

Association for Information Systems AIS Electronic Library (AISeL)

WHICEB 2013 Proceedings

Wuhan International Conference on e-Business

Summer 5-25-2013

A Situation of Economic Management in NTU Cooperative Fuzzy Games

Lijun Xu

College of Business Administration, Shenyang University, P. R. China

Wenbo Yang

School of Economics, Fudan University, P. R. China

Follow this and additional works at: <http://aisel.aisnet.org/whiceb2013>

Recommended Citation

Xu, Lijun and Yang, Wenbo, "A Situation of Economic Management in NTU Cooperative Fuzzy Games" (2013). *WHICEB 2013 Proceedings*. 1.

<http://aisel.aisnet.org/whiceb2013/1>

This material is brought to you by the Wuhan International Conference on e-Business at AIS Electronic Library (AISeL). It has been accepted for inclusion in WHICEB 2013 Proceedings by an authorized administrator of AIS Electronic Library (AISeL). For more information, please contact elibrary@aisnet.org.

A Situation of Economic Management in NTU Cooperative Fuzzy Games

Lijun Xu¹, Wenbo Yang²

¹College of Business Administration, Shenyang University, P. R. China

²School of Economics, Fudan University, P. R. China

Abstract: In economic management, we often use some (divisible) private resources to cooperative. Fuzzy coalitions always be used to describe this situation in cooperative fuzzy games. In this paper, we proposed two new solution concepts in NTU cooperative fuzzy games, and discussed their properties.

Keywords: Bargaining set; NTU; Cooperative Fuzzy game

1. INTRODUCTION

In economic management, businessmen often possess some (divisible) private resources such as commodities, time, money, and have to decide about the amount to be invested in the joint project. Cooperative games with fuzzy coalitions are introduced in [1] and [2] to model the above situations where businessmen have the possibility to cooperate with different participation levels, varying from non-cooperation to full cooperation, and where the obtained reward depends on the levels of participation.

In this paper, we proposed more reasonable concepts of the bargaining sets which are very important solution concepts in game theory underlying NTU cooperative fuzzy game, and prove the inclusion relation between them. It is necessary to claim that the results of this paper were based on the work of Tijjs^[3] Muto^[4] and Zhang Fengrong^[5].

2. PRELIMINARIES

Let $N = \{1, 2, \dots, n\}$ be a finite player set, and a fuzzy coalition is a vector $s \in [0, 1]^N$. The i -th coordinate s_i of s is the participation level of player i in the fuzzy coalition s . Instead of $[0, 1]^N$ we will also write Φ^N for the set of fuzzy coalitions on player set N .

For $s \in \Phi^N$ we define the carrier of s by $\text{car}(s) = \{i \in N \mid s_i > 0\}$ and call s a proper fuzzy coalition if $\text{car}(s) \neq \emptyset$. The set of proper fuzzy coalitions on player set N is denoted by $\Pi\Phi^N$, and the set of non-empty proper fuzzy coalitions on player set N by $\Pi\Phi_0^N$. For $s, t \in \Phi^N$ we use the notation $s \leq t$ expresses $s_i \leq t_i$ for each $i \in N$. We define $s \wedge t = (\min(s_1, t_1), \dots, \min(s_n, t_n))$ and $s \vee t = (\max(s_1, t_1), \dots, \max(s_n, t_n))$.

Let $R^{\text{car}(s)}$ indicates $|\text{car}(s)|$ -dimension Euclid space. Denoted $x \leq y$ means $x_i \leq y_i$ for each $i \in \text{car}(s)$; and $x << y$ means $x_i < y_i$ for each $i \in \text{car}(s)$ for any $x, y \in R^{\text{car}(s)}$.

An NTU fuzzy cooperative game is a pair (N, V) , where $N = \{1, 2, \dots, n\}$ is a non-empty finite set, and V , the characteristic function, assigns to every fuzzy coalition s a $V(s)$ which is a subset of $R^{\text{car}(s)}$, and satisfies the follow properties:

(1) $V(0) = \emptyset$, where $0 = (0, 0, \dots, 0)$.

(2) For each $s \in \Phi^N$, $V(s)$ is a non-empty subset of $R^{car(s)}$, and it is closed and comprehensiveness. (that is to say, if $x \in V(s)$, $y \in R^{car(s)}$, and $y \leq x$, then $y \in V(s)$, too).

