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A Model of Pricing in the Sharing Economy: Pricing Dynamics with Awareness Generating Adoptions

Completed Research Paper

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Abstract

We develop a model of providers operating on a Sharing Economy platform where consumers first become aware of a product or service, and then consider adopting it. Past adoptions increase the likelihood that future consumers will discover the provider, but awareness also decays over time. We exhibit that pricing dynamics for products and services of short life span are consistent with penetration pricing. For products with longer life spans, the relationship between population awareness and price is non-monotonic, and consistent with a tipping point in the adoption process. Providers price higher when they have either achieved high awareness or have been positioned in a market niche. Fees levied by the platform adhere to the double-marginalization effect. Our model positively explains empirical observations, including the significant price fluctuation and the emergence of niche and superstar providers.

Keywords: Sharing economy, collaborative economy, electronic markets, ecommerce, p2p, social influence, diffusion, dynamic programming

Introduction

Over the past few years we have been witnessing the emergence of *Sharing Economy platforms*, a new type of marketplace where transactions take place between individuals (peers), rather than between an individual and a firm. This marketplace structure, also referred to as online peer-to-peer marketplaces, has been made possible by a number of technological advances such as internet-enabled mobile devices, as well as by the fact that the nascent problems of online marketplaces are by now well-understood and can be largely addressed (e.g. online recommendation systems as the digital analogue of trust (Dellarocas 2003, Adomavicius and Tuzhlin 2005)).

Several features distinguish Sharing Economy platforms from their traditional counterparts (Sundararajan 2013). In this paper, we focus on the fact that the mechanism through which consumers discover providers in the Sharing Economy is one of *peer-induced awareness*: awareness about a provider's services or products is mainly generated by consumers, rather than by the provider. In fact, providers typically find themselves in a position where they are unable to explicitly control common

awareness-inducing mechanisms. As an illustrating example consider the case of Airbnb.[1] On the Airbnb platform, a provider can do little in terms of advertising to climb higher in the platform rankings, convince buyers to write reviews, and more generally attract the attention of a greater number of consumers. The rationale here is that Sharing Economy platforms typically assume the role of a neutral third party, tasked only with bringing the two transacting sides together and setting basic marketplace rules; offering advertising services can therefore be perceived as intervening in a way antithetical to market fairness. In the absence of advertising we expect greater variance in population awareness about products and, subsequently, the transactions record of a provider to be characterized by a pronounced degree of uncertainty. This situation stands in stark contrast to approaches that assume specific adoption patterns ex-ante (Bass, 1969).

The economic significance of the Sharing Economy ecosystem[2] combined with its distinct differences from standard markets, make understanding the drivers of the behavior of agents engaging in it an important endeavor. Toward that end, in this paper we present a model of a provider operating in a market where positive adoption decisions increase *awareness* in the population of potential consumers. We use the term adoption to refer to purchases of goods and services. For example, an adoption decision may be buying a new backpack on Etsy, but can also be hiring a worker on Taskrabbit. The provider has no advertising option, and can only control the price she sets. Higher awareness translates to consumers being more likely to discover the provider in the future. Our analysis assumes that the undecided segment of the population is disproportionately affected by consumers who have decided to adopt compared to those that considered the product or service and decided not to. This assumption is grounded on empirical evidence, such as the work of Banerjee et al. (2013) who show that a microfinance participant is seven times more likely to inform other households about microfinance loans than a knowledgeable non-participant.

While capturing the abovementioned effects, we abstract away from the exact nature of the underlying mechanisms of awareness diffusion. Nonetheless it is worth mentioning here some of these mechanisms, which may consist of both direct and indirect channels through which awareness flows from adopters to non-adopters, consciously or subconsciously. Direct channels include peer effects and posts on social media. Indirect channels are of equal importance, and include recommendation systems employed by online platforms to provide suggestions based on what other consumers bought in the past (often resulting in a rich-get-richer effect (Adamopoulos et al. 2014)), as well as displaying lists of the most popular products. Experimental evidence shows that a high search ranking for hotels on Expedia increases sales unconditional on click, but has no effect on sales conditional on a click (Ursu 2015). This suggests that a high ranking increases sales due to costs associated with ordered search. Thus, if more sales result in higher ranking, we would expect increased demand even when the ranking does not convey any information about the product's attributes. Examples outside the Sharing Economy context include word of mouth, internet search engines prioritizing results that other users have signaled interest in, and traditional media running stories about the product "everybody is talking about".

We first consider the finite horizon setting. Some products and services inherently have a limited life span due to factors such as seasonality, replacement by new technology, or because they are offered in marketplaces where fashion and fads are prevalent (Day, 1981). Providers who look into Sharing Economy platforms only for a temporary source of income also fall under the finite-time framework. We find that incorporating awareness dynamics results in a price path consistent with *penetration pricing* (Tellis, 1986). Penetration pricing policies are characterized by setting low prices in the beginning of the process to generate awareness through adoptions, followed by a positive time trend. In contrast to previous work, the time trend is not imposed exogenously and the evolution of prices depends on the adoption history and the properties of the awareness-generating process.

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[1] Airbnb is a peer-to-peer online marketplace for listing and renting lodging.

[2] Sharing Economy platforms have amassed more than $16 billion in funding and have spawned firms like Airbnb and Uber, currently valued at more than $25 billion each. Some of these platforms have even managed in their short lifetime to surpass long incumbent firms (http://goo.gl/VjeqOV, last accessed September 1st , 2016)

The infinite-time extension of our model captures products and services with longer life span, as well as providers whose long-term occupation is found on the platform. We find that prices are not monotonically increasing in awareness, and the dynamics of the adoption process are instead characterized by a *tipping point* delineating various degrees of vicious and virtuous cycles. Prices are at their lowest when awareness is close to the tipping point, as a result of the provider attempting to tip the level of awareness towards a high state. In contrast, prices are highest when the provider has either achieved high population awareness (superstar status) or when she has found her niche.

Our results are in line with phenomena commonly observed on Sharing Economy platforms. It is well-known that most providers fade to relative obscurity while only few remain popular over extended time periods. Common explanations involve bad timing, mismanagement, unappealing ideas and low quality. Although such arguments certainly have merit, our analysis exhibits awareness can as a complementary explanatory factor of a provider's path of success. These results also extend to more traditional online marketplaces, where we often observe substantial price fluctuation. In the example of Figure 1, prices and search interest are given for a specific product. Although the evidence of the relation is at best anecdotal in this example, we observe substantial fluctuations in both variables. Our model offers a positive explanation of such patterns, rooted in providers responding to the randomness inherent in consumer decisions. On the prescriptive side, we see that platforms are incentivized to charge higher fees for both niche and popular providers, while the variance in their fees is lower than compared to that of the prices. Our results also apply to other online or bricks-and-mortar marketplaces, where it is not advertising but peers that drive population awareness and demand.

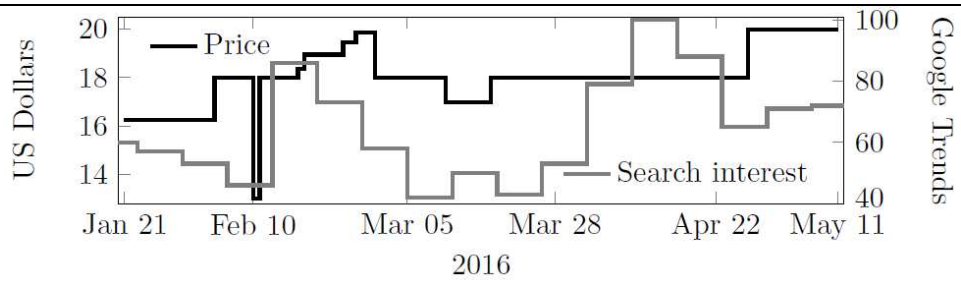


Figure 1. Price over time for a USB rechargeable electric lighter on Amazon.com (black, left axis, accessed through http://goo.gl/80dLF3 on May 11, 2016) and search interest score for the search term "electric lighter" on Google trends (gray, right axis, accessed through Google Trends on May 12, 2016).

