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ANALYSIS OF THE GROWTH OF SOCIAL NETWORKING SERVICES BASED ON THE ISING TYPE AGENT MODEL

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Abstract

In recent years, social networking services such as facebook, twitter and some kind of blogs have been growing, obtaining more and more users and having larger impact on many areas of social activities. In these services, it has been noted that network externality plays an important role. The utility of a service depends on the number of users that use the service. In general, the growth curve of the number of the users etc. is nonlinear. So far, however, this nonlinearity has not been understood quantitatively. By applying the nearest neighboring agent model called Ising type agent model, we analyze the growth of a blog type social networking service. It is shown that the growth curve is quantitatively reproduced covering the whole range from the initial service-starting stage to the final saturating stage, despite the fact that the model has only two parameters. This means that the model is promising as a starting point of analyzing the growth and the impact of social networking services.

Keywords: Social networking service, Network externality, Blog service, Ising model, Nearest neighboring interaction.

1. INTRODUCTION

In recent Web based social networking services (abbreviated to SNS hereafter) such as facebook, twitter, and some kinds of blogs, network externality plays an important role. In these kind of services, the more the number of the users increases, the more its utility increases, which leads to the more users. In the result of this cycle, it is known that the number of users increases drastically at a point.

A typical example that is affected by the network externality is telephone (Artle and Averous 1973). If there is only one user, telephone is useless, because the user can call nobody. If there are many users, however, the utility increases. As a result, the more users join the telephone service, and the more its utility increases. At a critical point, the number of the users starts increasing rapidly. Other examples are the rapid growth of the users of mobile phones in mid-1990s and the recent growth of SNS. In many cases, Web based services are more or less affected by the network externality. This is reasonably understood from the viewpoint that those services are based on the internet which makes sense when there are many users.

So far, there have been many works that studied the effect and the development of the network externalities. Roughly speaking, there were two kinds of approaches. The first one is analytical approach (Leibenstein 1950; Rohlfs 1974; Katz and Shapiro 1985; Farrell and Saloner 1985). In those works, it is assumed that the model system is spatially homogeneous. Therefore the mechanism to develop the nonlinear and spatially inhomogeneous effect of the network externality is not understood quantitatively. For example, it is not understood how the “critical point” of the rapid increase is determined from the parameter to describe the service providing company and its economic environment. Furthermore, it is not clear if there really exists the “critical point.” The growing curve might be just an exponential one, and there might not be any point of qualitative change.

The second one is the agent model approach which is based on the computer simulation. Recently, this approach has been well-developed due to the recent availability of high performing computing resource. In this kind of approach, one can investigate the macroscopic features based on the microscopic one such as the purchase of a service of an individual. Although there have been many kinds of agent model, the nearest neighboring interaction model called Ising type model (Ising 1925; Kapusta 2006) is used not only in analyzing the economical aspect of network externality but also in many areas of social science (Oh and Jeon 2007).

In either of these approaches, it has not been well understood how the qualitative feature of the network externality can be applied to quantitatively understand the recent growth of the IT related services such as social networking services.

In this report, we study the growth of the social networking service and see whether the agent model approach can explain it quantitatively. As an example, we pick up the growth of blog service in Japan. By applying the Ising type agent model, we see if it is possible to quantitatively explain the growth curve. The reason that we chose the blog service not the wide-spreading one such as facebook is that the blog service is already saturated and is a good sample as a first step to see if the Ising type agent model can reproduce the behaviour of its growth covering all the stages from the early developing stage to the last saturation one.

This report is organized as following. In the next section, we show the growing curve of blog service in Japan. In this section, we also explain the Ising type agent model. Since this kind of agent model is essentially the same as the previous works, we just briefly show the outline of the model. In the following section, we show the expression of the model and the way of carrying out the numerical simulation. In the next section, we show the results of the numerical calculation. It is seen that the growth curve of blog can be explained quantitatively for wide range of time scale and the dependency on the parameters within the model is small. The last section is devoted to the summary and the prospect of the future work.

2. THE GROWTH CURVE OF BLOG SERVICE AND THE OUTLINE OF THE ISING TYPE AGENT MODEL

In Figure 1, we show the number of written articles of blogs in Japan for each quarter year. Unfortunately, we could not obtain the number of the active users. An active user is the one that really uses the service. Since it does not cost anything to join the service, it is highly possible that there are many users that have their accounts but do not use the service at all. So, we instead show the number of the written articles which is expected to reflect the number of the active users.

