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Liu Xianghua

School of Finance, Zhongnan University of Economics and Law, xhliu2004@126.com

Xiao Xueping

School of Finance, Zhongnan University of Economics and Law, xxiao.123@163.com

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Pricing Model of Credit Default Swap Based on Jump-Diffusion Process and Volatility with Markov Regime Shift

*Liu Xianghua, Xiao Xueping**

School of Finance, Zhongnan University of Economics and Law, China

Abstract: By introducing the Jump-Diffusion Process and Markov Regime Shift, the paper explores Monte Carlo simulation to examine the pricing problem of single name Credit Default Swaps (CDS), which the price of CDS is affected by both unpredictable idiosyncratic risk and system risk caused by the macroeconomic change. The study shows that the price of CDS increases as the intensity and the amplitude of the Jump-Diffusion Process increase. Furthermore, the CDS price depends on the initial state and transition intensity of the volatility of the corporate value, which the former can reflect the influence of macroeconomic situation.

Keywords: Credit Default Swap, Jump-Diffusion Process, Markov Regime Shift, Monte Carlo Simulation

1. INTRODUCTION

Credit Default Swap (CDS) is one of the simplest and widely used credit derivatives, which the buyer of CDS pays the fees to the seller regularly to acquire protection from default. When the credit event occurs, the seller has to compensate the buyer for losses. In recent years, CDS develops rapidly and has become one of the most popular credit derivatives. In November 2010, Chinese CDS-- Credit Risk Mitigation (CRM) was put forward in inter-bank market. The CRM will have far-ranging applied space and applied outlook in risk management field in the future.

The primary problem of credit derivatives is pricing. There are two kinds of models to price credit derivatives: structural model and reduced model. Structural model uses structural variables such as change of asset value or debt value to model default. With strong economic background, this model provides endogenous default probability and recovery rate. Unlike structural model, reduced model assumes that default event cannot be predicted. Default event is one jump of the exogenous shock. The intensity of jump-diffusion, namely the default intensity, can be implied from market data. Responding to the default probability within the structural framework, we try to develop a modified model to price CDS. The first structural model is put forward by Merton (1974) which closed-form solutions for default probability and the price of credit derivatives can be achieved based on the Black-Scholes option pricing theory. Merton (1976) develops option pricing when stock price follows a jump-diffusion process. Black and Cox (1976) modify assumptions of Merton (1974)'s Model to allow default ahead of security maturity, which indicates that short-term credit spreads is closed to zero and default event can be predicted. Zhou(2001) comes up with a new model based on jump-diffusion process which solves the problem of default predictability to some extent and obtains positive short-term credit spreads. The newest study on structural framework is the introduction of Markov regime shift into pricing of credit derivatives. Elizalde (2007) assumes that default event is related to the external credit rating and the change of credit rating can be viewed as one regime shift of credit rating state when pricing CDS. Hackbarth, Miao and Morellec (2006) introduce a Markov regime shift for company's cash flow to discuss the influence of business cycle on company's cash flow and default policy. Chen (2010) develops a structural model and obtains closed-form solutions for endogenous default probability and losses under the opinion that periodic change of expected

* Corresponding author. Email: xhliu2004@126.com (Liu Xianghua), xxiao.123@163.com (Xiao Xueping)

economic growth, economic instability and risk premium affect company's financing and default policy. Based on all these research achievements, this paper tries to apply the jump-diffusion and Markov regime shift to model the influence of macro-economic conditions on the volatility of firm value. Under this condition, we compute company's default probability and use it directly to price single name CDS.

When modeling the volatility of asset value of the firm, most existing papers assume that it is unchangeable which may not be appropriate for the long term corporate securities. The volatility of asset price is affected by company's financing and strategy policies. During a long period, such as 10 years, company management will adjust its financing and strategy policies according to the change of external economy. Thus, it is reasonable to assume that the volatility of asset price depends on the change of economic situation. The influence of business situation can be viewed as systematic risk which all companies face. It causes homogeneous, lasting and ineludible influences on the whole market. Except for systematic risk, there is particular risk that one single firm has to face, which we can call unsystematic risk or specific risk. This kind of risk can be described as unpredictable and uncontrollable shock to companies, which is related to industry, geographic location and its own operations. Therefore, responding to company's specific risk, we introduce the jump-diffusion process into the model and the change of company's specific risk can be viewed as a jump of exogenous Jump-Diffusion. we consider and distinguish company's systematic and specific risk, assuming asset price follows jump-diffusion process and its volatility has Markov regime shift. Under this condition, we inspect the change of CDS price and default probability in different risk state to achieve more precise CDS price.

