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Abstract: This paper consider optimization decision-making of closed-loop supply chain based on revenue sharing contract and prospect theory, and study the effect of lose aversion on the performance of closed-loop supply chain. We establish a decision-making model for the closed-loop supply chain which consists a loss averse retailer and a loss averse manufacturer. Through the analysis of this model under both centralized and decentralized closed-loop supply chain, we show how the characteristic of loss aversion affect optimal order quantity and optimal wholesale price respectively, and display the relationship between recycling price and loss aversion coefficient. In the last part of the model analysis, we give the conditions which can realize perfect coordination of the closed-loop supply chain. Finally, we give a numerical example to verify all the conclusions we obtain.

Keywords: Loss averse; Closed-loop supply chain; Optimization decision

1. INTRODUCTION

With the growing understanding of sustainable development, the research of closed-loop supply chain gains widely attention from both industry and academic. Closed-loop supply chain is composed by different economic entities, each entity is aim to maximize his own revenue. While, this individual's rational behavior often result in poor performance. Previous studies have demonstrated that supply chain member's risk attitude will affect the performance of supply chain system. Loss aversion as one of closed-loop supply chain member's risk attitudes has not been studied.

Recently, the study of closed-loop supply chain has received widespread concerns. At present, most of researches in this area are concentrating on pricing strategy and optimizing coordination. Savaskan and Wassenhove[1][2] focus on the interaction effect between recycling strategy and new product's pricing strategy. Q and JH[3] use game theory to study pricing strategy and coordination mechanism of closed-loop supply chain. Da and Li[4] use game theory to test the impact of decision-making power's leadership on efficiency. To further research, a number of scholars extended closed-loop supply chain research to the case of multi-stage. Amaro and Barbosa-Povoaa[5] establish a multi-stage planning model which considering the uncertainty of recycling product demand and price. G and P[6] design a multi-gradient, multi-stage and multi-product closed-loop supply chain network based on the example of recycled lead-acid battery. With the increasing complexity of the model, different algorithms have been used to solve models and increase computational efficiency. Georgiadis and Vlachos[7], Kannan[6] and Ouyang[8] , Huang and Qiu[9] use different methods and algorithms to solve the problems. The above literatures regard the supply chain members as risk neutral, so Shi and Chen[10] use downside-risk measure theory to study optimization decision-making of the closed-loop supply chain which consists of one risk neutral manufacturer and one risk averse retailer. Gao and Chen[11] study two stage closed-loop supply chain which composed by one risk averse retailer and one risk averse manufacturer with conditional value at risk (CVaR).

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Loss aversion as an important basic principle of prospect theory has been widely adopted in newsboy problems and supply chain issues. S and Z[12] explain in detail what is loss aversion. Shen and Xu[13] point out that the manufacturer's purchasing quantity and procurement time have relationship with his loss averse attitude. Wen[14] find that the optimal order quantity is related to the degree of loss aversion. Lin and Cai[15] analysis supply chain model under loss aversion by sub-linear utility function, and find that the optimal order quantity and expected utility are decreasing with the increase of loss averse degree. In the existing literature of closed-loop supply chain, Loss aversion has not been mentioned.

Our paper analyze the closed-loop supply chain optimization decision-making based on prospect theory, and explore the effect of lose aversion on optimal order quantity, whole sale price and recycle price with theoretical analysis and numerical analysis. At last, we compared the size relationships of optimal order quantity between decentralized supply chain and integrated supply chain.

2. SYMBOLS AND ASSUMPTIONS

In this paper, we study a two-stage closed-loop supply chain, which consists of one loss averse retailer and one loss averse manufacturer. Manufacturer product one single short life production, retailer forecast the market demand to make orders. We assume retailer is responsible for waste product's recycling, and manufacturer receives all waste products retailer recovery. Related symbols and assumptions are:

C_m : unit cost of production; C_{rm} : unit cost of reproduction; w : unit wholesale price; p : unit retail price; q : order quantity; b_1 : manufacturer's unit recovery cost; b_2 : retailer's unit recovery cost; s : unit salvage value of the remaining products; λ_r : retailer's loss aversion coefficient; λ_s : manufacturer's loss aversion coefficient; λ_{sc} : supply chain's loss aversion coefficient; x : random demand; $f(x)$: density function of random demand; $F(x)$: distribution function of random demand.

