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The Win-win Mechanism of Loyalty Programs Partnerships: Considering the Customer Heterogeneity

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Abstract: Considering the heterogeneity of customer purchase behavior in loyalty programs partnerships, in the asymmetric situation of enterprise A who joins the loyalty programs partnerships while enterprise B who implements customer loyalty programs alone, discussing the win-win mechanism of loyalty programs partnerships through the two stage game model, and providing equilibrium price and asymmetric equilibrium existence conditions of enterprise A and B. The results show that the profits of enterprise A who joins loyalty programs partnerships is higher than the profits of the two enterprises who don't join loyalty programs partnerships, and the profits of enterprise B who doesn't join loyalty programs partnerships is higher than the profits of the two enterprises who both join the loyalty programs partnerships.

Keywords: loyalty programs partnerships, asymmetric equilibrium, customer heterogeneity, Hotelling model

1. INTRODUCTION

Loyalty programs partnerships is short for "customer loyalty programs partnerships", it is a marketing pattern which droved by exchanging the rewards points based on the enterprise alliance and the consumer alliance [1]. Loyalty programs partnerships provides a new way to improve customer loyalty for enterprises, a variety of consumption patterns for consumers, a innovation marketing method for alliance parties, and it also plays an important role in improving the popularity of enterprises and keeping sustainable competitive advantage [2]. Many scholars think that enterprises who implement customer loyalty programs can improve customer loyalty through the rewards of customer purchase, so as to create goodwill and improve profitability [3]-[4], Bolton (2004) think loyalty programs partnerships strengthen the customer loyalty and enhance the performance of the action because of its reinforcing effects on customer purchase and the higher commitment and switching costs which customer perceived [5]-[7]. Lemon and Wangenheim (2009) think that the mechanism enhance and achieve the goal of win-win when loyalty programs partnerships is in effective operation state [8]-[10]. Sookyoung & Heajung (2009) analysis the loyalty programs partnerships of different industries, the brand image of enterprises can enhance when multiple industries join loyalty programs partnerships [11].

A part of the research achievements have been made, however, the win-win mechanism of loyalty programs partnerships is needed to further study. Therefore, in this paper, according to the characteristics of loyalty programs partnerships and considering the customer heterogeneity, in the case of asymmetric enterprise marketing strategy (that is, enterprise A who joins loyalty programs partnerships while enterprise B who doesn't join the loyalty programs partnerships), to analyze the market equilibrium and optimal profits of enterprises through the discussion of the customer continuous purchase probability, the rewards of customer loyalty programs and the rewards of loyalty programs partnerships.

2. MODELING

Assume that enterprise A and B who provide homogeneous products or service are in duopoly market and the market size is 1, enterprise A who joins loyalty programs partnerships while enterprise B who implements

customer loyalty programs alone. Enterprise A and B are located in the ends of the line which length is 1 respectively. Consumers are uniform distributed in the line of 0 to 1 and choose which enterprise to consume mainly depends on the following three factors: the price of products or service, the distance of consumers and enterprises and the rewards of customer loyalty programs.

Alliance and enterprises reward the customers who purchase in the two continuous stages, the customer would get β of the alliance in rewards if he purchases in enterprise A in the two continuous stages or purchases in enterprise B in the first stage and purchases in enterprise A in the second stage; the customer would get α ($\beta > \alpha$) of enterprise B in rewards if he purchases in enterprise B in the two continuous stages; the customer would not get anything in rewards if he purchases in enterprise A in the first stage, and purchases in enterprise B in the second stage. Therefore, the whole market is divided into three segments, customers in the range of $[0, x_1]$ who purchase in enterprise A in the two stages, customers in(x_1, x_2) who purchase in the first stage and leave in the second stage, but there will be the same number of customers who come into the market in the second stage, customers in $[x_2, 1]$ who purchase in enterprise B in the two continuous stages. Customers in the middle of the range (x_2 - x_1) who purchase in enterprise A in the first stage, and continue to purchase in enterprise A in the second stage would get β of the alliance in rewards, if the customer switch to purchase in enterprise B he can not get anything in rewards. Assume that this part of the customers purchase in enterprise B in the first stage, and switch to purchase in enterprise A in the second stage, he would get a greater β in rewards after the two continuously purchase. Obviously, the switch of customers will only occur in the situation of enterprise B to A (see figure 1).