2. THE MODEL

2.1 Pricing principle of CDS

The cash flow of CDS consists of two parts. One part is the expected payments that the CDS buyer makes when the default event occurs or does not occur. The other part is the expected compensation that the CDS buyer obtains when the default event occurs. Suppose that the fee rate of CDS, s , as a fraction of notional in Basis Point per year is paid at dates $t_1 < t_2 < \dots < t_n = T$. $\Delta(t_{i-1}, t_i)$ denotes time interval between payment date t_{i-1} and t_i . When the reference entity does not default, the expected payments that CDS buyers make can be expressed as equation (1), where F is notional principal. $\beta(t_i)$ is the default probability before t_i under risk neutral measure. $B(0, t_i)$ is risk-free discount factor.

$$A = F \cdot s \cdot \sum_{i=1}^n \Delta(t_{i-1}, t_i) \cdot [1 - \beta(t_i)] \cdot B(0, t_i) \quad (1)$$

When the default event occurs, the payment made by CDS buyer can be defined as:

$$B = F \cdot s \cdot \sum_{i=1}^n \frac{\Delta(t_{i-1}, t_i)}{2} \cdot [\beta(t_i) - \beta(t_{i-1})] \cdot B(0, t_i) \quad (2)$$

where it is assumed that the default between the regular payments always occur exactly in the middle. The error form this approximation gets smaller as the time step gets smaller. When the default event occurs, the contingent leg payoff, C , can be described as

$$C = (1 - R) \cdot \sum_{i=1}^n [\beta(t_i) - \beta(t_{i-1})] \cdot B(0, t_i) \quad (3)$$

At the moment the contract is signed, according to no-arbitrage pricing principle, the cash flow of the CDS should be zero. That is,

$$C = A + B = s \cdot (L + M) \quad (4)$$

Therefore, the CDS price is

$$s = \frac{C}{L + M} \quad (5)$$

2.2 Model assumptions and derivation

Assumption 1: When the volatility of asset price locates in regime $\sigma_i (i = L, H)$, the dynamics of V_t is given by the following jump-diffusion process

$$dV_t/V_t = (\mu_i - \lambda\nu)dt + \sigma_i dZ + (\Pi - 1)dY \quad i = L, H \quad (6)$$

Where μ_i denotes the drift rate in different regime, λ is the intensity of jump diffusion process and dZ is a standard Brownian motion. dY is a Poisson process with intensity parameter λ , and Π is the jump amplitude with expected value equal to $\nu + 1$. We assume that Π is an i.i.d log-normal random variable, such that

$$\ln \Pi : N\left(\mu_\pi, \sigma_\pi^2\right) \quad (7)$$

This assumption implies that

$$\nu = E[\Pi - 1] = \exp\left(\mu_\pi + \frac{\sigma_\pi^2}{2}\right) - 1 \quad (8)$$

Assumption 2: Initial state of σ_i can be observed and during the whole period, σ_i at time t_{i+1} only depends on its value at time t_i . That is

$$P\left(\sigma_{t+1} = j \mid \sigma_t = i, \sigma_{t-1} = i_{t-1}, L, \sigma_{t_0} = i_{t_0}\right) = P\left(\sigma_{t+1} = j \mid \sigma_t = i\right) = p_{ij} \quad (9)$$

Where p_{ij} is the transition probability from state i to state j during time interval Δt . Then the regime shift of σ_i can be described as a Markov Chain with two states. The transition probability matrix is defined as follows

$$P = \begin{bmatrix} p_{HH} & p_{HL} \\ p_{LH} & p_{LL} \end{bmatrix} \quad (10)$$

Particularly, if the transition follows Poisson process, within very small time interval Δt , we have

$$p_{ij} = \begin{cases} 0, & \text{with prob. } \exp(-\lambda_i \Delta t) : 1 - \lambda_i \cdot \Delta t \\ 1, & \text{with prob. } 1 - \exp(-\lambda_i \Delta t) : \lambda_i \cdot \Delta t \end{cases} \quad (11)$$

Where $\lambda_i, i = L, H$ denotes the rate of leaving state i . Let l_i denote the time spent in state i , then the exponential law holds:

$$P(l_i > t) = e^{-\lambda_i t}, \quad i = L, H \quad (12)$$

In addition, the expected duration of regime L is $(\lambda_L)^{-1}$ and the average fraction of time spent in that regime is $\lambda_H (\lambda_L + \lambda_H)^{-1}$.

