bidding on one unit before its price reaches $\underline{v}(2) - \overline{v}(1)$ and remain active on the package of two units until its price reaches their respective valuations. The proposed equilibrium strategies are independent of the bidders' actual package valuations and therefore are rationalizable ex-post. **QED.**

Similar to the 2×2 *FPSB package auction,* the equilibrium bidding for the double-unit package in the *single-winner equilibrium* of the 2×2 *ascending package auction* corresponds to the equilibrium strategy of the well-known standard ascending (English) auction in which two units are sold as the sole package to two bidders. Again, any bidder can enforce the *single-winner equilibrium* by making the dual-winner outcome unprofitable for the opponent. However, the *dual-winner equilibrium* always strictly dominates the *single-winner equilibrium* in profit.

Theorem 8: *Any bidder i strictly prefers the dual-winner equilibrium to the single-winner equilibrium in the* 2×2 *ascending package auction in DWE.*

Proof: In the *dual-winner equilibrium* from **Theorem 6**, the bidder with the lowest value for one unit obtains the lowest profit of $\underline{v}(1)$ with certainty. According to **Theorem 7**, the highest possible profit achievable in the *single-winner equilibrium* is $\overline{v}(2) - \underline{v}(2)$. Using the definition of *dual-winner efficiency* and the fact that $\underline{v}(2) > \underline{v}(1)$, the lowest obtainable profit in the *dual-winner equilibrium* strictly exceeds the highest possible profit in the *single-winner equilibrium*. Therefore, profit in the *dual-winner equilibrium* is strictly greater than in the *single-winner equilibrium* for all possible bidders' valuations $v_i(1), v_j(1) \in [\underline{v}(1), \overline{v}(1)]$ and $v_i(2) \in [\underline{v}(2), \overline{v}(2)]$. **QED.**

In the *dual-winner equilibrium* of the 2×2 *FPSB package auction* from **Theorem 3** both bidders submit pooling prices for one unit and high veto-bids for two units that make the single-winner outcome unprofitable. In the ascending counterpart in **Theorem 6** both bidders only get active on one unit each and win at a prize of zero. The *single-winner equilibrium* of the 2×2 *FPSB package auction* from **Theorem 4** involves a low veto-bid on one unit that makes the dual-winner outcome unprofitable and an optimal bid on two units. Similarly, in **Theorem 7** in the ascending format both bidders submit high veto-bids on one unit to make the single-winner outcome more desirable. Due to the specific form of the *dual-winner equilibrium* in the 2×2 *ascending package auction* **Theorem 8** shows that it strictly dominates in payoff whereas in the FPSB format principals face an equilibrium selection problem as described in **Theorem 5**.

There is an additional reason why the *dual-winner equilibrium* serves as a natural focal point in the 2×2 *ascending package auction*. Bidders can observe their opponents' equilibrium choices and adjust accordingly. Especially, the nature of the *dual-winner equilibrium* as defined in **Theorem 6** allows a bidder to start aiming for one unit and adjust to the *single-winner equilibrium* from **Theorem 7** if he discovers his opponent aims for two units. To illustrate the last point, suppose bidder i plays the *dual-winner equilibrium* and let opponent j chose the *single-winner equilibrium*. As all package prices are publicly observable, bidder i is able to recognize that his opponent is playing a different equilibrium and can adjust his own equilibrium strategy to the *single-winner equilibrium*. Although neither bidder overcame the equilibrium selection problem in the beginning, they are able to coordinate on the *single-winner equilibrium* in the end.

In general, the agents' equilibrium behavior of not bidding on one unit leads to a conflict of interest with the principals' *dual-winner equilibrium*. In the next section, we discuss how to overcome this conflict via budget constraints in the symmetric information setting.

The Limits of Budget Constraints

Let us now analyze if a principal could set budget constraints in the FPSB package auction, so that the agent implements his strategy. The focus on budget constraints is well motivated by the extensive literature on this subject in auctions. In this subsection, we still assume that the supports of the prior distributions and the value draws are known to the principals, and we show that even then, budget constraints can be insufficient to implement the principal's equilibrium bidding strategy in the first-price auction with an agent.

