A Price Theory of Silicon Valley

Research-in-Progress

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Abstract

I develop a model of Silicon Valley engineer labor and venture capital (VC) markets. Software engineers choose between joining an established software company or founding a venture-backed startup. In equilibrium, they are indifferent between entrepreneurship and employment. The model predicts equilibrium markets wages, the terms of VC investments (i.e., the fraction of equity retained by the entrepreneur) and the minimum "quality" of the ideas pursued by startups, as represented by the VC funding "standard". This funding standard I inversely related to the number of new startups per period. When direct startup costs fall—e.g., as a result of technological advances or lowered interest rates—engineer wages rise and entrepreneurs can obtain better terms from VCs; VC funding standards drop, increasing the number of startups. Increases in the supply of engineers reduce wages, decrease retained equity and lower the VC funding standard. Increases in the supply of ideas raise wages, increase retained equity and raise the VC funding standard. Increases in demand in the product market for software raise wages, raise retained equity and lower the VC funding standard.

Keywords: Venture capital, innovation, engineer wages, entrepreneurship

Introduction

Brian Acton and Jan Koum were both engineers at Yahoo. After being turned down for a job at Facebook, Acton founded the mobile messaging service WhatsApp and soon recruited Koum as a co-founder.[1] WhatsApp was later acquired by Facebook for $19 billion. Although the outcome is remarkable and the employment near-miss is an interesting twist, the career trajectories of both Acton and Koum are commonplace in Silicon Valley: elite software developers often choose between joining an established tech company or founding a startup. It is easy to imagine Jerry Yang, Larry Page and Mark Zuckerberg working for—rather than founding—Yahoo!, Google and Facebook. Engineers weighing the relative costs and benefits of entrepreneurship and employment creates a connection between the labor market for top engineering talent and the market for venture funding of new startups; the implications of this labor/capital market connection are the focus of this paper.

In the equilibrium of my model, engineers are indifferent between employment and entrepreneurship, meaning that the expected returns of founding a startup equal the market wage for engineers at established tech firms.[2] Engineers that select entrepreneurship use VC funding to cover the direct costs of founding a startup. The opportunity cost of the startup for an engineer is the foregone wage at an

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[1] "Facebook turned down WhatsApp co-founder Brian Acton for job in 2009," The Guardian, February 20th, 2014.

[2] Hall and Woodward (2010) use data on venture-backed technology "exits" to compute the assets and risk-aversion needed to make an entrepreneur indifferent between a salaried job and entrepreneurship.

established tech firm. In the model, the established tech companies are those startups from the previous period that "won" the monopolistic competition with other startups. The assumption of monopoly is well-suited to the kind of venture-backed firms this paper considers, as software companies generally have rapidly declining average costs. For this reason, there is often a single big winner for a particular product category: Google in search, YouTube for videos, Facebook in social networks, and so on.

In the model, startup ideas are free for any entrepreneur to pursue. And while they differ in their quality—defined as their probability of taking hold in the product market—the expected success probabilities are equalized because of competition. This notion of ideas being public goods rather than fruits of private effort is at odds with the more conventional economic notion of ideas being patented and carefully protected. While viewing patents as proxies for innovation (Griliches, 1998) make sense in some contexts, this framework seems ill-fitting in the case of software startups and companies, for whom patents are generally unimportant.[3] There is a pervasive mindset among entrepreneurs and VCs alike that "ideas don't matter" and that what matters is execution.[4] Of course, ideas do matter, but the distilled wisdom in the VC perspective is that (1) there are lots of ideas "in the air" and (2) any good idea will attract lots of entrants. This paper offers a novel price theory of Silicon Valley. I use Silicon Valley as a metonym for the US software focused tech industry centered around the San Francisco Bay Area rather than a literal geographic area. I use the term "price theory" to highlight the distinction between my approach and extant theories of Silicon Valley, which focus on explaining why it exists and persists. Agglomeration economies (Duranton and Page, 2004), proximity to research universities (Jaffe, 1989), institutional factors—such as California's worker-friendly laws on non-compete agreements [5]—and cultural or managerial differences (Saxenian, 1996) are cited. While clearly important, these explanations have little to say about how change in key factors—such as the state of technology, the cost of real estate, the cost of capital, the size of the engineer workforce, innovation policy and the product market for what Silicon Valley produces—will directionally affect outcomes like the wages of engineers, the equity retained by entrepreneurs, and the number and quality of startups that receive funding.