Assumption 3: There exists a positive threshold value K for the reference entity at which financial distress occurs. The firm continues to operate and to be able to meet its contractual obligations as long as $V_t > K$. Otherwise, it defaults on all of its obligations immediately and some form of corporate restructuring takes place.

Define $X_t = V_t / K$ as the ratio between firm asset value and threshold value. The default event occurs when $X_t \leq 1$. Assumption 1 and the definition that $X_t = V_t / K$ yield immediately:

$$dX_t / X_t = (\mu - \lambda\nu)dt + \sigma_i dZ + (\Pi - 1)dY \quad i = L, H \quad (13)$$

Under risk neutral probability measure, using the Ito Lemma, we have

$$d \ln X_t = (r - \sigma_i^2 / 2 - \lambda\nu)dt + \sigma_i dZ + \ln \Pi dY \quad i = L, H \quad (14)$$

Under no arbitrage condition, we know that the price of a derivative security satisfies the partial differential equation (15) in one single state.

$$0.5\sigma_i^2 X^2 G_{XX} + (r - \lambda v) X G_X - rG + \lambda \cdot E_t [G(X\Pi, T) - G(X, T)] = G_T \quad i = L, H \quad (15)$$

where G_X, G_{XX} mean the first and the second partial derivatives respectively. Let τ represent the time when a default occurs. Mathematically,

$$\tau = \inf \{t | X_t \leq 1, t > 0\} \quad (16)$$

Without regard to the regime shift, Abrahams(1986) and Zhou (2001) point out, the closed-form solution for equation (15) does not exist, neither nor the default probability. Zhou(2001) and Wang and Chen(2003) provide the closed-form solution for a simplified model where it is assumed that the default only occurs at maturity date T . Namely, $\tau = T$. Under this condition, the default probability can be written as

$$F_T^Q(1 | X) = Q(X_T \leq 1) = \sum_{i=0}^{\infty} \frac{\exp(-\lambda T)(\lambda T)^i}{i!} N\left(-\frac{\ln X + (r - 0.5\sigma^2 - \lambda v)T + i\mu_\pi}{\sqrt{\sigma^2 T + i\sigma_\pi^2}}\right) \quad (17)$$

Where Q denotes risk neutral probability measure. Because there is no general closed-form solution for the equation (15) and the corresponding default probability, we turn to use Monte Carlo simulation to compute the firm default probability and the corresponding CDS price under jump-diffusion process that its price volatility has the property of Markov regime shift. Let $t_j = jT/n$, where T denotes the maturity date, n denotes the total number of time intervals. According to equation (14), we have

$$\ln X_{t_j}^* - \ln X_{t_{j-1}}^* = x_j + y_j \cdot \pi_j, \quad j = 1, 2, \dots, n \quad (18)$$

where

$$x_j : N\left(\left(r - \sigma_i^2/2 - \lambda v\right) \cdot T/n, \sigma_i^2 \cdot T/n\right) \quad (19)$$

σ_i shift its state with the probability presented before.

$$\pi_j : N\left(\mu_\pi, \sigma_\pi^2\right) \quad (20)$$

$$y_j = \begin{cases} 0, & \text{with prob. } 1 - \lambda \cdot T/n \\ 1, & \text{with prob. } \lambda \cdot T/n \end{cases} \quad (21)$$

Equation (21) is similar to equation (11). Both of them are the simplified form of Poisson distribution. They hold because in a very small time period, there is no more than one jump can occur and the diffusion process cannot move a large distance almost surely.

2.3 Monte Carlo simulation

We now describe a Monte Carlo approach to value CDS based on the former principle. Procedures of valuing the CDS as follows:

Step 1: Divide the time interval $[0, T]$ into n equal subperiods for sufficiently large n , n can be one day. It can be determined according to the maturity date T .

Step 2: Take Monte Carlo simulations by repeating the following sub-procedures for W ($j = 1, 2, \dots, W$) times to compute the default probability. Typically, one can choose W between 10,000 and 100,000.

1) For each time point t_j generate random variable σ_i as equation (11), then generate independent random vectors (x_j, π_j, y_j) , where x_j, π_j, y_j respectively follows the distribution expressed by equation (19), (20) \(\lambda\) (21).

2) At initial time, let $X_{t_0}^* = X$, X is exogenously given. The initial state of σ_{t_0} is observable. $\ln X_{t_i}^*$ can be computed as equation (18).

