THE EFFECT OF THIRD PARTY INVESTIGATION ON PAY-PER-CLICK ADVERTISING

Completed Research Paper

Min Chen

School of Management George Mason University 4400 University Drive, Enterprise Hall Fairfax, VA 22030, USA mchen15@gmu.edu

Suresh Radhakrishnan

Naveen Jindal School of Management University of Texas at Dallas 800 West Campbell Road Richardson, Texas 75080, USA sradhakr@utdallas.edu

Varghese S. Jacob

Naveen Jindal School of Management University of Texas at Dallas 800 West Campbell Road Richardson, Texas 75080, USA vjacob@utdallas.edu

Young U. Ryu

Naveen Jindal School of Management University of Texas at Dallas 800 West Campbell Road Richardson, Texas 75080, USA ryoung@utdallas.edu

Abstract

Click fraud is a critical problem in pay-per-click advertising. While both service providers (SPs) and advertisers employ technologies to identify fraudulent clicks, prior work shows that they cannot be induced to make further improvements to their respective technologies. We consider the use of third-party investigation to address this problem and examine whether the responsibility of investigation payments helps induce both parties to work towards improving their technologies unilaterally. Using a principal-agent setting, we show that the advertiser always has incentives to improve his verification technology and the SP will improve his detection technology only when the detection cost is not too large. Given that the cost of detection technology is likely to be small due to the use of inexpensive online filters, our result suggests that third-party investigation helps induce further enhancements to the technologies and is a good mechanism to address the incentive problems in the click fraud setting.

Keywords: Click fraud, double moral hazard, online advertising, incentives, game theory

Introduction

The past decade has witnessed extraordinary growth in the online advertising market. As the most popular payment model that accounts for more than 50% of the entire market (Hamner 2009), the payper-click model is widely accepted by both service providers (SP) and advertisers and is the primary source of income for many SPs. One primary advantage of this model is its simplicity and accountability: advertisers pay the SP only when someone clicks on the advertisement. However, because the pay-perclick model relies on the assumption that a person clicking on an ad has an interest in the advertised product or service, it is vulnerable to *click fraud*, a practice of imitating a legitimate user to click on an ad to generate a charge per click without having an actual interest in the target of the ad (Liu et al. 2009). Click Forensics (2010a) estimates the average click fraud rate to be 18.6% for the second quarter of 2010.

Both SPs and advertisers are taking actions against click fraud. Tuzhilin (2006) reports that SPs commonly employ online filters relying on click patterns to identify invalid/fraudulent clicks and charge the advertisers only for valid clicks. Many advertisers choose to examine the clicks classified as valid by the SP to verify that they are indeed valid and flag contestable clicks.¹ Currently, the contestable clicks are resolved by the SP with the help of advanced offline tools or human experts.² The advertiser pays for clicks that are identified as valid by both the advertiser and the SP.

Although both SPs and advertisers are concerned about click fraud, they do not disclose the methodologies used to identify invalid clicks for fear of the information falling into the wrong hands; as such, the qualities of the technologies in identifying invalid clicks are not directly observable and contractible. Since SPs are better off with more clicks and advertisers are worse off if many of the clicks are fraudulent/invalid, the incentives for click fraud identification are not aligned. It follows that the SP and the advertiser can choose a technology of low rather than high precision to identify invalid clicks.

The click fraud problem has drawn increasing attention in both academics and industry. The results from several recent research studies (e.g., Wilbur and Zhu 2009, Chen et al. 2012) suggest that pay-per-click industry would benefit from using other mechanisms, such as reputation and third party investigation, to deter click fraud. This is also in line with suggestions from many industry practitioners (see Arrington 2007, Hedger 2008, Megna 2008b, for examples).

In this paper, we focus on the third party investigation mechanism in particular and examine whether having a neutral third party to audit SPs' click fraud detection algorithms could help address the click fraud issue. In this setting, a click can be either valid or invalid, the SP and the advertiser have common knowledge of the probability that a click is valid but cannot observe the true nature of the click. Once a click occurs, three stages of identifying click fraud proceed in the following order: First, the SP classifies clicks using a click fraud *detection* system. Second, the clicks classified as valid by the SP's detection technology are verified by the advertiser using a *verification* technology. Finally, a neutral third party inspects the clicks using an *investigation* technology if there are disagreements between the classification results from the first two stages, and the outcomes from the investigation are considered binding to both parties. The advertiser pays for the clicks that are jointly identified as valid in all of the three stages.

The classifications by the detection, verification, and investigation technologies are imperfect: they do not identify all invalid clicks and may incorrectly classify some valid clicks as invalid (Kshetri 2010). We use the term "precision" to measure the quality of the technologies, where the precision of a technology is defined as the probability that a valid click is correctly identified as valid by the technology. The SP and advertiser can choose either a high-precision or a low-precision technology; as such, high-precision technologies are also more costly. The SP and the advertiser choose the high- or low-precision detection and verification technology based on the payment-per-click (PPC) to maximize their own expected profit. The choice of the high- or low-precision technology is not jointly observable and

¹ Many firms use click fraud auditing firms to help verify the clicks classified by the SP, see Click Forensics (2010b) and Megna (2008a) for examples.

² Google and Yahoo, for example, accept and investigate electronic traffic quality reports from Click Forensics on behalf of its advertisers (Olsen 2008; Perez 2008).

contractible which leads to a double moral hazard problem. On the other hand, the quality/precision of the neutral third-party's investigation technology is observable and known to both parties. We model the problem in a principal-agent setting with the advertiser as the principal and the SP as the agent.

In this paper, we consider a "loser-pay" scheme in which the party who loses the dispute pays for investigation cost. In other words, the SP pays for the investigation cost if a contestable click is classified as invalid by the third party; otherwise, the advertiser pays for investigation cost. The probability that a click is paid for, i.e., a click is identified by all the three technologies as valid, is higher with high-precision than low-precision detection technology and is lower with high-precision verification technology. Similarly, the probability that the SP (advertiser) loses the dispute and pays for the third party's investigation is lower with high-precision detection (verification) technology. Accordingly, both the SP and the advertiser have benefits of choosing the current high-precision technology over the low-precision technology, and they balance their choice with the cost and benefits from choosing the technology.

We show that either party's incentive problem can be more severe. Particularly, if the PPC is obtained from the SP's incentive compatibility constraint we refer to this as the SP's incentive problem being more severe than the advertiser's incentive problem. On the other hand, if the PPC is obtained from the advertiser's incentive constraint we refer to this as the advertiser's incentive problem being more severe. To keep the intuition straightforward, the SP's incentive problem is more severe if the cost of his high-precision detection technology is sufficiently large. Consequently, the PPC needs to be high enough to induce the SP to choose the high-precision detection technology. On the other hand, the advertiser's incentive problem is more severe if the cost of SP's detection technology is not very large. In this case, the PPC induces the advertiser's choice of the high-precision instead of low-precision verification technology, and the SP's high-precision detection technology can be automatically induced, which is a new result in double moral hazard models and arises because we consider a sequential action setting (see Hwang et al. 2006, Jayanth et al. 2011).

An important question on click fraud is whether the PPC model alone is sufficient to induce both SP and advertiser to *unilaterally* make improvements to their respective click fraud identification technologies. This is a vital question because if the answer is affirmative, then we can conclude that the two parties will work towards improving their technologies despite the double moral hazard problem. However, it has been shown that the PPC alone cannot induce unilateral improvements and other mechanisms are needed to help address the click fraud issue (Chen et al. 2012). Thus, a critical question addressed in this paper is when allowing for third party investigation in the PPC model, whether the responsibility of payments to the third party can induce the SP and the advertiser to *unilaterally* make improvements to the detection and verification technologies?³ In analyzing this question, we consider only improvements in a technology without a corresponding cost.⁴

Starting from the advertiser's verification technology, we show that the advertiser always has an incentive to improve verification technology because doing so increases advertiser's profit. This happens because as the high-precision verification technology improves, the advertiser has more incentive to choose it to reduce the probability that he pays for a click to the SP and the probability that he loses the dispute and pays for third party investigation. This leads to a lower PPC being sufficient to induce the high-precision technologies. As a result, the advertiser's profit increases as he further improves the high-precision verification technology.