The number of the written articles (in unit of million)

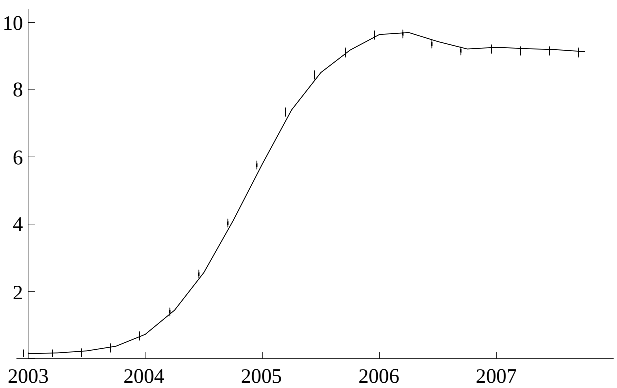

Figure 1. The growth of the number of the articles of blog services in Japan. Cited from "Reports on the usage of blog" published from the Information Communication Research Institute of the Ministry of Internal Affairs and Communication.

One can see that there are roughly three stages. The first one is the years before mid-2004, in which the count increases nonlinearly. The second one is between mid-2004 and early 2005 during which the count increases rapidly linearly from 2 million to 8 million. The third one is after mid-2005, in which the count saturates.

As is shown in the last section, we use the Ising type agent model to explain this overall growth curve. Although there have been many kinds of agent models, the Ising type agent model is the simplest one with only two parameters. Therefore, we chose this model as a first step and tried to see how this simple model can explain the real world.

Ising model was first introduced to explain the ferromagnetism of iron. Iron consists of iron atoms that have small magnetism. In Ising model, it is assumed that the interaction between atoms are restricted to the nearest neighboring atoms. Due to this interaction, it is energetically favourable (i.e. gives the low-energy stable state) that the direction of the magnetism is the same as the ones with the neighboring atoms. If the magnetism of all the atoms are aligned as is shown in Figure 2, the total magnetism is large. As the result, the iron is a permanent magnet.

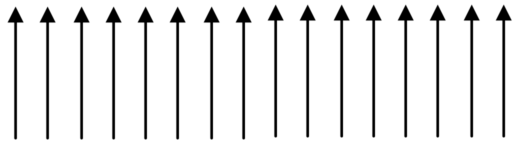

Figure 2. In the permanent magnet, all the magnetism of the iron atoms is aligned.

The feature that the nearest neighboring interaction gives the alignment has been applied to the agent model to analyze many social aspects in the previous works. The alignment is analogous to the behaviour of consumers that join the same service used by the acquaintances.

By interpreting that the agent with downward direction is the individual that does not purchase a service and that the agent with upward direction is the one that purchase the service, one can construct the Ising type agent model. The growth of a service can be interpreted to be a transition from the initial state shown in Figure 3 to the final state shown in Figure 2.

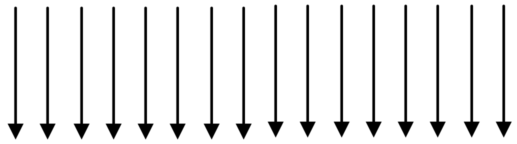

Figure 3. The initial state that no agent joins the service.

As was pointed out by Rohlfs (1974), the transition does not take place spontaneously. The initial incentive of joining the service is required. In the case of the mobile phone, this incentive is the utility that the owner can call and receive any time anywhere to/from the traditional home-telephone. In the Ising model, this utility is taken into account by the parameter called external magnetic field denominated by the parameter *h* in the model. As is shown in Figure 4, each agent (atom in the original model) has higher utility (lower energy in the original model) if its direction (magnetism) is in the same direction of the external field. Therefore, this effect gives the first incentive to join the service. The detail of the expression of this parameter is described in the next section.

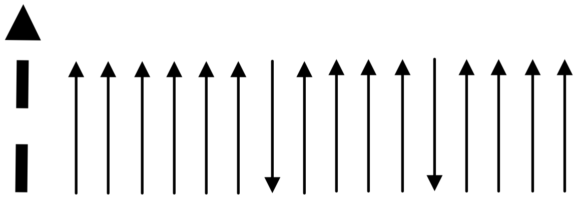

Figure 4. If there is an external field shown by the left big arrow, many of the atoms prefer having the same direction with the external field.

In the model, there is one more parameter J, which gives the strength of the interaction between the agents. This is also explained in the next section.