Figure 1. Market structure

The price decision-making of the enterprise and customer purchase behavior are divided into two stages. In the first stage, the enterprise A and B announce the price of the products, customers decide which enterprise to purchase; assume that the probability of customers who purchase in the two stage is γ , at the end of the first stage, there are 1- γ customers loss, in order to keep the total market size constantly, there will be added in the same amount of customers in the second stage, there are μ customers of this part who purchase in enterprise A, and 1- μ customers who purchase in enterprise B. In the second stage, the enterprise A and B announce the price of products, customers decide which enterprise to purchase; At the end of the second stage, the customers who purchase products in the two stage continuously will get rewards (see figure 2).

On the basis of above hypothesis, the enterprise two stage total revenue function is as follows:

$$\pi_{A} = \pi_{1a} + \pi_{2a} = p_{1a}x_{1} + [\gamma x_{1}p_{2a}(1-\beta) + \gamma p_{2a}(x_{2} - x_{1}) + (1-\gamma)\mu p_{2a}]$$
 (1)

$$\pi_{B} = \pi_{1b} + \pi_{2b} = p_{1b} (1 - x_{2}) + \gamma p_{2b} (1 - \alpha) (1 - x_{2}) + (1 - \gamma) (1 - \mu) p_{2b}$$
 (2)

 \mathcal{P}_{1a} is the price for customers who purchase in enterprise A in the first stage, \mathcal{P}_{2a} is the price for

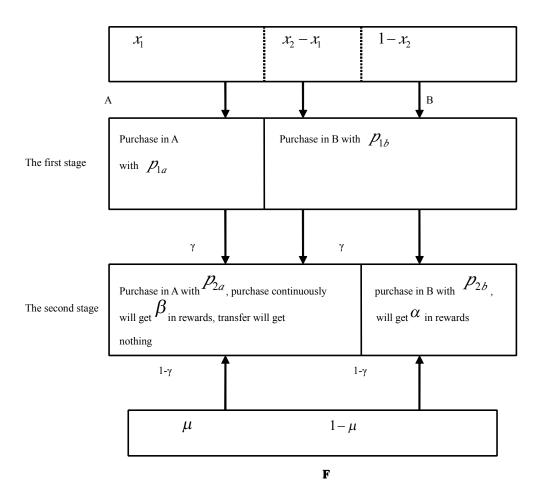


Figure 2. Customer purchase behavior in two stage model

customers who purchase in enterprise A in the second stage, P_{1b} and P_{2b} are the prices for customers who purchase in enterprise B in the two stages respectively.

3. SOLUTION METHOD FOR MODEL

In order to get the solution, we use reverse push induction in the two stage game.

First of all, according to the principle of enterprise profits maximization in the second stage, we find out the prices of products or service in the second stage;

$$p_{2a} = \frac{4\gamma (1+\alpha)}{3(1-\gamma)} x_2 - \frac{8\beta\gamma}{3(1-\gamma)} x_1 + \frac{2(-2\alpha\gamma - \gamma + 3)}{3(1-\gamma)}$$
(3)

$$p_{2b} = \frac{4\gamma (2\alpha - 1)}{3(1 - \gamma)} x_2 - \frac{4\beta\gamma}{3(1 - \gamma)} x_1 + \frac{2(-4\alpha\gamma + \gamma + 3)}{3(1 - \gamma)}$$
(4)

$$\mu = \frac{\gamma \left(2 - \alpha\right)}{3\left(1 - \gamma\right)} x_2 - \frac{\beta \gamma}{3\left(1 - \gamma\right)} x_1 + \frac{2\alpha \gamma - 5\gamma + 3}{6\left(1 - \gamma\right)} \tag{5}$$

put $^{\mathcal{X}_{l}}$, $^{\mathcal{X}_{2}}$, $^{\mathcal{P}_{2a}}$, $^{\mathcal{P}_{2b}}$, $^{\mu}$ into two stage profits equations (1) (2) ,solving $^{\mathcal{P}_{la}}$ and $^{\mathcal{P}_{lb}}$ respectively