We then examine whether the SP will have incentives to further improve the high-precision detection technology. We find that when the cost of SP's high-precision detection technology is not very large, i.e., advertiser's incentive problem is more severe and the PPC needs to be high enough to induce advertiser's high-precision verification technology, improving the detection technology increases the SP's expected

³ By effecting improvements to their respective high-precision technologies unilaterally, we imply that improvement in the high-precision technology increases their own expected profit.

⁴ In most cases, improvements in technology can occur without any additional cost. As such, this is a reasonable assumption. More importantly, if the two parties do not find it beneficial to improve their respective technologies even without a corresponding increase in costs, considering increase in costs will make the SP and advertiser not choose improvements more intensely.

profit. This happens because as the high-precision detection technology improves, the advertiser's benefits from choosing the high-precision verification technology decrease. Intuitively, when the SP does a good job in catching fraudulent clicks, the advertiser need not work hard (or can work less hard) with the verification technology. As a result, the advertiser has less incentive to choose the high-precision verification technology, i.e., the SP commands more rents due to a higher PPC. As such, the SP has an incentive to improve the detection technology in this case.

Nevertheless, we find that when the cost of SP's high-precision detection is sufficiently large, i.e., the SP's incentive problem is more severe, improving SP's detection technology will decrease the SP's expected profit. This happens because as the high-precision detection technology improves, the benefits from choosing it to increase SP's probability of receiving a PPC from the advertiser and to decrease SP's probability of paying for third party investigation also increase. As a result, a lower PPC is sufficient to induce SP to choose the improved high-precision detection technology and this decreases SP's profits.

Overall, we find when a third party handles inspection of contestable clicks, the advertiser would always have incentives to make further improvement to the high-precision verification technology while the SP gains higher profits from improving the detection technology only when SP's detection technology is not very costly. Given that the cost of detection technology is likely to be small due to the use of inexpensive and automated online filters, our results suggest that third party investigation indeed can help address the incentive problem in the click fraud setting.

We then examine the social welfare to benchmark whether improvement in technologies will be beneficial to the agency. We show that the social welfare increases with improvement in SP's detection technology only when the advertiser's verification precision is sufficiently high. Similarly, improving advertiser's verification would increase the social welfare only when the SP's detection precision is sufficiently high. The insight from the social welfare analysis along with the earlier result leads to the following insight: although the social welfare could be hurt in the beginning when the detection or verification precisions are not quite high, it will increase later once the two precisions are improved. As a result, third party investigation not only helps mitigate the agency problem by providing both parties with incentives to further improve their technologies but also increases the social welfare in the long run.

Related Literature

The issue of click fraud has drawn increasing attention in the recent years. Wilbur and Zhu (2009) analyze the effects of click fraud on the online advertising industry. They show that when advertisers know the level of click fraud, they will lower their bids to the point that click fraud has no impact on total advertising expenditures. Nevertheless, when the level of click fraud is uncertain, the SPs' revenues will rise when the keyword auction is less competitive and fall when the keyword auction is more competitive. Their result that SPs are sometimes helped and sometimes hurt by click fraud reinforces the need for a neutral third party to audit SPs' click fraud detection algorithms. Chen et al. (2012) examine the click fraud problem in a setting where the investigation is handled by SP and the quality of investigation is not observable to the advertiser. They find that when the cost of high-precision investigation is sufficiently high, even though both parties may be better off with improvements in both the detection and verification technologies, they will not unilaterally do so: a classic prisoner's dilemma result. Their results also suggest that other mechanisms such as third party investigation are needed to help mitigate the incentive problem in click fraud setting. However, the reasons for such implication are quite different from that from Wilbur and Zhu (2009): specifically, Chen et al. (2012) find that interaction among the detection, verification and investigation technologies makes it difficult for the PPC to induce both parties to make further improvements in their technologies.

Other works focus on the use of alternative payment methods to mitigate the click fraud problem. Mahdian and Tomak (2009) propose the pay-per-action (PPA) model. They examine the challenges in the design of incentive-compatible PPA mechanisms and suggest approaches to tackle some of them. The PPA model, however, creates an incentive for advertisers not to report all conversions occurred on their Web sites and hence produces a new fraudulent activity known as *action* fraud. Goodman (2005) proposes a pricing scheme that sells advertisers a particular percentage of all impressions rather than user clicks. Nevertheless, the per-per-percentage of impressions model, apart from being difficult to implement,

deviates from the developed industry standard that sells clicks, and risks a negative backlash in the marketplace (Immorlica et al. 2005).

This research also contributes to the literature of double moral hazard problems which have been examined extensively in economics and have been applied in a variety of settings. Balachandran and Radhakrishnan (2005) examine a double moral hazard case where both the supplier's and the buyer's qualities are unobservable. They consider not only whether contract payments and penalties based on either information from incoming inspection or information from external failures induces first-best quality, but also whether the penalty satisfies the fairness criterion. Hwang et al. (2006) examine a two-tier supply chain between a supplier and a buyer, with the supplier's quality and the buyer's inspection effort being unobservable. They compare the inspection regime with the certification regime and show that inspection leads to additional agency cost due to the presence of agency problems, which provides a rationale for the shift to the certification regime even though the direct cost of inspection is low.

In contrast to these studies we consider the interaction between the technologies employed for classifying clicks and extend previous research on double moral hazard to the click fraud setting by examining a sequential action setting. While Chen et al. (2012) consider a double moral hazard with three sequential actions similar to our setting, they do not consider a third party. Our results, therefore, highlight the impact of both the sequential actions and the presence of a third party investigator. The inclusion of a third party in this paper helps to provide insights into whether third party inspection along with the loser pay scheme can be a useful mechanism to mitigate the agency problems in click fraud setting, not only directly, but also by providing incentives for the SP and the advertiser to improve their high-precision technologies.

Model Description

We examine a double moral hazard problem between a risk-neutral advertiser and a risk-neutral service provider (SP), as shown in Figure 1. The advertiser's sponsored link receives x clicks enabled by the SP. Without loss of generality, we set x=1. A click could either be valid (non-fraudulent) or invalid (fraudulent). The information on whether a click is fraudulent is not observable to the advertiser or the SP. Both the advertiser and the SP assess a probability of α that the click is valid, i.e., with probability $(1-\alpha)$ the advertiser and the SP expect the click to be fraudulent.

The SP uses a detection technology that classifies a valid click as valid with probability q^S and an invalid click as invalid with probability $\phi^S q^S$, where $\phi^S \epsilon (0, 1/q^S)$.⁵ Correspondingly, the detection technology's type I error rate of classifying a valid click as invalid is $(1 - q^S)$, and the type II error rate of classifying an invalid click as valid is $(1 - \phi^S q^S)$. We refer to q^S as the precision of the detection technology because increasing q^S would result in a decrease in both type I and type II errors. The report of the classification by the detection technology is denoted $r^S = v$, f for valid and invalid/fraudulent clicks, respectively. The SP can choose a detection technology with either a high precision or low precision, i.e., $q^S \in \{q_H^S, q_L^S\}$ with $q_H^S > q_L^S$. The corresponding *technology procurement* cost of the high-precision and low-precision detection technology is $C^S(q^S) \in \{C_H^S, C_L^S\}$ with $C_H^S > C_L^S$. The choice of the detection technology is not observable and thus, is subject to moral hazard. In addition, there is a *unit cost of classifying clicks*, denoted by m^S , which captures the cost of performing classifications of clicks and is a variable cost. We assume that the unit cost is the same irrespective of whether the high or the low precision detection technology is chosen by the SP.

