

# NEWS RECOMMENDER SYSTEMS WITH FEEDBACK

*Completed Research Paper*

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## Abstract

*The focus of present research is widely used news recommendation techniques such as “most popular” or “most e-mailed”. In this paper we have introduced an alternative way of recommendation based on feedback. Various notable properties of the feedback based recommendation technique have been also discussed. Through simulation model we show that the recommendation technique used in the present research allows implementers to have a flexibility to make a balance between accuracy and distortion. Analytical results have been established in a special case of two articles using the formulation based on generalized urn models. Finally, we show that news recommender systems can be also studied through two armed bandit algorithms.*

**Keywords:** Algorithms, recommendation agent, business intelligence

## Introduction

In last decade a dramatic shift has been noted in the news industry primarily due to change in news consumption behavior of readers, driven by a bigger penetration of the internet and social media in society. *The Economist* has noted that there is growing trend towards using the internet as a primary source of news in US. Similar trend has been observed in Europe, where nearly 50 percent of Europeans visit newspaper websites (ComScore 2012). The increasing trend in the growth rate of online news readers has been also observed in developing countries (Gavane June 7, 2011).

In this changing environment of the news industry one thing appears to be widely adopted among almost all news websites - the display recommended articles such “most popular” or “most e-mailed” prominently on their front page. The prominence of these automated systems is well recognized in the context of news ecosystems (Weber December 19, 2010). This has also been noted in modern satire. *The Onion*, a popular site, for instance wrote in 2007 that “*online readers instinctively overlook harder news for the eye-catching Most E-Mailed box*” (April 7, 2007).

These news recommender systems (NRS) are often considered an important source of news articles for readers, articles which otherwise may get lost due to dynamic environment of news cycles driven by continuous arrival of news articles (Weber December 19, 2010). It has been also noted that once a story is promoted by the NRS, there is an abrupt increase in its popularity and viewership than other stories (Bilton April 1, 2012; Lerman et al. 2010).

In recent work it has been shown that these systems have a self-reinforcing nature and are easily susceptible to manipulation (Prawesh et al. 2011). The self-reinforcing nature comes from the fact that once an article makes it into a Top-N list, it gains even more popularity simply by virtue of being in such a prominent list. A manipulator can exploit the self-reinforcing nature of Top-N NRS with some effort in the initial period in the life span of the target article. Recently this problem has been noted in popular press articles and the problem of manipulation in particular is getting bigger attention in industry and policy makers (Rusli et al. March 15, 2012; Weber December 19, 2010).

To address the aforementioned issues with the Top-N NRS a *probabilistic* NRS has been proposed (Prawesh et al. 2011). In probabilistic NRS, the articles to be displayed as recommendation are updated at regular time intervals. At each time step,  $N$  articles are selected for display (as recommendation) using probabilistic selection without replacement, from the comprehensive list of articles in the system. The probability of an article being selected in the recommendation list is proportional to the counts (or clicks) it has received thus far.

This method still generates good recommendations, but permits all articles to have some chance of being recommended. Such a mechanism does not penalize the marginal next articles that might have just missed a hard cutoff in a traditional Top-N list. This mechanism is also more robust against manipulation since it does not suffer as much from the self-reinforcing nature of the hard cutoff lists.

However, there are some limitations of the probabilistic NRS presented in (Prawesh et al. 2011). For one, this approach may select some articles that are not as popular, thereby potentially sacrificing short-term clicks or readership. In an era where page views translate proportionally to advertising revenue this can be a concern in implementing this. Second, among a wide range of selection methods for recommendation, assigning an article a selection probability proportional to its count is just one approach of selection and it does not provide online media managers with much flexibility in implementing such a sampling scheme.

For instance, based on these findings a content manager can either choose between using the (current) Top-N framework, or replace it with one that samples probabilistically. But such a choice does not give them any control in operating in the “continuum” between these two options. For instance, due to short-term revenue goals, they might want to retain the probabilistic nature but give “much higher” weights to more popular articles. On the flip side if their objective is to use the recommendations to drive traffic to long-tail articles they might wish to tweak the probabilistic selection closer to even purely random (or

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<sup>1</sup> It is also known as Top-N NRS

even “worse than random” where they might choose to only promote un-noticed articles at the extreme).

The present research addresses the limitations of recommendation systems proposed earlier and makes novel contributions in NRS research in the following way:

1. We introduce a class of recommendation techniques with *feedback* (FNRS) and discuss various notable properties of it both analytically and through simulations. Feedback models (Drinea et al. 2002), are used in applications where the behavior of the system creates either positive or negative feedback that affects the future behavior of the system. In general the recommendation probability of an article with count  $n$  is proportional to  $f(n) = n^\gamma, \gamma \in \mathbb{R}$ . In this expression,  $\gamma$  acts as the feedback parameter of the model. The design of FNRS exhibits feedback behavior by causing people to process recommended information in different ways. We can reduce the richer get richer effects for articles, or amplify them, or steer them in a different direction (with articles with low counts becoming more popular) by help of the parameter  $\gamma$ . Hence, this parameter is now a practical way of controlling the amount of attention for the most popular articles that is considered desirable by the media owner. Note that we are not suggesting that one value is better than another; instead domain experts can choose to dynamically tune their Top-N lists based on whether they want more diversity or whether they want to favor exploitation of what is most popular right now. Further, urn models from probability theory have been used to discuss the various properties of the proposed recommendation mechanism in a special case (Drinea et al. 2002).
2. An optimization problem has been formulated for FNRS to discuss the trade-off between accuracy and distortion. In our context, accuracy is a measure that captures how close the recommended articles match the current most popular ones. Distortion captures how much the recommendation process alters the distribution of the natural counts or popularity of articles. Further, we show that for the given preference of an implementer and with knowledge of other parameters (discussed later) an optimal level of feedback exponent (denoted as  $\gamma^*$ ) can be obtained. The methodology to determine  $\gamma^*$  can be used by the media site to use the proposed FNRS for news recommendations.

Finally we show that the feedback phenomena in context of NRS can also be modeled through the two-armed bandit algorithm- a stochastic approximation procedure widely studied in mathematical psychology, sequential analysis and learning automata (Meyer et al. 1995; Norman 1968; Shapiro et al. 1969). However, we note that our proposed FNRS has advantages in terms of elegance in interpretation and implementation. The framework that we have proposed will help researchers to extend the application of the two-armed bandit algorithm (perhaps through multi-armed bandit algorithms) in the context of recommender systems..