(3) For each $x \in R^{car(s)}$, $V(s) \cap (x + R_+^{car(s)})$ is bounded.

(4) $V(s)$ is nonlevelness, *i.e.*, if $x, y \in V^0(s)$, and $x \geq y$, then $x = y$, where, $V^0(s) = \{x \in V(s) | \text{there is no } y \in V(s), \text{ such that } x \ll y\}$ is boundary of $V(s)$.

For any two disjoint fuzzy coalitions s and t , and two vectors $x \in R^{car(s)}$, $y \in R^{car(t)}$, we denote by (x, y) the vector in $R^{car(s) \cup car(t)}$ whose $car(s)$ -components are as in x and $car(t)$ -components are as in y . The game (N, V) is superadditive if for every two fuzzy coalitions s and t with $car(s) \cap car(t) = \emptyset$, and any two vectors $x \in V(s)$, $y \in V(t)$, we have $(x, y) \in V(s \vee t)$.

The preimputation set of NTU fuzzy game (N, V) is $P = V^0(N)$, imputation set is $I = V^0(N) \cap R_+^N$.

3. BARGAINING SET FOR COOPERATIVE FUZZY GAME WITH NON-TRANSFERABLE UTILITY

Wenbo Yang *etc.*^[6] firstly extended the concept of bargaining set which was proposed by Davis-Machler^[7] to TU fuzzy game. Here we will redefine the concept of bargaining set from the view point of Davis-Machler to NTU cooperative fuzzy game.

Definition 3.1 Let x be an imputation of the NTU fuzzy game (N, V) , then an objection of player k against player l with respect to x is a pair (y, s) , if $s_i y_i > s_l x_l$, for all $i \in car(s)$, $y \in V(s)$, where $s \in N$ with $car(s) \neq \emptyset$ and N , $s_l > 0$, $s_l = 0$, and y is a vector whose indices are the members of $car(s)$.

Definition 3.2 Let x be an imputation of the NTU fuzzy game (N, V) , and let (y, s) be an objection of player k against player l with respect to x . Then (z, t) is a counter objection of player l to the objection (y, s) of player k , if

$$\begin{aligned} t_i(z_i - x_i) &\geq s_i(y_i - x_i), \quad \text{for all } i \in car(s) \cap car(t) \\ t_i z_i &\geq t_l x_l, \quad \text{for all } i \in car(t) \setminus car(s) \\ z &\in V(t), \end{aligned}$$

where $t \in \Phi^N$ with $t_k = 0$, $t_l > 0$, and $t_i \geq s_i$, for all $i \in car(s) \cap car(t)$, and z is a vector whose indices are the members of $car(t)$.

Definition 3.3 An imputation x belongs to the bargaining set $M_{F^{ij}}(V)$ for a NTU fuzzy cooperative game, if for every objection, there exists a counter objection to it.

Mos-Colell^[8](1989) proposed another bargaining set, now we will redefine the concept to NTU fuzzy game as follows.

Definition 3.4 Let x be a preimputation of the NTU fuzzy game (N, V) , for a fuzzy coalition $s \in \Phi^N$, if

$car(s) \neq \emptyset$ and N , an objection on fuzzy coalition s with respect to x is a pair (y, s) , where $y \in R^{car(s)}$ and satisfying:

$$y \in V(s),$$

$$s_i y_i \geq s_i x_i, i \in car(s),$$

and at least one of the inequalities above is strict.

Definition 3.5 Let x be a preimputation of the NTU fuzzy game (N, V) , and let (y, s) be an objection to fuzzy coalition s with respect to x . Then (z, t) is a counter objection on fuzzy coalition $t \in \Phi^N$ with respect to x , here $t_i \geq s_i$, and for all $i \in car(s) \cap car(t)$, $z \in R^{car(t)}$ satisfying

$$z \in V(t)$$

$$t_i(z_i - x_i) \geq s_i(y_i - x_i), \text{ for all } i \in car(s) \cap car(t)$$

$$t_i z_i \geq t_i x_i, \text{ for all } i \in car(t) \setminus car(s)$$

and at least one of the inequalities above is strict.

An objection is called justified, if there is no counter objection.

Definition 3.6 A preimputation x belongs to the bargaining set $MB_F(V)$ of NTU fuzzy cooperative game if there is no justified objections with respect to x .