The clicks that are classified as valid by the SP are verified by the advertiser.⁶ The advertiser's verification

⁵ The SP typically uses large amounts of click stream data from many advertisers and for many keywords to develop algorithms to identify click fraud (Greenberg 2008). Besides advertisers' log data, SPs collect other data that can be used to assist in click fraud detection in a variety of ways, including JavaScript page tags (ClickFacts 2010), URL redirect service and real-time API (Click Forensics 2010c) and proprietary server-side data collection method (Clicklab 2006).

⁶ While some information, e.g., the IP addresses of clicks, is available to both the advertiser and SP, the advertisers have page-visit data after a visitor is directed to his Web site from the SP (Greenberg 2008). This is used by advertisers to provide the advertiser with more precise information to classify clicks.

E-Business and Competitive Strategy

technology classifies a truly valid click that is classified as valid by the SP, as valid with probability q^A . Also, the verification technology classifies a truly invalid click that is classified as valid by the SP, as invalid with probability $\phi^A q^A$, where $\phi^A \epsilon (0, 1/q^A)$. The precision of the verification technology is characterized by q^A . The report of the classification by the verification technology is denoted $r^A = v, f$, for valid and invalid/fraudulent clicks, respectively. The advertiser can choose a verification technology with either high or low precision, i.e., $q^A \in \{q^A_H, q^L_A\}$, where $q^A_H (q^L_L)$ denotes the precision of high- (low-) precision technology with corresponding procurement cost for verification technology $C^A \in \{C^A_H, C^A_L\}$, where $q^A_H > q^A_L$ and $C^A_H > C^A_L$. The advertiser's choice of verification technology is not observable and thus, is subject to moral hazard as well. In addition, there is a unit cost m^A for verifying each click that the SP classifies as valid. This is a variable cost that incurs only when verification technology classifies a click. We assume that the unit cost is the same irrespective of whether the high- or the low-precision verification technology is chosen by the advertiser. The probability that a click is verified by the advertiser is denoted by $I^A_i = \Pr(r^S_i = v)$ where the subscript *i* denotes that the report is conditioned on q^S_i the detection technology. Without loss of generality we assume that $C^S_L = C^A_L = 0$.



The disagreements in the SP's and advertiser's classification of invalid clicks are resolved by a neutral third party carefully investigating such disagreements. The investigation technology used by this third party correctly classifies a truly valid contestable click as valid with probability q^I , and a truly invalid contestable click as invalid with probability $\phi^I q^I$, where $\phi^I \epsilon (0, 1/q^I)$. The report of the classification by the investigation technology is denoted $r^I = v, f$ for valid and invalid/fraudulent clicks, respectively. The precision of the investigation technology used by the third party, i.e., q^I , unlike the detection precision

and verification precision which are not observable or contractible, is common knowledge to both players. The third party charges a unit cost m^{l} for each click investigated, i.e., when a click is classified as valid by the SP but as invalid by the advertiser.

The advertiser pays the SP θ for each click classified jointly by all the three stages as valid. This is equivalent to the advertiser paying θ for each click classified by the SP as valid, and then obtaining a refund of θ for clicks classified as invalid by the verification and investigation technologies. The sequence of events unfolds as follows. First, the advertiser and the SP agree on payment per click (PPC) θ . Second, the SP chooses the detection technology and the advertiser chooses the verification technology. Finally, the clicks occur, the technologies classify the clicks and the payment occurs when the clicks are classified as valid by the detection and verification technologies or the detection and the investigation technologies. We assume that the classification obtained from the technologies cannot be manipulated by the SP or the advertiser. In effect, the choice of the technologies, i.e., the program/code and the team who are dedicated to examine/monitor the clicks, constitutes the processes that spew out the classification. The events are depicted in Figure 1.

We denote by T_{ij} the probability that a click is either classified as valid by the detection and verification technologies or is inspected as valid by the investigation technology, where the subscripts $i, j \in \{H, L\}$ indicate the high- or low-precision detection and verification technologies chosen by the two parties. As such, T_{ij} is the probability that a click is paid for (with Pr denoting probability) and is given by:

$$T_{ij} = \Pr[a \text{ click is paid} | q_i^S, q_j^A, q^I]$$

= $\Pr[r_i^S = v, r_{ij}^A = v] + \Pr[r_i^S = v, r_{ij}^A = f, r^I = v]$
= $\alpha q_i^S [q^I + q_j^A (1 - q^I)] + (1 - \alpha)(1 - \phi^S q_i^S)(1 - \phi^A q_j^A \phi^I q^I).$

The advertiser expects to receive a benefit of γ from a valid click: the expected benefit of γ includes not only the revenues obtained from the customer who has clicked through to his site, but also the probability that the valid customer's visit to the advertiser's site results in a sale. Thus given that not all truly valid customers may buy the advertiser's product/services, the outcome of the sales cannot be used to provide information on truly valid clicks. The expected profit for the advertiser (*U*) for *i*, *j* \in {*H*,*L*} is given by

$$U_{ij} = \gamma \alpha - \theta T_{ij} - C_j^A - m^A I_i^A - m^I I_{ij}^{IA}.$$

 U_{ij} is the advertiser's expected benefit from the click, less (a) the expected payment to the SP, (b) the cost of verification technology, (c) the expected cost of verifying the click classified by the detection technology as valid, and (d) the advertiser's expected investigation cost. We assume that the expected benefit the advertiser receives from a click is high enough such that he has incentives to participate in PPC advertising. The advertiser's investigation payment probability, i.e., the probability that a contestable click is classified by the third party's investigation technology to be valid, is given by:

$$I_{ij}^{IA} = \alpha q_i^S (1 - q_j^A) q^I + (1 - \alpha) (1 - \phi^S q_i^S) (1 - \phi^I q^I) \phi^A q_j^A.$$

The expected profit for the SP (*V*) for $i, j \in \{H, L\}$ is given by

$$V_{ii} = \theta T_{ii} - C_i^S - m^S - m^I I_{ii}^{IS}.$$

 V_{ij} is the SP's expected click payment from the advertiser less (a) the cost of employing the detection technology, (b) the expected cost of performing detection, and (c) the SP's expected investigation cost. The SP is responsible for the investigation cost when a contestable click is deemed invalid by the investigation technology. The probability that SP pays for investigation cost is given by:

$$I_{ij}^{IS} = \alpha q_i^S (1 - q_j^A) (1 - q^I) + (1 - \alpha) (1 - \phi^S q_i^S) \phi^A q_j^A \phi^I q^I.$$

The detection and verification technologies use automated programs, so that unit costs of conducting detection and verification are likely to be very small. Hence, without loss of generality, we let $m^S = m^A = 0$ in our analysis.

We make some assumptions to ensure that the representation of the problem conforms to the standard

double moral hazard settings. First, we assume that the PPC payment probability (T_{ij}) and the investigation payment probability for both parties $(I_{ij}^{IS} \text{ and } I_{ij}^{IA})$ satisfy AI below.

AI. (a) $T_{Hj} > T_{Lj}$, (b) $T_{iL} > T_{iH}$, (c) $I_{Hj}^{IS} < I_{Lj}^{IS}$, and (d) $I_{iH}^{IA} < I_{iL}^{IA}$ for $i, j \in \{H, L\}$.

Assumption AI(a) states that compared to the low-precision detection technology, the high-precision detection technology results in an increase in probability that a click is paid for, i.e., $[dT/dq^S] > 0$. This will provide the SP the motivation to choose high-precision detection technology. Assumption AI(b) states that compared to the low-precision verification technology, the high-precision verification technology decreases the probability that the click is paid for, i.e., $[dT/dq^A] < 0$. This will provide the motivation for the advertiser to choose the high-precision verification technology over the low-precision verification technology. Assumptions AI(c) states that choosing the high-precision detection technology reduces the SP's probability of losing the dispute, i.e., $[dI^{IS}/dq^S] < 0$. Similarly, AI(d) states that the advertiser has a lower probability of losing the dispute if he chooses the high-precision verification technology, i.e., $[dI^{IA}/dq^A] < 0$. Overall, AI is similar to the assumption in the standard agency problems where high action is more productive than low action.

