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Jack S. Cook Ph.D
State University of New York (SUNY) -Geneseo

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Generalized Concentration Equipment Location Problem

Jack S. Cook, Ph.D.
State University of New York (SUNY) - Geneseo

1.0 Introduction

Deregulation of the US telecommunication market and the break-up of the Bell System in the 1980s, followed by the privatizing and deregulating of most economically advanced countries of Europe and the Asia-Pacific region, ushered in an era of market turbulence, technological uncertainty and regulatory confusion in the information technology sector [Dholakia and Dholakia 1994]. This transformation from a regulated to a deregulated industry was the impetus for the marked drop in long-distance rates and the ensuing rapid upsurge in usage, particularly in data communication. In response to competition, common carriers have greatly increased the number of services available. For example, since divestiture, AT&T responded with hundreds of new interstate services and features; indeed, the number of new AT&T pricing plans and services rose from 35 in 1984 to 195 in 1991 [Garfinkel 1993]. Competition of this magnitude makes it possible for businesses to economically create vast private telecommunication networks.

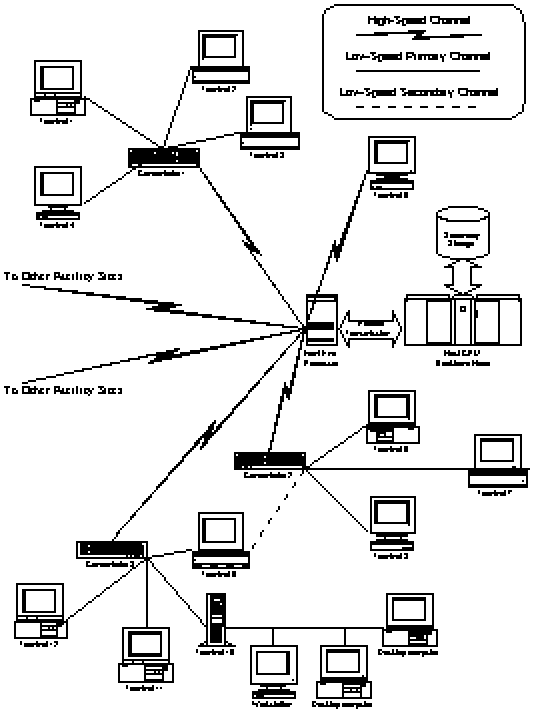
As society demands larger and more complex data communication networks, particularly to support such applications as end-user computing, multimedia, and electronic data interchange (EDI), designing cost effective networks and subsequently managing them becomes increasingly more difficult and consequential. One noteworthy topological issue in the design of data communication networks is how to connect large numbers of remote terminals to a central site. Historically, the usual design method utilized strictly concentrators [Mirzaian 1985]. However, with the advent of microelectronics, there is a host of data concentration equipment available that facilitate the economic utilization of transmission media. Despite many sophisticated alternatives supported by advanced technology, transmission will always constitute the most expensive component of telecommunication systems, requiring careful management to achieve the highest possible economy [Puzman and Kubin 1992].

This research addresses a network design problem referred to as the generalized concentrator equipment location problem (GCELP). GCELP may be defined as the problem of determining where to establish sites consisting of one or more concentrators, each of which are connected to a central site (e.g., a node of a backbone network or a processing site), and connecting terminals, often remote, to these concentrators. For completeness sake, variable concentrator coverage is considered. That is, each terminal will be connected to some concentrator for its primary coverage, possibly to another concentrator for its secondary coverage, and so forth [Pirkul *et al.* 1988, Narasimhan 1990]. In the next section, a mathematical formulation of GCELP is provided.

2.0 Problem Statement

The four distinctions of the generalized concentration equipment location problem (GCELP) that greatly improve its applicability over the traditional concentrator location problem are: (1) numerous concentrator types are available for potential selection at each established facility site, (2) more than one concentrator may be operated at an individual site, (3) the decision maker may limit the number of concentrators at any particular site, and (4) terminals may be connected to more than one concentrator for variable coverage. Hence, GCELP consists of the following decisions: (1) Out of a set of potential facility sites, which sites are to be established?; (2) At each selected site, what concentrator types are to be operated?; (3) How many of each concentrator type are to be operated at each selected site?; (4) Out of all possible concentrator/site combinations, how are terminals to be connected? These four decisions are made in such a way that the cost of establishing facility sites consisting of potentially multiple concentrators and connecting terminals to these concentrators is minimized while not exceeding each concentrator's capacities (i.e., number of circuit ports and processing capabilities). In addition, similar to Lee [1993], terminals can also be directly connected to the central processing site, which for modeling purposes is considered a distinguished concentrator located at a pre-established site (Figure 1). Practically speaking, although feasible, such direct connections are costly.