Model

Each period, I startup ideas are generated by society. These ideas are common knowledge and any engineer/entrepreneur is free to use them for their startup. After a period, all ideas expire. If pursued by a startup, an idea will "work" in the product market with probability $q \sim U[0,1]$. Product success is independent across ideas and all ideas have zero cross-price elasticities of demand. Conditional upon being a success, all startup ideas offer the same product demand curve and generate the same labor demand schedule. However, each idea offers a natural monopoly and only one startup that pursues a particular idea will "win" among all startups pursuing that idea. An idea with success probability q' will attract n' entrepreneurs until startup success probabilities are equalized across ideas:

$$\frac{q'}{n'} = \underline{q},\tag{1}$$

where $\underline{q}$ is the idea success probability of the worst idea funded by the VCs. The expected number of funded startups will be

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[3] Software patents are widely viewed with suspicion in Silicon Valley and some leading firms—which would presumably benefits as incumbents with vast patent portfolios—are advocates of weakening IP protections and claim to hold and patent primarily for defensive purposes. See "Patents and Innovations" on Official Google Blog, http://googleblog.blogspot.com/2011/04/patents-and-innovation.html

[4] Paul Graham, the former head of startup incubator Y Combinator points out that it is not possible to sell startup ideas, which potentially says something about the demand for ideas: See http://www.paulgraham.com/ideas.html.

[5] See Marx et al. (2009) for their importance on facilitating employee mobility across firms.

$$N_p = I\left(1 - F\left(\underline{q}\right)\right)\int_{\underline{q}}^1 \frac{q}{\underline{q}} f(q)dq = \left(1 - \underline{q}\right)\frac{I}{\underline{q}}E\left[q\middle|q \geq \underline{q}\right] = I\frac{(1-\underline{q})(1+\underline{q})}{2\underline{q}} = \frac{I}{2}\left(\frac{1-\underline{q}^2}{\underline{q}}\right), \tag{2}$$

where F is the cdf of the distribution of ideas (which with a uniform distribution is just F(q)=q). As each startup has the same probability of success, the expected number of scaling startups in any period will be

$$N_s = \underline{q}N_p = \frac{I}{2}\left(1 - \underline{q}^2\right). \tag{3}$$

A startup is founded by a single risk-neutral engineer/entrepreneur, implementing a single idea. The direct costs of creating a prototype are α. They include costs such as hardware, real estate, living expenses, etc., with time-based costs scaled by the time required to develop a prototype, given the state of technology. Engineers have no other source of funds and α can only be financed with a VC investment.[6] After the engineer/entrepreneur creates the prototype, they learn not only whether the idea is successful, but also whether their particular implementation of the idea will beat out all other competitors. For all startups, the probability of winning is thus $\underline{q}$.

Scaling a successful startup requires engineering labor. The amount of labor required to scale to serve x customers is $l(x)$. The startup's profits are

$$\pi(w) = argmax\ p(x,z)x - wl(x), \tag{4}$$

where $p(x, z)$ is the inverse demand curve for the startup's product, w is the market wage for engineers and z is an index for the "size" of the product market. I assume that $\frac{\partial p(x,z)}{\partial z} > 0$. This maximized profit is common knowledge among VCs and entrepreneurs. Whether a startup succeeds or fails is common knowledge and verifiable and so scaling costs can be financed with regular, non-VC capital.[7] Let $d(w) = \frac{\partial \pi}{\partial w}$, which is the scaling firm's demand schedule for engineer labor. After a single period, the successful startup shuts down.