Table 1 provides a glossary of notations used in this paper. We now proceed to analyze the model.

Table 1: Summary of Notations	
Notation	Definition
α	The probability that a click is valid, i.e., the valid click rate
γ	The advertiser's expected profit from a valid click
θ	The advertiser's PPC to the SP
q_i^S	The precision of the SP's detection technology $i \in \{H, L\}$
q_j^A	The precision of the advertiser's verification technology $j \in \{H, L\}$
q^I	The precision of the third party's investigation technology
ϕ^{s}	The ratio of true positive to true negative for the SP's detection technology
$\phi^{\scriptscriptstyle A}$	The ratio of true positive to true negative for the advertiser's verification technology
ϕ^{I}	The ratio of true positive to true negative for the neutral auditor's investigation technology
C_i^S	The SP's detection cost when choosing detection technology $i \in \{H, L\}$
C_j^A	The advertiser's verification cost when choosing verification technology $j \in \{H, L\}$
m^S	The unit cost of classifying a click by SP's detection technology
m^A	The unit cost of classifying a click by advertiser's verification technology
m^{I}	The unit cost of classifying a click by neutral auditor's investigation technology
I_{ij}^{IA}	The probability that the investigation cost is paid by the advertiser
I_{ij}^{IS}	The probability that the investigation cost is paid by the SP
T _{ij}	The probability that a click is paid to the SP

Analysis

We examine the case where the PPC induces the choice of high-precision detection and verification technologies from the SP and the advertiser, respectively. The optimization problem is provided in Program 1.

Program 1

subject to
$$\begin{aligned} \max_{\theta} U_{HH} & (OBJ) \\ V_{HH} \geq \overline{V} = 0 & (PCS) \\ V_{HH} \geq V_{LH} & (ICS) \\ U_{HH} \geq U_{HL} & (ICA) \end{aligned}$$

Program 1 is a standard double moral hazard model (see Demski et al., 2004; Hwang et al., 2006). The advertiser's expected profit function is given in (OBJ). The participation constraint (PCS) ensures that the SP receives at least the reservation profit which is set to zero. Constraint (ICS) is the incentive compatibility constraint for the choice of high-precision detection by the SP. Constraint (ICA) is the incentive compatibility constraint with respect to the advertiser's verification technology. We make an assumption on the unit cost of investigation.

AII. $m^{I} < C_{H}^{S}T_{LH}/(T_{HH}I_{LH}^{IS} - T_{LH}I_{HH}^{IS}).$

Assumption AII states the cost of investigating a click is not too large. This ensures that the incentive problem of inducing SP's high-precision detection technology is sufficiently severe and that the solution to Program 1 is not determined by the individual rationality constraint (PCS). Thus, the solution to Program 1 when AII is violated is to pay the SP the cost of employing the high-precision detection technology and the expected cost of third party investigation. Of course, this solution will not have any agency cost and thus incentive design problem is moot. Hence, we do not consider this solution.

Results

We characterize the solution to the Program 1 in the proposition below. Our analysis focuses on the parameter region where the advertiser and the SP are induced to choose their respective high-precision technologies. All proofs are presented in the Appendix.

Proposition 1

When the cost of high-precision detection technology is sufficiently large (small), then the incentive problem of inducing the high-precision detection technology is more (less) severe than the incentive problem of inducing the high-precision verification technology. Technically, if condition C1 is satisfied, constraint (ICS) is binding, and if condition C1 is not satisfied, constraint (ICA) is binding; and in particular,

- 1. if C1 is satisfied, the solution is $\theta^* = [C_H^S - m^I (I_{LH}^{IS} - I_{HH}^{IS})] / (T_{HH} - T_{LH}), U^* = \gamma \alpha - \theta^* T_{HH} - C_H^A - m^I I_{HH}^{IA},$ $V^* = \theta^* T_{HH} - C_H^S - m^I I_{HH}^{IS}.$
- 2. If C1 is not satisfied, the solution is $\theta^{**} = [C_H^A - m^I (I_{HL}^{IA} - I_{HH}^{IA})] / (T_{HL} - T_{HH}), U^{**} = \gamma \alpha - \theta^{**} T_{HH} - C_H^A - m^I I_{HH}^{IA},$ $V^{**} = \theta^{**} T_{HH} - C_H^S - m^I I_{HH}^{IS}.$

where condition C1 is given by

$$[C_{H}^{S} - m^{I}(I_{LH}^{IS} - I_{HH}^{IS})] / [C_{H}^{A} - m^{I}(I_{HL}^{IA} - I_{HH}^{IA})] > [\{T_{HH} - T_{LH}\} / \{T_{HL} - T_{HH}\}].$$

Proposition 1 characterizes the solution to the Program 1. It shows that either the advertiser's incentive problem or the SP's incentive problem can dictate the PPC, as determined by the condition C1. This is standard in binary action double moral hazard problems, i.e., one parties' agency problem is more severe than the other (see Jayanth et al. 2011; Arya et al. 2007). Particularly, if the PPC is obtained from the SP's incentive compatibility constraint we refer to this as the SP's incentive problem being more severe than the advertiser's incentive problem. On the other hand, if the PPC is obtained from the advertiser's incentive constraint we refer to this as the advertiser's incentive problem being more severe.

There are two drivers that determine the severity of the incentive problem. The first is the cost part: if the cost of the high-precision detection technology is sufficiently large, then the SP's incentive problem is likely to be more severe. The second is the technology part: the choice of the high or low precision technologies will influence both the PPC payment and investigation cost. In particular, we define *the effectiveness of a technology on PPC* as the increase in the probability that a click is paid for when SP (advertiser) chooses the high- (low-) instead of the low- (high-) precision technologies. Further, we define *the effectiveness of a technology on investigation* as the decrease in the probability that the SP (advertiser) pays for the investigation cost of a click when the high- instead of the low- precision detection (verification) technology is chosen. If the effectiveness of the detection (verification) technology on PPC and investigation, i.e., $[T_{HH} - T_{LH}]$ and $[I_{LH}^{LS} - I_{HH}^{LS}]$ ($[T_{HL} - T_{HH}]$ and $[I_{HL}^{LA} - I_{HH}^{LA}]$), are sufficiently large, then the SP (advertiser) will have more benefits from choosing the high-precision technology and hence SP's (advertiser's) incentive problem is likely to be less severe.

To keep the intuition straightforward, we focus our explanation on the cost part: if the cost of highprecision detection technology is sufficiently large then C1 is satisfied and the incentive problem of inducing the SP's high-precision detection technology is more severe; and vice versa. The advertiser should consider this incentive problem when considering his auction bids: if the bids are not incentive compatible in the sense represented in Program 1, then the SP may not have an incentive to detect frauds.

Proposition 1 is a standard solution and provides a benchmark of the characteristics of the PPC that incentivizes the SP and the advertiser to choose the high-precision technologies. If the PPC is not incentive compatible then even the choice of high-precision technology is infeasible and thus the question of improvements to high-precision technologies is a moot question. We now proceed to gain insights into whether the SP and the advertiser have incentives to improve their respective high-precision technologies.

Who has incentives to improve technologies? Some insights from comparative statics analyses

In this section, we examine how the profits of the advertiser and the SP change when each of the highprecision technologies is improved. The idea behind the analysis is to gain insights into improvement in which technology can help increase the profits of the advertiser and the SP. Will the SP improve the detection technology and improve his profits? Similarly, will the advertiser improve the verification technology and improve his profits? If both or any of these occur then each party will have an incentive to make improvements to their respective technologies. For this purpose, we allow for the costs of the technologies to be constant.

Before proceeding with examining this question we examine how improvements in the precision of one party's technology affect the effectiveness of the technology controlled by the other party. The set of observations below will help disentangle the interaction between improvements in precision and effectiveness and will be useful to understand the force/ intuition for the later propositions.