## Related Work

There is growing interest among researchers to study the different characteristics of articles that appear in a recommended list. For example, Hogg et al. (2012) have used a stochastic model of user behavior to predict the popularity of a story submitted on the Digg website. Through their user model they have also been able to distinguish between the effects of the ‘content visibility’ and ‘interestingness’ to users. In a slightly different approach Berger et al. (2011) have investigated how emotions shape virality (social transmission) of an article. They have taken a psychological approach to understand the diffusion of news articles. Their findings are based on *New York Times* articles which make into the “most e-mailed list”. They have also noted that “*some people look into the most emailed list every day to determine what articles to read*”.

The phenomena of count distortion created by the popular Top-N NRS have been discussed by Prawesh et al. (2011). Using simulation and analytical model based on urn formulation they have shown that Top-N NRS such as ‘most e-mailed’ or ‘most popular’ is susceptible to amplifying negligible initial differences in counts of  $N^{th}$  and  $(N + 1)^{th}$  article. We take a similar approach in the present research to introduce a feedback<sup>2</sup>-based recommendation technique and discuss various properties of it. In particular, we show that FNRS behaves as probabilistic and Top-N NRS in special cases – an important property that gives it the flexibility discussed earlier.

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<sup>2</sup> Feedback and reinforcement have been used interchangeably.

The urn models used in the special cases of FNRS (for analytical results) and two-armed bandit formulation have its origin in probability theory. The earliest urn model was proposed by Pólya (Pólya 1924) to model the spread of infectious diseases. Later, different variations of it have been introduced such as, Friedman urn, coupon collector urn, Ehrenfest urn, OK corral urn, Mabinogion urn and many others (Flajolet et al. 2008; Mahmoud 2008). Mathematical properties of urn models have been also investigated in the probability literature.

The application of urn models to study reinforcement growth processes, similar to the case of FNRS, can be found in statistics, biology, economics and computer science (Pemantle 2007). In particular, in the context of Information Systems, Metcalfe's law-used to characterize the network effect (or network externality) of information goods, can be considered an example of a generalized Pólya urn scheme (Bakos et al. 1999; Khanin et al. 2001).

Khanin et al. (2001) have studied a probabilistic model for the early stage of neuron growth, using urn processes. Their model replicates the growth process of one rapidly elongating axon out of several 'neurites'. The probability that a neurite grows at a time depends on its length at that time and the length of all other neurites. Our theoretical discussion follows closely the approach of Khanin et al. (2001), adapted in the context of FNRS.

The two-armed bandit algorithm can be understood through an urn formulation – a representation introduced by Lamberton et al. (2004), in special cases. Using this formulation and representing the count evolution process in case of FNRS through a stochastic differential equation, we illustrate that in some special cases, FNRS can be understood through two-armed bandit algorithm.

In the context of recommender systems, there are few examples of applications of urn models. Fleder et al. (2009) have used urn function defined as a combination of recommendation-acceptance and consumer's original choice to study the impact of recommender systems on sales diversity. Particularly in the context of NRS, Prawesh et al. (2011) have used Pólya urn and Friedman urn models to examine the count evolution of articles in the probabilistic NRS. We also note that though urn models have been widely used in other research areas. So far, their applications in information systems field are still limited.

The issue of tradeoff between diversity and accuracy has received wide attention in the field of recommendation systems research. *Diversity* in recommender systems is recognized at two levels, namely: *individual diversity* and *aggregate diversity*. Individual diversity measures the recommendation diversity from an individual user's perspective, whereas aggregate diversity is measured by the number of distinct items recommended across all users. Further, it has been noted that aggregate diversity is often neglected in the context of recommender systems (Adomavicius et al. 2012). For the sake of brevity, henceforth we refer aggregate diversity as diversity.

There is some research where the tradeoffs between accuracy and diversity have been analyzed. Zhang et al. (2008) have proposed an evaluation metric *ItemNovelty* based on binary optimization problem between matching function of item recommendation and average dissimilarity between recommended items. Adomavicius et al. (2012) have used "ranking threshold" coupled with recommendation ranking techniques on "ranking criterion". They have introduced several ranking techniques that generate recommendations with high diversity with maintaining comparable level of accuracy. Deselaers et al. (2009) have presented a method to jointly optimize *relevance* and *diversity* in image retrieval. They address this problem through information retrieval criterion applied with heuristics such as, clustering, greedy selection and dynamic programming.

We have taken a similar approach to discuss the tradeoff between *accuracy* and *distortion* (defined later) in the context of NRS. The techniques of recommendation introduced in this research will allow implementers to generate recommendations with low distortion while maintaining a sufficient level of accuracy.

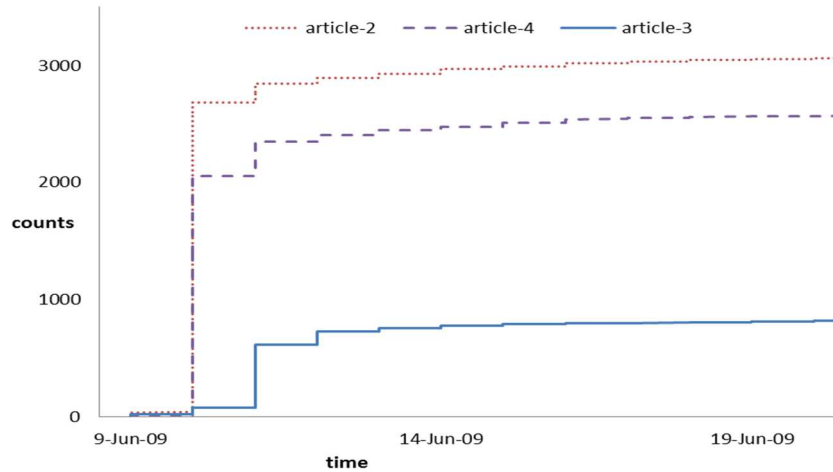
## Data Analysis

To understand the impact of recommendation on readers (albeit in a slightly different context), we present the evolution of the number of votes<sup>3</sup> of articles that has been promoted to the *front page* of news

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<sup>3</sup> In the context of Digg the number of 'votes' of a story and the number of clicks that the article has received are

aggregator website Digg. Digg<sup>4</sup> is a social news aggregator website with approximately 5 million visitors per month (as noted by Quantcast). It allows users to *submit* articles and *vote* (or digging) for the favorite submissions. Approximately 16,000 submissions of articles are received by Digg per day (Lerman et al. 2010). From these a number of articles are selected to be displayed on the front page as “Top news”. Newly submitted stories go to the upcoming list where it remains for 24 hours, or until it is promoted to the front page, whichever occurs earlier. Fifteen promoted (or “popular”) stories are displayed in the front page in reverse chronological order of their promotion.