FPSB Package Auction

So far, we assumed agent i's package budget constraint, $\alpha_i(l)$, to be an increasing function of the package value. However, agents would not bid on a single-unit package in equilibrium with these types of budget constraints.[9] We will therefore analyze budget constraints of the form $\hat{\alpha}_i(2) < \alpha(1)$ in which the principal restricts the budget for two units to be strictly smaller than the single-unit budget. In case of the "no overbidding" assumption, this would imply an agent i to be allowed only to bid up to a proxy value, $\hat{v}_i(2)$, strictly smaller than his true valuation $v_i(2)$: $\hat{v}_i(2) < v_i(2)$. In the general budget constraint notation the principal defines a proxy value $\hat{v}_i(2) \in [\underline{v}(2), v_i(2)]$ as an upper bound and assigns a respective budget constraint of $\alpha(\hat{v}_i(2))$, so that the final budget constraint for the agent is given by $\alpha(\hat{v}_i(2)) = \hat{\alpha}_i(2)$.

Theorem 9: *A principal i can direct his agent on the dual-winner outcome in the second stage of the principal-agent* 2×2 *FPSB package auction model by assigning him package-dependent budget constraints of the form* $\alpha_i = [\alpha_i(1), \hat{\alpha}_i(2)]$, *with* $\alpha_i(1) = \alpha(v_i(1)$, $\hat{\alpha}_i(2) = \alpha(\hat{v}_i(2))$ *and* $\hat{\alpha}_i(2) < \alpha_i(1)$ *that satisfy the following two conditions (I) and (II):*

(1.) $\alpha(v_i(1)) + \alpha(\underline{v}(1)) \geq \alpha(\hat{v}_i(2))$

(2.) $w(v_i(1)) \geq w(v_i(2)) \cdot F_1(\hat{v}_i(2))$

Proof: Assume both agents follow their equilibrium strategy of submitting their budget of $\alpha_i(1)$ for the single-unit package. They win the dual-winner outcome with certainty if the sum of both single-unit bids exceeds each agent's double-unit bid. Remember from **Lemma 1** that any agent always spends his entire

[9] It is straightforward to show that under the more general notation of budget constraints, $\alpha_i(l) = \alpha(v_i(l))$, as defined in footnote 6, **Theorem 1** and **Theorem 2** hold with single- and double-unit package budgets of the form $\alpha_i(1) \leq \alpha_i(2)$. According to **Lemma 1** and **Lemma 2** the agent submits a double-unit package bid of $\beta_i(2) = \alpha_i(2)$ and a single-unit package bid of $\beta_i(1) \in \{0, \alpha_i(1)\}$, respectively.

double-unit budget constraint. And according to **Lemma 2** if an agent submits a non-zero bid on one unit, he must bid his entire single-unit budget constraint.

In this equilibrium, any agent i pays his single-unit budget constraint, $\alpha_i(1)$, for sure and therefore his principal provides him with an optimal budget of $\alpha_i(1) = \alpha(v_i(1))$. In addition, the principal must chose a weakly reduced double-unit budget constraint $\alpha_i(2) = \alpha(\hat{v}_i(2))$ of the form $\alpha\big(\hat{v}_i(2)\big) \leq \alpha(v_i(2))$ with proxy valuation $\hat{v}_i(2) \leq v_i(2)$, so that condition **(1.)** is satisfied. Principal i does not know firm j's package value for one unit and therefore has to make sure his choice of the weakly reduced two-unit budget constraint $\alpha(\hat{v}_i(2))$ is below the sum of both single-unit budget constraints. This has to be true for all possible single-unit budget constraints of his opponent, especially the smallest budget constraint of $\alpha(\underline{v}(1))$. By symmetry, opponent j follows the same strategy and condition **(1.)** does in fact guarantee each agent will obtain the small package with certainty.

For agent i to be incentivized to bid on one unit, his utility from the dual-winner outcome must exceed his expected utility from the single-winner outcome. This is ensured in the incentive compatibility condition **(2.)**. The RHS represents agent i's expected payoff from the single-winner outcome when not bidding on one unit. Here, $F_1\big(\hat{v}_i(2)\big)$ is the probability with which his weakly reduced double-unit budget constraint, $\alpha(\hat{v}_i(2))$, exceeds opponent j's single-unit budget constraint $\alpha(v_j(1))$. Remember that opponent j is provided with budgets of $\hat{\alpha}_j(2) < \alpha(1)$. The proxy value $\hat{v}_i(2)$ is chosen so that $F_1\big(\hat{v}_i(2)\big)$ is low enough for the RHS to be lower than the LHS and the incentive compatibility condition satisfied. **QED**.