Observation 1

(a) The effectiveness of SP's detection technology on PPC, i.e., the difference in the probability that a click is paid for across high-precision and low-precision detection technologies, increases with improvements in the precision of verification technology. Technically, $\left[d[T_{Hj} - T_{Lj}]/dq_i^A\right] > 0$.

(b) The effectiveness of advertiser's verification technology on PPC, i.e., the difference in the probability that a click is paid for across low-precision and high-precision verification technologies, decreases with improvements in the precision of detection technology. Technically, $[d[T_{iL} - T_{iH}]/dq_i^S] < 0$.

Observation 1(a) shows that the effectiveness of the detection technology on PPC increases with increased precision of verification technology. This is because with improved verification technology, a valid click that is correctly classified by the detection technology is more likely to be correctly verified by the advertiser. Hence, this increases the SP's benefits of choosing the high-precision detection technology. Intuitively, if the advertiser chooses the high-precision verification technology, this will help induce the SP to choose the high-precision detection technology as well. This provides an insight into the question of why the advertiser would want to choose the high-precision verification technology. In other words, one benefit of the advertiser's choosing the high-precision verification technology is that the SP's benefit from

choosing the high-precision detection technology is also higher.

Observation 1(b) shows that the effectiveness of advertiser's verification technology on PPC decreases with increases in the precision of detection technology. This occurs because a contestable click is more likely to be correctly classified with a higher detection precision and this decreases the advertiser's benefits from choosing the high-precision verification technology. Therefore, improving detection technology helps exacerbate the advertiser's incentive problem.

Observation 2

(a) The effectiveness of SP's detection technology on investigation, i.e., the difference in the probability that the SP pays for third party's investigation of a click across low-precision and high-precision detection technologies, increases with improvements in the precision of verification technology. Technically, $\left[d[I_{Li}^{IS} - I_{Hi}^{IS}]/dq_{i}^{A}\right] > 0$.

(b) The effectiveness of advertiser's verification technology on investigation, i.e., the difference in the probability that the advertiser pays for third party's investigation of a click across low-precision and high-precision verification technologies, increases with improvements in the precision of detection technology. Technically, $[d[I_{iL}^{IA} - I_{iH}^{IA}]/dq_i^S] > 0$.

Observation 2(a) shows that as the verification precision improves, the effectiveness of SP's detection technology on investigation also increases. This occurs because with a better verification technology, a contestable click is more likely to be an invalid click that is misclassified as valid by the SP's detection technology, and hence the SP will likely have to pay for the investigation cost. As a result, the SP would have more incentives to choose the high-precision detection technology to reduce his misclassification rate, which in turn decreases his chances of losing the dispute and paying for investigation cost. Similarly, Observation 2(b) shows that as SP's detection technology improves, the advertiser's benefit from choosing the high-precision verification technology to reduce his probability of paying for third-party investigation also increases.

Does advertiser have incentives to improve verification technology?

We then examine the question of whether the PPC model provides adequate incentives for the SP and the advertiser to improve their respective high-precision detection and verification technologies when a third party handles investigation. We first examine the impact of improving advertiser's verification technology. The advertiser can improve the precision of verification technology by collecting more data from sources other than that on the advertiser's Web site and then conducting experiments on the data to tune parameters for the verification technology. The result is summarized in the following proposition.

Proposition 2. As advertiser's high-precision verification technology improves, (a) the PPC (θ) decreases, (b) the advertiser's profit (U) increases. Technically, (a) $d\theta^*/dq_H^A < 0$ and $d\theta^{**}/dq_H^A < 0$, (b) $dU^*/dq_H^A > 0$ and $dU^{**}/dq_H^A > 0$.

Proposition 2 shows that irrespective of whose incentive problem is more severe, i.e., whether condition C1 is satisfied or not, the advertiser will have incentives to further improve the high-precision verification technology. First, consider the case where the cost of SP's detection technology is not very high, i.e., the advertiser's incentive problem is more severe and the PPC is sufficiently high to induce advertiser's high-precision verification technology. The advertiser can increase his expected profit from improving the high-precision verification technology. This happens because a lower PPC is sufficient to induce the improved high-precision verification technology, i.e., the SP commands less rents due to a lower PPC. This is a standard result applicable to any moral hazard problem, in the sense that the effectiveness of the verification technology on PPC and on investigation is higher as its precision increases, and thus, less incentive is needed to induce the advertiser to choose the high-precision verification technology.

More interestingly, even when the cost of the detection technology is sufficiently large, i.e., the SP's incentive problem is more severe, the advertiser can still increase his expected profit from improving the high-precision verification technology. Note that in this case the PPC needs to be high enough to induce SP's high-precision detection technology. Thus, the high-precision verification technology is obtained for "free", and it could appear that improvements in verification may not benefit the advertiser. However, the seemingly counterintuitive result occurs because of the interaction between the high-precision detection

and verification technologies with respect to the probability that a click is paid for and the probability that the advertiser pays for investigation. Specifically, the laws of probability dictate that improvements in the verification technology make SP's detection technology more effective (see Observation 1(a) and 2(a)). Loosely speaking, when the SP knows that verification technology is going to yield more precise classifications, he will have more incentives to use the high-precision detection technology. As such, a lower PPC is sufficient to induce the high-precision detection technology when the high-precision verification technology is improved. Thus, overall the advertiser will have an incentive to further improve the high-precision verification technology.

Does SP have incentives to improve detection technology?

We now turn to examine the impact of improving SP's detection technology. The SP can improve the detection technology by incorporating more techniques into click fraud detection⁷ or by actively monitoring new click fraud activities and developing online filters to detect them. The result is summarized in the following proposition.

Proposition 3. (i) When the SP's incentive problem is more severe, i.e., the cost of high-precision detection technology is sufficiently large, as the high-precision detection technology improves, (a) the PPC (θ) decreases, (b) the SP's profit (V) decreases. Technically, (a) $d\theta^*/dq_H^S < 0$, (b) $dV^*/dq_H^S < 0$.

(ii) When the advertiser's incentive problem is more severe, i.e., the cost of high-precision detection technology is sufficiently small, as the high-precision detection technology improves, (a) the PPC (θ) increases, (b) the SP's profit (V) increases. Technically, (a) $d\theta^{**}/dq_H^S > 0$, (b) $dV^{**}/dq_H^S > 0$.

Proposition 3(i) shows that when SP's high-precision detection technology is sufficiently costly, PPC decreases with improvements in the detection technology. Note that in this case, PPC needs to be high enough to induce SP's high-precision detection technology, i.e., the solution is characterized in Proposition 1.1. As the detection technology improves, the effectiveness of the detection technology on PPC and on investigation also increases, and thus less incentive is needed to induce SP's choice of the high-precision detection technology. As a result, PPC decreases, and corresponding to the decrease in PPC, the SP's profit also decreases.

Proposition 3(ii) shows that when the cost of SP's high-precision detection technology is sufficiently small, the advertiser's payment increases as the SP further improves the high-precision detection technology. In this case, the advertiser's incentive problem is more severe and hence the PPC needs to be sufficiently high to induce advertiser's choice of high-precision verification technology. Improvement in high-precision detection technology has two effects on the advertiser. First, it decreases the effectiveness of the advertiser's verification technology on PPC (Observation 1(b)), and this helps exacerbate advertiser's incentive problem. Second, it increases the effectiveness of verification technology on investigation (Observation 2(b)), and this helps alleviate advertiser's incentive problem. The first effect dominates the second one and overall the PPC needs to increase to induce advertiser's choice of high-precision verification. Corresponding to the increase in PPC, the SP's profits increase with improvements in the high-precision detection technology. Therefore, SP can be induced to make improvements in the detection technologies only when the advertiser's incentive problem is more severe.

Does a better technology improve social welfare?