Our assumptions refer to the relevant assumptions in the paper of Gao and Chen[11]:

(1)The recovery amount of waste product $L(b_2,q)=\varepsilon b_2 q$, ε is the sensitivity coefficient and $0\le\varepsilon\le 1/b_2$ for the recovery quantity is less than the retailer's order quantity.

(2)We assume $p>w, w>s, C_m\ge s, \varphi(p-s)>w, C_m>C_{rm}+b_1>b_1>b_2>0$, $p(1-\varphi)<C_m$ to ensure the problem we research has practical significance.

(3)We do not consider the case of product return, and ignore the inventory and shortage costs.

(4) Manufacturer and retailer are rational; they make decisions to maximize their own expected utility.

(5)Operation process is: before the start of the sales season, manufacturer provides revenue sharing contract $T(w,b_1,\varphi,q,b_2)$ to retailer, φ is the revenue sharing ratio, retailer determine the optimal order quantity and the optimal recovery price of waste products.

(6)All the recovery waste products can be reproduced.

(7) Information is completely symmetrical.

In this paper, we adopt a simple piecewise-linear function to describe the characteristic of loss aversion. Let W_0 present the initial wealth of decision maker, the utility function that reflects supply chain member's loss aversion characteristic can be expressed as:

$$U(W)=\begin{cases}W-W_0, W\ge W\\ \lambda(W-W_0), W<W_0\end{cases} \quad (1).$$

λ is the decision maker's loss aversion coefficient and $\lambda\ge 1$, the greater of λ means the higher degree of loss aversion. We assume decision-maker's initial wealth equal to zero, that is $W_0=0$. The function we adopt can not only reflect the basic characteristic of S-shaped curve, but also simplify the calculation.

3. THE MODELS

We consider the effect of loss aversion characteristic on centralized and decentralized closed-loop supply chain respectively.

3.1 Centralized closed-loop supply chain

According to the above assumptions and related symbols, random revenue function of the closed-loop supply chain can be expressed as:

$$\Pi_{sc}=\begin{cases}\Pi_{sc,1}=(p-s)x-(C_m-s)q+(C_m-C_{rm}-b_2)\varepsilon b_2 q, & x<q\\ \Pi_{sc,2}=pq-C_m q+(C_m-C_{rm}-b_2)\varepsilon b_2 q, & x\geq q\end{cases}(2).$$

Obviously, the random revenue function is a piecewise function, $\Pi_{sc,1}$ presents the revenue when order quantity is less than demand, $\Pi_{sc,2}$ presents the revenue when order quantity is more than demand. According to the random revenue function, we can get the expected profit function:

$$\pi_{sc}=E\Pi_{sc}=E\Pi_{sc,1}+E\Pi_{sc,2}=\int_0^q((p-s)x+sq)f(x)dx+\int_q^{+\infty}pqf(x)-(C_m-C_{rm}-b_2)\varepsilon b_2 q. \quad (3)$$

In order to get utility function that can reflect lose aversion characteristic, let $\Pi_{sc,1}=0$ ($\Pi_{sc,2}>0$), we get the break-even demand x_{sc}^* of closed-loop supply chain:

$$x_{sc}^*=\frac{-s+C_m-(C_m-C_{rm}-b_2)\varepsilon b_2}{p-s}q,(4)$$

set $q_{sc}^*=x_{sc}^*$, we know that when market demand $X<q_{sc}^*$, the revenue of closed-loop supply chain is negative, when $X>q_{sc}^*$, the revenue is positive. Thus the utility function can be expressed as:

$$U_{sc}=\begin{cases}(p-s)x-(C_m-s)q+(C_m-C_{rm}-b_2)\varepsilon b_2 q, & q_{sc}^*\leq x\leq q\\ pq-C_m q+(C_m-C_{rm}-b_2)\varepsilon b_2 q, & x\geq q\\ \lambda_{sc}[(p-s)x-(C_m-s)q+(C_m-C_{rm}-b_2)\varepsilon b_2 q], & x<q_{sc}^*\end{cases} \quad (5)$$

According to the definition of expected utility function, we get the expected utility function of closed-loop supply chain:

$$EU_{sc}=(\lambda_{sc}-1)\int_0^{q_{sc}^*}[(p-s)x+(s-C_m)q+(C_m-C_{rm}-b_2)\varepsilon b_2 q]f(x)dx+\pi_{sc}.(6)$$

Through the above analysis, we can draw proposition 1.