3) Find the minimum $\ln X_{t_i}^*$, if $\ln X_{\min}^* \leq 0$, default event occurs.

4) Let W_0 denote the sum the routes in which default event occur, then the default probability can be

expressed as W_0/W .

Step3: Compute the CDS price according to the equation (1)、(2) (3) (5).

1) Follow step 2, calculate the default probability before t_j , present value of expected payments when default occurs or does not occur and expected reparation, where $B(0,t_i) = \exp(-rt_i)$.

2) Sum the present value at each point t_j , we obtain A, B, C and the CDS price can be computed by equation (5).

3. NUMERICAL ANALYSIS

3.1 Calibration of parameters

In order to make analysis easy, the related parameters are denoted as follows:

1) Mean value of lognormal distribution $\mu_\pi = 0$, the variance $\sigma_\pi^2 = 0.25$. According to equation (7)、(8), we have $\ln \Pi : N(0, 0.25)$ and $\nu = 0.1331$.

2) The risk-free rate $r = 0.05$. Recovery rate $R = 0.4$. The initial ratio between firm asset value and default threshold $X_{t_0}^* = 2$.

3) The volatility of asset price in different state: $\sigma_H^2 = 0.045, \sigma_L^2 = 0.0225$.

4) The transition intensity in different state, $\lambda_H = 0.8, \lambda_L = 0.4$. When the economy is in the downturn (high volatility state), the government has the motivation to take powerful macro-control measures to stimulate economic growth. Thus, it is reasonable to assume that the transition intensity is larger when the economy is in high volatility state. According to former content, the expected duration of regime L is $1/\lambda_L = 2.5$ and the expected duration of regime H is $1/\lambda_H = 1.25$. During 10 years, the average time spent in regime H is 3.3 years and the average time spent in regime L is 6.7 years.

5) The jump intensity $\lambda = 0.05$. $S_0 = 1$ means the initial value of volatility $\sigma_L^2 = 0.0225$, responding to economic boom period. $S_0 = 2$ means the initial value of volatility $\sigma_H^2 = 0.045$, responding to economic downturn period. $S_0 = 0$ means the situation where the influence of Markov regime shift is ignored. For comparison, we assume that when $S_0 = 0$, the volatility is time weighted mean of that with regime shift. Namely

$$\sigma_{S_0}^2 = \sigma_H^2 \cdot \frac{\lambda_L}{\lambda_L + \lambda_H} + \sigma_L^2 \cdot \frac{\lambda_H}{\lambda_L + \lambda_H} \quad (22)$$

According to equation (22), we have $\sigma_{S_0}^2 = 0.03$.

In the later analytical process, all parameters are determined as presented above without specification.

3.2 Jump diffusion and CDS price

3.2.1 Jump amplitude and CDS price

As shown in figure 1, when low volatility regime is the initial state, the default probability increases as σ_π^2 increases. According to equation (18) and (20), when $\sigma_\pi^2 = 0$, the asset price follows general diffusion process.

Given $X_{t_0}^* = 2$, it is known that a diffusion process has a continuous sample path and cannot cross a boundary from somewhere else instantaneously. Therefore, the probability of defaulting on very short term is zero, so is the CDS price. As the jump volatility increases, the jump size becomes bigger and it is possible the asset price drops dramatically. Then default event occurs, which makes the CDS price positive in a short-term period. Figure 2 shows that, due to the existence of jump diffusion, the CDS price is positive in short term period. In this way, the flaw of Merton(1976) and Black and Cox(1976) model that default event can be predicted and

credit spreads is zero in short term is modified.