Overall, Proposition 2 and 3 show that when a third party handles inspection of contestable clicks, the advertiser would always have incentives to make further improvement to the high-precision verification technology while the SP gains higher profits from improving the detection technology only when SP's detection technology is not very costly. Given that the cost of detection technology is likely to be small due to the use of inexpensive and automated online filters, our results suggest that the two parties will have incentives to continue improving their technologies. The next question is whether such improvements in technologies would be beneficial to the agency, i.e., increase the social welfare, where the social welfare in

⁷ For example, as Tuzhilin (2006) suggests, in addition to the rule-based and anomaly-based approaches, Google can also develop classifier-based filters based on well-known data mining methods to improve the performance of online filters.

the loser-pay scheme when the high precisions are induced is denoted by:

$$SW^{LP} = U_{HH} + V_{HH} = \gamma \alpha - C_{H}^{S} - C_{H}^{A} - m^{I} (I_{HH}^{IS} + I_{HH}^{IA})$$

The following proposition summarizes the result.

Proposition 4. (i) As SP's high-precision detection technology improves, the social welfare increases if the advertiser's high-precision verification is sufficiently high.

(ii) As advertiser's high-precision verification technology improves, the social welfare increases if the SP's high-precision detection is sufficiently high.

Proposition 4 suggests that further improving one party's technology could decrease the social welfare if the other party's technology is not sufficiently good. For the SP, improving the detection technology would lead to more contestable clicks if the advertiser's verification precision is not low because many truly valid clicks that pass through the detection systems would be misclassified as invalid by advertiser's verification system. As a result, more clicks will be subject to third party investigation and hence a higher investigation cost. Similarly, if the SP's detection system does not do a good job of identifying fraudulent clicks, then many truly invalid clicks will be verified by the advertiser and hence a more accurate verification technology would likely yield more contestable clicks that are subject to investigations.

Nevertheless, the social welfare will increase as the detection or verification technology improves if the precision of the other technology is sufficiently high. Given that we have shown the two parties will have incentives to continue improving their technologies, this suggests that though the social welfare could be hurt in the beginning when the detection or verification precisions are not quite high, it will increase in the long run once the two precisions are improved.

Managerial Implications and Concluding Remarks

Click fraud is a critical problem in PPC advertising industry. While both SPs and advertisers employ technologies to identify fraudulent clicks, prior studies have shown that they cannot be induced to make further improvements to their respective technologies and suggested using other mechanisms to address the click fraud problem. In this paper, we consider third party investigation as a mechanism to mitigate the incentive problem in click fraud setting. In particular, we examine whether the responsibility of payments to the third party allows the two parties to work towards improving their click fraud identification technologies unilaterally.

We examined a model of click fraud identification in a three stage process: (1) the SP classifies clicks using a detection technology; (2) the advertiser does the same to those classified as valid by the SP's detection technology using a verification technology; (3) a neutral third party auditor examines the disagreements using an investigation technology. The SP is paid for a click only when it is classified jointly as valid by all three stages. The choice of technologies by the advertiser and the SP are not jointly observable and cannot be used for contracting, but the quality of the third party's investigation technology is observable to both the SP and the advertiser. We modeled this problem in a double moral hazard, principal-agent setting with the advertiser as the principal and the SP as the agent.

We consider a "loser-pay" scheme for investigation cost in the sense that the party who loses the disputes pays for third party's investigation cost. In other words, if the third party's investigation technology classifies a contestable click to be valid, then the advertiser pays the inspection cost; otherwise, the SP pays the third party for inspection. Our analysis examines the effects of further improving the technologies used for identifying click fraud and provides insights into the question of whether third party investigation helps induce the SP and the advertiser to make further improvements to the detection and verification technologies unilaterally.

We show that the advertiser would always have incentives to improve the verification technology. This happens because improvements in verification technology make both detection and verification technologies more effective. As such, both parties would have more incentives to choose their high-precision instead of low-precision technologies to increase their expected profits. Thus, a lower PPC is sufficient to induce the high-precision technologies. Corresponding to the decrease in PPC, advertiser's profit increases.

We find that the SP has an incentive to improve the detection technology only when the advertiser's incentive problem is more severe, i.e., the cost of SP's detection technology is not very high. This happens because improving detection technology makes the advertiser's verification technology less effective. The intuition is when the SP's detection technology does a good job in catching fraudulent clicks, the advertiser can work less hard with the verification technology. This leads to a higher PPC being needed to induce the high-precision verification technology, i.e., the SP commands more rents.

Nevertheless, the SP will not improve the detection technology when his incentive problem is more severe, i.e., the cost of SP's detection technology is sufficiently large. This is because improving detection technology increases its effectiveness and hence less incentive is needed to induce SP's high-precision detection technology, leading to a decrease in PPC. Consequently, the SP's profit will fall if he further improves the detection technology.

Since the detection technology involves the use of inexpensive and automated online filters, it is likely not to be very costly. Therefore, our finding suggests that when allowing a third party to handles inspection of contestable click, the responsibility of payment to the third party helps PPC model induce the SPs and advertisers to make unilateral improvements to their click fraud identification technologies.

We show that the social welfare increases with improvement in SP's detection technology only when the advertiser's verification precision is sufficiently high. Similarly, improving advertiser's verification would increase the social welfare only when the SP's detection precision is sufficiently high. The insight from the social welfare analysis along with the earlier result leads to the following insight: although the social welfare could be hurt in the beginning when the detection or verification precisions are not quite high, it will increase later once the two precisions are improved. As a result, third party investigation not only helps mitigate the agency problem by providing both parties with incentives to further improve their technologies but also increases the social welfare in the long run. Hence, third party investigation can be used in PPC advertising to address the incentive problems in the click fraud setting.

References

- Arrington, M. 2007. Anchor Intelligence to Audit Click Fraud. http://techcrunch.com/2007/12/09/anchor-intelligence-to-audit-click-fraud/
- Arya, A., Glover, J., Radhakrishnan, S. 2007. The Controllability Principle in Responsibility Accounting: Another Look, Essays in Accounting Theory in Honour of Joel S. Demski. Springer, New York, pp. 183–198.
- Balachandran, K. R., Radhakrishnan, S. 2005. "Quality Implication of Warranties in a Supply Chain," *Management Science* (51:8), pp. 1266–1277.
- Chen, M., Jacob, V.S., Radhakrishnan, S., Ryu, Y.U. 2012. "Can Payment-per-click Induce Improvements in Click Fraud Identification Technologies?" *Working Paper*. University of Texas at Dallas.
- ClickFacts. 2010. ClickFacts Ad Network Suite for Advertising Networks and Publishers. http://www.clickfacts.com/ad-buddy.html
- Clicklab. 2006. Clicklab Click Fraud Detection Frequently Asked Questions.
- http://www.clicklab.com/Clicklab-click-fraud-detection-FAQ.pdf
- Click Forensics. 2010a. Click Fraud Rate Rises Slightly in Q2 2010 to 18.6 Percent. http://www.clickforensics.com/newsroom/press-releases/165-click-fraud-rate-rises-slightly-in-q2-2010-to-186-percent.html.
- Click Forensics. 2010b. Current Click Forensics for Advertisers. http://www.clickforensics.com/products/for-advertisers-and-agencies/advertiser-advertisers.html.
- Click Forensics. 2010c. Collecting Click Data. http://www.clickforensics.com/whatwedo/how-we-doit/getting-data.html.
- Demski, J. S., Frimor, H., Sappington, D. E. M. 2004. "Efficient Manipulation in a Repeated Setting," *Journal of Accounting Research* (42:1), pp. 31–49.
- Goodman, J. 2005. "Pay-per-Percentage of Impressions: An Advertising Method that is Highly Robust to Fraud," *Workshop on Sponsored Search Auctions*, Vancouver, BC, Canada.
- Greenberg, A. 2008. Yahoo! Cozies Up To Its Click-Fraud Critics. http://www.forbes.com/2008/03/17/click-fraud-yahoo-tech-security-cx_ag_0317click.html.