 $\frac{\partial \pi_{_{A}}}{\partial p_{_{1}a}} = 0 \quad \frac{\partial \pi_{_{B}}}{\partial p_{_{1}b}} = 0$ when $\frac{\partial \sigma_{_{B}}}{\partial p_{_{1}b}} = 0$, $\frac{\partial \sigma_{_{B}}}{\partial p_{_{1}b}} = 0$, $\frac{\partial \sigma_{_{B}}}{\partial p_{_{1}b}} = 0$, $\frac{\partial \sigma_{_{B}}}{\partial p_{_{1}b}} = 0$, when $\frac{\partial \sigma_{_{B}}}{\partial p_{_{1}b}} = 0$, $\frac{\partial \sigma_{_{B}}}{\partial p_{_{1}b}} = 0$, $\frac{\partial \sigma_{_{B}}}{\partial p_{_{1}b}} = 0$, when $\frac{\partial \sigma_{_{1}b}}{\partial p_{_{1}b}} = 0$, $\frac{\partial \sigma_{_{1}b}}{\partial p_{_{1}b}} = 0$, $\frac{\partial \sigma_{_{1}b}}{\partial p_{_{1}b}} = 0$, $\frac{\partial \sigma_{_{1}b}}{\partial p_{_{1}b}} = 0$, when $\frac{\partial \sigma_{_{1}b}}{\partial p_{_{1}b}} = 0$, $\frac{\partial \sigma_{_{1}b}}{\partial p_{_{1}b}} = 0$, when $\frac{\partial \sigma_{_{1}b}}{\partial p_{_{1}b}} = 0$, $\frac{\partial \sigma_{_{1}b}}{\partial p_{_{1}b}} = 0$,

$$p_{1a} = \frac{BE + C}{1 - BD} \tag{6}$$

$$p_{1B} = \frac{DC + E}{1 - BD} \tag{7}$$

4. THE CONDITIONS OF ASYMMETRIC EQUILIBRIUM EXISTENCE

In order to prove that the existence of asymmetric equilibrium, we need to prove that the profits of enterprise A who joins loyalty programs partnerships alone is higher than the two enterprises who don't join loyalty programs partnerships; And we have to prove that the profits of two enterprises A and B, when enterprise A joins loyalty programs partnerships alone while enterprise B doesn't join loyalty programs partnerships is higher than the two enterprises who both join the loyalty programs partnerships. So we need to prove the following equations:

$$\pi_{A}^{*}(Asym) - \frac{-2\alpha\gamma^{2} - 2\gamma + 3\alpha^{2}\gamma^{2} - 4\alpha\gamma - \gamma^{2} + 6}{3(1 - \gamma)} > 0$$
 (8)

$$\pi_{B}^{*}(Asym) - \frac{-2\beta\gamma^{2} - 2\gamma + 3\beta^{2}\gamma^{2} - 4\beta\gamma - \gamma^{2} + 6}{3(1 - \gamma)} > 0$$
(9)

If the equations are satisfied at the same time, it means that the equilibrium solution is existence in the asymmetric condition. Obviously, it depends on three parameters: rewards of loyalty programs partnerships, rewards of enterprise who implements loyalty programs partnerships alone and the probability of customers who purchase in the two continuous stages. Due to the complexity of the expression of equilibrium profits, it is difficult to solve directly, so we use numerical analysis to prove the conditions of asymmetric equilibrium existence. First of all we need to consider the range of the parameters, in our study the range of the three parameters is (0, 1). For the solution of the inequality, we calculated the value of the expressions before the greater-than sign, if the value is positive, the inequality is established, if the value is negative, the inequality is not established. Before the calculation we fixed the continuous purchase probability γ , make it as a known variable, then take γ in 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 respectively, and take values of α and β in the range of [0,0.8].