Proposition 1 Expected utility function is a concave function on order quantity q, and optimal order quantity Q_{sc}^* satisfy the first order condition equation.

$$-(\lambda_{sc}-1)(p-s)F(q_{sc}^*(Q_{sc}^*))\frac{-s+C_m-(C_m-C_{rm}-b_2)\varepsilon b_2}{p-s}+(s-p)F(Q_{sc}^*)+p-C_m+(C_m-C_{rm}-b_2)\varepsilon b_2=0.(7)$$

Proof: The first order and second order derivatives of expected utility function on the order quantity q are:

$$\frac{\partial EU_{sc}}{\partial q}=-(\lambda_{sc}-1)(p-s)F(q_{sc}^*)\frac{-s+C_m-(C_m-C_{rm}-b_2)\varepsilon b_2}{p-s}+(s-p)F(Q_{sc}^*)+p-C_m+(C_m-C_{rm}-b_2)\varepsilon b_2,$$

$$\frac{\partial^2 EU_{sc}}{\partial q^2}=-(\lambda_{sc}-1)f(q_{sc}^*)\frac{(-s+C_m-(C_m-C_{rm}-b_2)\varepsilon b_2)^2}{p-s}-(p-s)f(q).$$

Obviously, the second order derivative is negative, according to the definition of concave function, we know EU_{sc} is the concave function on order quantity q, let the first order derivative equal to zero, we can get the expression of the optimal order quantity, so proposition 1 is proved.

Proposition 2 Optimal order quantity is decrease with the loss aversion degree, the higher the level of loss aversion, the less the optimal order quantity.

Proof: As optimal order quantity function is an implicit function, it is not easy to analyze the impact of loss aversion coefficient λ_{sc} on the optimal order quantity. To simplify, we refer to the idea of the Lin and Cai [15], set $\beta_{sc}=\frac{-s+C_m-(C_m-C_{rm}-b_2)\varepsilon b_2}{p-s}$, then the optimal order quantity function can be simplified as:

$$-(\lambda_{sc}-1)\beta_{sc}F(\beta_{sc}Q_{sc}^*)-F(Q_{sc}^*)+1-\beta_{sc}=0. \quad (8)$$

Set $V_{sc}=-(\lambda_{sc}-1)\beta_{sc}F(\beta_{sc}Q_{sc}^*)-F(Q_{sc}^*)+1-\beta_{sc}$, according to the derivation rules of the implicit function, we can get formula (9):

$$\frac{dQ_{sc}^*}{d\lambda_{sc}} = -\frac{\partial V_{sc}/\partial\lambda_{sc}}{\partial V_{sc}/\partial Q_{sc}^*} = -\frac{F(q_{sc}^*(Q_{sc}^*))}{(\lambda_{sc}-1)\beta_{sc} f(q_{sc}^*(Q_{sc}^*)) + f(Q_{sc}^*)} < 0. \quad (9)$$

From the formula (9), we can derive the conclusion that in the centralized closed-loop supply chain, the optimal order quantity is decrease with loss aversion coefficient. This indicates that loss aversion characteristic does affect the system's optimal order quantity. Prospect theory tell us that when the marginal shortage cost is less than the marginal depreciation cost, the retailer's optimal order quantity will increase with the loss aversion coefficient. In this paper, we assume that shortage cost is zero and salvage value is less than the wholesale price, this assumption meet with the above condition, so proposition 2 is consistent with previous conclusion.

Proposition 3 Recycling price is decrease with loss aversion coefficient if recycling price is higher than optimal recycling price and is increase with the loss aversion coefficient if recycling price is lower than optimal recycling price. Loss aversion has no effect on the optimal recycling price.