In the market for venture capital, the amount of investment sought by engineer/entrepreneurs is α; the market is cleared by s, the share of the firm purchased by the VC (and thus the claim to the profits if the startup succeeds). The expected payoff from being a funded entrepreneur is thus $(1 - s)\underline{q}\pi$. For an entrepreneur/engineer to be indifferent between working for a scaling startup and founding a startup, expected payoff from entrepreneurship must equal the market wage, or:

$$(1 - s)\underline{q}\pi = w. \tag{5}$$

There is free entry in the VC market which pushes investment profits to zero. In sharp contrast, I assume the supply of software engineers is fixed and that engineers supply labor inelastically on the extensive margin. Although this is a simplification—presumably finding good VCs to review startup pitches is challenging and top engineers can be lured to Silicon Valley—the global nature of capital but the localized nature of labor justifies my characterization. More capital can readily and quickly be inversed in VC firms and then made available to startups, whereas software engineers take longer to create. Although many elite software developers are quite young, this reflects the fact that many began programming when they were still children, not that it is an easy skill to acquire quickly.

Given free entry in the VC market, the return to the capital invested in a startup equals the share of expected profits:

$$(1 + r)\alpha = s\underline{q}\pi, \tag{6}$$

where r is the interest rate. Note that the ratio of entrepreneur equity to VC equity is the ratio of wages— the entrepreneur's opportunity cost—to the VCs opportunity cost, or

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[6] We might think that only VCs have the expertise to know q and that regular capital institutions would be quickly driven out of market by adverse selection. We might also think of VC capital as necessary to attract the market interest needed to learn whether or not a startup has succeeded.

[7] Later round investments by VC firms often have this flavor.

$$\frac{1-s}{s} = \frac{w}{(1+r)\alpha}.\tag{7}$$

The engineer's market wage is

$$w = \pi\underline{q} - (1+r)\alpha,\tag{8}$$

which is the expected profit from a startup, minus the VC's opportunity costs of financing the idea.

Although startups are developed in "periods" I assume that the market is sufficiently large and that periods are sufficiently short that it is reasonable to model equilibrium non-dynamically. In other words, the demand for engineers in a period is the number of startups (each using one entrepreneur) *plus* the expected number of successful startups *times* the labor demand of each one. For the market for software engineers and entrepreneurs to clear,

$$S = N_p + N_s d(w)$$

$$= \frac{I}{2}\left(\frac{1-\underline{q}^2}{\underline{q}}\right) + d(w)\frac{I}{2}\left(1-\underline{q}^2\right)$$

$$= \frac{I}{2}\left(1-\underline{q}^2\right)\left(\frac{1}{\underline{q}}+d(w)\right).\tag{9}$$

Model Prediction

An Equilibrium is defined by three endogenous variables: w, the wage rate for engineers; s, the share of equity sold to VCs; and $\underline{q}$, the worst idea that is funded (which is also the probability of success for any startup). I assume an interior solution exists. All proofs are in Appendix. Model predictions are summarized in Table 1, with table entries of " ↘ " for when $\frac{\partial COL}{\partial ROW} < 0$ and " ↗ " for $\frac{\partial COL}{\partial ROW} > 0$.

	Engineer wage, w	Share of equity retained by entrepreneur, $1-s$	Success probability of the worst-funded idea, $\underline{q}$
α , direct startup costs	↘	↘	↗
r , interest rate	↘	↘	↗
z , product market size	↗	↗	↘
S , supply of engineers	↘	↘	↘
I , new startup idea per period	↗	↗	↗

Table 1. Model Comparative Statics Predictions

Notes: This table contains the comparative static predictions from the mode. The endogenous variables are listed as columns and the exogenous variables are listed in the rows. Table entries of " ↘ " indicate $\frac{\partial COL}{\partial ROW} < 0$ and " ↗ " indicate $\frac{\partial COL}{\partial ROW} > 0$.

Proposition 1.

Increases in either the interest rate or the direct costs of a startup decrease engineer wages and reduce retained entrepreneur equity. These increases also increase the VC funding standard for new startups, reducing the number of new startups.

When the direct cost of doing a startup goes up, entrepreneurs require more capital and have to give up more equity. This is turn makes employment more attractive, which puts downward pressure on wages—a higher interest rate has an analogous effect. When costs are higher, VC firms and entrepreneurs are less willing to pursue more speculative ideas and so the VC funding standard increases. The logic is reversed when startup costs go down: wages rise, entrepreneurs get better terms and more speculative startups get funded.