First of all, we calculated values of 3.1. The range of γ is 0.1 to 0.6, values of the expression are negative, and inequality 8 is not established. That is to say, when the two stages continuous purchase probability γ is less than or equal to 0.6, in the condition of $\beta > \alpha$, the profits of enterprise A who joins customer loyalty programs partnerships alone is less than who doesn't join. At this time the optimal decision of enterprise A is not joining customer loyalty programs partnerships, and implementing the customer loyalty programs independently like enterprise B.

When the two inequalities 8 and 9 above are established at the same time, values of α and β are shown in table 1.

	γ=0.7	γ =0.8							
α	(0.57,0.69)	(0.6,0.7)	(0.01,0.09)	(0.11,0.19)	(0.21,0.29)	(0.31,0.39)	(0.41,0.49)	(0.51,0.59)	(0.61,0.69)
β	(0.58,0.70)	(0.7,0.8)	(0.08,0.10)	(0.18.0.20)	(0.28,0.30)	(0.38,0.40)	(0.48,0.50)	(0.58,0.60)	(0.68,0.70)

Table 1. The value condition of inequalities are established at the same time

In the range of table 1, as for enterprise A, the profits of joining customer loyalty programs partnerships is higher than the two enterprises who don't join customer loyalty programs partnerships; As for enterprise B, when enterprise A joins customer loyalty programs partnerships alone, the profits of enterprise B who doesn't join customer loyalty programs partnerships is higher than the two enterprises are both joining the customer loyalty programs partnerships. So at this time, the optimal decision of enterprise A is to join the customer loyalty programs partnerships, the optimal decision of enterprise B is not to join the customer loyalty programs partnerships but to implement customer loyalty programs independently.

The calculation and analysis show that asymmetric equilibrium is existed and the equilibrium is not single. Obviously, among the several equilibriums there are low efficiency equilibrium solutions, the integral rewards which the enterprises give to customers is the main costs of enterprises when implement customer loyalty programs, the higher rate of rewards, the higher costs of enterprises is. So among several equilibriums, when the value of γ is equal to 0.8, take α in the range of (0.01, 0.09), take β in the range of (0.08, 0.10), compared with other equilibrium solutions, the costs of enterprises who pay for the rewards is the lowest, so as for the enterprises, this is the best symmetrical balance.

5. CONCLUSIONS

This paper consider the heterogeneity of purchase behavior of customer in the loyalty programs partnerships, in the asymmetric situation of enterprise A who joins the loyalty programs partnerships while enterprise B implements the customer loyalty programs alone, examine the win-win mechanism of loyalty programs partnerships through the two stage game model.

We found that, under certain conditions, the asymmetric equilibrium exists in the market, namely the profits of enterprise A who joins loyalty programs partnerships is higher than two enterprises who don't join loyalty programs partnerships, the profits of enterprise B who doesn't join loyalty programs partnerships is higher than two enterprises who both join loyalty programs partnerships. At this point, the two enterprises have higher profits.

If the customer purchase in the two stages continuously, the adaption of different market positioning for enterprises is profitable, such as a enterprise provides loyalty programs partnerships and another enterprise adapts the low price competition strategy. The asymmetric equilibrium makes customer segment through the purchase, and makes the two enterprises have higher profits.

We can use empirical methods in the future, to distinguish target markets through the enterprise customer transaction data, and to make the enterprises adapt more targeted marketing. We can measure the comprehensive costs of implementation of loyalty programs partnerships according to the actual operation data, the settlement rules of enterprises in loyalty programs partnerships and the rewards of customer loyalty programs , and then to optimize the corresponding rate of rewards.