Literature

The Sharing Economy

The Sharing Economy, alternatively known as online peer-to-peer marketplaces, can be broadly defined as platforms facilitating the trade of products and services between individuals (Gansky 2010, Sundararajan 2013). An impressive variety of products and services can be found on these platforms, ranging from housing (Airbnb, Couchsurfing) and transportation (Lyft, Uber, Sidecar) to labor (oDesk, Taskrabbit) and risk capital (Kickstarter, Indiegogo).

The economic impact of the Sharing Economy is still unclear. Its proponents have pointed out that Sharing Economy platforms promise easier access to goods and services, increased efficiency of underutilized resources, and higher welfare gains for below-median income consumers. The models and counterfactuals of Fraiberger and Sundararajan (2015) and Horton and Zeckhauser (2016) both confirm these claims, and point out that the majority of the surplus created is allocated to these social strata. On the other hand, there is also a large degree of skepticism regarding the Sharing Economy. Critics claim that value is mainly created through the avoidance of costly regulation (Malhotra and Van Alstyn, 2014;

Avital et al 2015)), and that lower-end providers may be disproportionately affected. Zervas et al. (2015) conduct an econometric analysis and find evidence that Airbnb impacts low-priced hotels more, but it also benefits all consumers regardless of whether they are Sharing Economy participants.

Our work adds to the Sharing Economy literature by developing a model of provider pricing that closely resembles the conditions often encountered on these platforms. Our analysis provides a positive explanation to how providers behave on Sharing Economy platforms, as well as prescriptive advice on platform fees. We also aspire to offer a simple and easily extendable framework on which subsequent studies about similar marketplaces can be based.

Awareness and Adoption

Our assumptions about consumer awareness are rooted in findings from the psychology and marketing literature. Consumers' ability to remember brands decays in time (Hutchinson and Moore, 1984), or because other brands competitively interfere (Burke and Srull, 1988). Advertising impressions may sometimes not even reach the consumers; a related phenomenon in online advertising is *banner blindness* (Pagendarm and Schaumburg, 2006; Hervet et al., 2011).

The endogenous source of awareness in our model is past adoptions: when the stock of adopters increases, we expect - ceteris paribus - an increase in the probability of consumers becoming aware of the provider. The seminal paper of Bass (1969) uses a set of differential equations to describe a similar diffusion process in a social system consisting of innovators (early movers) and imitators. Kalish (1985) extends the Bass model to include pricing, quality uncertainty and advertising. As in our model, adoption is a two-step process where consumers first become aware through word-of-mouth or advertising, and then decide to adopt or not conditional on awareness. Uncertainty about a product's quality decreases in the number of past adopters, and simultaneously raises willingness-to-pay due to risk aversion. (Krishnan and Jane, 2006) study a dynamic advertising setting and find that the optimal direction of the advertising intensity over time depends on several interacting factors. Our work departs from Bass-type diffusion models in several aspects. While consumers are more likely to discover a provider following a history of many adoptions, we do not ex ante distinguish between consumers as innovators or imitators, and the order of adoption decisions is random. In the Bass family of models the diffusion process is to a large extent predetermined by model parameters. In contrast, luck and random events play an important part in a provider's success or failure, and the optimal pricing policy is not pre-determined but instead a function of the history of adoptions.

A related stream of research studies the long tail and superstar phenomena observed in online markets (Bar-Isaac et al. 2012, Brynjolfsson et al. 2010, Brynjolfsson et al. 2011). Advances in information technologies have resulted in some product markets being dominated by a few, immensely successful products (superstars), reinforcing what has historically been the case. On the other hand, the very same technologies have also facilitated the rise of a flourishing niche market that in aggregate holds a substantial market share (long tail). Our paper positively explains the rise of both superstar and niche products, as a result of consumer awareness created and directed by search and personalization technologies commonly employed in online markets. An important question is how the varying degrees of popularity affect prices. In our model, differences in a product's (endogenous) level of awareness, which translates to differences in expected sales, command different prices. Both niche and mass-market products command a price premium compared to products of intermediate levels of awareness. The intuition is that the high awareness associated with the most popular products is likely to be self-sustained, hence the provider does not need to lower prices in order to raise sales, and in turn, future awareness of the product. On the other hand, providers set higher prices for niche products for the opposite reason: since there are few potential customers, the boost in awareness following a sale is likely to drop shortly after, which makes self-sustained awareness to be extremely unlikely. Therefore, it is a better strategy to set a high price for the few consumers that do become aware of the product. The largest price discounts are found in intermediate levels of awareness: products that are neither niche nor superstars. Prices are lower both to avoid falling into obscurity, and because these products have a realistic chance of becoming a "superstar" with self-sustained awareness levels and higher prices.

A central result in our paper is that prices fluctuate over time. We are not the first ones to explain such pricing behavior; Board (2008) and Garret (2013) consider durable-good monopolies with varying demand, and exhibit ways in which a provider can take advantage of these fluctuations to price-

discriminate. Mixed pricing strategies due to partial consumer inertia in the model of sales of Varian (1980) explain fluctuating prices in competitive environments. Monopoly models of word-of-mouth also find price fluctuations. Campbell (2015) studies a model where customers inform their friends about a product. A non-negligible stock of informed consumers with valuations below the current price is eventually formed, which incentivizes the provider to lower the price for a short amount of time in order to skim informed low-value customers. Ajorlou et al. (forthcoming) find similar results in a model over a Poisson random graph. Unlike these papers, we consider a situation in which a given consumer can only adopt once, and where a consumer's valuation is independent of both time and history. Thus, there is no room for price discrimination. Our similar result on fluctuating prices is driven by a different mechanism rooted in the randomness in the consumer types which causes the provider to continuously update the price. Our model therefore also shares some features with the work of Arthur (1989) and Bikhchandani et al. (1992) in which the random order of consumer arrivals is an important determinant of market outcomes.

In our model, more adoptions imply increased demand facing the firm. In that regard, the underlying mechanism generates demand-side externalities similar to network effects. Unlike network effects, this increase in demand is due to higher probability of consumers becoming aware of the product, and not due to consumers' utilities being affected by increased adoption. Despite this difference, our results resemble some of those found for network goods. Lower introductory pricing as an equilibrium pricing strategy has been identified by Cabral et al. (1999), and our results regarding tipping points in adoption are similar to the notion of a critical mass in relation to a multiplicity of equilibria in adoption of network goods exhibited by Katz and Shapiro (1985) and Economides (1996).