Following backward induction, we need to analyze whether the budget constraint scheme from **Theorem 9** actually permits the implementation of the principals' *dual-winner equilibrium*. In other words, we need to understand when these budget constraints violate the principals' equilibrium bid functions from **Theorem 3**. In **Theorem 10**, we derive a condition under which the principals cannot direct their agents on truthfully revealing their profit-maximizing equilibrium strategies.

Theorem 10: *There is no vector of budget constraints $\alpha_i = [\alpha_i(1), \hat{\alpha}_i(2)]$ that constitutes a Perfect Bayesian Equilibrium for the principal-agent 2×2 FPSB package auction model under DWE in which the principal implements his dual-winner equilibrium strategies if inequality $2 \cdot \big(\overline{v}(2) - \underline{v}(1)\big) > \overline{v}(1)$ is true.*

Proof: In **Theorem 3**, every principal chooses the same pooling price of $\beta(1)$ in the *dual-winner equilibrium*. Thus, according to **Theorem 9**, any principal i has to provide his agent with the same single-unit budget constraint in height of the pooling bid: $\alpha\big(v_i(1)\big) = \beta(1)\ \forall\, i \in I$. Moreover, any principal i must select a weakly reduced double-unit budget constraint that corresponds to his equilibrium bid on two units: $\alpha\big(\hat{v}_i(2)\big) = \beta(v_i(2))$. Due to **Lemma 1**, every agent always truthfully bids his entire weakly reduced double-unit budget constraint for two units $\alpha\big(\hat{v}_i(2)\big)$. The following three conditions **(I)** to **(III)** then have to be satisfied:

$$2 \cdot \beta(1) \geq \alpha\big(\hat{v}_i(2)\big) \quad \textbf{(I)}$$

$$\alpha\big(\hat{v}_i(2)\big) \geq G(v_i(2), \beta(1)) \quad \textbf{(II)}$$

$$w(v_i(1)) \geq w(v_i(2)) \cdot F_1\big(\hat{v}_i(2)\big) \quad \textbf{(III)}$$

Condition **(I)** ensures that the auctioneer will choose the dual-winner outcome in equilibrium and corresponds to condition **(1.)** from **Theorem 9**. The last two conditions **(II)** and **(III)** ensure that no deviation from obtaining the single-unit package will be profitable for principal and agent in equilibrium. They correspond to conditions **(5.)** from **Theorem 3** and **(2.)** from **Theorem 9,** respectively. Let us now check whether the above three conditions can be satisfied for the focal point pooling bid of $\beta(1) = \overline{v}(2) - \underline{v}(1)$. Suppose firm i has the highest possible value for the package of two units $v_i(2) = \overline{v}(2)$. In this case, condition **(II)** becomes **(II')** by definition of $G(v_i(2), \beta(1))$:

$$\alpha\big(\hat{v}_i(2)\big) \geq 2 \cdot (\overline{v}(2) - \underline{v}(1)) \quad \textbf{(II')}$$

Given **Theorem 2 (4.)**, for a package value of $v_i(2) = \overline{v}(2)$ it must be true that $\beta_i(\overline{v}(2)) = 2 \cdot (\overline{v}(2) - \underline{v}(1))$. Agent i's bid on two units equals twice the pooling bid. Remember, principal i chooses a double-unit package budget constraint in the amount of his respective equilibrium bid: $\alpha(\hat{v}_i(2)) = \beta_i(\overline{v}(2))$. This implies $\alpha(\hat{v}_i(2)) = 2 \cdot (\overline{v}(2) - \underline{v}(1))$ which satisfies condition **(I)** and forces condition **(II')** to hold with equality. Moreover, note that condition **(III)** implies $\hat{v}_i(2) \leq \overline{v}(1)$ because $\overline{v}(1)$ is the highest possible value in the support of $F_1(\cdot)$. Applying this insight to the assumption of $\alpha(\hat{v}_i(2)) \leq \hat{v}_i(2)$ we obtain condition **(III')**:

$$\alpha(\hat{v}_i(2)) \leq \overline{v}(1) \qquad \textbf{(III')}$$

Finally, combining conditions **(II')** and **(III')** is not possible if condition $2 \cdot \left(\overline{v}(2) - \underline{v}(1)\right) > \overline{v}(1)$ is true. In this case, any firm i with package value of $v_i(2) = \overline{v}(2)$ cannot implement budget constraints that satisfy two restrictions: they correspond to its principal's equilibrium strategy and at the same time direct its agent to bid truthfully on both packages. Hence, the *dual-winner equilibrium* cannot be supported as a Perfect Bayesian equilibrium of the *principal-agent* 2×2 *FPSB package auction model*. **QED.**

It follows that a Perfect Bayesian equilibrium in which principal and agent are incentive aligned, cannot always exist. Intuitively, any firm faces the following trade-off: the principal has to bid high enough on two units to prohibit him from making a profit by deviating from the *dual-winner equilibrium* and winning. The agent can only be directed on bidding for one unit if his budget constraint on the double-unit package is low enough. Consequently, both requirements cannot always be met simultaneously by the budget constraints. In such a case, the principals cannot enforce the *dual-winner equilibrium*.

Ascending Package Auction

Unlike its sealed-bid counterpart, the principals' *dual-winner equilibrium* can easily be implemented as a Perfect Bayesian equilibrium of the *principal-agent* 2×2 *ascending package auction model*.

Theorem 11: *The vector of budget constraints* $\alpha_i = [\alpha_i(1), \hat{\alpha}_i(2)]$, *with* $\alpha_i(1) = v_i(1)$ *and* $\hat{\alpha}_i(2) = 0$, *constitutes a Perfect Bayesian equilibrium for the principal-agent* 2×2 *ascending package auction model in which any principal* i *implements the dual-winner equilibrium*.

Proof: In **Theorem 6**, any principal i does not bid on the package of two units and remains active on the package of one unit until the price reaches his corresponding valuation of $v_i(1)$. To implement the principal's *dual-winner equilibrium* strategy for his agent, the principal provides a zero budget constraint on the large package. This eliminates the agent's possibility to win the double-unit package and he is in fact competing in a package auction that consists solely of one unit. As a consequence of **Lemma 1**, the agent then truthfully bids up to his budget constraint for the single-unit package and the principal can simply provide his agent with a budget constraint in the amount of his valuation for one unit. **QED.**

In summary, the principal can always set budgets in the form of $\alpha_i = [\alpha_i(1), \hat{\alpha}_i(2)]$, *with* $\alpha_i(1) = v_i(1)$ *and* $\hat{\alpha}_i(2) = 0$ to implement his *dual-winner equilibrium* in the 2×2 *ascending package auction model*. In its sealed-bid counterpart, budget constraints of $\alpha_i = [\alpha_i(1), \hat{\alpha}_i(2)]$ with $\alpha(v_i(1)) = \beta(1)$ and $\alpha(\hat{v}_i(2)) = \beta(v_i(2))$ as defined in **Theorem 3** only implement the *dual-winner equilibrium* if $2 \cdot \left(\overline{v}(2) - \underline{v}(1)\right) \leq \overline{v}(1)$ is true. The nature of the *dual-winner equilibrium* in the 2×2 *ascending package auction* allow for the optimal delegation with budgets to solve the adverse selection problem. The difficulty to set budget constraints in equilibrium in the first-price package auction is an argument for the ascending auctions in our model that is different to traditional arguments for ascending auctions such as the linkage principle (Milgrom and Weber 1982).

Extensions

In this section, we first demonstrate how our benchmark *principal-agent* 2×2 *package auction model* can be used to analyze general second-price sealed-bid auction mechanisms. We then extend our findings to larger $m \times n$ markets.

Second-Price Mechanisms

The VCG mechanism is the generalization of the well-known second-price sealed-bid (Vickrey) auction to combinatorial package auctions. The VCG mechanism is composed of the Clarke Pivot payment rule that is strategy proof for profit-maximizing bidders (principals) and incentivizes the bidders to truthfully report their package valuations. The VCG mechanism further contains an allocation rule that allocates the package of items with the sum of highest reports to the corresponding bidders and therefore achieves welfare-maximization.

