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Summer 6-1-2014

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### Recommended Citation

Mei, Ying and Tu, Yiliu, "The Study and Prospects of the Dynamic Pricing Model for One-of-a-Kind Production Supply Chain" (2014). *WHICEB 2014 Proceedings*. 86.  
<http://aisel.aisnet.org/whiceb2014/86>

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# The Study and Prospects of the Dynamic Pricing Model for One-of-a-Kind Production Supply Chain

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**Abstract:** In this research paper we modeled a DPS when the firm's capacity is limited and a due-date guarantee may be required or favored by some customers. We suggest a promising approach to integrate a two-tier one-of-a-kind production supply chain with no dominant tier. We formulated a dynamic Bellman model to compute the optimal price quotes. By comparing DPS with SPS and analyzing the computational complexity of the proposed dynamic method, we show that when the firm dynamically changes the prices for each type of orders, both the firm and the customers are better off. This finding strongly supports the superiority of DPS. In the introduction of the future work, we also propose the OKP whole production supply chain model's optimal targets.

Keywords: dynamic pricing, OKP, supply chain coordination

## 1. INTRODUCTION

OKP is intensely customer focused such that every product is based on specific customer requirements, and products differ on matters of colors, shapes, dimensions, functionalities, materials, processing times, and so on. Because it is usually the case that an OKP firm receives discrete orders which arrive sequentially and are heterogeneous in lead-time requirements, the firm need a proper strategy to solve the problem of how to allocate capacity to customers from different priority classes. That is, whether to allocate capacity to the current customer or save it for future arrivals that might be from higher priority classes and hence generate higher profits (Keskinocak and Tayur 2004).

In this paper, we focus on the case where some customers have strict requirements on lead-time guarantee while the others do not. In this case the firm offers two types of orders: due-date-guaranteed orders (G-orders) and due-date-unguaranteed orders (U-orders), and only the G-orders are promised due-date delivery. We compare two pricing strategies: a dynamic pricing strategy (DPS) and a static pricing strategy (SPS). G-orders and U-orders are priced differently, but in DPS the firm dynamically changes the price for each type of order to maximize its profit. That is, the price quotes to a customer are also determined by its arrival time. We analyze how the supply chain benefits when each firm prices each order accounting for the order's arrival time together with the firm's production capacity and schedule.

We design numerical experiment to test how the proposed DPS influences the performance of the supply chain. Our results suggest a promising approach to integrate (or coordinate) a two-tier supply chain with no dominant tier.

The rest of this paper is organized as follows: Section 2 gives a brief literature review. In Section 3, we describe the problem and define the notation and assumptions. In Section 4, a dynamic pricing method is presented with a polynomial algorithm to find the optimal solution. Section 5 draws conclusions. Section 6 proposes future work.

## 2. LITERATURE REVIEW

In the literature, we can find a number of articles studying dynamic pricing, but most research focus on

pricing in make-to-stock (MTS) firms, e.g., Transchel and Minner (2009), Ray et al. (2005), etc. In contrast, we use the Bellman equation (Bellman 1957) to compute the price quote for OKP firms, which are often MTO firms. The earliest work we have found on pricing using Bellman equations is Kinacaid and Darling (1963), and more recent studies used similar methods to model supply chain dynamics, e.g., Stadje (1990), Gallego and Ryzin (1994) and Zhao and Lian (2011), etc. To the best of our knowledge, <sup>[1]</sup> was the first to study priority queues in which the priorities are associated with the prices paid by the customers. However, he focuses on the customers' behavior rather than the priority pricing scheme. Well-cited research on priority pricing in priority queues can be found in <sup>[2]</sup>, <sup>[3]</sup> and <sup>[4]</sup>. <sup>[5]</sup> showed congestion-based lead-time quotation for heterogeneous customers with convex-concave delay costs. <sup>[6]</sup> studied the pricing for different leadtime options when the firm faces customers that are heterogeneous both in price sensitivity and leadtime sensitivity. In her work, the distribution of customer choices for all leadtime options is induced by the prices. Due to the problem complexity, she only studied the case where the firm faces two classes of customers and offers two different leadtime options. <sup>[7]</sup> studied the general leadtime pricing for an arbitrary number of customer classes where they guarantee the promised leadtime by expediting as an extra cost. They presented the theoretical form of the price for each leadtime option without constraining the number of leadtime options. However, the problem complexity remains as they also only presented a two-class example in their numerical test. Indeed, <sup>[8]</sup> has made clear the difficulties of pricing more than two priority options. According to the best of our knowledge, an implementable method for solving the leadtime pricing problem for more than two leadtime options is still an open research question.