Model Setup

We begin by introducing the finite horizon model. At each time period $t \in \{0,1,\dots,T-1\}$ a one-period lived consumer is born. The consumer, to whom we refer as consumer t to economize on notation, has a one-time chance to discover and potentially adopt. Consumer t *arrives* (or *becomes aware of* or *discovers* the provider) with probability $Q(a_t) \in [0,1]$, where a_t is the *population awareness* about the product at time t. Population awareness follows the law of motion

$$\mathbf{a}_{t+1} = \boldsymbol{\delta}(\mathbf{a}_t + \mathbf{k}\mathbf{b}_t) \tag{1}$$

where $b_t \in \{0,1\}$ is a binary variable indicating whether an adoption was made in period t, $\delta \in [0,1]$ is a discount factor which captures awareness decay in the population, and $k > 0$ is the awareness impact of a positive adoption decision. Letting a_0 be the initial population awareness, Equation (1) can recursively be written as $a_t = \delta^t a_0 + k \sum_{s=0}^{t-1} \delta^{t-s} b_s$. After consumer t becomes aware of the product, he learns his privately observed valuation $\theta_t \in [0, \theta_h]$, drawn from a publicly known distribution $F(\theta)$, i.i.d. across consumers. For a given price p_t consumer t decides to adopt ($b_t = 1$) if $\theta_t > p_t$. Therefore, consumer t adopts with probability $Q(a_t)(1 - F(p_t))$, which is the joint probability of both becoming aware of the provider and having a valuation that exceeds the price demanded.

The Provider's Problem

The provider faces a constant marginal cost c such that $c > \theta_h$. At time t the provider decides on a price that maximizes the future-discounted sum of her expected profits:

$$\boldsymbol{E}[\,\boldsymbol{\Sigma}_{s=t}^{t-1}\, \boldsymbol{\beta}^{s-t} \boldsymbol{Q}(\boldsymbol{a}_s)\big(\mathbf{1} - \boldsymbol{F}(\boldsymbol{p}_s)\big)(\boldsymbol{p}_s - \boldsymbol{c})] \tag{2}$$

The provider's optimization problem is cast in terms of the dynamic programming framework by the following Bellman equation:

$$\begin{aligned} \boldsymbol{V}_t(\boldsymbol{a}_t) = \mathbf{max}_{\boldsymbol{p}_t}\ & \boldsymbol{Q}(\boldsymbol{a}_t)\left[\big(\mathbf{1} - \boldsymbol{F}(\boldsymbol{p}_t)\big)\left(\boldsymbol{p}_t - \boldsymbol{c} + \boldsymbol{\beta}\boldsymbol{V}_{t+1}\left(\boldsymbol{a}_{t+1}^{1}(\boldsymbol{t}+\mathbf{1})\right) + \boldsymbol{F}(\boldsymbol{p}_t)\boldsymbol{\beta}\boldsymbol{V}_{t+1}\big(\boldsymbol{a}_{t+1}^{0}\big)\right)\right] \\ & +(\mathbf{1} - \boldsymbol{Q}(\boldsymbol{a}_t))\boldsymbol{\beta}\boldsymbol{V}_{t+1}(\boldsymbol{a}_{t+1}^{0}) \end{aligned} \tag{3}$$

where $a_{t+1}^1 = \delta(a_t + k)$ is the population awareness following a positive adoption decision ($b_t = 1$) and $a_{t+1}^0 = \delta a_t$ is the population awareness in the opposite case ($b_t = 0$). The Bellman equation is interpreted as follows: at time t with a given population awareness a_t , a consumer becomes aware of the product with

probability $Q(a_t)$. At price p_t the consumer's valuation θ_t exceeds $p_t$with probability $1 - F(p_t)$, in which case the provider earns period t profits of $p_t - c$ and a continuation value of $V_{t+1}(a_{t+1}^1)$ since the number of adopters increase by one unit. Consumer t does not adopt if she becomes aware but her valuation does not exceed the price, or she does not become aware of the product at all. In either case the continuation value is $V_{t+1}(a_{t+1}^0)$. The first-order-condition of the maximization problem is:

$$\boldsymbol{Q(a_t)\left[\left(1 - F(p_t)\right) - f(p_t)\left(p_t - c + \beta V_{t+1}(a_{t+1}^1) + f(p_t)V_{t+1}(a_{t+1}^0)\right)\right] = 0} \qquad (4)$$

Conditional on a consumer arriving, a marginal increase in p_t has the following effects on expected revenue: a positive price effect $1 - F(p_t)$ and a negative expected output effect $-f(p_t)p_t$. In addition, higher prices decrease the probability of receiving a higher continuation value $V_{t+1}(a_{t+1}^1)$. This creates an incentive for the provider to decrease the price compared to the case where past adoptions and population awareness are unrelated. The *virtual valuation* formulation the first order condition can be written as

$$\boldsymbol{p_t - \frac{1-F(p_t)}{f(p_t)} = c - \beta[V_{t+1}(a_{t+1}^1) - V_{t+1}(a_{t+1}^0)\,]} \qquad \textbf{(4)}$$

where the fraction $[1 - F(p_t)]/f(p_t)$ is the reciprocal of the hazard ratio of the valuation distribution, and the left hand side can be interpreted as the (present) expected marginal revenue conditional on the consumer becoming aware of a product. Since $V_{t+1}(a_{t+1}^1) - V_{t+1}(a_{t+1}^0)$ is clearly non-negative, the present expected marginal revenue can be lower than the marginal cost if this difference is sufficiently large. We impose the following standard condition on the probability distribution F

$$\boldsymbol{1 - \frac{d(1-F(p))/f(p)}{dp} > 0} \qquad \textbf{(5)}$$

Equation (5) ensures that (4) admits a unique solution, and therefore a global maximum. The assumption is a weaker version of the increasing hazard rate assumption, frequently encountered in economics (e.g. the auction theory literature) and satisfied for most unimodal distributions, including the normal and uniform distributions. Although this assumption is sufficient for obtaining a unique solution, a strictly increasing hazard rate ensures that changes in right-hand-side variables in (5) have a less-than-unity effect on the price. This stricter assumption is satisfied for the uniform distribution which we assume in several cases in this paper.

Finite Horizon Analysis

The 2-period Model

We start our analysis by focusing on the two-period model ($t \in \{0,1\}$. The two-period model is the simplest finite horizon case that captures the dynamics of awareness generating adoption while admitting analytic solutions. Every result derived here clearly extends to the last two periods for any $T > 1$. At period 1 the price is given by

$$p_1^* - \frac{[1 - F(p_1^*)]}{f(p_1^*)} = c.$$

Note that the price of the last period is independent of population awareness (end of the world effect). The corresponding value function is:

$$V_1(a_1) = Q(a_1)[1 - F(p_1^*)](p_1 - c),$$

where $a_1 = \delta(a_0 + k\, b_0)$. The difference in the probability that the consumer of period 1 will discover the provider (equivalently arrive) if one more consumer has adopted in the past is denoted by:

$$\Delta Q(a_1) = Q(\delta[a_0 + k]) - Q(\delta a_0).$$

Since p_1^* is independent of adoptions in $t = 0$, p_0^* only depends on $\Delta Q(a_1)$. Equation (4) then gives us:

$$p_0^* - \frac{[1 - F(p_0^*)]}{f(p_0^*)} = c - \Delta Q(a_1)\, \beta\left(1 - F(p_1^*)\right)(p_1^* - c)$$

The following result formalizes the intuition that when adoptions increase arrival probabilities more, prices are lower

Proposition 1. *Optimal price is decreasing in* $\Delta Q(a_1)$.

Proof: The result follows by differentiation and (5), since $dp_0^*/dQ(a_1) < 0$.

We can also explicitly relate population awareness with the functional form of Q.