Proof: According to the above implicit function (8), we can get formula (10) as follow:

$$\frac{db_2}{d\lambda_{sc}} = -\frac{\partial V_{sc}/\partial\lambda_{sc}}{\partial V_{sc}/\partial b_2} = -\frac{\beta_{sc}F(\beta_{sc}Q_{sc}^*)}{\frac{\varepsilon(2b_2 - C_m + C_{rm})}{p-s}[(\lambda_{sc}-1)F(\beta_{sc}Q_{sc}^*) + (\lambda_{sc}-1)\beta_{sc}Q_{sc}^* f(q_{sc}^*(Q_{sc}^*)) + 1]}. (10)$$

Formula (10) is positive or negative depends on the size of $2b_2 - C_m + C_{rm}$, if $2b_2 - C_m + C_{rm} > 0$, then $\frac{db_2}{d\lambda_{sc}} < 0$, if $2b_2 - C_m + C_{rm} < 0$, then $\frac{db_2}{d\lambda_{sc}} > 0$. This suggest that if retailer provide a higher recycling price, recycling price is decrease with loss aversion coefficient, and if retailer provide a lower recycling price, recycling price is increase with the loss aversion coefficient. $(C_m - C_{rm})/2$ is the optimal recycling price, it can be derived as following:

Let $\frac{\partial EU_{sc}}{\partial b_2} = \varepsilon b_2 q - (C_m - C_{rm} - b_2)\varepsilon q = 0$, it's easy to get the formula of the optimal recycling price, that is

$$b_{2sc}^* = \frac{C_m - C_{rm}}{2}, \quad (11)$$

formula (11) tell us the optimal recycling price has nothing to do with loss aversion coefficient under centralized closed-loop supply chain, and retailer's optimal recycling price is determined by manufacturer's produce cost and reproduce cost.

3.2 Decentralized closed-loop supply chain system

In decentralized closed-loop supply chain system, the retailer's random revenue function can be expressed as following:

$$\Pi_r = \begin{cases} \Pi_{r,1} = \varphi[px + s(q-x)] + b_1\varepsilon b_2 q - \varphi\varepsilon b_2^2 q - wq, & x < q \\ \Pi_{r,2} = \varphi pq + b_1\varepsilon b_2 q - \varphi\varepsilon b_2^2 q - wq, & x \ge q \end{cases} (12).$$

$\Pi_{r,1}$ is the revenue when demand is less than the order quantity and $\Pi_{r,2}$ is the revenue when demand is more than the order quantity. The expected revenue function of retailer π_r is:

$$\pi_r = E\Pi_r = E\Pi_{r,1} + E\Pi_{r,2} = \varphi[pq - \int_0^q (p-s)(q-x)f(x)dx] + b_1\varepsilon b_2 q - \varphi\varepsilon b_2^2 q - wq. (13)$$

Obviously $\Pi_{r,2} > 0$, set $\Pi_{r,1} = 0$, we get the break-even demand x_r^* of retailer, and $x_r^* = \frac{\varphi\varepsilon b_2^2 + w - b_1\varepsilon b_2 - \varphi\varepsilon}{\varphi(p-s)} q$.

Set $q_r^* = x_r^*$, this means when demand $X < q_r^*$, the revenue of retailer is negative, when demand $X > q_r^*$, the revenue of retailer is positive. So the utility function U_r of retailer can be expressed as:

$$U_r = \begin{cases} \varphi[px + s(q-x)] + b_1\varepsilon b_2 q - \varphi\varepsilon b_2^2 q - wq, & q_r^* \le x < q \\ \varphi pq + b_1\varepsilon b_2 q - \varphi\varepsilon b_2^2 q - wq, & x \ge q \\ \lambda_r[\varphi[px + s(q-x)] + b_1\varepsilon b_2 q - \varphi\varepsilon b_2^2 q - wq], & x < q_r^* \end{cases} (14).$$

According to function (14), we can get the formula of expected utility function EU_r :

$$EU_r = (\lambda_r - 1)\int_0^{q_r^*} \{\varphi[px + s(q-x)] + b_1\varepsilon b_2 q - \varphi\varepsilon b_2^2 q - wq\} f(x)dx + \pi_r. \quad (15)$$

Similarly, we get manufacturer's expected revenue function π_s and expected utility function $E\Pi_s$:

$$\pi_s = E\Pi_s = (1-\varphi)[pq - \int_0^q (p-s)(q-x) f(x)dx + (C_m - C_{rm} - b_2)\varepsilon b_2 q - (1-\varphi)\varepsilon b_2^2 q + wq - qC_m], \ (16)$$

$$EU_s = (\lambda_s - 1)\int_0^{q_s^*} \{(1-\varphi)[px + s(q-x)] + (C_m - C_{rm} - b_2)\varepsilon b_2 q - (1-\varphi)\varepsilon b_2^2 q + wq - qC_m\} f(x)dx + \pi_s. \ (17)$$

And the break-even demand is $x_s^* = \dfrac{(1-\varphi)\varepsilon b_2^2 - w + C_m - (C_m - C_{rm} - b_2)\varepsilon b_2 - (1-\varphi)s}{(1-\varphi)(p-s)} q$.

