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NFC based service innovation in retail: An explorative study

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GAME-THEORETIC ANALYSIS OF PAY-AS-BID MECHANISMS

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Abstract

Enterprises are facing a challenging dilemma. In order to be able to accommodate peak loads on their IT systems, they must maintain large computing clusters, which lie idle most of the time. At the same time, IT departments are under constant pressure to cut down on hard- and software expenses. Grid technology offers a promising way out of this dilemma by allowing the dynamic sharing both within enterprises as well as across organizational boundaries. This sharing approach, however, requires proper economic incentives. This paper is concerned with the determination of dynamic market-based prices. Due to their simplicity, so-called pay-as-bid mechanisms have become popular. This paper is novel as we provide an in-depth analysis of two such pay-as-bid mechanisms – Proportional Share and a discriminatory pay-as-bid mechanism – for the case of three users, thus extending previous work by Sanghavi and Hajek (2004) and Stößer et al. (2008). This analysis is important as we show that the nice results for two users cannot be retained once three or more users are present. Even worse, we show that these results can even be reversed if we move to games with more than two players.

Keywords: Pay-as-bid Mechanisms, Game-theoretic Analysis, Distributed Resource Allocation.

1 INTRODUCTION

Increasingly complex applications require massive amounts of processing power while exhibiting fluctuating utilization patterns. One example for such applications are CAx applications such as Computer Aided Engineering and Computer Aided Design or so-called "digital factory applications", e.g. for production planning. As a result, companies pile up resources to accommodate few peak loads on their systems. In a meta-study, Carr (2005) reports that data centers are only using between 10-35% of their available processing power, leading to tremendous inefficiencies. This development is exacerbated by intra-organizational boundaries. Even business units and departments within one enterprise are unwilling to share idle resources with other units.

Grid technology offers a promising way out of this dilemma. Using virtualization technologies, physical resources can be dynamically assigned to applications according to these applications' priority. Thus, if a unit experiences a peak load on its system, it can accommodate this peak load on remote resources, and analogously, in case an organizational unit temporarily has idle resources, it can host applications of other units. These applications can be forced out and moved to other resources as soon as the unit falls short of resources itself. In essence, Grid technology yields more efficient resource utilization as the system load is distributed across all organizational units. This sharing approach, however, requires proper economic incentives. Organizational units will only contribute their resources to the Grid if they get something in return. At the same time, units need to charge resource usage to avoid excessive usage. Both issues can be addressed by introducing dynamic prices. This paper is concerned with the determination of dynamic prices that are simple enough to be useful in Grids. Due to their simplicity, so-called pay-as-bid mechanisms have become popular – the user simply pays what he bid, and receives a share of the resource according to some specific allocation rule which relates this user's bid to the other users' bids.

This paper is novel as we provide an in-depth analysis of two such pay-as-bid mechanisms – Proportional Share and a discriminatory pay-as-bid mechanism – for the case of three users, thus extending previous work by Sanghavi and Hajek (2004) and Stößer et al. (2008). This analysis is important as we show that the nice results for two users cannot be retained once three or more users are present. Even worse, we show that these results can even be reversed if we move to games with more than two players.

This paper is structured as follows. In Section 2, we introduce a sample scenario which illustrates the business case for Grids. Section 3 discusses previous work on Grid market mechanisms. At the core of this paper, Section 4 provides a game-theoretic analysis of Proportional Share and the discriminatory pay-as-bid mechanism introduced by Sanghavi and Hajek (2004). In Section 5, we discuss our analytic results. Subsequently, Section 6 concludes the paper and points to future research directions.

2 MOTIVATIONAL SCENARIO

In the following, we showcase a scenario where market mechanisms create value in real business cases. The scenario is TXTDemand, an application by TXT e-solutions[1] for forecasting demand and replenishment within a supply chain. The application combines CPU intense forecasting algorithms as well as algorithms for the analysis of historic data and current sales data with interactive revision tools. In a typical business scenario, a customer is running the application daily for defining demand and replenishment strategies. During the night batches process the previous days' sales data to define initial forecast plans. For example, the replenishment module generates optimized replenishment plans, taking historic inventory levels, future demand forecasts and the type of the market into

[1] http://www.txtgroup.com/

consideration. The plans are analyzed and refined by the user at the other day in the interactive mode. The rationale for user interaction in the planning process is that users have more in-depth knowledge about the context. For example, if a certain fair takes place for the first time, the replenishment plan generated from historical data are most likely ignoring this. The user can, hence, significantly improve the replenishment plan by conducting as-if scenario analyses, refining the automatically produced plan.