Proposition 2. *The price is increasing (resp. decreasing) in population awareness* a_0*, if* $Q(\delta a_0)$ *and* $Q\big(\delta(a_0+k)\big)$ *are concave (resp. convex) in their arguments.*

Proof: Consider the effect of a marginal increase in a_0 on the optimal price

$$\frac{dp_0^*}{da_0} = -\frac{1}{1-\frac{d\big(1-F(p_0^*)\big)/f(p_0^*)}{dp}}\big(1-F(p_1^*)\big)p_1^*\beta\frac{d\Delta Q(a_1)}{da_0}$$

where $\frac{d\Delta Q}{da_0} = \frac{dQ(\delta(a_0+k))}{da_0} - \frac{dQ(\delta(a_0))}{da_0}$. If Q is concave in both a_0 and a_0+k then $\frac{d\Delta Q}{da_0} < 0$, which implies that p_0^* is increasing in a_0. For convex Q the inequality is reversed.

The exact effect of population awareness hence depends on the shape of Q. The intuition behind this result is straight-forward. At the convex part, we have increasing returns to awareness in the sense that positive adoption decisions lead to larger changes in future arrival probabilities, which in turn incentivizes the provider to set a lower price. At the concave part there are diminishing returns to awareness, therefore the provider puts relatively higher weight on current profits; no adoption today has little negative effect on future adoption probabilities. We resume this result again and study its robustness in the T-period case.

We now derive an additional result in the case where consumer's valuations are drawn from the standard uniform distribution, i.e. $F(\theta)\sim U[0,1]$. This is an assumption that we will impose in most of what follows since it allows us to derive analytic results while not being restrictive. From (4), the price p_0 is given by:

$$p_0 = \max\{\frac{1+c}{2} - \beta\frac{\Delta Q(a_1)}{8}(1-c)^2, 0\}.$$

The following proposition shows that even if an adoption at $t=0$ leads to a large increase in the future arrival probability, it is never optimal for the provider to sell at a loss.

Proposition 3. *Price never drops below the marginal cost.*

Proof: The inequality $p_0 < c$ is equivalent to $\beta\Delta Q(a_1) > \frac{4}{1-c}$. Since $\beta\Delta Q(a_1) \in [0,1]$, $\frac{4}{1-c} < 1$ must hold for below-marginal cost pricing to occur in equilibrium. This implies $c < -3$, which violates the non-negative marginal cost assumption.

Proposition 3 establishes the existence of cases where increasing future profit is never sufficiently valuable to incur losses today. We will later show numerically that this result does not extend in the T-period case, where pricing below marginal cost may occur for sufficiently large T.

T periods

For more than two periods analytic solutions are hard to obtain even with very specific restrictions on the form of the distribution Q. This difficulty is common in dynamic programming problems and its source is the complexity caused by the dependence of optimal prices on continuation values. Nevertheless, it is important to see whether our analysis for the two-period model holds in the $T > 2$ case. Toward that end, we employ here numerical methods to test the robustness of our results. An exception is the case in which Q is uniformly distributed, and $Q(a_t)$ is strictly within the (0,1) interval for all attainable values of a_t.

Proposition 4. *Let* $Q(a)\sim U[0,h]$ *and assume that* $\delta a_t, \delta a_{t+k} \in (0,h)$ *for all attainable values of the population awareness* a_t*. Then the optimal price* p_t *depends only on time.*

Proof: The proof relies on $\Delta Q(a_{t+1}) = \delta k/h$ beung constant across time and states for the uniform distribution in the (0,1)range. In period $T-1$ the price is clearly independent of the current state. In period $T-2$ the price is given by equation (4) and we have

$$\boldsymbol{p^*_{T-2} - \frac{1-F(p^*_{T-2})}{f(p^*_{T-2})} = c - \frac{\delta k}{h}\beta[1 - F(p^*_{T-1})\,](p^*_{T-1} - c)},$$

which is independent of the current population awareness a_{T-2}. Iterating the argument proves the claim.

Figure 2 illustrates the implication of Proposition 4. Panel (a-1) shows the paths arrival probabilities depending on whether an adoption occurred in the previous period. Observe that the difference in the arrival probabilities of the next period is equal and independent of the current population awareness. As a consequence, there is a unique price path which is independent of the adoption history. Consequently, price is deterministic and gradually increasing over time. In Figure 2 (b-1), the differences in future arrival probabilities are not constant since the arrival probability reaches its upper bound of 1 following a sequence of adoptions. In other words, it is feasible to reach the concave part of the function $Q(a)$. The price path in that case is not unique, and price depends on the adoptions history. In the next subsection we analyze the functional form of Q further.

Functional Form of $Q(a_t)$

As is evident from the two-period analysis, the functional form of Q is an important determinant of the connection between population awareness and prices. We numerically show here that the results of Proposition 2 extend to the T −period case. In other words, if Q is convex (concave) then prices are lower (higher) following an adoption compared to no adoption. The intuition remains the same. An adoption increases population awareness relative to a non-adoption. Therefore, if Q is convex (concave), then higher population awareness leads to a larger (smaller) difference in future arrival probabilities. The convexity (concavity) of Q leads to lower (higher) prices for a given awareness and a given time.

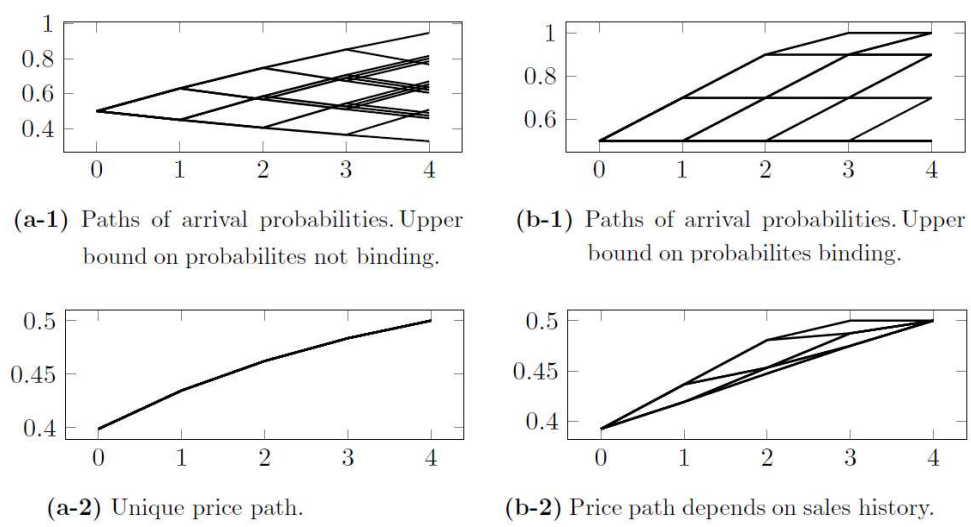
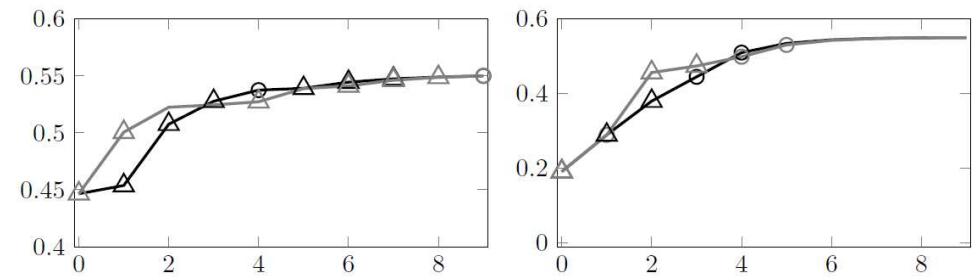


Figure 2. Paths of arrival probabilities and prices when Q is uniform. Parameters: $= 5$ $a_0 = 0.5, k = 0.2, \beta = 1, c = 0, \theta \sim U[0,1], Q \sim U[0,1]$. Figures (a): $\delta = 0.9$ (b): $\delta = 1$

Price simulations are shown in Figure 3. In Figure 3 (a), the arrival probability $Q(a)$ follows a Pareto distribution (strictly concave over its domain). Notice that at any given time, higher awareness corresponds to higher prices. For example, in period $t = 0$, an adoption occurs in the simulation corresponding to the gray line, while there is no consumer arrival in the simulation corresponding to the black line. As a consequence, the price at the gray line is higher in the next period because of higher

awareness. The price in the black simulation eventually exceeds the one in the gray one when the population awareness exceeds that of the gray simulation. Furthermore, the price difference between the two eventually gets smaller over time after they both achieve to generate high awareness and as the end-of-the-world effect becomes stronger (time approaches T).