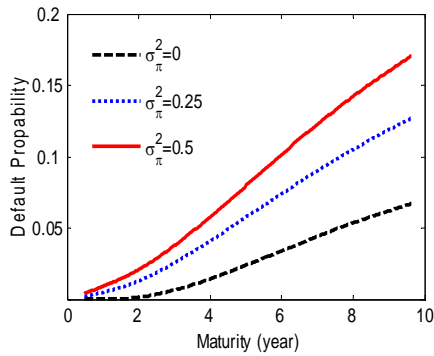


Figure 1 Jump amplitude and default probability

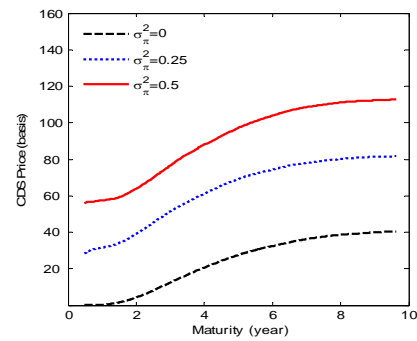
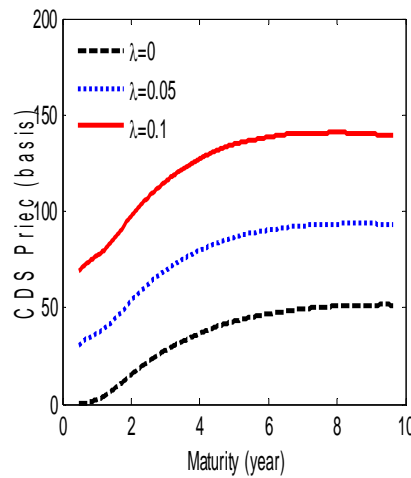


Figure2 Jump amplitude and CDS price



3.2.2 Jump intensity and CDS price

Assuming high volatility regime is the initial state, Figure 3 illustrates that the larger the jump intensity, the higher the CDS price. The influence of jump intensity on CDS price is similar to that of jump amplitude. When $\lambda = 0$, asset price follows diffusion process. Under this condition, the short term credit spreads is zero. As jump intensity becomes larger, times of jump increases and short term credit spreads becomes larger.

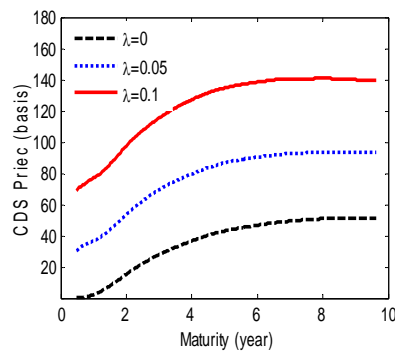


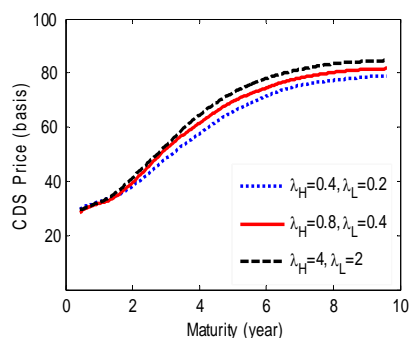
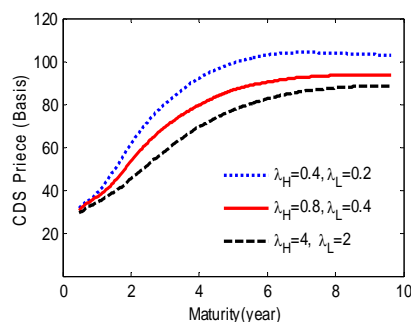
Figure 3 Jump intensity and CDS price

3.3 Regime shift and CDS price

3.3.1 State transition intensity and CDS price

As shown in Figure 4, given the initial state $S_0 = 1$, the CDS price increases according to the increase of

state transition intensity. While in Figure 5, given the initial state $S_0 = 2$, the CDS price decreases according to the increase of state transition intensity. In Figure 4, given $S_0 = 1$, as the state transition intensity is small, the initial value of low volatility will continue affect asset price for a long time. Thus, the probability of default is small and so is the CDS price. As the transition intensity increases, the influence of initial volatility state on asset price becomes weak and the CDS price increases. Similar with the analysis of Figure 4, with the initial state of high volatility, the increase of transition intensity weakens the influence of initial state of high volatility and the CDS price becomes lower.

Figure 4 CDS price when $S_0 = 1$ Figure 5 CDS price when $S_0 = 2$

3.3.2 Comprehensive relation between initial state, transition intensity and CDS price

Figure 6,7 and 8 show the influence of initial state on CDS price under the situation of different transition intensity. In Figure 6, the CDS price is the largest with initial state of high volatility, the second is the price without regime shift with its volatility value equal to the average of regime L and H weighted by time spent in each regime. The price is smallest with initial state of low volatility. Compared with Figure 6, Figure 7 shows that as the transition intensity increases, the difference between different initial state decreases. Comparing Figure 6 and 7 with Figure 8, a interesting finding is that with further increase of transition intensity, the difference between them becomes smaller but the price without regime shift becomes smallest of all. In Figure 6, the average transition times is between 4 and 8, the initial state dominates the CDS price. When transition intensity increases, as shown in Figure 7, the influence of initial state becomes weak and the effect of regime shift increases. Ulteriorly, in Figure 8, the average transition times is between 20 and 40. The impact of regime shift prevails over that of initial state and the CDS price with regime shift is larger than that without regime shift. This phenomenon can be explained as follows: when the asset price volatility has the property of regime shift, whatever the initial state, the firm default probability is always larger that the situation without regime shift as long as the transition intensity is large enough. As we know, regime shift of state can be viewed as the volatility of asset price. If the transition intensity is large enough, the probability that asset price changes dramatically is larger than the situation of constant volatility. Under this condition, it is easier to produce lower extremes. Thus, both the default probability and the CDS price become larger.