The direct costs of founding a software startup have fallen dramatically in recent years. Although software tools have gotten better—and more of them are free—the main innovation has been the reduction in hardware capital requirements. Web-focused startups previously had to purchase their own physical servers and house them in a warehouse. Now, would-be entrepreneurs can easily "host" their web software on remote servers that are rented by usage—if rented at all, as most providers have a significant free tier. Examples include Amazon Web Services, Microsoft's Azure, Heroku and Google App Engine.

Although technological direct costs of funding a startup have fallen, other direct costs have risen. Living expenses—particularly real estate—has become far more expensive in the last decade. The cost of real estate/cost of living is a double hit for engineers: both employees and entrepreneurs generally have to live in Silicon Valley but high real estate costs increase the direct costs of doing a startup, which in turn lowers engineer wages. Although employers do have to react to increases in cost of living with wage increases, software developers already work near their intensive margin limit and existing firms seem to believe the stock of elite engineers is already present in Silicon Valley.

The negative effect of real estate costs might explain a stylized fact about Silicon Valley that is not a prediction of the model *per se* but is understandable in the logic of the model. A common piece of startup advice is to get "Ramen profitable" as quickly as possible—meaning that the startup brings in enough revenue to cover the entrepreneur's (minimal) living expenses. If an entrepreneur does not need money, they can obtain better terms from the VC. Similarly, working more quickly and having fewer family or financial obligations reduces the amount that needs to be borrowed from VCs. This could explain some of the preference for younger entrepreneurs: not having a family or being accustomed to higher standard of living is a way to keep direct costs down.

In addition to affecting wages and retained equity, direct startup costs also affect the VC funding standard. When startups cost little to found, VCs are willing to fund marginally worse startup ideas with lower success probabilities. This prediction could explain the frequent hand-wringing that too many startups are pursuing "frivolous" applications or are chasing slight variations on an existing product. It is fashionable to ask whether the word needs another photo sharing application and wonder why more smart people are not trying to pursue "big" ideas as in Silicon Valley days of lore that created and commercialized the transistor. However, it is the low cost of founding a startup rather than unimaginative entrepreneurs less talented than their forbearers that make the marginal funded idea "worse". But these bad ideas are not crowding out better ideas—those ideas are also being funded.

Proposition 2.

An increase in the supply of engineers lowers wages, lowers entrepreneur retained equity and lowers the VC funding standard.

As we would expect, Proposition 2 predicts that more engineers lowers engineer wages. In terms of the desirability of increasing the supply of engineers—say through increased STEM education or eased restriction on high-skilled migration—Proposition 2 suggests policy makers must make decisions about how much weight to put on the interest of various parties. Unlike the case with low-skilled immigration, there is stronger evidence that high-skilled foreign workers are substitutes for their native counterparts (Bound et al., 2014). Although increasing supply reduces native wages, it also increases the number of successful firms in the product market (by lowering the VC funding standard).

Proposition 3.

An increase in the supply of ideas raises wages, raises entrepreneur retained equity and raises the VC funding standard.

Startup ideas are a complement to engineers and more ideas tend to raise wages. When there are many ideas, workers are pulled away from the established tech companies and the number of startups increases. To keep engineers indifferent, firms need to raise wages. When ideas are more plentiful, the VC funding standard increases because entrepreneurs and firms have a larger stock of ideas to choose from, making the previously worst but still funded idea infra-marginal.

Proposition 4.

As the product market demand for software increases, engineer wages and retained equity increases while the quality of the marginal startup ideas falls.

The market for software has expanded enormously in the last 15 years. In 2000, the percentage of the global population using the internet was approximately 7% and as of 2014, it was 39%. Over a shorter time frame, there has been enormous growth in the demand for mobile applications (Ghose and Han, 2014). Proposition 4 predicts how such an expansion—modeled as a shifting out of the demand curve—affects the VC and labor markets. Unsurprisingly, profits increase, as do engineer wages. Less obviously, it also increases the retained equity by entrepreneurs and it lowers the equity of the marginal startup.