(a) $Q(a) = (1 - 1/(a+1)^2)$ (Pareto). $a_0 = 2, k = 1, \delta = .85$

(b) $Q(a) = \min\{a^2, 1\}$. $a_0 = 1, k = .4, \delta = .7$

Figure 3. Price simulations. Δ: adoption O: arrival without adoption. Higher awareness leads to higher prices when there are diminishing returns to awareness (concave) and lower prices when the returns are increasing (convex). Parameters: $\boldsymbol{T = 10, \theta_t \sim U[0,1], c = 0.1, \beta = 1}$.

Figure 3 (b) plots simulations with $Q(a) = \min\{a^2, 1\}$, strictly convex for all feasible a_t. In this case, higher awareness leads to lower prices, as states in which awareness is high have more potential to attract consumers. This incentivizes providers to set lower prices today in order to increase the probability of getting awareness generating adoptions. In situations where the probability restriction $Q(a_t) \leq 1$ binds for some a_t, this result does no longer hold, as there are a "decreasing returns to awareness" at future states. Nonetheless, Proposition 2 from the two-period case appears to be robust for the strict concavity and convexity cases.

Infinite Horizon Analysis

The finite-horizon model we developed in the previous sections aided us in studying the relation between population awareness and prices. However, the impact of the positive time trend inherent in finite-period models is hard to disentangle from the effect of population awareness. While end-of-the-world effects are present in a considerable number of real-life situations (e.g. products and services for which seasonality effects are prevalent), it is also important to study what happens in their absence. Toward that end we extend our model to the infinite horizon case and carry out a numerical analysis.

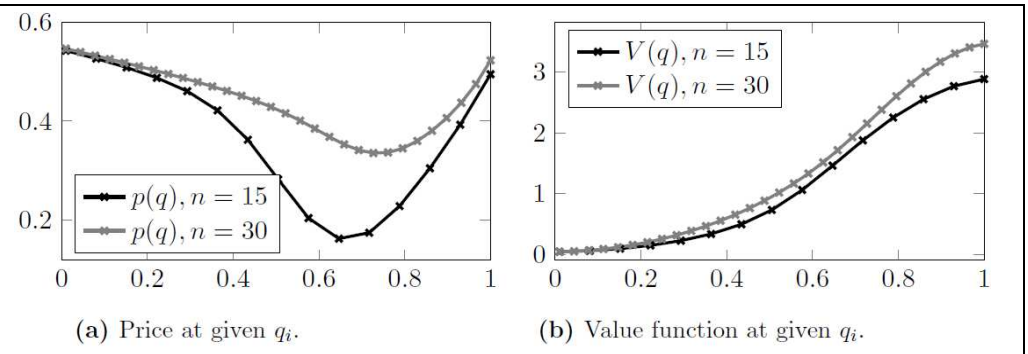
We begin by assuming that our system can be in a finite number of states, each characterized by an arrival probability. Formally, let the arrival probability in period t be denoted by $q_t \in \{q_0, q_1, \dots, q_{n-1}\}$, with $0 \leq q_0 \leq \dots \leq q_{n-1} \leq 1$. While not too restrictive, this assumption circumvents the curse of dimensionality and allows us to computationally approach the problem. The population awareness law of motion is replaced by the following state transition rule:

$$\boldsymbol{If\ q_t = q_i,\ then\ q_{t+1} = \begin{cases} q_{i+1} & if\ b_t = 1\ and\ i < n-1 \\ q_i & if\ b_t = 1\ and\ i = n-1 \\ q_{i-1} & if\ b_t = 0\ and\ i > 0 \\ q_i & if\ b_t = 0\ and\ i = 0 \end{cases}} \quad \textbf{(6)}$$

Equation 6 can be interpreted as the discrete state analog of the law of motion in equation 1. Arrival probabilities increase following an adoption, and otherwise decrease. For clarity of exposition we also assume in what follows that $\Delta q_i = q_{i+1} - q_i = (q_{n-1} - q_0)/(n-1)$, with our results extending to every other case we tried. The value function of the provider is no longer a function of time, and the Bellman equation becomes:

$$\boldsymbol{V(q_i) = \max_{p_i} q_i\big[\big(1 - F(p_i)\big)\big(p_t - c + \beta V(q_{i+1}) + F(p_i)\beta V(q_{i-1})\big)\big] + (1 - q_i)\beta V(q_{i-1})} \quad \textbf{(7)}$$

At the boundary states ($i = 0, i = 1$) the value function is naturally handled by the transition rule of equation (6) (i.e., $V(q_{-1}) \coloneqq V(q_0)$ and $V(q_n) \coloneqq V(q_{n-1})$. We can now solve the Bellman equation numerically. Details are provided in the Appendix. In Figure 4 we plot the price and value functions for all states (equivalently, for all arrival probabilities q_i) in the cases of $n = 15$ and $n = 30$.



(a) Price at given q_i. (b) Value function at given q_i.

Figure 4. Price and value function against q for $n = 15, 30$. Parameters: $q_0 = 0.01$, $q_{n-1} = 1$, $\beta = 0.95$, $c = 0.1$, $\theta \sim U[0, 1]$.

From Figure 4 (a) it is immediately clear that price is not monotonic in q. For q close to 0 and 1 and as n grows larger the price converges to 0.55, which is the optimal price when adoption decisions do not affect future arrival probabilities ($\Delta Q = 0$). Intuitively, providers set lower prices when adoptions substantially increase expected future profits, given by the slope of the value functions in panel (b). For low arrival probabilities, an adoption today is not likely to increase future arrival probabilities to a large degree, and providers set a high price to maximize profit in the rare event that an arrival occurs. For high arrival probabilities an arrival not leading to adoption does not decrease future profit by much since arrivals occur in the near future with a high probability; it is therefore optimal for the provider to set a high price.

Tipping Points

Interestingly, price attains its minimum for intermediate arrival probabilities ($q = 0.64$ and $q = 0.73$ for $n = 15$ and $n = 30$ respectively). The benefit of a low price is more pronounced at these states since failing to generate population awareness may lead to a sequence of non-arrivals which can in turn push the product into obscurity. The state that corresponds to the lowest price can be interpreted as a *tipping point* of the adoption process, above (resp. below) which arrivals are increasingly more (resp. less) frequent. Consequently, a low price is set as an attempt to tip population awareness in the provider's favor.

Figure 5 provides simulations of the arrival probabilities for $n \in \{15,30\}$. Observe that when awareness decreases to a sufficient extent below the tipping point the product quickly falls into obscurity. This effect is more rapid when changes in population awareness are more abrupt.

The degree of change in population awareness following adoption decisions also has a large effect on prices. As is evident from Figure 4, prices are lower across all states for smaller n since the impact of a bad sequence of non-adopters is more pronounced. In that case, low-value consumers are more important and we expect the adoption process to be more unstable.