The calculations of the replenishment module are particularly resource demanding when used in real business due to the very large amounts of data surpassing several hundred millions of entries. Grid computing can help by distributing the calculations on several machines. By pooling resources of several providers, the costs for IT infrastructure can be significantly reduced. Since the calculations are so demanding, the total cost of ownership for one company alone would make the use of the forecasting module unprofitable. It is the pooling of resources in a Grid that amortizes the use of the replenishment module. As the pure sharing of resources without compensation is hampered by free-riding behavior, market mechanisms seem to work well, establishing the right incentives for the participants.

The market mechanisms need to cope with the peculiarities of the domain. We identify two main requirements for the market mechanism. In case of interactive applications, the requests for Grid resources depend on daily human interactions. Thus there can be unpredictable peaks of requests, occurring at any second. This requires from the mechanism to *attain an allocation of resources in near real-time*. Furthermore, from a technical viewpoint, *avoiding starvation* is an important objective. In scheduling theory, starvation denotes the fact that low-priority tasks are prevented from doing any progress because all resources are assigned to other higher-value tasks. Combining the economic and the technical viewpoint, it can be desirable to give “better” service to high-value applications but to also give at least “some” service to low-value applications.

3 RELATED WORK

A bulk of mechanisms has been proposed for Grid resource allocation (see Wolski et al. (2001) and Neumann et al. (2008) for surveys). In AuYoung et al. (2004), Bapna et al. (2006) and Schnizler et al. (2006), the scheduling problem in Grids is formalized as a combinatorial allocation problem. Bapna et al. (2006) and Stößer et al. (2007) present greedy heuristics to mitigate the resulting computational complexity. While these mechanisms allow for dependencies between multiple Grid resources (e.g. CPU and memory), they are based on strong informational assumptions, such as complete knowledge about the time constraints and resource requirements of applications. However, in practice the users themselves will only have fuzzy knowledge about this information, as can be seen when comparing the user estimates and actual runtimes and resource requirements in workload traces from supercomputers.

A fundamentally different approach is taken by mechanisms that continuously assign *resource shares* to applications. These shares are continuously updated as new user requests are submitted or tasks are completed, thus allowing for real-time allocations. With an allocation rule purely based on economic reasoning (e.g. the prominent Vickrey auction), all available resources would be given to one single user, with the highest valuation. However, this will lead to the starvation of other tasks with lower value. Consequently, considering the requirements of our scenario, in this work we will consider two different allocation mechanisms: Proportional Share (Chun and Culler 2000, Lai et al. 2004) and the discriminatory pay-as-bid mechanism by Sanghavi and Hajek (2004).

Proportional Share is well understood and has been implemented by Hewlett Packard in its Tycoon system (Lai et al.). If user i reports a valuation of w_i, he will receive a fraction of the resources amounting to $\frac{w_i}{\sum_{j=1}^{n} w_j}$.

The discriminatory pay-as-bid mechanism has been proposed by Sanghavi and Hajek (2004) for bandwidth allocation in computer networks. It works slightly different than Proportional Share, giving a discount to the high-value user, thus resulting in a lower unit price than the low-value users. The idea is to encourage the high-value user to bid close(r) to his true valuation rather than to shade down his bid. We will denote the allocated share for each user i of n users for Proportional Share with $\tau_i^{ps} := \frac{w_i}{\sum_{j=1}^{n} w_j}$ and the Sanghavi-Hajek allocation rule with $\tau_i^{sh} := \frac{w_i}{w_{max}} \int_0^1 \prod_{j \neq i} \left(1 - s\frac{w_j}{w_{max}}\right) ds$. Sun Microsystems is considering to integrate the latter mechanism into their Sun Grid Engine (Stößer et al. 2008).

4 THE MODEL

One of the main goals of this work is to compare the performance of the two given allocation mechanisms and thus to conclude which of these may be more appropriate for use in typical Grid scenarios. Clearly our work is abstract and can be applied to other domains as well though in this work we restrict our attention to Grids as the mechanisms can be applied there reasonably.