We also develop a set of practical parameter estimation methods for our proposed pricing strategy. In the literature, most authors assume that the distribution of the customers "impatience factors", which can be defined as how much it costs a customer for each time unit that the lead time of its order is increased, is exogenous and known by the firm. Based on this distribution of impatience factors, optimal prices are computed following the discipline of "third degree price discrimination" (Perlo 2009). However, in practice, this impatience factor is usually hard to measure or obtain. In the supply chain management literature the distribution of random variables are usually obtained from a learning process. For example, Chen and Plambeck (2008) develop a learning method to obtain the probability of a customer choosing a substitute, and Tomlin (2009) uses a Bayesian learning process to dynamically update the supplier's yield distribution. In our model, we develop a maximum-likelihood estimation (MLE) based learning method to estimate the distribution of the customer's willingness to pay (WTP) as well as the distribution of the impatience factor. An extensive literature can be found relating to the use of MLE and MNL. The MNL function has been widely used in the research on customer choices between multiple substitutable products. A number of studies can be found which apply MNL function in operations management. For example, <sup>[9]</sup> studied the product line selection and pricing problem in which the objective is to maximize the expected profit. <sup>[10]</sup> studied an application of MNL in inventory management, which is close to the capacity management problem we study. There are also several articles that focus on estimating the distribution of the customer's WTP. Bishop and Heberlein (1979), Hanemann (1984) and Cameron (1988) proposed methods to estimate the mean of the customer's WTP when the distribution type is known. Kristrom (1990) studied the case where the distribution type is not known and proposed a non-parametric estimator of the distribution of the customer's WTP, which requires larger sample sizes. Due to sample size limitations in OKP supply chains, we extend the previous literature and study the case where the distribution type is known, but the distribution parameters, such as the mean and variance of a normal distribution, must be estimated. This is realistic when the firm has some rough information on its customers' WTP distribution. As for the customer's impatience factor, we have not found any work studying the estimation of its distribution.

### 3. NOTATION and ASSUMPTIONS

We study an OKP firm which accepts two types of orders, G-orders and U-orders. Our first study target is to integrate a two-tier supply chain with no dominant tier. Our problem is to decide the optimal price quote for each type of order. Every newly received G-order is guaranteed delivery by the end of the next period. To guarantee the promised due date of the G-orders, the firm dispatches a higher priority to the G-orders in production. The quantity of unallocated capacity is used to guarantee the delivery of G-orders. We use the term available capacity to represent the quantity of unallocated capacity. The firm does not accept G-orders when it has no available capacity. In addition, the firm stops accepting G-orders at the beginning of the next period, which we name as deadline. After the deadline, the production schedule in that period is frozen. The firm does not allow new orders to be inserted into a frozen schedule because at the beginning of each production period, the firm needs to reallocate the available resources, e.g., the number of workers at each machine, internal and external logistics, etc. Inserting an order usually incurs extra cost. The superiority of freezing production schedules has been proven by Sridharan et al. (1987).

#### 3.1 Notation

We denote the two different price quotes the firm orders for G-order and U-order by  $p^G$ ;  $p^U$ ;  $p^G, p^U \in R^+$ , respectively. The available capacity, which represents the number of capacity units (i.e., man-day, man-hour, etc.), is denoted by  $m$ ;  $m \in Z^+$ . The number of future arrivals before the deadline is denoted by  $n$ ;  $n \in Z^+$ , which is adaptively estimated, noting that each arrival does not necessarily result in an accepted order.  $(m, n)$  represents the case in which the firm has  $m$  available capacity and expects  $n$  future arrivals.

We use  $r$ ;  $r \in R$  to represent a customer's WTP for a G-order. A customer's WTP is determined by two factors, i.e., its valuation on the firm's product and the substitutes from the firm's competitors. Supposing that a customer values the firm's product at  $V$ ;  $V \in R$  and the profit it can obtain by choosing the best substitute is  $S$ ;  $S \in R^+$ , then we define a customer's WTP as  $r = V - S$ . Because  $V$  and  $S$  are both random variables, then  $r$  is a random variable. We use  $v$ ;  $v \in R^+$  to represent the customer's impatience factor. We define the impatience factor as the cost incurred to the customer when there is no due-date guarantee. Without loss of generality, we use  $f(r, v)$  to denote the joint probability density function (JPDF). We also use the notation  $f(r, v, \vec{\theta})$  to represent the JPDF of  $r$  and  $v$  when the form of the distribution functions depends on the vector  $\vec{\theta}$ .