Ignoring the agents in the second stage of our *principal-agent* 2×2 *VCG package auction model*, the VCG mechanism implements the dual-winner outcome for the principals in the *DWE* setting. The payment rule forces the principals to truthfully reveal their package valuations and *DWE* guarantees the allocation rule to select the dual-winner outcome. In the second stage of the *principal-agent* 2×2 *VCG package auction model* the payment rule does not affect the agents by definition. Thus, their optimal reporting strategy in the VCG mechanism corresponds to the equilibrium from the second stage of the *principal-agent* 2×2 *FPSB package auction model* described in **Theorem 1**. By a similar logic as in **Theorem 11** the principals can then provide their agents with the same optimal budgets to implement the strategy-proof and welfare-maximizing dual-winner outcome as a Perfect Bayesian equilibrium in the *principal-agent* 2×2 *VCG package auction model.*

In summary, the VCG mechanism is outcome equivalent to the ascending auction in our *principal-agent* 2×2 *package auction model.* As an advantage it does not result in an equilibrium selection problem for the principals, although this selection problem is very weak in the ascending format anyway. However, note that the VCG mechanism is hardly used in practice at all.

Larger Markets

The results for 2×2 markets depend on the power of each bidder to veto a split. In more complex $m \times n$ environments this veto-power can either be strong or weak. Suppose, for example, the number of units strictly exceeds the number of bidders. In this setting, individual bidders' veto-power against "splitting" allocations might still be strong and coordination hard to achieve. Here, our results from the 2×2 market will extend wlog to larger $m \times n$ auctions.

Now assume the number of bidders strictly exceeds the number of units. Each bidder's veto-power against "splitting" allocations is low. The higher the number of bidders compared to the number of units, the more likely at least some of them will coordinate. To demonstrate this point let us focus on the second stage of the *principal agent* $2 \times n$ *ascending package auction model* under *DWE*. Making use of the "no-overbidding" assumption there is an ex-post equilibrium in which all agents bid on one unit and the two bidders with the highest valuation for the singleton win one unit each. Unlike **Theorem 2** all but the bidder with the highest valuation for the double-unit package let the price for one unit rise until they overbid the other bidder on two units (under *DWE* this must eventually happen). The latter will then also start bidding on one unit. Given the other bidders coordinate, any one bidder will certainly lose if he does not participate.

However, ignoring this technical example (that crucially depends on *DWE* and two units) any individual bidder can never be entirely certain about his competitors' willingness to coordinate. Hence, Bayesian Nash Equilibria in which bidders do not coordinate can still be supported with appropriate believes and the fundamental trade-offs described in the *principal-agent* 2×2 *package auction model* remain valid. As an example, we now focus on the optimal principal strategy in the $2 \times n$ *FPSB package auction* under *DWE*. Suppose all opponents employ a veto strategy as defined in **Theorem 4 (2.)** to exclude the dual-winner outcome and try to win the large package. In this case, any one bidder cannot win a single unit as it is blocked by his opponents' veto strategies. Thus, he has to participate in the *dual-winner equilibrium* too. Regarding the agents in the second stage of the *principal-agent* $2 \times n$ *FPSB package auction model,* especially bidders with high valuations for the large package might try to use their veto-power against "splitting" allocations in order to win larger bundles on their own.

Conclusions

In summary, the information asymmetries in bidding firms can make it very difficult for principals to incentivize the agents. This has implications for auctioneers and their choice of auction format as well. There is an ongoing policy discussion about sealed-bid vs. ascending package auctions. For example, in a spectrum auction in Norway in 2014, the allocation of a FPSB combinatorial auction was such that some items remained unsold, whereas one incumbent did not win sufficient spectrum and had to leave the market later. There can be many reasons for such an outcome. However, our analysis suggests that principal-agent relationships can be an important reason why bidders do not coordinate in a FPSB package auction. For a principal, it is easier to direct his agent to coordination in an ascending auction as compared to a FPSB auction. If principals cannot set budget constraints appropriately, there is a danger that agents will overbid and that they will not bid on smaller packages, even in situations in which this is payoff-maximal for the firm. While auctions are usually described as a mechanism to achieve high efficiency, we have shown that this is not necessarily the case anymore with principal-agent relationships in the bidding firm.

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