In general the performance of mechanisms can be measured analytically, through simulations or with laboratory experiments. In this work though we will focus on analytical evaluation since it is well known that a common metric for measuring a mechanisms performance analytically is given by computing the *performance ratio* in its *Nash equilibrium*. Therefore we outline the setting used in our work and give a short introduction into Nash equilibria followed by the main analysis of the Nash equilibria of the two allocation mechanisms, Proportional Share and Sanghavi-Hajek. We will analyze under which conditions (multiple or unique) Nash equilibria exist for the case of more than two users. It is well known that the results from 2-player games cannot always be generalized to n-player games. Since this applies to our analysis as well, we will consider three players first, showing the tremendous challenges and difficulties as well as the rigorous conditions one encounters even for the simple case of one additional player.

4.1 The Setting

We assume there is an auction with one seller who offers a perfectly divisible good (i.e. there are no constraints in how many parts the good may be split up nor the size of these parts) and n buyers, each one offering a bid w_i and receiving a share through the utilized allocation mechanism τ. Let $w = (w_1, \ldots, w_n)$ be the vector of bids given by the users. We also assume $w_i \geq 0$ for $i = 1, \ldots, n$ with at least one $w_i > 0$, since a positive w_i represents the payment made by user i. At this point we may already say, that we can exclude any Nash equilibria, where negative bids w_i are submitted, since this would mean that user i would receive a payment by the seller for his allocated part of the good and thus would turn into a seller, also leading to a contradiction to the assumption of a given non-negative payment vector. The vector $x = (x_1, \ldots x_n)$, $x_i \in \mathbb{R}_+^0$, $\sum_{i=1}^{n} x_i = 1$, denotes the actual allocation, where the shares x_i for each user are allocated according to the allocation mechanism τ^{sh} for the Sanghavi-Hajek mechanism and τ^{ps} for Proportional Share respectively, where each mechanism depends on the payment vector w. Furthermore we assume in line with auction theory (Mas-Colell et al. 1995) that each user has a quasi-linear utility function $u_i(x) = v_i x_i - c(x_i)$, with $v_i \in \mathbb{R}_+$ denoting the valuation of user i for the good and $c(x_i) = p_i x_i$ the linear price function, where p_i denotes user i's unit price, i.e. the price user i would have to pay if she were awarded with the whole resources. As was shown in Johari and Tsitsiklis (2004), when evaluation the worst-case performance ratio in our setting, it is enough to look at linear utility functions.

Additionally we use in our setting a so-called *pay-as-bid pricing scheme*, so that we have $c(x_i) = p_i x_i = w_i$, $i = 1, \ldots, n$. Thus we can reformulate the utility functions to $u_i(w) = v_i \tau_i(w) - w_i$.

4.2 Nash Equilibria

Since the main goal of this section is to analyze the possible Nash equilibria for the several given allocation mechanisms for two, three and n users, the concept and the idea behind this equilibrium needs to be clear. One of the main goals of mechanism design is to construct these mechanisms in a way that they are incentive compatible, i.e. each user bids his true valuation of the good and has no advantage of reporting false valuations, because it will not lead to any improvement for him. In this situation the concept of Nash equilibria is widely used (cf. Sanghavi and Hajek 2004).

Definition 1. (Nash equilibrium) A bid vector w^{NE} is a *Nash equilibrium*, if $u_i(w_i^{NE}) \geq u_i(w_i, w_{-i}^{NE})$ for all $i = 1, \dots, n$, meaning that no user i can benefit by unilaterally deviating from his equilibrium bid w_i^{NE}.

For a given allocation mechanism there may exist no, one unique or several Nash equilibria. We will analyze under what conditions we may retrieve no equilibrium but by changing the realizations of the users bids and/or valuations slightly we obtain a unique or several Nash equilibria. Also we have not only to worry about multiple Nash equilibria but also acceptable (i.e. feasible) ones, meaning that we will not accept any solution resulting in a contradiction to our assumptions as already pointed out above. Having said this we will speak of a feasible Nash equilibrium if an equilibrium chosen out of a given set of several Nash equilibria fulfills all taken assumptions and leads to no contradiction at all. Therefore for any possible solution we have to check afterwards if the foregoing assumptions and conditions still hold.