The adaptive control process is described as the following: the dynamic pricing module computes the prices  $p^G$  and  $p^U$ , and then the firm quotes the prices to arriving customers. The sample collecting module gathers customers' choices and their arrival rate, and the production monitoring module monitors the workload of the firm's each production line in real time. The sample collecting module and the production monitoring module periodically pass the information to the parameter estimating module. Based on the current firm's production status, customer arrival rate and customer choices, the parameter learning module adjusts the estimation of  $m$ ,  $n$ ,  $\vec{\theta}_r$  and  $\vec{\theta}_v$ , and then passes the new estimators back to the dynamic pricing module.

#### 3.2 Assumptions

We make the following assumptions to form our model:

Assumption 1: The firm makes a take-it-or-leave-it offer.

We assume that the firm makes price quote to each customer. If the customer accepts the price quote, then it places the order; otherwise the customer leaves and does not come back. A similar assumption can be found in previous research, e.g., Gallego and Ryzin (1994), where they assumed that customers do not act strategically by

adjusting their buying behavior in response to the firm's pricing strategy. This is also common in practice. As an OKP firm usually keeps little or no parts inventory and only makes orders on demand, but when the firm orders parts, it requires fast delivery. Therefore, if the customer does not accept the current offer, then a substitution has to be found immediately and hence long term strategic behavior does not happen.

Assumption 2: The processing time of an order is constant.

Here we assume that the workload of a single order is constant and equal to unity, which we take as a capacity unit. This assumption is consistent with the characteristic of OKP that the batch sizes of an order are small, or even just a single unit. For the case where the orders are heterogeneous in processing time, we can approximate the optimal prices through our model. We abstract from the issues of differing set-up costs between orders.

Assumption 3: The variable cost of production is zero.

As mentioned in earlier, we treat the labor cost as a fixed cost, that is, an added order does not incur additional labor cost. We do not consider material cost as the pricing strategy is our focus recognizing that the problem can be easily generalized by subtracting a constant unit production cost from the unit price. A similar assumption can be found in the literature on production planning and scheduling when pricing is considered, for example, Chen and Hall (2010) and Deng and Yano (2006). There is little loss of generality as with a constant processing time, we can take the variable cost of each order to be fixed, and reinterpret prices as net of costs.

#### 4. DYNAMIC PRICING STRATEGY (DPS)

In this section, we modeled a DPS when the firm's capacity is limited and a due-date guarantee may be required or favored by some customers. We formulated a dynamic Bellman model to compute the optimal price quotes.

Suppose that a customer arrives and receives price quotations  $p^G$  for G-order and  $p^U$  for U-order. If the customer chooses a G-order, then its net gain, denoted by  $\xi^G$ , is  $\xi^G = r - p^G$ . Otherwise, if the customer chooses a U-order, then its net gain, denoted by  $\xi^U$ , is  $\xi^U = r - p^U - v$ . The customer chooses the option that creates the higher net gain and only purchases when its net gain is non-negative. Otherwise it leaves without purchasing and its net gain (from the firm) is zero. First, we compute the probabilities of the customer choosing each type of order.

The customer chooses the G-order only if  $\xi^G \geq 0$  and  $\xi^G \geq \xi^U$ . Thus, given  $p^G, p^U$  and the distribution of  $r$  and  $v$ , the probability of the customer choosing the G-order, denoted by  $P_G(p^G, p^U)$ , can be obtained as

$$\begin{aligned} P_G(p^G, p^U) &= \Pr ob(\xi^G \geq 0 \text{ and } \xi^G \geq \xi^U) \\ &= \Pr ob(r - p^G \geq 0 \text{ and } r - p^G \geq r - v - p^U) \\ &= \int_{p^G}^{+\infty} \int_{p^G - p^U}^{+\infty} f(r, v) dv dr \end{aligned} \quad (3.1)$$

The customer chooses the U-order under two circumstances, i.e.,  $\xi^G \geq 0$  and  $\xi^G < \xi^U$ , or  $\xi^G < 0$  and  $\xi^U \geq 0$ . Thus, given  $p^G, p^U$  and the distribution of  $r$  and  $v$ , the probability of the customer choosing the U-order, denoted by  $P_U(p^G, p^U)$ , can be obtained as

$$\begin{aligned} P_U(p^G, p^U) &= \Pr ob(\xi^G \geq 0 \text{ and } \xi^G < \xi^U) + \Pr ob(\xi^G < 0 \text{ and } \xi^U \geq 0) \\ &= \Pr ob(r - p^G \geq 0 \text{ and } r - p^G < r - v - p^U) + \Pr ob(r - p^G < 0 \text{ and } r - v - p^U \geq 0) \\ &= \int_{p^G}^{+\infty} \int_0^{p^G - p^U} f(r, v) dv dr + \int_{p^U}^{p^G} \int_0^{r - p^U} f(r, v) dv dr \end{aligned} \quad (3.2)$$

The Bellman equation used to compute the optimal price quotes is based on the probabilities obtained from (3.1) and (3.2). Suppose that when a customer arrives, the firm faces an  $(m, n)$  case. For clarity of expression, we let  $n$  include the current customer. We examine the optimal price quote under two different situations: when capacity is greater or equal to the number of future arrivals, and when capacity is less, i.e.,  $m \geq n$  and  $m < n$ .