**Figure 1: Counts for the Article Promoted to the Front Page (Digg)**

The dataset used in the present analysis was collected by Lerman et al. (2010) to study the effect of network structure on the dynamics of information flow and the vote evolution process of submitted stories. It contains a record of 3,018,197 votes made by 139,409 readers for 3553 popular articles which has been promoted to the front of Digg in June, 2009. The following information can be obtained about each article: (1) time stamp for each vote (2) voter id (3) article id and (4) the time stamp of promotion to the front page.

Figure 1 presents the voting pattern for some of the articles. In the figure we see an abrupt increase in the number of votes of articles corresponding to the time when they are promoted to the front page. While the exact ranking procedure by Digg to promote articles in the front page is unknown, the figure illustrates the self-reinforcing nature of these lists - once an article appears as “Top News” it becomes visible to a large number of readers and gets even more popularity. Lerman et al. (2010) has also observed that stories that are never promoted to the front page receive very few votes, leading to ‘inequality of popularity’ with relatively few stories becoming very popular.

## Model

We setup a simulation model as follows. A comprehensive list (CL) of articles and their corresponding clicks are maintained. From CL,  $N$  articles are selected for display as “recommendations”. Before the simulation starts, articles are assigned random counts in a given interval (e.g. 0 to 1000). Articles are sorted in decreasing order of their counts and the articles with high counts are selected for the display list (DL). Further, the  $(N + 1)^{th}$  article was deliberately assigned a count of exactly one less than the count of  $N^{th}$  article to examine how negligible initial differences play out over time under different mechanisms.

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different. For example, a reader may read a story but he may not ‘vote’ it.

<sup>4</sup> [www.digg.com](http://www.digg.com)

The selection of articles in the DL is updated at a pre-selected time step, and the selection of article is based on two different selection processes namely, *count based* and *probabilistic selection* based on probability function described below. Count-based selection is a “hard cutoff”, which selects N articles for display corresponding to highest counts. This is how typically most news websites implement the most popular (or most-emailed) recommender systems. Probabilistic selection with feedback exponent, is the mechanism proposed here, selects articles probabilistically for display based on counts received thus far.

Probabilistic selection of articles is based on probabilistic sampling without replacement for N articles. The probability that an article will be selected in DL is given by,

$$p_a(t) = \frac{c_a^\gamma(t)}{\sum_j c_j^\gamma(t)} \quad (1)$$

Where  $C_a(t)$  represents the count of an article ‘a’ at a given time  $t$  and  $\sum_j C_j^\gamma(t)$  represents the sum of counts of articles (those are not yet selected for DL) at time  $t$  to the power  $\gamma$ . This sampling process is repeated N times to generate the N recommendations in DL. Pseudo code for the implementation of these selection processes is discussed later in this section. In the pseudo code,  $n$  represents the number of articles in the system and  $N$  represents number of articles displayed in the recommended list.

Significantly, the probabilistic selection mechanism with feedback (equation 1) can be considered as a unified model of different selection techniques, used for news recommendation based on the count of articles.

Let us consider the different values of  $\gamma$  to understand the behavior of FNRS. For ( $\gamma = 0$ ) and ( $\gamma = 1$ ) the selection mechanism described by equation-1 corresponds to random and probabilistic selection of articles for recommendation, respectively. The feedback mechanism in the case of ( $\gamma \rightarrow \infty$ ), is identical to Top-N recommender – i.e. only few articles corresponding to highest counts will get recommended by FNRS. For  $\gamma > 1$ , the system generates recommendation such that the articles with high counts will have even higher probability of being selected in the recommended list at the next time step (positive feedback).

Whereas on the other end for  $\gamma < 1$ , articles with low counts will have higher chance of being selected in the recommended list (negative feedback). By promoting such (possible “new”) articles we are then able to generate “data” on whether users read it or not. This may therefore be a useful mechanism to address issues related to “cold-start” in recommender systems.

A reader upon arrival is assumed to select an article either from DL with some probability  $p$  or from the remaining list  $RL = (CL - DL)$  with probability  $1 - p$ . A reader selects an article from either list (DL or RL) randomly. For ease of exposition we have intentionally left out the other complicated factors related with news arrival process and reader behavior.

### **Implementation of NRS**

“Select” can be count based or probabilistic (feedback) while “choose” is random selection of the article.

For each reader

Sort the updated count and **select** N articles for DL

If selected article is from DL (i.e. with probability  $p$ )

**Choose** an article from DL and increase its count by 1

Else

**Choose** an article from RL; ( $RL = CL - DL$ ) and increase its count by 1

end for.

### Probabilistic Selection with Feedback

1. The  $\gamma^{th}$  power of article counts are  $c^\gamma[1], c^\gamma[2], \dots, c^\gamma[n]$
2.  $cum\_count[1] = 0$
3. for  $x = 2$  to  $n + 1$ 

$$cum\_count[x] \leftarrow cum\_count[x - 1] + c^\gamma[x - 1]$$
4. end for
5. for  $y = 1$  to  $N$ 
  - a. generate a random number (R) between 0 and  $cum\_count[n + 1]$
  - b. determine the indices between which R lies, as  $(i, i + 1)$
  - c. select article corresponding to  $c^\gamma[i]$  for  $DL$
  - d. Remove  $cum\_count[i + 1]$  and  $i$  th article
  - e.  $j \leftarrow c^\gamma[i]$
  - f. While ( $i$  is less than  $n + 1$ )
$$cum\_count[i + 1] = cum\_count[i + 1] - j$$
  - g. end while
  - h.  $n = n - 1$
6. end for

### Recommendation Boundary Amplification

To study the impact of NRS on boundary amplification, we use a measure (denoted as  $M1$ ) based on the counts of  $N^{th}$  and  $(N + 1)^{th}$  article. It is defined as the logarithmic-ratio of the counts of  $N^{th}$  and  $(N + 1)^{th}$  articles at each time-step.