Given the term of a Nash equilibrium we may define the performance ratio of an allocation mechanism:

Definition 2. (Performance ratio) Suppose a set of n users each having a utility function u_i and a provider selling the perfectly divisible good with a utility function u_P. Let the bid vector w^{NE} be a Nash equilibrium. Then the *performance ratio* of a given allocation mechanism τ is given by

$$\frac{u(\tau(w^{NE}))}{u^*} = \frac{\sum_{i=1}^{n} u_i\big(\tau(w^{NE})\big) + u_P(\tau(w^{NE}))}{u^*}$$

where u^* denotes the theoretical optimum giving the whole good to the user with the highest bid.

The subsequent analysis will be along the following lines: We will compute the resulting Nash equilibria for n users ($n \in \{2, 3\}$) for the Sanghavi-Hajek as well as the Proportional Share mechanism. We will show under which conditions these equilibria exist and to what outcome (utility function) they entail. For this we use the following, well known proposition:

Proposition 3. For a set of given users i, $i = 1, \dots, n$, the payment vector w^{NE} is a Nash equilibrium, if and only if $\frac{\partial u_i(w^{NE})}{\partial w_i^{NE}} = 0$ for all $i = 1, \dots, n$.

Proposition 3 simply means that every agent maximizes his own utility in conjunction with all bids given by the other agents.

4.3 Two Users

Stößer et al. (2008) analyze the case with two requesters. We will briefly report the main results for the sake of completeness. We assume that the two users have the valuations v_1 and v_2 and without loss of generality user 2's valuation is not less than user 1's, i.e. $v_1 \leq v_2$.

Lemma 4. In the Nash equilibrium w^{NE} of the pay-as-bid mechanism τ^{sh}, user 1 bids $w_1^{NE} = \frac{v_1^2}{2v_2}$ and receives a share of $\tau_1^{sh}(w^{NE}) = \frac{v_1}{2v_2}$, whereas user 2 bids $w_2^{NE} = \frac{v_1}{2}$, thus receiving $\tau_2^{sh}(w^{NE}) = 1 - \frac{v_1}{2v_2}$.

Inserting the above results in the utility functions u_i for every user i, shows that the low-value user receives zero utility, while the high-value user has $u_2(w^{NE}) = v_2 - v_1$.

Lemma 5. In the Nash equilibrium w^{NE} of the pay-as-bid mechanism τ^{ps}, user 1 bids $w_1^{NE} = \frac{v_1 v_2}{v_1+v_2} - v_1\left(\frac{v_2}{v_1+v_2}\right)^2$ and receives a share of $\tau_1^{ps}(w^{NE}) = \frac{v_1}{v_1+v_2}$, whereas user 2 bids $w_2^{NE} = v_1\left(\frac{v_2}{v_1+v_2}\right)^2$, thus receiving $\tau_2^{ps}(w^{NE}) = \frac{v_2}{v_1+v_2}$.

Consequently, for two users there exists a unique Nash equilibrium without implying further restrictions on the valuations v_i of the users. Unfortunately, as we will show below, this does not hold any longer for three or more users.

4.4 Three Users

Now we will show our results for three users followed by additional examples demonstrating that our argumentation is not based on an empty set but that there exist scenarios allowing us to apply our analytic results. To illustrate our approach for each proposition we sketch the proofs which are straightforward but very in the appendix. We begin with the results of our analysis for the proportional Share mechanism for three users and subsequently move to the Sanghavi-Hajek mechanism.

4.4.1 Proportional Share

In this section we will analyze whether there exist a unique Nash equilibrium when there are three users present. Our main result is captured by Proposition 6, which is quite powerful, as it shows the conditions for which feasible Nash equilibria exist.

Proposition 6. For three users with quasi-linear value functions there exists no unique Nash equilibrium for the Proportional Share allocation mechanism. Given the restrictions

$$v_3 \geq \frac{v_1 v_2}{v_1+v_2},\ v_2 \geq \sqrt{v_1 w_3} \text{ and } v_1 \geq w_2 + w_3$$

there exists a feasible equilibrium.