From the above analysis, we derive proposition 4.

Proposition 4 EU_r is a concave function on order quantity q , optimal order quantity Q_r^* satisfy the first order condition equation

$$-(\lambda_r - 1)\varphi(p-s)F(q_r^*(Q_r^*))\frac{\varphi\varepsilon b_2^2 + w - b_1\varepsilon b_2 - \varphi s}{\varphi(p-s)} + \varphi p - \varphi(p-s)F(Q_r^*) + b_1\varepsilon b_2 - \varphi\varepsilon b_2^2 - w = 0. \quad (18)$$

Proof : The first order and second order derivatives of expected utility function (15) on order quantity q are:

$$\frac{\partial EU_r}{\partial q} = -(\lambda_r - 1)\varphi(p-s)F(q_r^*)\frac{\varphi\varepsilon b_2^2 + w - b_1\varepsilon b_2 - \varphi s}{\varphi(p-s)} + \varphi p - \varphi(p-s)F(Q) + b_1\varepsilon b_2 - \varphi\varepsilon b_2^2 - w,$$

$$\frac{\partial^2 EU_r}{\partial q^2} = -(\lambda_s - 1) f(q_r^*)\frac{(\varphi\varepsilon b_2^2 + w - b_1\varepsilon b_2 - \varphi s)^2}{\varphi(p-s)} - f(q)\varphi(p-s).$$

So, the second order derivative is negative, EU_r is a concave function on order quantity q . Set the first order derivative equal to zero, we can get the function of Q_r^* , that is formula (18), proposition 4 is proved.

Proposition 5 Optimal order quantity is decrease with the loss aversion coefficient, that is the more of loss aversion, the less of optimal order quantity.

Proof: As the optimal order quantity function is an implicit function, it is not easy to analyze the impact of loss aversion coefficient λ_r on the optimal order quantity. We refer to the idea of the Lin and Cai [15], set $\beta_r = \dfrac{\varphi\varepsilon b_2^2 + w - b_1\varepsilon b_2 - \varphi s}{\varphi(p-s)}$. The optimal order quantity function can be expressed as:

$$-(\lambda_r - 1)\beta_r F(\beta_r Q_r^*) - F(Q_r^*) + 1 - \beta_r = 0. \qquad (19)$$

Let $V_r = -(\lambda_r - 1)\beta_r F(\beta_r Q_r^*) - F(Q_r^*) + 1 - \beta_r$, according to the derivation law of implicit function, we can derive the following formula:

$$\frac{dQ_r^*}{d\lambda_r} = -\frac{\partial V_r / \partial\lambda_r}{\partial V_r / \partial Q_r^*} = -\frac{F(q_r^*(Q_r^*))}{(\lambda_r - 1)\beta_r f(q_r^*(Q_r^*)) + f(Q_r^*)} < 0.$$

Obviously, optimal order function is decrease with loss aversion coefficient, that means optimal order quantity is decrease with the retailer's loss aversion coefficient. Since we assume shortage cost is zero and salvage value is less than the whole sale price, proposition 5 is consistent with the existing conclusion.

Proposition 6 Wholesale price is decrease with the retailer's loss aversion coefficient, and increase with the manufacturer's loss aversion coefficient.

Proof: First we consider retailer, according to function (19), we get the following formula:

$$\frac{dw}{d\lambda_r} = -\frac{\partial V_r / \partial\lambda_r}{\partial V_r / \partial w} = -\frac{\beta_r F(\beta_r Q_r^*)}{\dfrac{1}{\varphi(p-s)}[(\lambda_r - 1)F(\beta_r Q_r^*) + (\lambda_r - 1)\beta_r Q_r^* f(q_r^*(Q_r^*)) + 1]} < 0.$$

We can see that the wholesale price is decrease with the retailer's loss aversion coefficient. This is consistent with the proposition 5, because from proposition 5 we know the optimal order quantity is decrease with the retailer's loss aversion coefficient. In order to increase profits, manufacturer will decrease the wholesale price to encourage retailer to increase the order quantity.