In the next section, we show the detailed expression of the Ising model and see how to formulate the growth of blog service by Ising model. We also show how we can carry out numerical simulations.

3. DETAILS OF THE NUMERICAL SIMULATION METHODS

For simplicity, we show the case in one dimension. In the original Ising model, atoms are discriminated by the index i. The energy of the atom i (E_i) is given by the sum of the potential energy and the interaction energy:

$$E_i = -h \cdot S_i - \frac{J}{2}(S_i \cdot S_{i+1} + S_i \cdot S_{i-1}) \quad (1),$$

where S_i=+1 or –1 is the direction of the magnetism of the atom i, h is the strength of the external magnetic field and J is the parameter to give the strength of the interaction between the neighboring atoms. In the same way of applying the model to the social science in the previous works, we interpret the term "energy" as the negative of the utility in the term of economics: i.e. $-E$ is the utility. As the system is stable when the energy is low in the original model, the system with higher utility is preferred in the Ising type agent model. If $h>0$, the first term in equation (1) means that the energy is lower (utility is higher) for $S_i=1$ than $S_i=-1$. This means that each agent prefers being in the state of upward direction as for the first term. For the second term, the energy is lower (utility is higher) for $S_i=S_{i+1}$ than $S_i=-S_{i+1}$ if $J>0$. This means that the agents prefer being in the same direction with the neighboring agents. Since the initial state is $S_i=-1$ for all the agents as is shown in Figure 3, the second term gives the effect to have the agents stay $S_i=-1$. If all the agents move to be in $S_i=1$, the state is most stable because it is favorable both in the first term and the second term. If $J>h$, however, the initial state is metastable because flipping some part of agents shown in Figure 5 gives higher energy (lower utility) due to the second term.

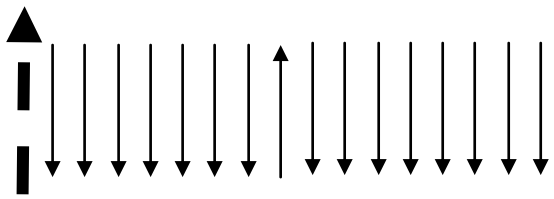

Figure 5. Flipping one atom gives larger interaction energy than the initial state.

According to the statistical mechanics, the probability that a state with energy E_i is realized is proportional to

$$\exp(-E_i) \quad (2).$$

(More precisely, there is a thermal parameter in the original Ising model. But it is omitted because it can be reinterpreted to be a part of the parameters h and J.) Thus even if the state shown in Figure 5 is unfavourable, there is a small possibility that the state is realized. If the state of Figure 5 happens to be realized, the agents neighboring to the upward agent prefer to be aligned to the agent i.e. upward direction. Through this process, the initial state gradually moves to the state shown in Figure 4.

From equations (1) and (2), the probability for a state is given by

$$\exp\left\{-\left(-h\cdot S_i-\frac{J}{2}(S_i\cdot S_{i+1}+S_i\cdot S_{i-1})\right)\right\} \qquad (3).$$

The parameters in the model are h and J. As is shown above, the state $S_i=-1$ is interpreted to be not joining the service and $S_i=1$ means that the agent specified by the index i is joining the service. The transition from the initial state shown in Figure 3 to the state shown in Figure 4 is the process that the service grows under the effect of network externality. As is used in the previous works, this process can be stochastically simulated in the following way:

1: All the state are set $S_i=-1$ as the initial state.

2: By putting $S_{i+1}=-1$ and $S_{i-1}=-1$ to the equation (3), the probability for $S_i=-1$ and $S_i=1$ is given after normalization.

3: Make a random real number between 0 and 1.

4: If the number is below the probability of $S_i=-1$ given by the step 2, $S_i=-1$ is chosen and $S_i=1$ is chosen otherwise.

5: This process is carried out for all the atoms and the new set of $\{S_i\}$ is given.

6: By putting the new value of S_{i+1} and S_{i-1} to the equation (3), the new probability for $S_i=-1$ and $S_i=1$ is given and the steps from 3 to 5 are repeated.

As an example with numerical value, let us show the case of $h=1$ and $J=2$. The process 2, 3, 4 and 5 above is:

2: $-E_i=-h+J=1$ for $S_i=-1$ and $-E_i=h-J=-1$ for $S_i=1$ for all the i's. This gives exp(1)=2.72 and exp(–1)=0.38 for the equation (2), and after normalization the probability for $S_i=-1$ and 1 is 0.88 and 0.12 i.e. 88% and 12%.