Proof. See Appendix.

Proposition 6 illustrates clearly that there does not exist a unique but two Nash equilibria, from which only one is feasible as it avoids payments from the seller to the user. The restrictions imply certain conditions (i.e. lower bounds) on the realizations of the valuations of the users but are not unlikely to happen in real-world scenarios, since they though being strong are nevertheless easy to check and fulfill. For the sake of simplicity we state them as a composition of constraints between the bids and the valuations, but since the bids depend unambiguously on the valuations only, more complex terms may be stated just using the users valuations. For more details we refer the reader to the appendix.

Example 7. We will give two simple examples, which illustrate this proposition. Assume there is a provider offering a certain amount of Terabytes of storage and three users having their own valuation and willing to pay a certain amount to achieve a share of the storage. The valuations are common knowledge and each user bids strategically trying to maximize his own utility.

- **a)** We assume the valuations to be $v_1 = 1$, $v_2 = 2$ and $v_3 = 3$. Using the results above we can compute the only payment vector $w = (\frac{-12}{121}, \frac{60}{121}, \frac{84}{121})$. The negative value of w_1 is the result of the third condition of the above proposition not being fulfilled, as $w_2 + w_3 = \frac{144}{121} > 1 = v_1$. Thus there exists no Nash equilibrium even for this simple case.
- **b)** Changing the valuations given in **a)** slightly, yields us a feasible outcome and Nash equilibrium: Assuming that we have the realization of the valuation vector $v = (2, 3, 4)$ this

yields $w = (\frac{24}{169}, \frac{120}{169}, \frac{168}{169})$ and the following vector of utilities $u = (\frac{2}{169}, \frac{75}{169}, \frac{196}{169})$ thus amounting to a total revenue for the provider of $\sum_{i=1}^{3} w_i = \frac{273}{169}$.

4.4.2 Sanghavi-Hajek Mechanism

In this section we analyze Nash equilibria for the Sanghavi-Hajek mechanism. These results differ very much from the ones we achieved for the Proportional Share mechanism.

Proposition 8. For three users, the Sanghavi-Hajek allocation mechanism combined with a *pay-as-bid pricing scheme* and quasi-linear utility functions, we have

a) A unique Nash equilibrium for the realization $v_3 = \frac{1}{3}\frac{v_1 v_2(3(v_1+v_2)+4\sqrt{2}\sqrt{v_1 v_2})}{9(v_1^2+v_2^2)-14 v_1 v_2}$ and the condition $\frac{8}{9}v_1 \le v_2 \le \frac{9}{8}v_1$, resulting in zero utility for the lower-bidding users and non-negative utility for the highest-bidding user.

b) Multiple Nash equilibria for $v_3 > \frac{1}{3}\frac{v_1 v_2(3(v_1+v_2)+4\sqrt{2}\sqrt{v_1 v_2})}{9(v_1^2+v_2^2)-14 v_1 v_2}$ and the realizations of the valuations of the three users being so that $2w_{3\pm} \le v_i \le 3w_{3\pm}$ for $i = 1, 2$ holds.

Proof. See Appendix.

As we learn from Proposition 8, for the Sanghavi-Hajek mechanism a unique Nash equilibrium may exist, but occurs under very restrictive assumptions on the users valuations. We can state immediately that in a real-world scenario the exact realization of v_3 as given above for a unique equilibrium is very unlikely to happen and tends to converge to zero. This is due to the fact that users usually will have expectations only about the valuations of the other users. Multiple Nash equilibria may also exist under less restrictive conditions though these conditions as seen in Proposition 8 depend on the highest given bid w_3. A strict dependency on the valuations of the users only can't be given since v_3 is not determined but can be chosen freely above the limit.

Example 9. To clarify our results, we give two short examples assuming the same scenario as already given in Example 7.

a) Taking into account the conditions involved for finding a unique Nash equilibrium, we assume the realizations of the valuations of the lower-bidding users to be $v_1 = v_2 = 1$ and $v_3 = \frac{1}{2} + \frac{1}{3}\sqrt{2}$. Thus the users may bid the strategy $w_3 = \frac{1}{4}\frac{4+3\sqrt{2}}{3+2\sqrt{2}}$ and $w_1 = w_2 = \frac{3}{8}\frac{(2+\sqrt{2})(4+3\sqrt{2})}{(3+2\sqrt{2})^2}$ resulting in a utility vector $u = (0, 0, \frac{1}{12(3+2\sqrt{2})})$.

b) If given the valuation vector $v = (3, 3, 5)$ all necessary conditions are satisfied, resulting in a zero utility for the lower-bidding users and a positive utility for the highest bidder.