Theorem 3.1 Let (N, V) be a superadditive NTU fuzzy game, then

$$M_F^{(N)}(V) \subseteq MB_F(V).$$

Proof. Suppose, for contradiction, that $x \in M_F^{(N)}(V) \setminus MB_F(V)$. Let (y, c) be a justified objection in the sense of $MB_F(V)$ at x . Chosen c such that $c \geq s$, where s is an any fuzzy coalition which can cause a justified objection in $MB_F(V)$ at x . Let $c_k > 0$, and $c_k y_k > c_k x_k$. From the concept of the bargaining set $MB_F(V)$, we have $car(s) \neq N$. Let $l \in N \setminus car(c)$.

By the comprehensiveness and nonlevelness of $V(c)$, we can modify y , and y is still in $V(c)$. Let the k -component of y will decrease, but is still great then x_k , and the other components of y will increase. So, we obtain a vector $\tilde{y} \in V(c)$ satisfying

$$c_k \tilde{y}_k > c_k x_k$$

$$c_i \tilde{y}_i > c_i y_i, i \in car(c) \setminus \{k\}$$

Clearly, (\tilde{y}, c) is an objection of k against l at x . Since $x \in M_F^{(N)}(V)$, there exists a counter objection (z, d) to (\tilde{y}, c) satisfying $d_i \geq c_i, i \in car(d) \cap car(c), d_k = 0, d_l > 0$. So we have

$$d_i(z_i - x_i) \geq c_i(\tilde{y}_i - x_i) > c_i(y_i - x_i), i \in car(d) \cap car(c) \tag{1}$$

$$d_i z_i \geq d_i x_i, i \in car(d) \setminus car(c) \tag{2}$$

Thus, (z, d) satisfies almost all the requirements to be a counter objection to (y, c) in the sense of $MB_F(V)$, except one condition, *i.e.*, at least one strict inequality in (1) and (2) is true. Since (y, c) is justified, and in the inequality (1), $car(c) \cap car(d)$ must be an empty set. But from the property of superadditivity, we know that $((y, z), c \vee d)$ is a justified objection in the sense of $MB_F(V)$ at x , so $c < (c \vee d)$, contradicting our assumption. The conclusion is true.

4. CONCLUSIONS

NTU cooperative game can be converted to TU cooperative game under specific conditions. In the general sense, NTU cooperative game can be viewed as the extension of TU cooperative game. But, many properties of NTU cooperative game can help to study the properties of the TU cooperative games. In this paper we extended two concepts of the bargaining set for TU cooperative game to NTU cooperative fuzzy game, and proved that the inclusion relation between these two bargaining sets is still satisfied. The problem of the bargaining sets how can be used in economic management effectively is still open^[10]. We will work on it in future.

REFERENCES

- [1] Aubin, J.P.: Coeur et valeur des jeux flous a paiements lateraux, Comptes Rendus de l' Academie des Sciences Paris 279 A, 891-894(1974).
- [2] Aubin, J.P.: Cooperative fuzzy games, Mathematics of Operations 6, 1-13(1981)
- [3] Tijs, S., R. Branzei, S. Ishihara, and S. Muto: On cores and stable sets for fuzzy games, Fuzzy Sets and Systems 146, 285-296(2004)
- [4] Muto, S., S. Ishihara, E. Fukuda, S. Tijs, and R. Branzei: Generalized cores and stable sets for fuzzy games, International Game Theory Review 8, 95-109(2006)
- [5] Zhang Fengrong, Zhang Shengkai, Xie liyang, The Fuzzy NTU-game, Systems Engineering, Vol. 22, No. 10, Oct., 2004
- [6] Wenbo, Y., Jiuqiang L., Xiaodong L.: Aubin Cores and Bargaining Sets for Convex Cooperative Fuzzy Games, International Journal of Game Theory, Online first 2010
- [7] Aumann, R.J., Maschler, M.: The bargaining set for cooperative games. In: Dresher, M., Shapley, L.S., Tucker, A.W.(eds) Advances in game theory, Princeton University Press, Princeton, N.J., pp. 443-476(1964)
- [8] Mas-Colell, A.: An equivalence theorem for a bargaining set. J Math Econ 18, 129-138(1989)
- [9] Branzei, R., D. Dimitrov, and S. Tijs: Convex fuzzy games and participation monotonic allocation schemes, Fuzzy Sets and Systems 139, 267-281(2003)
- [10] Xie Shiyu, Economics Game Theory, Fudan university press,(1997)