Now we consider manufacturer, set $\beta_s = \frac{(1-\varphi)\varepsilon b_2^2 - w + C_m - (C_m - C_{rm} - b_2)\varepsilon b_2 - (1-\varphi)s}{(1-\varphi)(p-s)}$, we get the optimal order function for manufacturer, the function is:

$$V_s = -(\lambda_s - 1)\beta_s F(\beta_s Q_s^*) - F(Q_s^*) + 1 - \beta_s. \quad (20)$$

Derivative of wholesale price on loss aversion coefficient is:

$$\frac{dw}{d\lambda_s} = -\frac{\partial V_s / \partial \lambda_s}{\partial V_s / \partial w} = -\frac{-\beta_s F(\beta_s Q_s^*)}{\frac{1}{(1-\varphi)(p-s)}[(\lambda_s - 1)F(\beta_s Q_s^*) + (\lambda_s - 1)\beta_s Q_s^* f(q_s^*(Q_s^*)) + 1]} > 0.$$

So wholesale price function is increase with loss aversion coefficient, this means wholesale price is increase with the manufacturer's loss aversion degree. If manufacturer is loss averse, he would like to increase the wholesale price to ensure his profits.

Proposition 7 If recycling price is higher than optimal recycling price, recycling price is decrease with the retailer's loss aversion coefficient. If recycling price is lower than optimal recycling price, recycling price is increase with the retailer's loss aversion coefficient. Loss aversion has no effect to the optimal recycling price.

Proof: According to function (19), we derive the following formula:

$$\frac{db_2}{d\lambda_r} = -\frac{\partial V_r / \partial \lambda_r}{\partial V_r / \partial b_2} = -\frac{\beta_r F(\beta_r Q_r^*)}{\frac{\varepsilon(2\varphi b_2 - b_1)}{\varphi(p-s)}[(\lambda_r - 1)F(\beta_r Q_r^*) + (\lambda_r - 1)\beta_r Q_r^* f(q_r^*(Q_r^*)) + 1]}. \quad (21)$$

Formula (21) is negative or positive depends on the size of $2\varphi b_2 - b_1$, if $2\varphi b_2 - b_1 > 0$, then $\frac{db_2}{d\lambda_r} < 0$,and if $2\varphi b_2 - b_1 < 0$, then $\frac{db_2}{d\lambda_r} > 0$.This implies that if retailer offer higher price to consumers, recycling price is decrease with the retailer's lose aversion coefficient, and if retailer offer lower price, recycling price is increase with the retailer's lose aversion coefficient. This can be explained, when recycling price b_2 is higher, the unit profit of recycling waste products $(b_1 - b_2)$ is small, so when the loss aversion of retailer increases, retailer tends to reduce b_2. When recycling price b_2 is lower, the unit profit of recycling waste products is large, so when the loss aversion of retailer increase, retailer will reduce wholesale price to increase recycling products to obtain higher profits. $\frac{b_1}{2\varphi}$ is the optimal recycling price of retailer, it can be derived as:

$$\text{let } \frac{\partial EU_r}{\partial b_2} = b_1 \varepsilon q - 2\varphi\varepsilon b_2 q = 0, \qquad b_{2r}^* = \frac{b_1}{2\varphi}.$$

Retailer's optimal recycling price function is a linear function of manufacturer's recycling price, and has nothing to do with loss aversion. Manufacturer can encourage retailer to try hard to recycle by choose appropriate recycling price.

3.3. Comparison of two kind of closed-loop supply chain system

Through the above analysis, whether it is centralized closed-loop supply chain or decentralized closed-loop supply chain, the optimal order quantity is decrease with the loss aversion coefficient λ. In these two cases, we derive two optimal order quantity, now we compare these two kinds of optimal order quantity.