- Hamner, S. 2009. Pay-Per-Click Web Advertisers Combat Costly Fraud. *The New York Times*. http://www.nytimes.com/2009/05/13/business/media/13adco.html.
- Hedger, J. 2008. Click Fraud and the Need for Third Party Verification. http://www.searchenginejournal.com/third-party-click-fraud-verification/6647/
- Hwang, I., Radhakrishnan, S., Su, L. 2006. "Vendor certification and appraisal: Implications for supplier quality," *Management Science* (52:10), pp. 1472-1482.

Immorlica, N., Jain, K., Mahdian, M., Talwar, K. 2005. "Click Fraud Resistant Methods for Learning Click-through Rates," In *Lecture Notes in Computer Science*. Springer-Verlag, New York, pp. 34–45.

Jayanth, R., Jacob, V.S., and Radhakrishnan, S. 2011. "Vendor/Client Interaction for Requirements Assessment in Software Development: Implications for Feedback Process," *Information Systems Research* (22:2), pp. 289-305.

Kshetri, N. 2010. "The Economics of Click Fraud," IEEE Security and Privacy (8:3), pp. 45-53.

- Liu, D., Chen, J., and Whinston, A. B. 2009. "Current Issues in Keyword Auctions," In Business Computing (Handbooks in Information Systems). Emerald Group Publishing Limited. Volume 3, pp. 69-97.
- Mahdian, M., Tomak, K. 2009. "Pay-Per-Action Model for On-line Advertising," *International Journal of Electronic Commerce* (13:2), pp. 113–128.
- Megna, M . 2008a. Click Forensics Submitting Click Fraud Reports to Google. http://www.ecommerce-guide.com/news/news/article.php/3777551.

Megna, M. 2008b. eBiz News: Study Says Search Firms Profit from PPC Fraud. http://www.ecommerceguide.com/article.php/3738776/eBiz-News-Study-Says-Search-Firms-Profit-from-PPC-Fraud.htm

- Olsen, S. 2008. Click Forensics, Yahoo Take on Click-fraud Cases. http://news.cnet.com/8301-1023_3-9986373-93.html.
- Perez, J. C. 2008. Google allies with click-fraud-detection firm Click Forensics. http://www.computerworld.com/s/article/print/9116932/Google_allies_with_click_fraud_detection_n_firm_Click_Forensics?taxonomyId=17&taxonomyName=Security
- Tuzhilin, A. 2006. The Lane's Gifts v. Google Report. http://googleblog.blogspot.com/pdf/Tuzhilin Report.pdf.

Wilbur, K. C., Zhu, Y. 2009. "Click Fraud," *Marketing Science* (28:2), pp. 293–308.

Appendix

Proof of Proposition 1

From (PCS) note that $\theta > 0$. The solution to Program 1 is given by θ such that either (PCS), (ICS), or (ICA) is binding. Rearranging constraints (PCS) and (ICS) requires $\theta \ge (C_H^S + m^I I_{HH}^{IS})/T_{HH} = A1$ and $\theta \ge [C_H^S - m^I (I_{LH}^{IS} - I_{HH}^{IS})]/(T_{HH} - T_{LH}) = A2$, respectively. Using Assumption AI, it follows $A2 \ge A1$. Hence, whenever (ICS) is satisfied, (PCS) will also be satisfied, and (PCS) is not binding. Thus, the solution to Program 1 is determined by either (ICS) or (ICA). Solving for θ using (ICS) yields θ^* and using (ICA) yields θ^{**} . Thus, the solution to Program 1 is given by max{ θ^*, θ^{**} }. Using condition C1, it can be verified that $\theta^* > \theta^{**}$ when C1 is satisfied, and vice versa. The other expressions are derived by substituting optimums θ^* and θ^{**} in U(.) and V(.), respectively.

Proof of Observation 1

(a) The difference in probability that a click is paid for across high-precision and low-precision detection technology is given by:

$$\begin{split} T_{Hj} - T_{Lj} &= (q_H^S - q_L^S) \{ \alpha \big[q_j^A + \big(1 - q_j^A \big) q^I \big] - (1 - \alpha) \phi^S (1 - \phi^A \phi^I q_j^A q^I) \}. \\ & \left[d \big[T_{Hj} - T_{Lj} \big] / dq_j^A \big] = (q_H^S - q_L^S) [\alpha (1 - q^I) + (1 - \alpha) \phi^S \phi^A \phi^I q^I] > 0. \end{split}$$

(b) The difference in probability that a click is paid for across low-precision and high-precision verification technology is given by:

$$\begin{split} T_{iL} - T_{iH} &= (q_H^A - q_L^A)[(1 - \alpha)(1 - \phi^S q_i^S)\phi^A \phi^I q^I - \alpha q_i^S(1 - q^I)].\\ [d[T_{iL} - T_{iH}]/dq_i^S] &= -(q_H^A - q_L^A)[\alpha(1 - q^I) + (1 - \alpha)\phi^S \phi^A \phi^I q^I] < 0. \end{split}$$

Proof of Observation 2

(a) The difference in probability that the SP pays the third party for investigation cost across low-precision and high-precision detection technology is given by:

$$\begin{split} I_{Lj}^{IS} &- I_{Hj}^{IS} = (q_H^S - q_L^S)[(1 - \alpha)\phi^S \phi^A \phi^I q_j^A q^I - \alpha (1 - q_j^A)(1 - q^I)].\\ \left[d \left[I_{Lj}^{IS} - I_{Hj}^{IS} \right] / d q_j^A \right] &= (q_H^S - q_L^S)[\alpha (1 - q^I) + (1 - \alpha)\phi^S \phi^A \phi^I q^I] > 0. \end{split}$$

(b) The difference in probability that the advertiser pays the third party for investigation cost across lowprecision and high-precision verification technology is given by:

$$\begin{split} &I_{iL}^{IA} - I_{iH}^{IA} = (q_H^A - q_L^A) [\alpha q_i^S q^I - (1 - \alpha)(1 - \phi^S q_i^S)(1 - \phi^I q^I)\phi^A]. \\ &[d[I_{iL}^{IA} - I_{iH}^{IA}]/dq_i^S] = (q_H^A - q_L^A) [\alpha q^I + (1 - \alpha)(1 - \phi^I q^I)\phi^S\phi^A] > 0. \end{split}$$

Proof of Proposition 2

Differentiating the optimum in Proposition 1.1 with respect to q_{H}^{A} , we have the following.

$$\begin{split} [d\theta^*/dq_H^A] &= -\left[\frac{c_H^S - m^I(I_{LH}^{IS} - I_{HH}^{IS})}{(T_{HH} - T_{LH})^2} \frac{d(T_{HH} - T_{LH})}{dq_H^A} + \frac{m^I}{(T_{HH} - T_{LH})} \frac{d(I_{LH}^{IS} - I_{HH}^{IS})}{dq_H^A}\right] < 0, \text{ where the inequality follows from observation 1(a) that } [d(T_{HH} - T_{LH})/dq_H^A] > 0 \text{ and observation 2(a) that } [d(I_{LH}^{IS} - I_{HH}^{IS})/dq_H^A] > 0. \\ [dU^*/dq_H^A] &= -\left[T_{HH}[d\theta^*/dq_H^A] + \theta^*[dT_{HH}/dq_H^A]\right] - m^I[dI_{HH}^{IA}/dq_H^A] \\ &= -[d\theta^*/dq_H^A]T_{HH} - \theta^*[dT_{HH}/dq_H^A] - m^I[dI_{HH}^{IA}/dq_H^A] > 0, \text{ where the inequality is obtained from } [d\theta^*/dq_H^A] < 0, \text{ and assumption AI(b) and AI(d) which implies that } [dT_{HH}/dq_H^A] < 0 \text{ and } [dI_{HH}^{IA}/dq_H^A] < 0. \end{split}$$

Differentiating the optimum in Proposition 1.2 with respect to q_H^A , we have the following.