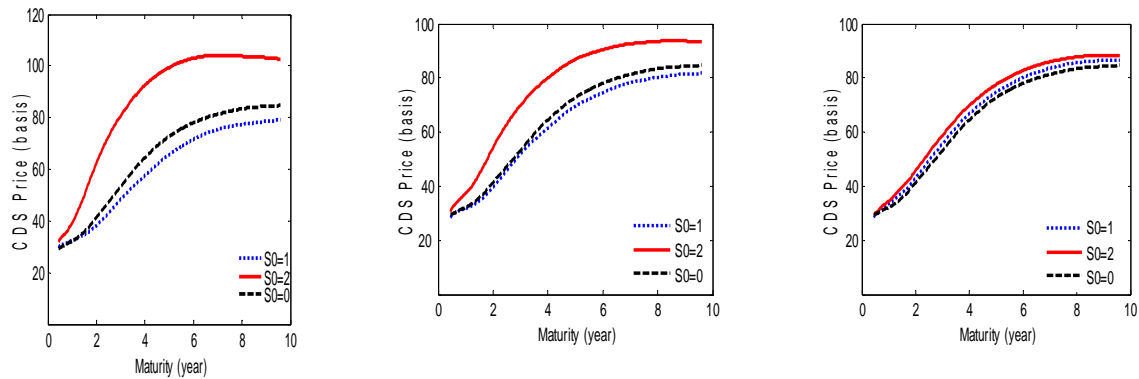


Figure 6 CDS price when $\lambda_H=0.4, \lambda_L=0.2$ Figure 7 CDS price when $\lambda_H=0.8, \lambda_L=0.4$ Figure 8 CDS price when $\lambda_H=4, \lambda_L=2$

4. CONCLUDING REMARKS

This paper develops a new CDS pricing model under structural framework. In our model, the asset price of the reference entity follows a jump-diffusion process and its volatility has the Markov regime shift. We provide complete Monte Carlo procedure to simulate CDS price. The simulation results show that the jump-diffusion process makes default unpredictable and short term credit spreads become positive which is different from the diffusion process. Compared with the CDS price under the situation where the volatility is constant, the CDS price under Markov regime shift is much different. The distance between them depends on initial state and transition intensity. The former can reflect the influence of business situation changes on CDS price. This is realistic when valuing a medium-and long-term CDS. It can be a good reference for CDS pricing.

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REFERENCES

- [1] Abrahams J..(1986). A survey of recent progress on level crossing problems. Communications and Networks: A Survey of Recent Advances, Springer-Verlag.
- [2] Black F., Cox J..(1976). Valuing corporate securities: some effects of bond indenture provisions. Journal of Finance, 31(2): 351-367.
- [3] Chen H.. (2010). Macroeconomic conditions and the puzzles of credit spreads and capital structure. Journal of Finance, 65(6): 2171-2212.
- [4] Dai Q., Singleton K.J. and Wei Y.. (2007). Regime shifts in a dynamic term structure model of U.S. Treasury Bond yields. The Review of Financial Studies, 20(5):1669-1706.
- [5] Elizalde A.. (2006). From Basel I to Basel II: An analysis of the Three Pillars. Workingpaper 2007_wp0704, CEMFI.
- [6] Hackbarth D., Miao J. and Morellec E..(2006). Capital structure, credit risk, and macroeconomic conditions. Journal of Financial Economics, 82(3): 519-550.
- [7] Merton R..(1974). On the pricing of corporate debt: the risk structure of interest rates. Journal of Finance, 29(2): 449-469.
- [8] Merton R..(1976). Option pricing when underlying stock returns are discontinuous. Journal of Financial Economics, 3(1-2): 125-144.
- [9] Wang Q., Chen J.. (2003). A study on pricing model of credit default swap based on jump-diffusion process. Systems Engineering, 21: 79-83. (in Chinese)
- [10] Zhou C..(2001).The term structure of credit spreads with jump risk. Journal of Banking and Finance,25(11):2015-2040.