$M1(t) = \ln(count_{Nt}) - \ln(count_{(N+1)t}) = \ln \frac{count_{Nt}}{count_{(N+1)t}}$  at the  $t^{th}$  iteration of the simulation, for a given

NRS. This measures the relative change in counts of  $N^{th}$  and  $(N + 1)^{th}$  article. As mentioned in the Table-2, at the start of the simulation,  $count(N) \sim count(N + 1)$ , hence  $M1(0) \sim 0$ .

### Top-N Reinforcement

For any given NRS (Top-N or FNRS) we measure the percentage of new clicks (i.e., after  $t = 0$ ), received by the articles outside the Top-10 list - determined through ranking based on decreasing order of the counts of each article at a given time step. This measure is denoted by  $M2$  and its mathematical value is given by,

$$M2(t) = \left\{ 1 - \left( \frac{\sum_{top-10 \text{ list at time } t} (\# \text{ new clicks})}{\# \text{ total new clicks}} \right) \right\} * 100$$

If  $M2$  decreases towards its value in a hard-cutoff scenario, it suggests reinforcement in the current top-N list (because more of the new clicks into DL go to the current top-N articles). This helps us to examine the behavior of FNRS for the difference choice of the feedback value  $\gamma$ . In particular, this gives us an opportunity to examine how closely we can replicate FNRS as Top-N NRS.

For example, when there is a high likelihood of only reading articles in the recommended list, then under the Top-N (hard cut-off) selection we would expect  $M2$  to be zero since all new clicks will only go to the same articles in the Top-N list. This measure will help us determine, under feedback mechanisms, how quickly the FNRS behaves like a traditional Top-N list.

### The Update Rule

At each time period, the model proceeds as follows. One reader arrives at each time step. Upon arrival reader selects probabilistically to read an article either from displayed list ( $DL$ ) or the remaining list ( $RL$ ) of articles. The probability of selection of an article either from  $DL$  or  $RL$  is controlled in the simulation. If a reader selects an article from  $DL$  (or  $RL$ ), then random selection of an article is performed. The count of

the selected article is increased by 1.

For two different NRS, count-based and probabilistic (with feedback), the selection of  $N$  articles is made for  $DL$ , and  $DL$  is updated at each time step.

## Results

The analysis of findings is based on three sections where we compare the proposed FNRS with the “most popular” NRS based on (1) count amplification in  $N^{th}$  and  $(N + 1)^{th}$  article- through M1 (2) the path followed by M2 for FNRS in comparison with Top-N NRS and (3) tradeoff between accuracy-loss and distortion in FNRS.

### Simulation Parameters

To choose the number of time steps in the simulation, we conducted a brief analysis of number of page viewed at popular media sites per day. The estimates on page views- per day are given in the Table-1.

We estimated page views (per month), for news websites from sources such as [quantcast](#) and [comScore](#). This in turn helped us to get the approximate counts (in millions) of daily page views. For Huffington Post, the estimate has been obtained from quantcast. In case of New York Times, Wall Street Journal, Washington Post and NYDailyNews.com the estimates are directly obtained from comScore (2010). For Mail-Online and The Guardian, data on page views were not available. So we made a rough estimate based on the assumption that monthly average page views per visitor, remains constant during the time period of 2010-2011, in newspapers category. We acknowledge that, the statistics given here for the last two media sites are just for an illustration. Actual figures can be much higher. As according to the Economist<sup>5</sup>, in January 2012 Mail Online had more readers than New York Times (March 17, 2012). Following observations have been used in last two cases to come up with the estimate: (1) The number of total visitors in the given month for Mail-Online and The Guardian (comScore September 29, 2011) and (2) monthly average pages per visitor for newspaper categories (comScore 2010). Usually information from the past 24 hours is used by news websites to display articles as ‘most-emailed’ or most-popular’. Based on this we have chosen the main parameters in our simulation model for analysis.

**Table 1: An Estimate of the Number of Views for Media Sites**

Media sites	Time period	Page views (per day)
Huffington post	February 2012	23.3
New York Times	May 2010	24
Wall Street Journal	May 2010	4
WashingtonPost.com	May 2010	6
NYDailyNews.com	May 2010	4
Mail Online	August 2011	4.1
The Guardian	August 2011	2.6

We study the behavior of FNRS for different choice of the feedback parameter  $\gamma$ . In the simulation it has been varied with different integer values between 1, 2... 10. However, we have summarized our findings with few selected parameters. The value of simulation parameters used are listed the Table 2. We have chosen a specific choice of reading probability  $p = 0.9$  to examine the case of an influential (where probability of reading a recommended article is high) FNRS.

<sup>5</sup> Other sources are: (<http://www.guardian.co.uk/media/greenslade/2012/jan/25/dailymail-internet>, [http://www.huffingtonpost.com/2012/01/25/daily-mails-website-traffic-new-york-times\\_n\\_1231795.html](http://www.huffingtonpost.com/2012/01/25/daily-mails-website-traffic-new-york-times_n_1231795.html)).



**Table 2: The Model Parameters Used in the Simulation**

Parameter	Value
Number of readers	100,000
Number of article in DL	10
Number of articles in CL	200
Initial counts of articles	Random integer between 0 and 1000 <sup>6</sup>
Probability of selection of an article from DL ( $p$ )	0.9
Exponent ( $\gamma$ )	1, 2, 3...10
$\beta$ (will be defined in page-11)	0.5

### Properties of FNRS

First we summarize our findings based on the measure M1, tracked over the complete simulation for each NRS. It can be observed from the path followed by M1 (Figure-2) that for an ‘influential’ Top-N NRS, even negligible initial difference between the counts of  $N^{th}$  and  $(N + 1)^{th}$  article gets amplified heavily and quickly. This phenomenon is often encountered in real world, as noted by the economist Matthew O. Jackson. As he states, “being 11<sup>th</sup> in a top 10 list on the app store is a lot different than being 10<sup>th</sup> on that list,” (Bilton April 1, 2012). As expected, for the higher feedback (here  $\gamma = 10$ ), M1 closely follows the path in Top-N NRS after certain time, albeit to a lesser extent-as initially both articles compete. For the lower values for the feedback exponent ( $\gamma = 5$ ) the phenomenon of artificial amplification due to Top-N NRS can be addressed very efficiently (see Figure-2).