The simulations in Figure 6 illustrate this point. In panel (a), the provider eventually experiences a sequence of non-adoptions and reacts by reducing price to tip the awareness back to the higher state. In the depicted simulation the attempt fails, and price subsequently increases as arrivals become less frequent and the product fades into obscurity; the niche of consumers that will discover the product will be charged a high price to obtain it. In panel (b) the changes in awareness are smaller, and the provider reacts less aggressively; awareness remains relatively high and streaks of unlucky draws have smaller

impact on her profit. Prices are lower when the provider has neither achieved high awareness nor found her niche.

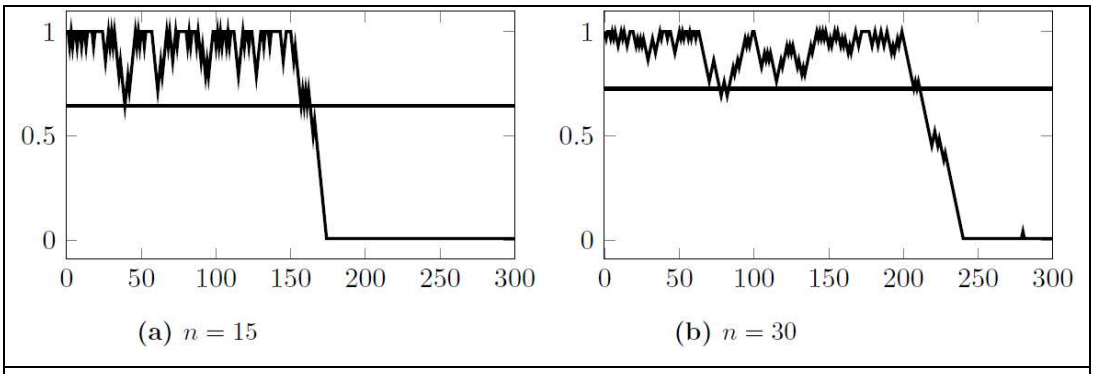


Figure 5. Simulations of arrival probabilities. The horizontal lines represent the arrival probability q corresponding to the lowest price. Parameters: $q_0 = 0.01$, $q_{n-1} = 1$, $\beta = 0.95$, $c = 0.1$, $\theta \sim U[0,1]$.

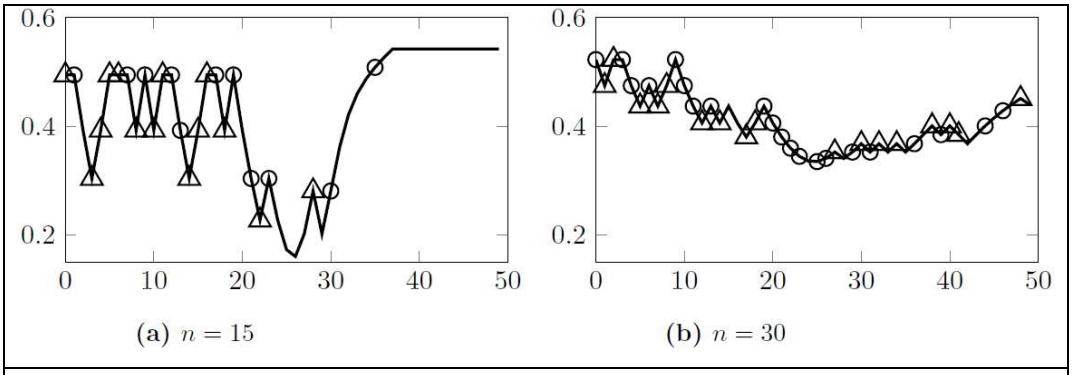


Figure 6. Price simulations with infinite time horizon. Δ: adoption O: arrival without adoption. Initial value of $q = q_{n-1} = 1$. Parameters: $q_0 = 0.01$,$q_{n-1} = 1$, $\beta = 0.95$, $c = 0.1$, $\theta \sim U[0,1]$.

Platform Fees

In Sharing Economy platforms such as Etsy, but also in other forms of online marketplaces such as eBay and Google Play we often observe a vertically related market structure: the platform allows providers to set the price of their products, and sets a fixed percentage fee (a tax-type commission) on the final price.[3] Although the platform technically is in the role of the downstream firm and providers in that of the upstream firm, this pricing model reverses this scheme; it is the platform that first imposes a fee to which providers subsequently react by adjusting their price. For clarity, we refer to the benchmark cases of the preceding sections as the platform and seller being *vertically integrated*, in which case maximization of total profits simplifies to the seller operating free-of-charge on the platform. The present case with sales fees imposed by the platform is thus referred to as the platform and the seller being *vertically separated*. In what follows we study how platform fees are related to population awareness.

[3] For example, the percentage fee on Etsy is 3.5%, on eBay it is 10%, and on Google Play 30%.

For a flat per-transaction fee τ, the first order condition (4) changes slightly, and the price set by the provider is now given by

$$\boldsymbol{p_t(\tau) - \frac{1-F(p_t(\tau))}{f(p_t(\tau))} = c + \tau - \beta[V_{t+1}(a^1_{t+1}) - V_{t+1}(a^0_{t+1})]} \quad \textbf{(8)}$$

Note that we are using a flat fee in lieu of a percentage one to make our derivations shorter, but the results are identical (as can be easily verified). The platform's objective is to maximize its expected profit, and the corresponding Bellman equation is given by

$$\boldsymbol{W_t(a_t) = \max \quad Q(a_t)\Big[\Big(1 - F(p_t(\tau_t))\Big)\Big(\tau_t + \beta W_{t+1}(a^1_{t+1})\Big) + F(p_t(\tau_t))\beta W_{t+1}(a^1_{t+1})\Big)] + [1 - Q(a_t)]\beta W_{t+1}(a^0_{t+1}),} \quad \textbf{(9)}$$

where the interpretation is equivalent to that of (3). The optimal fee τ_t is then given by the first-order condition:

$$\boldsymbol{\tau_t - \frac{1-F(p(\tau_t))}{f(p(\tau_t))dp/d\tau_t} = -\beta[W_{t+1}(a^1_{t+1}) - W_{t+1}(a^0_{t+1})]} \quad \textbf{(10)}$$

It is immediately clear from the above that the interaction between future adoptions and awareness has a dampening effect on platform fees; the platform is concerned about future profits. Due to vertical separation, the platform takes the reaction of the provider into account through the term $dp/d\tau > 0$.

Two-period Analysis

Assume that consumers' valuations are uniformly distributed ($\theta \sim U[0,1]$). The platform fee in period t is:

$$\boldsymbol{\tau^*_t = \frac{1}{2}[1 - c + \beta(\Delta V_{t+1} - \Delta W_{t+1})]} \quad \textbf{(11)}$$

where $\Delta V_{t+1} = V_{t+1}(a^1_{t+1}) - V_{t+1}(a^0_{t+1})$ and $\Delta W_{t+1} = W_{t+1}(a^1_{t+1}) - W_{t+1}(a^0_{t+1})$. The provider then sets price to $p(\tau^*) = \frac{1}{4}[3 + c - \beta(\Delta V_{t+1} + \Delta w_{t+1})$ for a price margin of $p_0(\tau^*_t) - \tau^*_t - c = \frac{1}{4}[1 - c - 3\beta\Delta V_{t+1} + \beta\Delta W_{t+1}$.