It is obvious that the conditions necessary for having a Nash equilibrium for both allocation mechanisms are different. The question which arises now is if the conditions given for one mechanism are stronger than the ones of the other meaning that if we have given one Nash equilibrium of one mechanism we would obtain automatically an equilibrium of the second allocation mechanism and thus allowing us to compare these mechanisms one to another. Fortunately we have the following result:

Corollary 10. Assume a unique Nash equilibrium w^{NE} for the Sanghavi-Hajek mechanism. Then w^{NE} is also a feasible equilibrium for Proportional Share.

Proof. See Appendix.

With this we may now compare the performance ratio of the two given allocation mechanisms assuming we have the conditions fulfilled for a unique Nash equilibrium for Sanghavi-Hajek. Since the proof is a simple computation and straightforward it is omitted. We just state the final result of our paper:

Corollary 11. Assume a unique Nash equilibrium w^{NE} for the Sanghavi-Hajek mechanism. Then the performance ratio for the Proportional Share mechanism exceeds the ratio of the Sanghavi-Hajek mechanism for all feasible realizations of v_1 and v_2.

This outcome is very surprising indeed as it is in opposition to the result obtained by Stößer et al. (2008) for two users and will be discussed in the following section.

Corollary 12. Assume a unique Nash equilibrium w^{NE} for the Sanghavi-Hajek mechanism. Then the revenue for the Proportional Share mechanism is exceeded by the revenue of the Sanghavi-Hajek mechanism for all feasible realizations of v_1 and v_2.

This result coincides with the work for 2-players done by Stößer et al (2008).

5 DISCUSSION

Our goal is to compare market mechanisms which can be integrated into existing Grid schedulers to achieve efficient resource allocations and dynamic prices. As pointed out earlier, there are three possible techniques to do such a comparison: game-theoretic analysis, numerical experiments, and laboratory experiments. In this paper, we report the results of our game-theoretic approach.

Earlier work on Proportional Share and the discriminatory pay-as-bid mechanism for the two user case has shown that the Sanghavi and Hajek mechanism is superior to Proportional Share with respect to overall efficiency and, given sufficiently close valuations of the two users, also as regards provider's revenue (Stößer et al. 2008). The aim of our analysis in this paper was to generalize these results for more than two users. To this end, we need to model the user behavior, i.e. how the users bid based on their true valuations. We chose to ground our analysis on the assumptions of quasi-linear utility functions and the prominent solution concept of Nash equilibria. We were able to derive conditions on the existence of such equilibria for both the Proportional Share and the Sanghavi and Hajek mechanism. However, the interpretation of these equilibria for the Sanghavi and Hajek mechanism gives rise to four issues:

- The two low-bidding users are pushed to zero utility while the high-bidding user only achieves a slightly positive utility, meaning that the provider reaps almost all the welfare generated by the mechanism. This corresponds to the results of Stößer et al. (2008) for the two user case. Consequently, the users are indifferent to not participating in the mechanism. One possibly remedy could be that the provider gives a small kickback to the low-bidding users to encourage them to participate. It can be an interesting question for future research to investigate how this impacts the users' strategic considerations.
- The user with the lowest valuation ends up being the highest bidder. We explain this somewhat paradoxical result by the fact that in order to achieve a (unique or feasible) Nash equilibrium we had to set up some (rigorous) restrictions on the valuations of the users. Obviously this cannot be done in real-world scenarios and though this procedure is legitimate from a mathematical point of view, it may result in odd economic interpretations.
- The unique Nash equilibrium comprises some interesting cross-dependencies between the given bids. Assume two users bid according to their equilibrium strategy. If one of the low-value users deviates from his bid, this will not affect his own utility, but instead result in a decrease of the other users' utilities.
- Interestingly, as shown in Corollary 11, the analytic results for the performance ratio reverse if we move from a scenario with two users to a scenario with three users. While with two users the Sanghavi-Hajek mechanism always generates higher efficiency than Proportional Share, with three users Proportional Share dominates the Sanghavi-Hajek mechanism.