 $\begin{bmatrix} d\theta^{**}/dq_{H}^{A} \end{bmatrix} = \begin{bmatrix} \frac{C_{H}^{A} - m^{I}(I_{HL}^{IA} - I_{HH}^{IA})}{(T_{HL} - T_{HH})^{2}} \frac{dT_{HH}}{dq_{H}^{A}} + \frac{m^{I}}{(T_{HL} - T_{HH})} \frac{dI_{HH}^{IA}}{dq_{H}^{A}} \end{bmatrix} < 0, \text{ where the inequality is obtained from assumption AI(b) and assumption AI(d) which states <math display="block">\begin{bmatrix} dT_{HH}/dq_{H}^{A} \end{bmatrix} < 0 \text{ and } \begin{bmatrix} dI_{HH}^{IA}/dq_{H}^{A} \end{bmatrix} < 0.$

 $[dU^{**}/dq_{H}^{A}] = -[T_{HH}[d\theta^{**}/dq_{H}^{A}] + \theta^{**}[dT_{HH}/dq_{H}^{A}]] - m^{I}[dI_{HH}^{IA}/dq_{H}^{A}]$

= $-[d\theta^{**}/dq_H^A]T_{HH} - \theta^{**}[dT_{HH}/dq_H^A] - m^I[dI_{HH}^{IA}/dq_H^A] > 0$, the inequality is obtained from $[d\theta^{**}/dq_H^A] < 0$, and assumption AI(b) and AI(d).

Proof of Proposition 3

(i) Differentiating the optimum in Proposition 1.1 with respect to q_H^S , we have the following.

 $\begin{bmatrix} d\theta^*/dq_H^S \end{bmatrix} = -\frac{c_H^S - m^I (l_{LH}^{IS} - l_{HH}^{IS})}{(T_{HH} - T_{LH})^2} \frac{dT_{HH}}{dq_H^S} + \frac{m^I}{(T_{HH} - T_{LH})} \frac{dl_{HH}^{IS}}{dq_H^S} < 0, \text{ where the inequality follows from assumption}$ AI(a) that implies $\begin{bmatrix} dT_{HH}/dq_H^S \end{bmatrix} > 0$ and AI(c) which suggests $\begin{bmatrix} dI_{HH}^{IS}/dq_H^S \end{bmatrix} < 0.$

$$[dV^*/dq_H^S] = \left[T_{HH}[d\theta^*/dq_H^S] + \theta^*[dT_{HH}/dq_H^S]\right] - m^I[dI_{HH}^{IS}/dq_H^S]$$

 $= -\frac{[C_{H}^{S} - m^{I}(l_{LH}^{IS} - l_{HH}^{IS})]^{T_{LH}}}{(T_{HH} - T_{LH})^{2}} \frac{dT_{HH}}{dq_{H}^{S}} + \frac{m^{I}T_{LH}}{(T_{HH} - T_{LH})} \frac{dI_{HH}^{IS}}{dq_{H}^{S}} < 0, \text{ where the inequality is obtained from assumption AI(a)}$ that $[dT_{HH}/dq_{H}^{S}] > 0$ and $(T_{HH} - T_{LH}) > 0$, and AI(c) which suggests $[dI_{HH}^{IS}/dq_{H}^{S}] < 0.$ (ii) Note that condition C1 is not satisfied when the cost of detection technology is sufficiently small. Rearranging the inequality for C1 is not satisfied gives:

$$C_{H}^{S} < [(T_{HH} - T_{LH})/(T_{HL} - T_{HH})][C_{H}^{A} - m^{l}(I_{HL}^{IA} - I_{HH}^{IA})] + m^{l}(I_{LH}^{IS} - I_{HH}^{IS}).$$

Rearranging assumption AII yields $C_{H}^{S} > m^{I} (T_{HH} I_{LH}^{IS} - T_{LH} I_{HH}^{IS}) / T_{LH}$. To ensure the parameter region for C_{H}^{S} is nonempty, the upper bound should be larger than lower bound, which requires:

$$C_{H}^{A} > m^{I} \big[(I_{HL}^{IA} - I_{HH}^{IA}) + (T_{HL} - T_{HH}) I_{LH}^{IS} / T_{LH} \big].$$

Differentiating the optimum in Proposition 1.2 with respect to q_H^S , we have the following.

$$\begin{split} \left[d\theta^{**} / dq_{H}^{S} \right] &= -\left[\frac{\left[C_{H}^{A} - m^{I} (I_{HL}^{IA} - I_{HH}^{IA}) \right]}{(T_{HL} - T_{HH})^{2}} \frac{d(T_{HL} - T_{HH})}{dq_{H}^{S}} + \frac{m^{I}}{(T_{HL} - T_{HH})} \frac{d(I_{HL}^{IA} - I_{HH}^{IA})}{dq_{H}^{S}} \right] \\ &= -\left[\frac{d(T_{HL} - T_{HH})}{dq_{H}^{S}} \right] / [T_{HL} - T_{HH}]^{2} \left[C_{H}^{A} - m^{I} \left[(I_{HL}^{IA} - I_{HH}^{IA}) - (T_{HL} - T_{HH}) \left[\frac{d(I_{HL}^{IA} - I_{HH}^{IA})}{dq_{H}^{S}} \right] / \left[\frac{d(T_{HL} - T_{HH})}{dq_{H}^{S}} \right] \right] \\ &> -\left[\frac{d(T_{HL} - T_{HH})}{dq_{H}^{S}} \right] / [T_{HL} - T_{HH}]^{2} \left[C_{H}^{A} - m^{I} \left[(I_{HL}^{IA} - I_{HH}^{IA}) + (T_{HL} - T_{HH}) I_{LH}^{IS} / T_{LH} \right] \right] > 0 \end{split}$$

where the first inequality is obtained from $\left[I_{LH}^{IS}/T_{LH}\right] > -\left[\frac{d(I_{HL}^{IA}-I_{HH}^{IA})}{dq_{H}^{S}}\right] \frac{d(T_{HL}-T_{HH})}{dq_{H}^{S}} > 0$, and the second inequality follows from observation 1(b) that $\left[d(T_{HL}-T_{HH})/dq_{H}^{S}\right] < 0$ and $C_{H}^{A} > m^{I}\left[(I_{HL}^{IA}-I_{HH}^{IA}) + (T_{HL}-T_{HH})/dq_{H}^{S}\right] < 0$

 $[dV^{**}/dq_H^S] = [T_{HH}[d\theta^{**}/dq_H^S] + \theta^{**}[dT_{HH}/dq_H^S]] - m^I[dI_{HH}^{IS}/dq_H^S] > 0, \text{ where the inequality is obtained}$ from $[d\theta^{**}/dq_H^S] > 0$, assumption AI(a) that $[dT_{HH}/dq_H^S] > 0$, and AI(c) that $[dI_{HH}^{IS}/dq_H^S] < 0$.

Proof of Proposition 4

The social welfare under "loser-pay" scheme when high actions are induced is given by:

$$SW^{LP} = U_{HH} + V_{HH} = \gamma \alpha - C_{H}^{S} - C_{H}^{A} - m^{I} (I_{HH}^{IS} + I_{HH}^{IA})$$

(i) Differentiating the social welfare with respect to q_H^S yields:

$$\operatorname{sign}\left[\frac{dSW^{LP}}{dq_{H}^{S}}\right] = \operatorname{sign}\left[(1-\alpha)\phi^{S}\phi^{A}q_{H}^{A} - \alpha(1-q_{H}^{A})\right] > 0 \text{ if } q_{H}^{A} > \frac{\alpha}{\left[\alpha + (1-\alpha)\phi^{S}\phi^{A}\right]^{2}}$$

(ii) Differentiating the social welfare with respect to q_H^A yields:

$$\operatorname{sign}\left[\frac{dSW^{LP}}{dq_{H}^{A}}\right] = \operatorname{sign}\left[\alpha q_{H}^{S} - (1-\alpha)(1-\phi^{S}q_{H}^{S})\phi^{A}\right] > 0, \text{ if } q_{H}^{S} > \frac{(1-\alpha)\phi^{A}}{\alpha + (1-\alpha)\phi^{S}\phi^{A}}$$