3: Assume that the random number is 0.9 for $i=3$.

4: Since 0.9 > 0.88, $S_3=1$ is chosen.

5: Assuming the random numbers are less than 0.88 for all the other i's, $S_i=-1$ is chosen. Namely the news set of $\{S_i\}$ is { –1, –1, 1, –1, –1, …}.

And the next cycle of these steps starting with the new set S_i={ –1, –1, 1, –1, –1, …} is:

2: For $i=2$ and 4 (i.e. the neighboring agents of the $i=3$ agent with $S_i=1$), the second term of the equation (2) is $S_i\cdot(S_{i-1}+S_{i+1})=S_i\cdot(-1+1)=0$ because the agent of the $i=3$ has $S_i=1$ and the other neighboring agent with $i=1$ or 5 has $S_i=-1$. Therefore $-E_i$ is now $-h=-1$ for $S_i=-1$, and $-E_i=1$ for $S_i=1$, which gives the probability 0.12 for $S_i=-1$ and 0.88 for $S_i=1$.

3: Assume that that the random number is 0.3 and 0.4 for $i=2$ and 4.

4: Since the random numbers are larger than 0.12, S_i=1 is chosen for i=2 and 4.

All the other process for the agents $i \neq 2$ and 4, the steps 2 to 5 is all the same as the first cycle. By repeating this process, the number of the agents with S_i=1 gradually increases.

Since this process is stochastic, there is statistical error and the result of the numerical simulation differs each time. To reduce the statistical error, we iterated the simulation 10000 times and took the average, which leads to the errors within 1%.

4. RESULTS OF NUMERICAL SIMULATIONS AND THE COMPARISON WITH BLOG

In Figures 6,7 and 8, we show the ratio of users of the service to all the agents (i.e. the ratio of upward agents to all the agents) for the parameters h=1.0, J=1.8; h=1.0, J=2.0 and h=2.0, J=2.2 respectively. The horizontal axis is the count of the iteration of the process 3-6 in the last section, which can be interpreted to be the time elapsed (called "steps" hereafter). Roughly, it is seen that the growth curve are all alike and they look the same as the growth curve of the blog service shown in Figure 1. The dependency on the parameter set gives only the time scale (steps). This feature can be understood reasonably. If J is large, it means the interaction between the nearest neighboring agents is large. Therefore all the agents favor to be in the same state with the nearest neighbors and the state remains the initial state for longer time. By comparing the Figures 6 and 7, one can see that in Figure 7 more steps are required to reach the final state, namely the growth is slow. If h is large, it means the utility is large and more agents would join the service regardless of the state of the neighbors. This leads to fast growth of the service. One can confirm this by comparing Figures 7 and 8.

For these three figures, the steps at the time that the ratio of the users is 50% (call the time A), 20%(C), 10%(D) are calculated by interpolating the dots linearly. The time of saturation (B) is also investigated. The time of saturation is defined to be the time that the speed of the growth is half of that when the ratio of the users is 50%. The results are shown in Table 1.

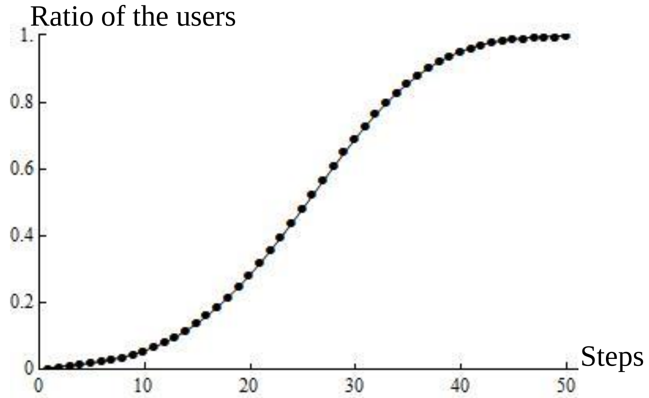

Figure 6. The growth of the service for h=1.0, J=1.8.

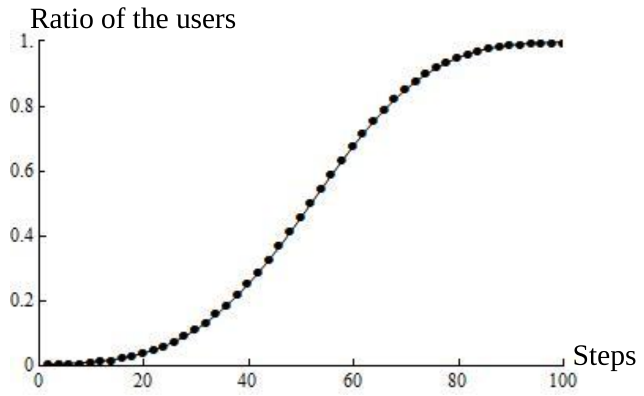

Figure 7. *The growth of the service for h*=1.0, *J*=2.0.