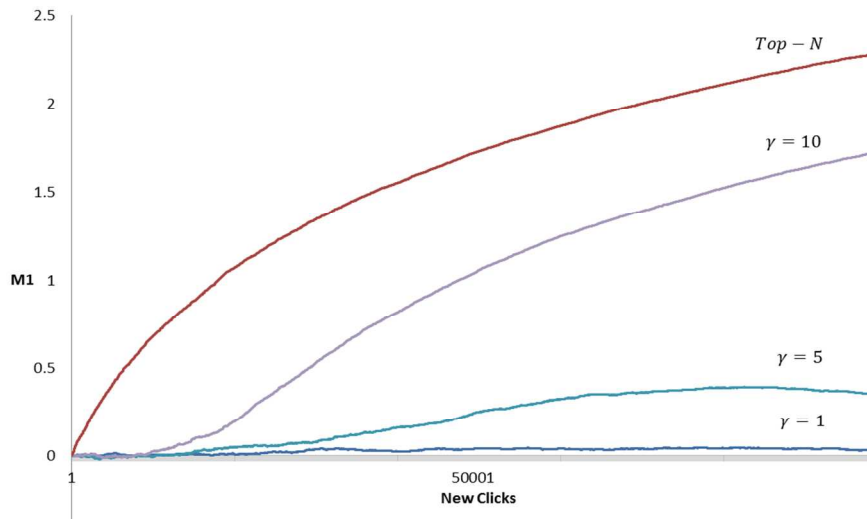
**Figure 2: Boundary Amplification of Articles**

Figure-3 represents the findings based on measure M2 defined earlier for different NRS (Top-N and FNRS with different values of  $\gamma$ ). In the figure, the top line corresponds to the probabilistic selection of articles for recommendation (i.e.  $\gamma = 1$ ), and the bottom line corresponds to the Top-N NRS (hard-cutoff). We observe that as the value of  $\gamma$  increases, the performance of FNRS becomes closer to the Top-N NRS. Further, the FNRS with positive feedback eventually may behave as Top-N after sufficient number of time steps (for example see the path of  $\gamma = 10$ ). We have also examined these properties in detail in the section

<sup>6</sup> Except  $N^{th}$  and  $(N + 1)^{th}$  articles. Counts for these articles were assigned such that  $count(N + 1) = count(N) - 1$ . This was done deliberately to test how the hard cutoff treats very small initial differences in quality between articles.

follows on the analytical results.

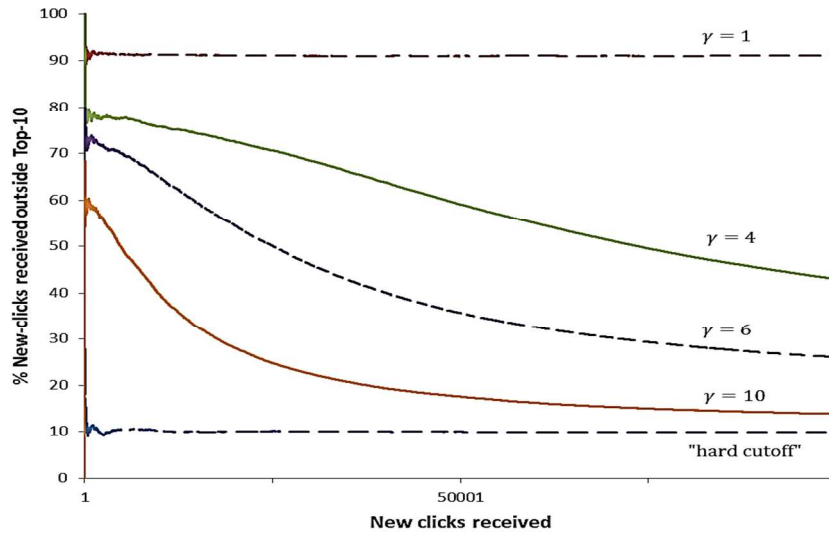


Figure 3: Reinforcement Behavior of FNRS

## Analytical Results

To present analytical results we consider a simple setting of two articles for both Top-N NRS and FNRS. The assumptions made to establish results are summarized below:

1. Two articles are available for recommendation (article-1 and article-2).
2. As in the simulation model, a reader upon arrival reads the recommended article (DL) with probability  $p$  or reads the other (RL) with probability  $1 - p$ .
3. An initial count of articles before the recommender system was implemented is given by  $c_1(0)$  and  $c_2(0)$  respectively.
4. NRS has fairly strong influence on reader's reading behavior, i.e.  $p$  takes value close to 1 ( $p \sim 1$ ).

The case of Top-N NRS in the above setting is trivial as the count evolution process can be easily analyzed through a simple binomial process. In the remaining discussion we focus on FNRS.

The count of two articles at time  $t$  has been denoted by  $c_1(t)$  and  $c_2(t)$  respectively. Let us denote the discrete time points by integer values. At each time, upon arrival of a reader, an article is read and its count is increased by 1. The total count of articles in the system at time  $t$  is deterministic and it is given by  $c_1(0) + c_2(0) + t$ . We focus on the article '1' for subsequent derivation; we also note that theoretical results for article '2' can be obtained in similar way. The probability  $p[c_1(t + 1) = c_1(t) + 1]$  is denoted as  $p_{1t}(read)$  – i.e. if article '1' is read at time  $t$  its count increases by one.

**Proposition 1 (Monopoly Result):** For the two-article FNRS considered above whose feedback-strength for article "1" is given by  $\frac{c_1(t)^\gamma}{c_1(t)^\gamma + c_2(t)^\gamma}, \gamma > 1$  one of the articles will receive all counts after certain finite time.

**Proof:** Here we present outline of the proof, complete derivation is omitted due to lack of space (will be made available online on request).

In FNRS (with two articles), article-1 can be read in two ways. The article is in the recommended list (with probability  $\frac{c_1(t)^\gamma}{c_1(t)^\gamma + c_2(t)^\gamma}$  and the reader chooses to read the recommended article (with probability  $p$ ). Or, article-1 can be in the other list =  $(CL \setminus DL)$  (with probability  $1 - \frac{c_1(t)^\gamma}{c_1(t)^\gamma + c_2(t)^\gamma}$ ) and the reader chooses to read the un-recommended article (with probability  $1 - p$ ).