As the demand in the product market has spiked, the wages of software developers have grown, but probably not as much as they would have in a truly competitive market. Recently, evidence has emerged of widespread collusive behavior by established tech firms to suppress wages.[8] That the legal risks of anti-competitive behavior did not deter these agreements is suggestive of how tight the labor market for elite software engineers has become.

Conclusion

One obvious direction for future research would be to use the emerging datasets on startups and equity investments to test the predictions of the model. For example, CrunchBase collects and makes public detailed data on the funding of startups. There are several sources of data on engineer wages, such as Glassdoor as well as more conventional sources like the BLS. At least for companies that go public, it should be possible to learn s through financial disclosures. It should be possible to obtain data on w and s and, over a long enough time frame, q. Obviously any actual estimation would need to complicate the model—for one, not all equity investments are the same size—but the basic framework should be adaptable to the context.

Although this paper explores how a number of first order policy-relevant factors affect the labor and VC markets, it does not explore the welfare consequences of the software industry and associated labor market. Putting aside the inefficiently low supply inherent in any monopoly, the question is whether the surplus generated by a new successful software company exceeds the full cost. Society is likely better off, as the assumption of zero profit in the VC market means that startups are funded only until the return investment equals expected reward. Further, more startups—even when of lower quality—improve welfare. However, the structure of the competition among startups is inefficient in a mechanical sense: ideas either succeed or fail independent of the number of startups pursuing that idea. As such, it would be optimal to have only one startup working on each idea. However, whether or not this would motivate the kind of efforts required to actually bring an idea to market is an unanswered question.

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Proofs

Lemma 1. *Any small change in an exogenous variable that raises w decreases $\underline{q}$ and vice-versa.*

Proof. The labor market clearing condition in Equation 9 can be written as

$$d(w) = -\frac{1}{\underline{q}} + \frac{2S}{I\left(1-\underline{q}^2\right)}. \tag{10}$$

Let γ be some exogenous variable. Differentiating with respect to γ, we have

$$d'(w)\frac{\partial w}{\partial \gamma} = \frac{\left(\frac{4\underline{q}^3 s}{I\left(\underline{q}^2-1\right)^2}+1\right)}{\underline{q}^2}\frac{\partial \underline{q}}{\partial \gamma} \tag{11}$$

And since $d'(w) < 0$,

$$sign\frac{\partial w}{\partial \gamma} = -sign\frac{\partial \underline{q}}{\partial \gamma} \tag{12}$$

Proposition 1 Claim: *Increases in either the interest rate or the direct costs of a startup decrease engineer wages and reduce retained entrepreneur equity. These increases also increase the VC funding standard for new startups, reducing the number of new startups.*

Proof. If it becomes costlier to do a startup via an increase in α, the effect on wages is:

$$\frac{\partial w}{\partial \alpha} = \pi\frac{\partial \underline{q}}{\partial \alpha} + \underline{q}\frac{\partial \pi}{\partial \alpha} - (1+r). \tag{13}$$

Since changes in profits are only influenced directly by wages, then by the envelope theorem,

$$\frac{\partial w}{\partial \alpha}\left(1 + d(w)\underline{q}\right) = \pi\frac{\partial \underline{q}}{\partial \alpha} - (1+r). \tag{14}$$

Assume $\frac{\partial w}{\partial \alpha} > 0$. Then by Lemma 1, $\frac{\partial \underline{q}}{\partial \alpha} < 0$ and hence the entire RHS would be negative (since $\pi > 0$), contra our assumption. Therefore, $\frac{\partial w}{\partial \alpha} < 0$. By the same logic $\frac{\partial w}{\partial r} < 0$ By Lemma 1, $\frac{\partial \underline{q}}{\partial \alpha} > 0$ and $\frac{\partial \underline{q}}{\partial r} > 0$. For the effects on the amount of equity sold, consider Equation 7:

$$\frac{1-s}{s} = \frac{w}{(1+r)\alpha}. \tag{15}$$

Taking the partial derivative with respect to α, we have

$$\frac{\partial}{\partial \alpha}\left[\frac{1-s}{s}\right] = \frac{\alpha}{1+r}\frac{\partial w}{\partial \alpha}, \tag{16}$$

and since $\frac{\partial w}{\partial \alpha} < 0$, $\frac{\partial}{\partial \alpha}\left[\frac{1-s}{s}\right] < 0$ as well and so $\frac{\partial s}{\partial \alpha} > 0$. By the same logic, $\frac{\partial s}{\partial r} > 0$.