When  $m \leq n$ , the firm estimates that the number of future arrivals will not exceed the current available capacity. The optimal price quotation for problem  $(m; n)$ , denoted by  $\{p_{mn}^G, p_{mn}^U\}$ , can be obtained by

$$\{p_{mn}^G, p_{mn}^U\} = \arg \max_{\{p^G, p^U\}} [p^G P_G(p^G, p^U) + p^U P_U(p^G, p^U)] \quad (3.3)$$

which can be solved through the first-order conditions. Because there are  $n$  future arrivals, the maximum expected total profit, denoted by  $\pi_n^m$ , is

$$\pi_n^m = n [p_{mn}^G P_G(p_{mn}^G, p_{mn}^U) + p_{mn}^U P_U(p_{mn}^G, p_{mn}^U)] \quad (3.4)$$

When  $m < n$ , the optimal price quote for the current customer,  $\{p_{mn}^G, p_{mn}^U\}$ , can be obtained by solving the following Bellman equation

$$\pi_n^m = \max_{\{p^G, p^U\}} [P_G(p^G, p^U) [p^G + \pi_{n-1}^{m-1}] + P_U(p^G, p^U) [p^U + \pi_{n-1}^m] + [1 - P_G(p^G, p^U) - P_U(p^G, p^U)] \pi_{n-1}^m] \quad (3.5)$$

When the firm has no available capacity, the customers can only purchase U-orders. Thus for any  $n$  can be obtained as  $\pi_n^0$  for any  $n$  can be obtained as

$$\begin{aligned} \pi_n^0 &= \max_{p^U} n p^U \Pr ob(\xi \geq 0) = \max_{p^U} n p^U \Pr ob(r - v - p^U \geq 0) \\ &= \max_{p^U} n p^U \int_{p^U}^{+\infty} \int_0^{r-p^U} f(r, v) dv dr \end{aligned} \quad (3.6)$$

Let  $T(m; n)$  be the number of recursions required in computing  $T_n^m|_{n>m}$  when the tabu list is incorporated. it is not difficult to obtain:

$$T(m, n) = 2m(n - m) + 1 \in O(mn) \quad (3.7)$$

Then we can conclude that when the tabu list is employed,  $\pi_n^m|_{n>m}$  can be computed within polynomial time, which means that solving  $p_{mn}^G$  and  $p_{mn}^U$  is not an NP-hard problem.

## 5. CONCLUSIONS

We analyze the computational complexity of the proposed dynamic method, proving that the complexity of the dynamic method to compute the optimal price quotes is polynomial.

We also analyze the firm's profit and the customers' welfare when different pricing strategies (DPS and Static pricing strategy) are employed. Usually it is believed that if the firm dynamically changes prices to maximize its profit, the market demand is exploited and so the customers' welfare is lowered. However, by comparing DPS with SPS, we show that when the firm dynamically change the prices for each type of orders, both the firm and the customers are better off. This finding strongly supports the superiority of DPS. In the literature, we have not found any research studying how the DPS increases benefits to the entire supply chain.

Both the firm's and the customer's net welfare are increased, the net welfare of the global supply chain is

increased, and that is the value of DPS over SPS. In our study, we propose a learning method to estimate the required parameters, which are complementary to the proposed pricing strategy.

## 6. FUTURE WORK

Our future study of the one-of-a-kind production supply chain coordination focuses on the whole supply chain network with the dominant tier. It's the dynamic and multi-objective optimization model. For satisfying the higher requirements of the customization in one-of-a-kind production, we use dynamic pricing strategy to deal with the complicated contradictions that are caused by the random demand information and the complex relationship in the OKP supply chain coordination. We need to find a model that can reflect the customer satisfaction level of the customization services and embody consolidated income of the members of the supply chain coordination. We conclude that the optimal targets of the model should include four factors: decrease of the customization production cost, increase of the customization time service levels, the increase of customization level and the strengthening of customization cooperation relation.

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