Specifically, the probability that an article '1' is being read and hence its count being increased by 1, at time  $t$  is given by:

$$p_{1t}(read) = p * \frac{c_1(t)^\gamma}{c_1(t)^\gamma + c_2(t)^\gamma} + (1 - p) * \frac{c_2(t)^\gamma}{c_1(t)^\gamma + c_2(t)^\gamma} \quad (2)$$

The case of influential FNRS considered here corresponds to  $p \sim 1$  however, to maintain the tractability (with slight loss of mathematical completeness) for the remaining discussion we consider  $p = 1$ . This assumption gives us the following form of equation-2:

$$p_{1t}(read) = \frac{c_1(t)^\gamma}{c_1(t)^\gamma + c_2(t)^\gamma} \quad (3)$$

The process defined by the equation-3 in FNRS context, gives rise to the processes that has similar characteristic as of one rapidly growing axon out of several neurites (Khanin et al. 2001). These processes correspond to the generalized Pólya scheme- urn models studied by learning theorists (Davis 1990).

More precisely, we can say that there exists a random time  $T^*$  such that for all  $t > T^*$ , one article will get recommended with probability 1 (as in Top-N NRS). This finding is based on embedding generalized Pólya processes into time- a representation from Herman Rubin (Davis 1990) and the Kolmogorov’s “three series theorem”.

The embedding process relies on the lack of memory of the exponential distribution as well as the fact that if  $U$  and  $V$  are independent exponential random variables with means  $u^{-1}$  and  $v^{-1}$  respectively, then the probability  $P(U < V) = \frac{u}{u+v}$  and vice-versa for  $P(V < U)$ . Whereas, Kolmogorov’s three series theorem gives a criterion for the *almost sure convergence* of the growth model, for the generalized Pólya processes.

As expected, the probability that one of the articles will get recommended also depends on the initial state of the system( $c_1(0), c_2(0)$ ). The detailed discussion on determining this monopoly probability (probability of one of the article gets recommended position) is presented by (Mitzenmacher et al. 2004). Mitzenmacher et al. (2004) have also established an approximate closed form expression for the eventual monopoly.

From the simulation findings of Mitzenmacher et al. (2004) we have also noted that the feedback mechanism is very effective to smoothen the ‘negligible’ initial differences between the count of article-1 and article-2, (e.g.  $c_1(0) = c_2(0) + 1$ ). But, if the difference between  $c_1(0)$  and  $c_2(0)$ , is above some threshold (say  $c_1(0) > c_2(0)$ ), then article-1 will attain eventually monopoly as in the case with Top-N NRS.

## Trade-off between Accuracy and Distortion

Without FNRS, using only the Top-N and probabilistic alternatives the designer faces an accuracy-distortion tradeoff. If counts of articles can be considered as a quality surrogate, then the Top-N recommender has high “accuracy” since it only picks articles with highest counts. However this results in other articles receiving substantially fewer counts in the future. If we consider the initial share of counts of articles to be the natural share or preference of readers then any change from this state creates “distortion”. Therefore the Top-N recommender creates high distortion as well. The probabilistic recommender on the other hand maintains the share of articles, create negligible distortion, but since it often recommends articles that are not the “highest” counts the accuracy is lower.

Since the proposed FNRS here can operate in a broader continuum compared to the other methods it is particularly effective as a method to balance this tradeoff.

In this section we present a technique to balance accuracy and distortion based on the designer’s preference. But first, let us define the metrics used for this purpose:

**Accuracy-Loss.** At any given time, *accuracy loss* is defined for FNRS (for a given  $\gamma$ ) assuming as a benchmark the counts of articles that appear as recommendations in Top-N NRS (at the corresponding time when implemented in parallel). We define accuracy loss metric in the following way at time  $j$ :

$$accuracy\ loss(E_j) = \frac{1}{N} \ln \left( \frac{\sum_{i=1}^N C_{ij}^h}{\sum_{i=1}^N C_{ij}^p} \right) \quad (4)$$

Obviously for the definition of the accuracy loss metric we make an assumption that readers will have little or no interest in articles with ‘low’ counts.

In the equation-4,  $N$  represents, the number of articles appearing in Top-N (or probabilistic) “recommended” list.  $C_{ij}^h$  represents the count of  $i^{th}$  article, appearing in the Top-N (hard-cutoff) NRS, at the  $j^{th}$  time step. Whereas  $C_{ij}^p$  represents the count of  $i^{th}$  article appearing in the FNRS, at the  $j^{th}$  time step. Hence,  $\sum_{i=1}^N C_{ij}^h$  and  $\sum_{i=1}^N C_{ij}^p$  represent the sum of counts of all articles that appear in Top-N NRS and FNRS respectively, at the  $j^{th}$  time step. This metric has been averaged over the number of iterations, as the simulation progresses.

$$avg\ accuracy\ loss\ (\bar{E}_t) = \frac{1}{t} \sum_{j=1}^t \frac{1}{N} \ln \left( \frac{\sum_{i=1}^N C_{ij}^h}{\sum_{i=1}^N C_{ij}^p} \right) \quad (5)$$

It can be easily noted that the accuracy loss metric attains the least value when FNRS behaves like a Top-N NRS and it takes maximum value for the completely random selection of articles (for  $\gamma \geq 0$ ).

Here one issue is noteworthy that, we have measured  $\sum_{i=1}^N C_{ij}^h$  and  $\sum_{i=1}^N C_{ij}^p$ , at every time step in the simulation by implementing Top-N NRS and the corresponding FNRS (for a given  $\gamma$ ), in parallel.

**Distortion.** For distortion we use the *Kullback – Leibler* distortion measure (Kullback et al. 1951) averaged over the number of iterations ( $t$ ) in the simulation. We denote the probability distribution of articles in the system in presence of FNRS, at the time step  $j$  as  $q_j(x_i)$ . Then the mean *KL* distortion for the articles  $\{x_1, x_2, \dots, x_n\}$  is given by

$$avg\ distortion(\bar{KL}_t) = \frac{1}{t} \sum_{j=1}^t \sum_{i=1}^n p(x_i) \ln \left( \frac{p(x_i)}{q_j(x_i)} \right) \quad (6)$$

Where  $n$  is the total number of articles in the system. The above expression represents the inefficiency of the distribution  $q_j$ , when the true distribution of articles is given by  $p$  (given initially).

To fuse the above two criteria in the recommendation we define a new metric  $f(.)$  as follows:

$$f_t(\beta, \gamma, p) = \beta * \bar{E}_t + (1 - \beta) * \bar{KL}_t \quad (7)$$

$\bar{E}_t$  and  $\bar{KL}_t$  in the equation-7 represent *average accuracy loss* and *average Kullback Leibler distortion* respectively, as defined in equations 5 and 6. The parameter  $\beta \in [0,1]$  represents the designer’s preference between accuracy and distortion. Higher value of  $\beta$  means, that the accuracy of FNRS is more important than the distortion created on article counts (natural shares) by the presence FNRS; and vice-versa for the lower value of  $\beta$ . The particular value of  $\beta$  chosen for the present discussion is 0.5.