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APPENDIX

$$p_{1a} = \frac{BE + C}{1 - BD}$$
 $p_{1B} = \frac{DC + E}{1 - BD}$

where

$$B = \frac{(6) - (3)(5)(14) - (14)(4)(2) - 2(15)(4)(5) - 2(3)(2)(16)}{2\{(5) - (14)(2)(5) - (15)(5)^2 - (2)^2(16)\}}$$

$$C = \frac{1 - (4) + (12)(5) + (13)(2) + (4)(2) - 2(15)(4)(5) - 2(16)(1)}{2\{(5) - (14)(2)(5) - (15)(5)^2 - (2)^2(16)\}}$$

$$D = \frac{(2) - (9)(4)(2) - (9)(1)(5) - 2(10)(1)(2) - 2(11)(6)(5)}{2(3) - 2(8)(3)(6) + (10)(3)^2 + 2(11)(6)^2}$$

$$E = \frac{(1) - (7)(3) - (8)(6) - (9)(1)(6) - (9)(3)(4) - 2(10)(1)(3) - 2(11)(4)(6)}{2(3) - 2(8)(3)(6) + (10)(3)^{2} + 2(11)(6)^{2}}$$

where

$$^{(1)=}\frac{8\left(\beta\gamma+\alpha\beta\gamma\right)\!\left(\beta\gamma^2+\alpha\beta\gamma^2\right)\!+2\!\left(3-\gamma-2\alpha\gamma+2\alpha^2\gamma\right)\!\left(3\beta\gamma-\beta\gamma^2-2\alpha\beta\gamma^2\right)}{4\!\left(9\!-\!12\gamma\!-\!6\alpha\gamma+6\alpha^2\gamma+3\gamma^2+6\alpha\gamma^2-6\alpha^2\gamma^2+6\beta^2\gamma^2-4\beta^2\gamma^3-6\alpha\beta^2\gamma^3+3\alpha^2\beta^2\gamma^3\right)}$$

$$(2) = \frac{3\left(3 - \gamma - 2\alpha\gamma + 2\alpha^2\gamma\right)\left(1 - \gamma\right)}{4\left(9 - 12\gamma - 6\alpha\gamma + 6\alpha^2\gamma + 3\gamma^2 + 6\alpha\gamma^2 - 6\alpha^2\gamma^2 + 6\beta^2\gamma^2 - 4\beta^2\gamma^3 - 6\alpha\beta^2\gamma^3 + 3\alpha^2\beta^2\gamma^3\right)}$$

$$(3) = \frac{3(3 - \gamma - 2\alpha\gamma + 2\alpha^{2}\gamma)(1 - \gamma)}{4(9 - 12\gamma - 6\alpha\gamma + 6\alpha^{2}\gamma + 3\gamma^{2} + 6\alpha\gamma^{2} - 6\alpha^{2}\gamma^{2} + 6\beta^{2}\gamma^{2} - 4\beta^{2}\gamma^{3} - 6\alpha\beta^{2}\gamma^{3} + 3\alpha^{2}\beta^{2}\gamma^{3})}$$

$$(4) = \frac{8(\beta \gamma + \alpha \beta \gamma)(\beta \gamma^{2} + \alpha \beta \gamma^{2})(4\alpha^{2} \gamma - 3\alpha \gamma - 3\alpha - \gamma + 3)}{4(9 - 12\gamma - 6\alpha \gamma + 6\alpha^{2} \gamma + 3\gamma^{2} + 6\alpha \gamma^{2} - 6\alpha^{2} \gamma^{2} + 6\beta^{2} \gamma^{2} - 4\beta^{2} \gamma^{3} - 6\alpha \beta^{2} \gamma^{3} + 3\alpha^{2} \beta^{2} \gamma^{3})(3 - \gamma - 2\alpha \gamma + 2\alpha^{2} \gamma)} + \frac{2(3\beta \gamma - \beta \gamma^{2} - 2\alpha \beta \gamma^{2})(\beta \gamma + \alpha \beta \gamma)}{4(2\alpha + 2\alpha \gamma + 2\alpha \gamma^{2} +$$