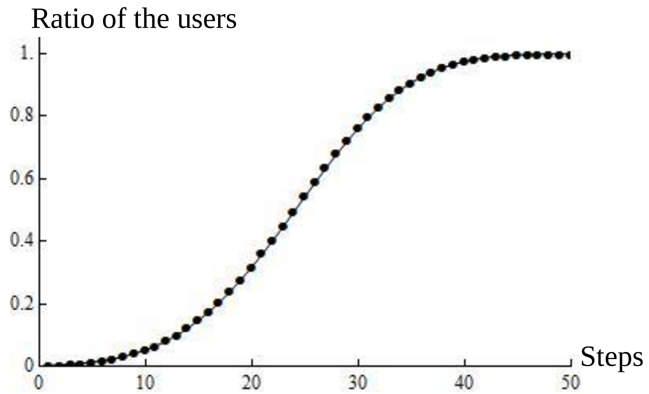

Figure 8. *The growth of the service for h*=2.0, *J*=2.2.

	50% (A)	Saturation (B)	20% (C)	10% (D)
Figure 6	25.4	37.1	17.3	13.1
Figure 7	51.9	73.6	36.8	28.8
Figure 8	24.0	34.6	16.7	12.9
Figure 1	10/11/2004	25/8/2005	11/5/2004	1/2/2004

Table 1. *Steps and the dates at the time that* 50%, 20%, 10% *of all the agents joined and the time of saturation*

To compare the values in Table1, we took the ratios: (B-A)/(A-C) and (B-C)/(C-D). The first one is the ratio of the period between saturation and the time of 50% to the period between the time of 50% and 20%. The second one is the ratio of the period between saturation and 20% to the period from 10% to 20%. The results are shown in Table 2. One can see that the results are identical and the growth of the blog service can be reproduced regardless of the parameter sets.

From this, one can also say that the market saturates after the period of 1.5 (4.6) times of the period required to reach 50% (20%) from 10% (20%).

	(B-A)/(A-C)	(B-C)/(C-D)
Figure 7	1.45	4.60
Figure 8	1.44	4.65
Figure 9	1.45	4.66
Figure 1	1.47	4.69

Table 2. The relative values of the results in Table 1.

5. DISCUSSION AND CONCLUSION

We studied the growth of blog service and compared the growth curve to the results obtained by the Ising type agent model. It was found that the growth rate can be quantitatively explained by this model, covering the whole range from the initial state to the final saturating state. This means that the service providing firms can estimate the growth of the market from the growth curve in the past and optimize the investment in the future.

The fact that the growth curve is reproduced in spite of the fact that the model has only two parameters means that the model is a promising one as a first step to analyze the social network and the effect of network externality. In the next step, we will proceed to analyzing the growth of facebook and twitter that are before the saturating stage.

References

Leibenstein,H. 1950, "Bandwagon, Snob, and Veblen Effects in the Theory of Consumers' Demand", The Quarterly Journal of Economics, (64:2), pp.183-207.

Artle, R. and Averous, C. 1973, "The Telephone System as a Public Good: Static and Dynamic Aspects," Bell Journal of Economics and Management Science 4, pp. 89-100.

Rohlfs, J. 1974, "A Theory of Interdependent demand for a Communications Service," Bell Journal of Economics and Management Science 5: pp. 16-37.

Katz, M.L. and C. Shapiro 1985, "Network Externalities, Competition, and Compatibility," American Economic Review (75:3), pp. 424-440.

Farrell, J. and G. Saloner 1985, "Standardization, Compatibility, and Innovation," Rand Journal of Economics (16:1), pp.70-83.

Oh, W. and Jeon, S. 2007, "Membership Herding and Network Stability in the Open Source Community: The Ising Perspective", Management Science (53:7), pp.1086-1101.

Ising, E 1925. "Beitrag zur theorie des ferromagnetismus," Zeitschrift für Physik (31), pp.253-258.

Kapusta, J. and Gale, C. 2006, "Finite-Temperature Field Theory: Principles and Applications", Cambridge University Press.