For the sake of brevity, we only have represented only those parameters in  $f_t(.)$  (equation-7) that are of interest for the discussion that follows.

Our objective is to minimize the metric  $f_t(.)$  obtained at the end of simulation and to determine the corresponding optimum exponent value  $\gamma^*$ , as mentioned in the Table-2  $t = 100,000$ .

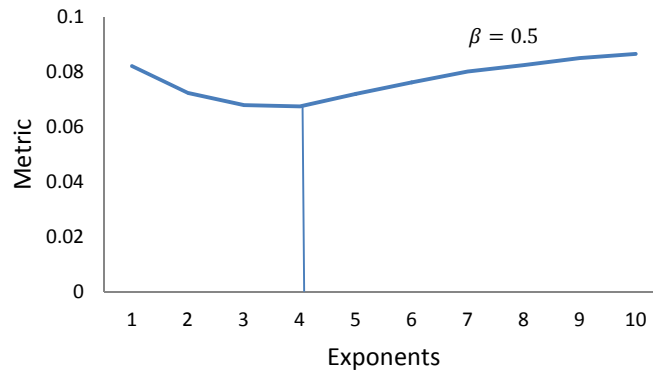


Figure 4: Metric v/s Feedback

For the simulation model considered in the present research, we obtain  $\gamma^*$  close to 4 (Figure-4). It is the

optimal choice of the feedback-exponent when accuracy loss and distortion in counts both are the important factors.

## NRS analysis via Two-Armed Bandit Problem

There has been growing attention towards creating a new framework for innovation in journalism (Phelps 2012). In which it has been argued that various computing concepts such as machine learning and computational modeling could apply to journalism. Also, in the panel discussion organized in the “ACM Conference on Recommender Systems-2011 (RecSys’11, Chicago)” there was consensus about promoting the use of machine learning techniques for diverse recommendations and the “Multi-Armed Bandit” algorithms in particular. In light of this growing recognition in this section we have tried to map the model of NRS through a ‘Two-Armed Bandit’ algorithm. Further, we show that the two-Armed bandit formulation of NRS can be considered as Pólya’s urn problem in a special case. Our attempt has been to provide researchers a platform to explore the application of these machine learning techniques in future research.

We follow the approach of Lamberton et al. (2004) closely, to present formulation of the two-armed bandit algorithm in the NRS context. The feedback mechanism applied in the context of ‘Two-Armed Bandit’ algorithm, differs slightly from the technique we have discussed earlier.

Consider a case with two articles  $A$  and  $B$ . At each time period one of them is selected to display. Ideally the implementer would like to allocate the recommended position to the most efficient article. But, he does not know which one it is. Simultaneously he also takes advantage of the performances of the best article as soon as possible. So he devises a periodic re-allocation procedure, of the ‘share’ of articles based on their periodic performances.

We model the recommendation process of articles as follows. We have presented the following discussion in the context of article  $A$ ; however similar analysis can be extended for the article  $B$ . Let  $X_t$  and  $1 - X_t$  be the shares of articles  $A$  and  $B$  during time  $t$  respectively. Every time unit the counts of articles are observed. If  $A$  has received the click in that period, then it is awarded an extra allocation of  $\gamma_{t+1}$  times the share of article  $B$ . So article  $A$  will have a share of  $X_t + \gamma_{t+1}(1 - X_t)$  in the next period for the recommendation. Similarly, if  $B$  has received the click at time  $t$ , it is awarded an extra allocation of  $\gamma_{t+1}$  times the share of article  $A$  at time  $t$  so that, at time  $t + 1$  share of article  $A$  will be reduced to  $X_{t+1} = X_t - \gamma_{t+1}X_t$ . Clearly, cases with  $\gamma_t = \{0,1\}$  are trivial. So, we consider cases when  $\gamma_t \in (0,1)$ .

The remaining setup of the NRS is same as discussed before, where a reader upon arrival reads the recommended article with probability  $p$  and the un-recommended article with probability  $1 - p$ . This assumption has been made for completeness. However, for a case of ‘influential’ recommender system the model with  $p = 1$  suffices for the analysis, as we expect  $p \sim 1$ . This property has been used later to make the model tractable.

Let us define the sequence of events when recommended and un-recommended articles are chosen by viewer as  $(R_t)_{t \geq 1}$  and  $(L_t)_{t \geq 1}$ , defined as follows:  $R_t = \{\text{recommended article being read at time } t\}$  and  $L_t = \{\text{un-recommended article being read at time } t\}$ .

To make an article appear as the recommended article at each time we toss a biased coin, so that the probability of article  $A$  and  $B$  being recommended is given by  $X_t$  and  $1 - X_t$  respectively. This biased coin can be generated through a sequence of random numbers  $(U_t)_{t \geq 1}; U_t \in [0,1]$  in the following manner:

$\{A \text{ is recommended at time } t\} = \{U_{t+1} \leq X_t\}$  and  $\{B \text{ is recommended at time } t\} = \{U_{t+1} > X_t\}$

$A$  will be allocated extra share of counts at time  $t$  if, (1)  $A$  being recommended and reader chooses to read the recommended article or (2)  $A$  is un-recommended and reader chooses to read the un-recommended article. Similarly,  $B$  will be allocated extra share at time  $t$  if, (1)  $B$  being recommended and reader chooses to read the recommended article or (2)  $B$  is un-recommended and the reader chooses to read the un-recommended article:

The formulation leads us to the following dynamics of  $X_t$ .

$$X_{t+1} = X_t + \gamma_{t+1}[(1 - X_t) * (1_{\{U_{t+1} \leq X_t\} \cap R_{t+1}} + 1_{\{U_{t+1} > X_t\} \cap L_{t+1}}) - X_t * (1_{\{U_{t+1} > X_t\} \cap R_{t+1}} + 1_{\{U_{t+1} \leq X_t\} \cap L_{t+1}})] \quad (8)$$

With  $X_0 = x \in [0,1]$

Where  $1_{\{U_{t+1} \leq X_t\}}$  is an indicator function defined as follows:

$$1_X = \begin{cases} 1, & \text{if } X \text{ holds} \\ 0, & \text{otherwise} \end{cases}$$

and  $(\gamma_t)_{t \geq 1}$  is a sequence of gain parameters such that  $\forall t \in \mathbb{N}^*, \gamma_t \in (0,1)$ .