$$(5) = \frac{3(1-\gamma)(\beta\gamma + \alpha\beta\gamma)}{4(9-12\gamma - 6\alpha\gamma + 6\alpha^2\gamma + 3\gamma^2 + 6\alpha\gamma^2 - 6\alpha^2\gamma^2 + 6\beta^2\gamma^2 - 4\beta^2\gamma^3 - 6\alpha\beta^2\gamma^3 + 3\alpha^2\beta^2\gamma^3)}$$

$$(6) = \frac{3(1-\gamma)(\beta\gamma + \alpha\beta\gamma)}{4(9-12\gamma - 6\alpha\gamma + 6\alpha^{2}\gamma + 3\gamma^{2} + 6\alpha\gamma^{2} - 6\alpha^{2}\gamma^{2} + 6\beta^{2}\gamma^{2} - 4\beta^{2}\gamma^{3} - 6\alpha\beta^{2}\gamma^{3} + 3\alpha^{2}\beta^{2}\gamma^{3})}$$

$$(7) = \frac{2\left(-2\alpha\gamma - \gamma + 3\right)\left(\gamma - \gamma\beta - 1\right)}{3\left(1 - \gamma\right)} + \frac{2\gamma\beta\left(-2\alpha\gamma - \gamma + 3\right)}{9\left(1 - \gamma\right)} - \frac{4\gamma\beta\left(2\alpha\gamma - 5\gamma + 3\right)}{9\left(1 - \gamma\right)}$$

$$(8) = \frac{2(-2\alpha\gamma - \gamma + 3)}{3(1-\gamma)} + \frac{2\gamma(2-\alpha)(-2\alpha\gamma - \gamma + 3)}{9(1-\gamma)} + \frac{2\gamma(1+\alpha)(2\alpha\gamma - 5\gamma + 3)}{9(1-\gamma)}$$

$$(9) = \frac{4\gamma^{2}(1+\alpha)(1-\beta)-4\gamma^{2}(1+\alpha)-8\beta\gamma^{2}}{3(1-\gamma)} - \frac{8\beta\gamma^{2}(2-\alpha)+4\beta\gamma^{2}(1+\alpha)}{9(1-\gamma)}$$

$$(10) = \frac{-8\beta\gamma^{2}(1-\beta) + 8\beta\gamma^{2}}{3(1-\gamma)} + \frac{8\beta^{2}\gamma^{2}}{9(1-\gamma)}$$

$$(11) = \frac{4\gamma^2 (1+\alpha)(4-\alpha-2\gamma+\gamma\alpha)}{9(1-\gamma)}$$

$$(12) = \frac{4\gamma^2 \left(2\alpha - 1\right)\left(1 - \alpha\right) + 2\gamma \left(1 - \alpha\right)\left(-4\alpha\gamma + \gamma + 3\right)}{3\left(1 - \gamma\right)} - \frac{2\gamma \left(2 - \alpha\right)\left(-4\alpha\gamma + \gamma + 3\right)}{9\left(1 - \gamma\right)} + \frac{2\gamma \left(2\alpha - 1\right)\left(-2\alpha\gamma - \gamma + 3\right)}{9\left(1 - \gamma\right)}$$

$$(13) = -\frac{4\beta\gamma^2(1-\alpha)}{3(1-\gamma)} + \frac{2\beta\gamma(-4\alpha\gamma + \gamma + 3)}{9(1-\gamma)} - \frac{4\beta\gamma(-2\alpha\gamma - \gamma + 3)}{18(1-\gamma)}$$

$$(14) = -\frac{8\beta\gamma^2(2-\alpha)}{9(1-\gamma)}$$

$$(15) = \frac{4\gamma^2 (2\alpha - 1)(1 - \alpha)}{9(1 - \gamma)}$$

$$(16) = -\frac{4\beta^2 \gamma^2}{9(1-\gamma)}$$