From function (8) and (9) we know, the relationship of Q_r^* and Q_{sc}^* is determined by the size of β and λ.We assume $\lambda_{sc} = \lambda_r$, then the size of β_{sc} and β_r determine the size of Q_r^* and Q_{sc}^*. Let $\beta_{sc} = \beta_r$, we get the optimal wholesale price function:

$$w^* = \varphi C_m - \varphi(C_m - C_{rm} - b_{2sc})\varepsilon b_{2sc} + \varepsilon b_{2r}(b_1 - \varphi b_{2r}). \quad (22)$$

This means that when $w = w^*, Q_r^* = Q_{sc}^*$. Through the analysis of the 3.1, we know Q_{sc}^* has nothing to do with wholesale price w, while according to the implicit function we get from 3.2, we know that the retailer's optimal order quantity has negative correlation with the wholesale price. That is:

$$\frac{dQ_r^*}{dw} = -\frac{\partial V_r / \partial w}{\partial V_r / \partial Q_r^*} = -\frac{-(\lambda_r - 1)[F(q_r^*(Q_r^*)) + \beta_r Q_r^* f(q_r^*(Q_r^*))] - 1}{-(\lambda_r - 1)(w - \varphi s)\beta_r f(q_r^*(Q_r^*)) - \varphi(p-s) f(Q_r^*)} < 0. \quad (23)$$

This suggests that in the decentralized closed-loop supply chain, the optimal order quantity function is decrease with wholesale price. So we draw the conclusion that if $w = w^*$ then $Q_r^* = Q_{sc}^*$, and if $w > w^*$ then $Q_r^* < Q_{sc}^*$.

Let $w = w^*, b_{2r}^* = b_{2sc}^*$, the closed-loop supply chain can achieve perfect coordination because the revenue function of closed-loop supply chain is:

$$\Pi_{sc} = \begin{cases} \Pi_{sc,1} = (p-s)x + sq + (C_m - C_{rm} - b_2)\varepsilon b_2 q - C_m q, & x < q \\ \Pi_{sc,2} = pq + (C_m - C_{rm} - b_2)\varepsilon b_2 q - C_m q, & x \geq q \end{cases} \quad (24),$$

and the revenue function of retailer is:

$$\Pi_r = \begin{cases} \Pi_{r,1} = \varphi[(p-s)x + sq] + \varphi(C_m - C_{rm} - b_2)\varepsilon b_2 q - wq, & x < q \\ \Pi_{r,2} = \varphi pq + \varphi(C_m - C_{rm} - b_2)\varepsilon b_2 q - wq, & x \geq q \end{cases} \quad (25).$$

Compared function (24) and (25), we find that the decision that maximize retailer's revenue also maximize the whole closed-loop supply chain system, and the decision has nothing to do with revenue sharing ratio φ, so when $w = w^*, b_{2r}^* = b_{2sc}^*$, the closed-loop supply chain can achieve perfect coordination.

4. NUMERICAL ANALYSIS

The above theoretical analysis reveals the interaction between decision variables and objective functions. In this section, we use numerical analysis to verify the conclusions we derived above. We assume market demand D conform uniform distribution U ~ [0,100], set $p = 15, C_m = 8, C_{rm} = 5, \varphi = 0.55, s = 3, \varepsilon = 0.05$. In this example, we assume recycling price is the optimal recycling price, and let $b_{2r}^* = b_{2sc}^*$. Substituting the above parameter values to the relevant model, we can derive $b_1^* = 1.65$ ' $b_2^* = 1.5$. From the above proposition, we know that the expected utility function is a concave function on order quantity Q under loss aversion. In figure 1, we describe the expected utility function of manufacturer, retailer and the closed-loop supply chain system. Figure 1 is consistent with our conclusions. Figure 1 is obtained when $\lambda = 1.8, w = 4$.

In figure 2, we describe the retailer's and supply chain system's expected utility function under both risk neutral and loss aversion. From figure 2, we can see that for both decentralized and centralized closed-loop supply chain system, expected utility under loss averse is less than under risk neutral.