Consider a given set of terminals (end-user nodes) indexed by $\mathbf{T}=\{1,2,...,\}$, a set of p potential facility sites, which is usually a subset of $\mathbf{T}$, indexed by $\mathbf{S}=\{1,2,..., p\}$, and a set of m concentrator types indexed by $\mathbf{I}=\{1,2,...,m\}$. Furthermore, since more than one concentrator of the same type may be operated at any established site, let nsi denote the maximum number of type i concentrators allowed to be operated at site s, and $\mathbf{J}$ $si=\{1,2,...,nsi\}$ the corresponding index set. Each concentrator j of type i at site s is characterized by its fixed cost, $fsij$, and connecting terminal t to such a concentrator results in a communication cost of $ctsij$. Furthermore, each type i concentrator has a finite number of circuit ports (i) as well as a limit on its



processing capabilities (*i*). In addition, if concentrator *j* of type *i* at site *s* is designated as the *k*th concentrator to serve terminal *t*, then terminal *t* demands a known amount of the concentrator's processing capability, *ptsijk*. However, in most situations, the traffic intensity is strictly a function of the terminal, the concentrator type and whether this concentrator connection is primary or not [Narasimhan 1990]. Hence, *ptsijk* can often be replaced by *ptik*. More realistically though, the communication load between terminal *t* and its *k*th backup is a function of the load between a terminal and its primary concentrator. Therefore, let *tik* denote the average percentage of the normal communication load between terminal *t* and its primary concentrator (assuming it's of type *i*) which is system critical given a concentrator of type *i* is designated as its *k*th backup. Consequently, *ptik* can be obtained by multiplying *pti1* by *tik* where *ti*1 is set equal to one. Hereinafter, when referring to *pti*1, the last subscript is dropped. In addition to the above notation, the decision variables for GCELP are interpreted as follows:

xtsijk = 1 if concentrator j of type i at site s is designated as the kth concentrator to serve terminal t;

0 otherwise.

and

ysij = 1 if concentrator j of type i at site s is to be operated;

0 otherwise.

The former decision variables are referred to as terminal-to-concentrator link variables and the latter, concentrator location variables. To support the following model formulation and subsequent discussions, relevant notation is summarized in Table 1.

GCELP)

$$\text{MIN } Z = \sum_{t\in \mathbf{T}} \sum_{s\in \mathbf{S}} \sum_{i\in \mathbf{I}} \sum_{j\in \mathbf{J}_{si}} \sum_{k\in \mathbf{K}_t} c_{tsij} x_{tsijk} + \left(\sum_{s\in \mathbf{S}} \sum_{i\in \mathbf{I}} f_{si} \left(\sum_{j\in \mathbf{J}_{si}} y_{sij} \right)\right) \tag{2.1}$$

s.t.

$$\sum_{s\in \mathbf{S}} \sum_{i\in \mathbf{I}} \sum_{j\in \mathbf{J}_{si}} x_{tsijk} = r_t \qquad \forall\, t\in \mathbf{T}, k\in \mathbf{K}_t \tag{2.2}$$

$$\sum_{t\in \mathbf{T}} \sum_{k\in \mathbf{K}_t} x_{tsijk} \le \alpha_i \qquad \forall\, s\in \mathbf{S}, i\in \mathbf{I}, j\in \mathbf{J}_{si} \tag{2.3}$$

$$\sum_{t\in \mathbf{T}} \sum_{k\in \mathbf{K}_t} \mu_{tik} p_{ti} x_{tsijk} \le \beta_i \qquad \forall\, s\in \mathbf{S}, i\in \mathbf{I}, j\in \mathbf{J}_{si} \tag{2.4}$$

$$\sum_{i\in \mathbf{I}} \sum_{j\in \mathbf{J}_{si}} y_{sij} \le \eta_s \qquad \forall\, s\in \mathbf{S} \tag{2.5}$$