Lemma 2. *Engineer wages, entrepreneur retained equity and the VC funding standard are all decreasing in the ratio of the supply of engineers to ideas.*

Proof. The ratio of engineers to startup ideas is $e = S/I$. Using Equation 9 we can write labor demand as a function of the VC funding standard:

$$d(w) = \frac{1}{\underline{q}} + e\frac{2}{(1-\underline{q})^2}. \tag{17}$$

Differentiating with respect to e we have

$$d'(w)\frac{\partial w}{\partial e} = \frac{1}{1-\underline{q}^2} + \frac{1}{\underline{q}}\frac{\partial q}{\partial e}. \tag{18}$$

Assume that $\frac{\partial w}{\partial e} > 0$. Since $d'(w) < 0$, the entire RHS is negative, which implies that $\frac{\partial q}{\partial e} < 0$. If we differentiate the expression for engineer wages by e, we get

$$\frac{\partial w}{\partial e}\left(1 + d(w)\underline{q}\right) = \pi\frac{\partial \underline{q}}{\partial e}, \tag{19}$$

which implies that $\frac{\partial w}{\partial} < 0$ *and* $\frac{\partial q}{\partial e}$ have the same sign, contra our original assumption. Therefore, $\frac{\partial w}{\partial e} < 0$.

Proposition 2 Claim: *An increase in the supply of engineers lowers wages, lowers entrepreneur retained equity and lowers the VC funding standard.*

Proof. If $e = \frac{S}{I}$, $\frac{\partial e}{\partial S} > 0$, and so by Lemma 2 $\frac{\partial w}{\partial S} < 0$, $\frac{\partial s}{\partial S} < 0$ and $\frac{\partial \underline{q}}{\partial S} > 0$.

Proposition 3 Claim: *An increase in the supply of ideas raises wages, raises entrepreneur retained equity and raises the VC funding standard.*

Proof. If $e = \frac{S}{I}$, $\frac{\partial e}{\partial I} < 0$, and so by Lemma 2 $\frac{\partial w}{\partial I} < 0$, $\frac{\partial s}{\partial I} < 0$ and $\frac{\partial \underline{q}}{\partial I} > 0$.

Proposition 4 Claim: *As the product market demand for software increases, engineer wages and retained equity increases while the quality of the marginal startup ideas falls.*

Proof. If we differentiate Equation 4 (the successful startup's profit maximization problem) with respect to z, and applying the envelope theorem, we have

$$\frac{\partial \pi}{\partial z} = \frac{\partial p(x,z)}{\partial z}x > 0. \tag{20}$$

From Lemma 1 $w'(z)$ and $\underline{q}'(z)$ have opposite signs. Next we differentiate Equation 7 by z to give

$$\frac{\partial}{\partial z}\left[\frac{1-s}{s}\right] = \frac{1}{(1+r)\alpha}\frac{\partial w}{\partial z}, \tag{21}$$

which implies that -sign$\frac{\partial s}{\partial z}$=sign$\frac{\partial w}{\partial z}$. Assume that $\frac{\partial w}{\partial z} < 0$. This implies that $\frac{\partial s}{\partial z} > 0$ and $\frac{\partial q}{\partial z} > 0$. However, if we take the natural log of Equation 6 and differentiate by z, we get

$$\frac{1}{\pi}\frac{\partial p(x,z)}{\partial z}x + \frac{1}{s}\frac{\partial s}{\partial z} + \frac{1}{\underline{q}}\frac{\partial \underline{q}}{\partial z} = 0, \tag{22}$$

And since the sum of three positive terms of the LHS cannot equal zero, $\frac{\partial w}{\partial z} > 0$. It immediately follows that $\frac{\partial s}{\partial z} < 0$ and $\frac{\partial \underline{q}}{\partial z} < 0$.