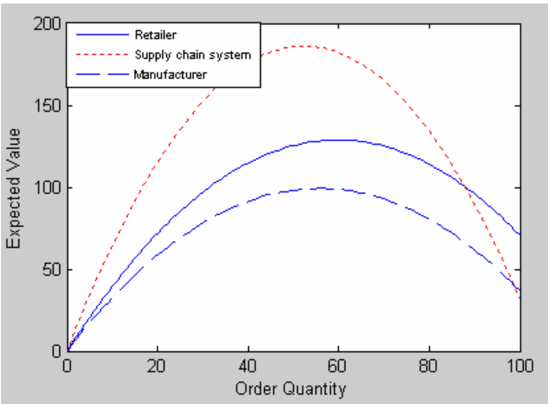

Figure 1 Expected utility function under loss averse

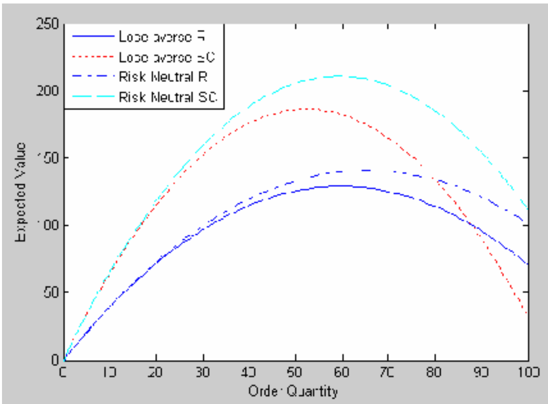

Figure 2 The impact of loss aversion on expected utility

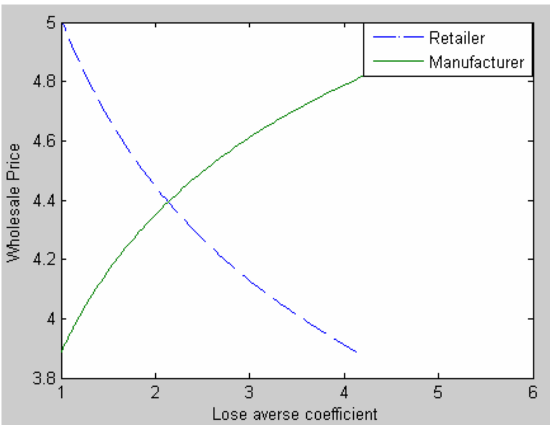

Figure 3 The impact of loss averse on wholesale price

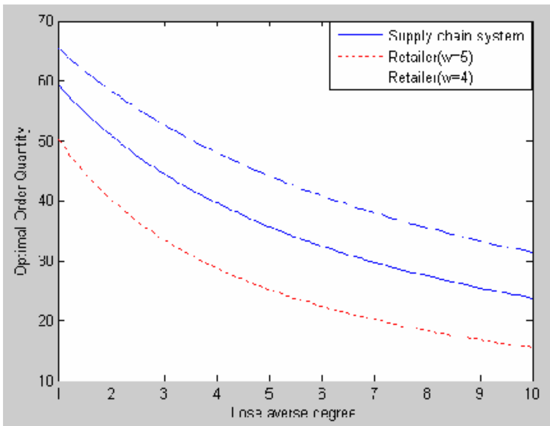

Figure 4 The impact of loss averse on optimal order quantity

From above proposition, we know the wholesale price is decrease with the retailer's loss aversion coefficient and increase with the manufacturer's loss aversion coefficient. Loss aversion coefficient $\lambda > 1$, figure 3 describe the change of wholesale price figure on λ, we assume order quantity Q=50.

In section 3.3, we proved that if $w = w^*$, then $Q_r^* = Q_{sc}^*$. In our example, we compute the optimal wholesale price $w^* = 4.39$, that means when $w = 4.39$, then $Q_r^* = Q_{sc}^*$. In figure 4, we figure the function of optimal order quantity on loss averse degree when w=4.39, w=4 'w=5 respectively. From figure 4 we also can see Q_r^* and Q_{sc}^* are decrease with loss aversion coefficient.

5. CONCLUSIONS

In this paper, we assume that recycling quantity is effected by the interaction of recycling price and sales quantity, and study the two-stage closed-loop supply chain under loss aversion. This paper is based on revenue sharing contact, describing supply chain member's risk attitude with loss aversion and researching the impact of loss aversion on the optimal decision of supply chain member and system, this is the significance of this study. In our paper, we establish the model of closed-loop supply chain, give the function of optimal order quantity and wholesale price, and reveal the impact of loss aversion on expected utility, order quantity, wholesale price and recycling price. The conclusions we get in this paper indicate that the retailer's optimal recycling price and manufacturer's optimal recycling price have nothing to do with loss aversion coefficient. We also compared the size relationship of the optimal order quantity in two kinds of closed-loop supply chain, and give the coordination condition.

This research need to be further improved and perfected. In this paper, we only considered the closed-loop supply chain consisted by a single manufacturer and a single retailer, it can be extended to the case considering multiple supply chain members. Another possible extension is to consider other recycling channels since we just consider the retailer responsible for recycling. What's more, in this paper, we use revenue sharing contract to coordination, future study can consider other contract to coordination.

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