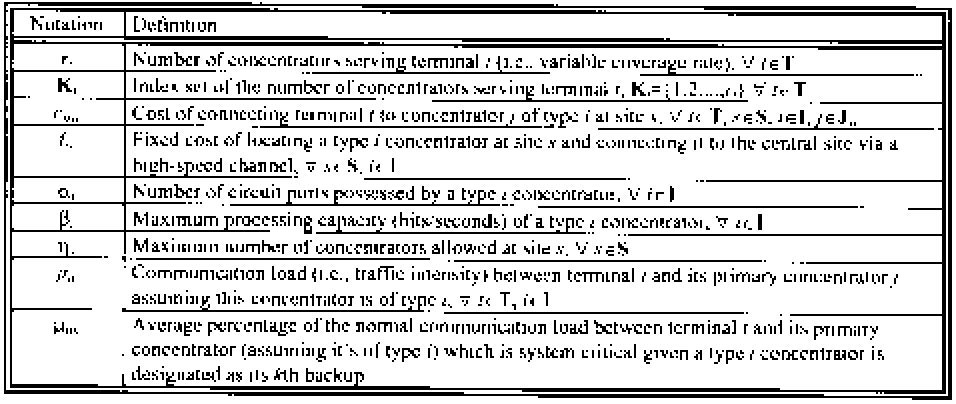
$$\sum_{k\in \mathbf{K}_t} x_{tsijk} \le y_{sij} \qquad \forall\, t\in \mathbf{T}, s\in \mathbf{S}, i\in \mathbf{I}, j\in \mathbf{J}_{si} \tag{2.6}$$

$$x_{tsijk} \in \{0,1\} \qquad \forall\, t\in \mathbf{T}, s\in \mathbf{S}, i\in \mathbf{I}, j\in \mathbf{J}_{si}, k\in \mathbf{K}_t \tag{2.7}$$

$$y_{sij} \in \{0,1\} \qquad \forall\, s\in \mathbf{S}, i\in \mathbf{I}, j\in \mathbf{J}_{si} \tag{2.8}$$

Hence, the mathematical formulation of GCELP can be written as ((2.1)-(2.8)):

Table 1 Notation For GCELP



Notation	Definition
r_t	Number of concentrators serving terminal t (i.e., variable coverage rate), $\forall\, t\in \mathbf{T}$
$\mathbf{K}_t$	Index set of the number of concentrators serving terminal t, $\mathbf{K}_t=\{1,2,\ldots,r_t\}$ $\forall\, t\in \mathbf{T}$
c_{tsij}	Cost of connecting terminal t to concentrator j of type i at site s, $\forall\, t\in \mathbf{T}, s\in \mathbf{S}, i\in \mathbf{I}, j\in \mathbf{J}_{si}$
f_{si}	Fixed cost of locating a type i concentrator at site s and connecting it to the central site via a high-speed channel, $\forall\, s\in \mathbf{S}, i\in \mathbf{I}$
α_i	Number of circuit ports possessed by a type i concentrator, $\forall\, i\in \mathbf{I}$
β_i	Maximum processing capacity (bits/seconds) of a type i concentrator, $\forall\, i\in \mathbf{I}$
η_s	Maximum number of concentrators allowed at site s, $\forall\, s\in \mathbf{S}$
p_{ti}	Communication load (i.e., traffic intensity) between terminal t and its primary concentrator j assuming this concentrator is of type i, $\forall\, t\in \mathbf{T}, i\in \mathbf{I}$
μ_{tik}	Average percentage of the normal communication load between terminal t and its primary concentrator (assuming it's of type i) which is system critical given a type i concentrator is designated as its kth backup

The objective function consists of two components: (1) cost of establishing the communication links between terminals and concentrators (as well as possibly connecting some terminals to the central processing site), and (2) cost of concentrator acquisition, installation and operation at selected sites plus the cost of connecting selected concentrators to the central site via high-speed channels. Historically, researchers have assumed that concentrator capacity could be purchased in units [Pirkul 1987, Pirkul *et al.* 1988, Narasimhan 1990]. In fact, these researchers recognize that this assumption is untrue; they reasonably argue that this approximation is reasonable since inaccuracies introduced by this assumption are insignificant compared to others interjected during the modeling process [Pirkul *et al.* 1988, Narasimhan 1990]. However, this results in their work finding good but not necessarily optimal solutions. Rather than make the simplifying assumption that capacity is scalable, this model explicitly recognizes that capacity is purchased in chunks.

Constraint set (2.2) ensures that each terminal *t* is connected to exactly *rt* concentrators. Constraint set (2.3) guarantees that the number of terminals connected to a concentrator does not exceed its circuit port capacity whereas constraint set (2.4) assures the total processing demand, based on the traffic intensity of connected terminals, does not exceed each concentrator's processing capablities. Constraint set (2.5) ensures that no more than *s* concentrators are located at site *s*. Constraint set (2.6) guarantees that each concentrator serving at least one terminal is acquired and that a concentrator does not serve a particular terminal in more than one capacity (i.e., a concentrator cannot provide both primary and backup coverage for a particular terminal). Furthermore, the integrality condition of (2.7) implies that partial connections between terminals and concentrators are not allowed. Lastly, partial purchase of concentrators is prohibited by integrality condition (2.8).

This research is part of an ongoing project designed to provide decision support tools for the design of telecommunication networks. The author will provide a review of relevant research as well as insights into potential solution methodologies.

References available upon request.