Now, consider the case with  $p = 1$ , the case for an influential NRS with two articles (here  $A$  and  $B$ ). In this case, equation-8 will be given by,

$$X_{t+1} = X_t + \gamma_{t+1}[(1 - X_t) * (1_{\{U_{t+1} \leq X_t\}}) - X_t * (1_{\{U_{t+1} > X_t\}})] = X_t + \gamma_{t+1}(1_{\{U_{t+1} \leq X_t\}} - X_t)$$

Hence,

$$X_{t+1} = X_t + \gamma_{t+1}(1_{\{U_{t+1} \leq X_t\}} - X_t) \quad (9)$$

Now, we show that how in a special case the two-armed bandit formulation for NRS can be viewed as Pólya urn problem.

### **Bandit Formulation as a Pólya Urn Model**

In a similar setup as described earlier consider an urn with  $c_1(0)$  red balls and  $c_2(0)$  black balls at time  $t = 0$ . At every time  $t$  one ball is drawn at random from the urn and it is replaced back with another ball of the same color. Then the share of red balls at time  $t$  is given by,

$$X_t = \frac{c_1(t)}{c_1(0) + c_2(0) + t} ; t \geq 0$$

Let  $(U_t)_{t \geq 1}$  be the sequence of identically independently distributed ( *i. i. d.* ) random variables uniformly distributed over  $[0,1]$ . The model formulation as follows: if  $U_{t+1} \leq X_t$ , the ball drawn at time  $t + 1$  is red, otherwise it is black. Then we have:

$$c_1(t + 1) = c_1(t) + 1_{\{U_{t+1} \leq X_t\}}$$

Hence,

$$X_{t+1} = X_t + \frac{1}{c_1(0) + c_2(0) + t + 1} (1_{\{U_{t+1} \leq X_t\}} - X_t) \quad (10)$$

Comparing equations 9 and 10, it can be observed that Pólya's urn problem is a special case of NRS formulation with  $X_0 = \frac{c_1(0)}{c_1(0)+c_2(0)}$  and  $\gamma_t = \frac{1}{c_1(0)+c_2(0)+t}$ .

Having established the connection between two-armed bandit formulation and Pólya urn formulation, we now discuss some properties of the equation-9, which can be viewed as a stochastic approximation procedure (Lamberton et al. 2004). One natural question arises is the convergence behavior of the random variable  $X_t$ , which has been addressed in detail by Lamberton et al. (2004), for different conditions on the sequence of gain parameters  $(\gamma_{t+1})_{t \geq 1}$ . Further, the issue of rate of convergence of the proposed algorithm has been also analyzed.

To implement the two-armed bandit framework in the context of NRS, the natural approach will be to extend the algorithm for an  $n$  –article case. However, at present it remains an open question and we hope that future research will bring us more insights about the implementation of multi-armed bandit algorithms in news recommender systems. Currently, our proposed FNRS algorithm has the advantage in terms of easy implementation, interpretation, as well as for the flexibility it offers as noted earlier in this paper.

## **Conclusion**

The issue of diversity in information consumption has been well noted with changing habits of people toward news reading behavior. As the author of “The Information Diet” Clay A. Johnson; refers to “*healthy information diet as seeking out diversity, both in topic area and perspective*”. Wide implementation of recommenders such as the Top 10 list, often leads to less choice on the web. The

limitations of Top-N recommenders are getting increasingly recognized. As in a recent article New York Times writes, “Once at the very top of those iTunes charts, it takes a long time to fall off. And with good reason. Would you rather sift through 600,000 apps in the App Store or quickly browse the top 25 list?” (Bilton April 1, 2012).

We find similar issues with the Top-N NRS from our simulation, analytical and empirical work. To address the limitation of Top-N NRS, we have presented a novel solution in the context of news recommender systems through a class of feedback functions. One notable property of FNRS is the absence of “hard-cutoff” that creates artificial boundary amplification in the Top-N NRS. In our context, we have also established theoretical results based on urn formulation. In the simulation model a new metric has been introduced to allow implementers to control for the level of accuracy and distortion based on their preferences. We also presented the modeling of FNRS from the two-armed bandit frame work (albeit in a special case), where feedback is introduced through a gain parameter. We also believe analyses presented in the present research can be easily adopted by news web sites and it also makes fundamental theoretical contributions in the field of news recommender systems research.

The simple setting that we have provided for NRS can be extended in multiple ways. So far, we have not addressed issue of manipulation in FNRS- the problem often encountered with Top-N recommender systems (Weber December 19, 2010). In several cases it has been observed or suspected, that manipulators artificially inflate the counts of target article (or app) (Rusli et al. March 15, 2012). It would be interesting to examine the impact of manipulation resistant algorithms (e.g. Resnick et al. (2007)), in context of FNRS to control the feedback exponent in presence of manipulation. In our present ongoing research we are trying extend the analysis of FNRS in this direction to examine the impact of manipulation in FNRS.

It would also be interesting to compare the performances of FNRS and the multi-armed algorithm, based on different computational requirements. Further, the issue of similarity (if it exists) between feedback process in FNRS and Two-Armed Bandit algorithm remains an open question and we hope future research will bring more insights.

Finally, investigating the impact of feedback parameter ( $\gamma$ ) on consumer satisfaction and advertising revenue will also facilitate our understanding regarding the implementation of FNRS on news websites. User satisfaction may be measured in different ways. Some users may favor accuracy, while others may value novelty more. While hard to do in simulation, this can be measured through actual user surveys. Structuring Top-N lists for optimizing advertising revenue is another interesting direction. It is possible that a relatively unpopular article that has very high value for niche advertisers may provide more click-through revenues than a most popular article that is not as easily monetizable for an advertiser. These issues are promising to examine in future work.

It should be also noted that in the new paradigm that we are proposing for recommendations the “most popular” term will not hold. Instead, it has to be replaced by more appropriate term such as “popular” articles, since articles are being selected into lists probabilistically.

Increasingly both governments and policy think-tanks are paying closer attention to the news ecosystem – including how it is generated, curated and distributed. News recommender systems are increasingly common, and in a world, in which algorithms shape what news people read, studying their properties is important and ours is one of the first such work to the best of our knowledge.

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