In the two-period case ($t \in \{0,1\}$) expected platform and provider profit at $t = 1$are:

$$W_1 = (1 - p_1(\tau^*_1))\,\tau^*_1 = Q(a_1)\,\frac{1}{8}(1-c)^2$$

$$V_1 = (1 - p_1(\tau^*_1))(p_1(\tau^*_1) - c - \tau^*_1) = Q(a_1)\,\frac{1}{16}(1-c)^2$$

respectively. The fee in $t = 0$ is then

$$\tau^*_0 = \frac{1}{2}(1-c)\left[1 + \frac{\beta\Delta Q(1-c)}{16}\right]$$

and the price charged by the provider is

$$p(\tau^*_0) = \frac{3+c}{4} - \frac{3}{64}\beta\Delta Q(1-c)^2.$$

Note that the price is higher compared to its vertical integration counterpart, due to the inefficiencies caused by the well-known double marginalization effect (Spengler, 1950). Moreover, price is less sensitive to differences in future arrival probabilities as compared to vertical integration, as shown by the following result.

Proposition 5. *Initial period price is less sensitive to changes in the difference of future arrival probabilities under vertical separation than under vertical integration.*

Proof: The derivatives of the price with respect to ΔQ under vertical integration ($\tau = 0$) and under vertical separation ($\tau = \tau^*(\Delta Q)$) are:

$$\frac{dp_0(0)}{d\Delta Q} = -\frac{1}{8}\beta(1-c)^2 \quad \text{and} \quad \frac{dp_0(\tau^*)}{d\Delta Q} = -\frac{3}{64}\beta(1-c)^2$$

respectively. The inequality $\left|\frac{dp_0(\tau^*)}{d\Delta Q}\right| < \left|\frac{dp_0(0)}{d\Delta Q}\right|$ then simplifies to $\frac{3}{8} < 1$.

For a given $\tau > 0$ the price is therefore less sensitive to the difference in continuation values because future profits are split between the provider and the platform. The platform's continuation value also depends positively on the provider's continuation value and enters the price indirectly through the fee τ, which makes the price more sensitive to changes in future profits. Nonetheless, the effect of a change in ΔQ becomes smaller under vertical separation. This makes intuitive sense, as vertical separation decreases total profits, which in turn decreases the differences in potential future total profits for a given ΔQ.

Infinite Horizon Analysis

We utilize the infinite-horizon framework we developed to obtain numerical results on pricing and platform fees in relation to population awareness. Examples for $n = 15, 30$ are shown in Figure 7.

Prices follow a similar pattern what we saw in the previous section, with larger price movements for lower n. Consistent with Proposition 5, prices are less sensitive to changes in awareness relative to the results of the previous section, since future expected profits are lower, both because profit is split between the provider and platform, and because of the inefficiencies due to double marginalization.

A second observation is that prices vary more than fees. While the future expected profits of the platform and the provider are highly correlated, their short-run incentives do not align. The provider wishes to set a lower price at intermediate levels of awareness, but the platform's *short-run* optimal response to a low price is to set a higher fee to skim the provider. For two-period case, this can be seen in equation (12), where, *ceteris paribus*, platform fees are increasing in ΔV. Nonetheless, provided that both sides put equal weight on future profits, the correlated outcomes in the two agent's value function dominate the short-run incentive to raise the platform fee.

We should briefly mention that platforms oftentimes combine sales fees with other contractual arrangements like two-part tariffs and resale price maintenance. Under such circumstances we would expect the results with respect to the final sales price to be closer to the non-fee case analyzed in the previous sections of this paper.

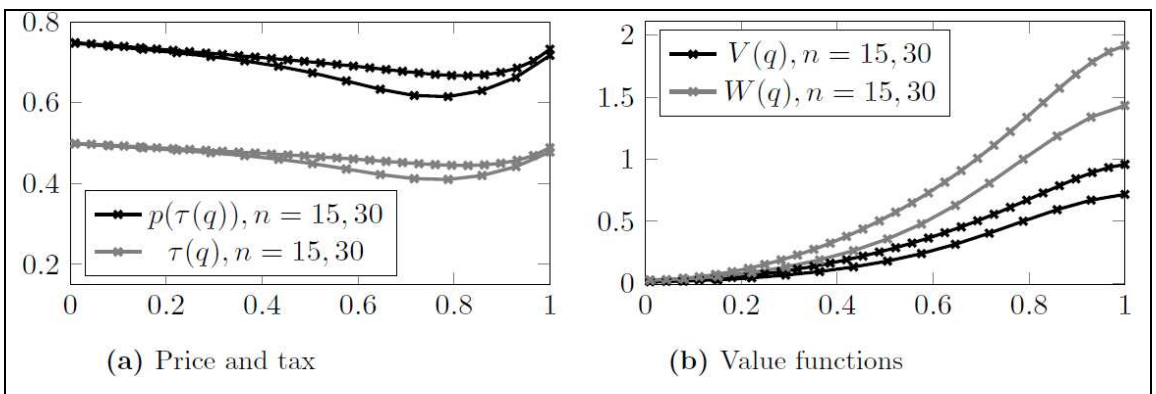


Figure 7. Price, platform fees, and value function of the provider and the platform against q for $n = 15, 30$. Parameters: $q_0 = 0.01, q_{n-1} = 1, \beta = 0.95, c = 0, \theta \sim U[0, 1]$.

Conclusion and Future Directions

In this work we attempt to shed light on the drivers of the behavior of providers operating on Sharing Economy platforms. We develop a dynamic pricing model with awareness generating adoptions, where the provider can only control price and lack other common awareness-promoting options (e.g.

advertising). We believe that the increasing interconnectedness of our world makes understanding mechanisms of awareness propagation an important task not only for Sharing Economy platforms, but also for traditional online or bricks-and-mortar marketplaces.

Our modeling approach has the provider solve a dynamic program to optimally readjust prices depending on whether she has achieved a satisfactory level of awareness. Population awareness is analogous to the probability that the product will be discovered by future consumers, and obeys two intuitive laws: past adopters generate positive externalities by increasing awareness, and awareness decays over time. In marketplaces for products and services of limited lifespan we find behavior consistent with penetration pricing, and influenced by end-of-the-world effects. In the infinite horizon setting, we exhibit the existence of tipping points. Population awareness and prices are related in a non-monotonic fashion, with mass-market and niche products being priced higher. We also investigate optimal platform fees in this setting; we show that these fees exhibit an inverted U-shape, and are lower for both mass-market and niche products. Those results exhibit the relation between awareness and the emergence of superstar and niche products in online marketplaces. Future research may focus on expanding on these ties and attempt to uncover the role of awareness in long tail phenomena (Bar-Isaac et al. 2012, Brynjolfsson et al. 2010, Brynjolfsson et al. 2011).

Our results have important implications for the emerging area of the Sharing Economy. Sharing Economy platforms typically do not offer advertising services, and an important lever for providers to control population awareness is through price. Admittedly, there are other factors that contribute to awareness, such as the heterogeneity in products and services, which presents providers with more dimensions to use in controlling awareness. Our model is therefore most applicable in settings where the marketplace has reasonably matured (and has filtered out the worst performing providers), or marketplaces where the products or services that are offered are relatively interchangeable. Our model is therefore a fitting initial framework for such settings, and showcases population awareness as an important explanatory factor for the pricing patterns often observed therein. Keeping the consumer population aware of a product while optimizing profits results in substantial price fluctuation and lower-than-monopoly prices even in one-provider markets. Moreover, optimally adjusting prices is not pre-determined but instead depends on the very nature of the awareness effects.

The effect of product awareness on demand and price patterns closely resembles classical results in the network effects literature (Katz and Shapiro, 1985; Economides, 1996; Cabral et al., 1999). However, the underlying mechanisms are dissimilar: network effects assume that consumers' utilities are a function of number of adopters, while awareness only affects the likelihood of a product being discovered. Toward that end, we point out the need account for both these parallel forces, as well future empirical work that attempts to decouple them and measure their relative influence.

It is important to point out that even though our model may be applicable to more general online marketplaces, we believe that it captures aspects unique to the Sharing Economy. We discussed in the introduction two factors ubiquitous in peer-to-peer marketplaces: the absence of advertising capabilities, as well as the role of recommendation systems and internet search in the emergence of consumer awareness as a moderator of economic activity. In addition, a model where providers respond to awareness attempts to capture the core characteristics of provider behavior on Sharing Economy platforms. The providers participating in such platforms are often less experienced (Hall and Krueger, 2015); we would therefore expect them to not possess information typically assumed in pricing models, such as accurate demand forecasts or a complete understanding of the underlying market competition dynamics. Even if they do, their limited experience may result in suboptimal utilization. Providers that simply respond to awareness – approximated through proxies such as observed demand (sales), rank in platform searches, or measures of interest provided by specialized services (e.g. Google Trends) - is a step towards a more realistic model of behavior on Sharing Economy platforms.

A limitation of our analysis can be found in the lack of a competition setting. In our model, competition can be partly captured by appropriately adjusting our model's parameters (since many firms compete for attention, more intense competition should imply both sharper time-decay in awareness and lower awareness returns due to adoptions). Game-theoretic approaches have been proposed in the literature, but have mostly been restricted to one-shot games (Bar-Isaac et al. 2012). We acknowledge that a game-theoretic formulation may help uncover additional insight. In future work, we plan to extend our current framework toward that direction.

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Appendix

Finite Horizon

Our objective is to compute the value function of the finite-horizon dynamic programming problem. Toward that end we have to take into account all feasible population awareness states, given in the matrix $\boldsymbol{A} \in \mathcal{R}^{T\times 2^T}$. More specifically, each column corresponds a time period $t \in \{0, \dots, T-1\}$, and each row $r \in \{0, \dots, 2^{T-1}-1\}$ corresponds to a given sequence of adoptions $\{b_0, b_1, \dots\}$. Note that the order of adoption decisions matters due to the time-depreciation of awareness in the model. Since there are two feasible events each period (adoption or no adoption), there exist 2^{T-1} population awareness states at time t. The initial period is $t = 0$.

The elements of the matrix are constructed using the law of motion (equation 1), which yields the following relation:

$$\boldsymbol{a_{r,t}} = \begin{cases} \boldsymbol{\delta a_{r,t-1}} & \boldsymbol{if\ t < 2^{t-1} - 1} \\ \boldsymbol{\delta\left(a_{r+2^{T-1},t-1} + k\right)} & \boldsymbol{if\ t \geq 2^{t-1} - 1} \end{cases} \quad \textbf{(12)}$$

For example $a_{1,2} = \delta a_{1,1}$ and $a_{2,2} = \delta a(_{0,1} + k)$. For any given column, the top half of the non-zero elements represents population awareness when an adoption did not occur the preceding period ($b_{t-1} = 0$) while for the bottom half an adoption did occur ($b_{t-1} = 1$). For each time period, awareness values that are not attainable are set to 0.

We can now use $\boldsymbol{A}$ to construct the matrix $\boldsymbol{Q} \in \mathcal{R}^{T\times 2^T}$ in which an element $q_{r,t} = Q(a_{r,t})$ denotes the arrival probability associated with awareness-period pair. Using $\boldsymbol{Q}$ we can now derive the value function matrix $\boldsymbol{V}$ and the optimal price matrix $\boldsymbol{P}$. Each element $v_{r,t}$ and $p_{r,t}$ is given by the Bellman equations:

$$\boldsymbol{v_{r,t}} = \mathbf{max}_{\boldsymbol{p_{r,t}}}\, \boldsymbol{q_{r,t}}\left[\left(\mathbf{1} - \boldsymbol{F(p_{r,t})}\right)\left(\boldsymbol{p_{r,t}} - \boldsymbol{c} + \boldsymbol{\beta v_{r+2^T,t+1}}\right) + \boldsymbol{F(p_{r,t})\beta v_{r,t+1}}\right] + \left[\mathbf{1} - \boldsymbol{q_{r,t}}\right]\boldsymbol{\beta v_{r,t+1}} \quad \textbf{(13)}$$

$$\boldsymbol{p_{r,t}} = \mathbf{argmax}_{\boldsymbol{p_{r,t}}}\, \boldsymbol{q_{r,t}}\left[\left(\mathbf{1} - \boldsymbol{F(p_{r,t})}\right)\left(\boldsymbol{p_{r,t}} - \boldsymbol{c} + \boldsymbol{\beta v_{r+2^T,t+1}}\right) + \boldsymbol{F(p_{r,t})\beta v_{r,t+1}}\right] + \left[\mathbf{1} - \boldsymbol{q_{r,t}}\right]\boldsymbol{\beta v_{r,t+1}} \quad \textbf{(14)}$$

We can compute $\boldsymbol{V}$ and $\boldsymbol{P}$ using backward induction starting with $v_{r,T-1}$ for $r \in \{0, 2^{T-1}-1\}$, where $v_{r,T} = 0$ for all r by construction. As in our problem there are 2^{T-1} feasible paths, the computational

power required to solve for value functions and prices increases exponentially in the number of periods. This limits us in analyzing the case for a large number of periods; we circumvent this difficulty by carrying out simulations in the infinite horizon.

Simulations

Having constructed matrices $\boldsymbol{Q}$ and $\boldsymbol{P}$ simulations are conducted as follows. For each period t, we draw $\theta_t \sim F$ the valuation of consumer t, and $X_t \sim U[0,1]$. Exactly one of the following four outcomes occurs:

1. $X_t \leq Q(a_{r,t})$ and $\theta_t \geq p_{r,t}$: A consumer arrives and adopts.
2. $X_t \leq Q(a_{r,t})$ and $\theta_t < p_{r,t}$: A consumer arrives and does not adopt.
3. $X_t > Q(a_{r,t})$ and $\theta_t \geq p_{r,t}$: A consumer does not arrive, but would have adopted if he arrived.
4. $X_t > Q(a_{r,t})$ and $\theta_t < p_{r,t}$: A consumer does not arrive and would not have adopted if he arrived.

In the price simulations, the first event means moving from position $p_{r,t}$ to position $p_{r+2^{T-1},t+1}$ of the matrix $\boldsymbol{P}$, while the three latter events correspond to a horizontal move from $p_{r,t}$ to $p_{r,t+1}$. Adoptions are represented by a triangle, arrivals and non-adoptions by a circle, and non-arrivals by the absence of marks (see Figure 2).

Infinite Horizon

In the infinite horizon case, we will approximate the solution by constructing the matrix $\boldsymbol{V} \in \mathcal{R}^{n \times T}$ and applying backward induction. Since we know that $\lim_{T \to \infty} V_t(q_i) = V(q_i)$ for $0 < \beta < 1$ convergence is guaranteed (Stokey and Lucas with Prescott, 1989). More specifically, backward induction here is equivalent to implementing value function iteration from initial states $V(q) = 0$ for all q.

Let $\boldsymbol{V}_t$ denote the $t+1$'th column of $\boldsymbol{V}$ which corresponds to time t. Our convergence criterion is then given by the Euclidean norm of the difference of the first two value functions:

$$||\boldsymbol{V_0} - \boldsymbol{V_1}||_2 < \epsilon$$

where ϵ is sufficiently close to zero. In our simulations we set $\epsilon = 10^{-6}$.