In summary, our analysis showed that complexity increases tremendously already when adding only one more user. We need to introduce strong restrictions on the users' valuations in order to obtain feasible Nash equilibria, let alone unique equilibria. While the analytic approach is attractive due to its elegancy, this strikingly shows the limitations of the analytic approach, as these equilibria are unlikely to occur in real-world scenarios.

This is exacerbated by the fact that we assume a scenario with complete information. In practice, users will at most have a rough estimate about the valuations of other users. Including such Bayesian solution concepts into our model would surely drive complexity beyond analytic feasibility.

In conclusion, we need to rely on numerical simulations and/or laboratory experiments when trying to analyze these mechanisms in more complex and realistic settings.

6 CONCLUSION & FUTURE WORK

In this article, we have approached the concept of scheduling in Grid systems from an economic viewpoint. Due to the strategic nature which is inherent to these systems, market-based schedulers are deemed promising to increase the efficiency of such systems, while at the same time providing incentives to contribute resources to the Grid. So-called pay-as-bid mechanisms score with their ease-of-use. Furthermore, they impose only a very low communicational and computational burden on the scheduling process and consequently allows for real-time allocations.

In this paper, we analyzed the Proportional Share mechanism and the discriminatory pay-as-bid mechanism by Sanghavi and Hajek (2004), two prominent proxies of such pay-as-bid mechanisms, thus extending previous work by Sanghavi and Hajek (2004) and Stößer et al. (2008). In order to study the performance of these mechanisms, e.g. as regards efficiency and provider's revenue, we modeled the strategic behavior of rational users using the prominent solution concept of Nash equilibria. The result of this paper is twofold. We show that for three users, Nash equilibria can only be obtained under strong assumptions. Moreover, in case we have a Nash equilibrium which holds for both mechanisms, Proportional Share achieves higher efficiency than the Sanghavi-Hajek mechanism, thus reversing the results for two users.

This shows the limitations of the analytic approach in more complex and realistic settings, and strengthens the case for alternative techniques. In the future, we will analyze the mechanisms by means of numerical experiments. This will allow us to model agents with learning capabilities. It will also be interesting to compare these simple pay-as-bid mechanisms to combinatorial approaches to Grid resource allocation, which have been proposed by several authors (cf. Neumann et al. 2008 for an overview). Moreover, our analysis is set within a real-world scenario, the Biz2Grid project which we introduced earlier. We thus plan a prototypical implementation of a market-based scheduler, which will serve as proof of our concept.

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7 APPENDIX

Proof of Proposition 6. Applying the conditions given in Proposition 3 leads us to the following system of equations, where for the sake of simplicity we write w_i instead of w_i^{NE} for the components of the strategic choosen Nash equilibrium payment vector:

$$w_1 = \sqrt{v_1(w_2+w_3)} - (w_2+w_3),\ w_2 = \sqrt{v_2(w_1+w_3)} - (w_1+w_3) \text{ and}$$
$$w_3 = \sqrt{v_3(w_1+w_2)} - (w_1+w_2).$$

Solving this system leads us to a unique value of w_3 and two possible options of $w_1$and w_2:

$$w_3 = \frac{2v_1v_2v_3(v_1v_3+v_2v_3-v_1v_2)}{(v_1v_2+v_1v_3+v_2v_3)^2},\ w_{2\pm} = \frac{v_2\left(v_1v_2 \pm \sqrt{v_1v_2(v_1v_2+4w_3(v_1+v_2))}\right)-2v_1w_3(v_1+v_2)}{2(v_1+v_2)^2} \text{ and}$$
$$w_{1\pm} = \sqrt{v_1(w_{2\pm}+w_3)} - (w_{2\pm}+w_3).$$

Clearly we have multiple Nash equilibria. In order to achieve a unique equilibrium we have to imply one condition on the valuation of user 3, i.e. $v_1v_2\big(v_1v_2+4w_3(v_1+v_2)\big)=0$ yields the condition $v_3 = \frac{1}{3}\frac{v_1v_2}{v_1+v_2}$. Furthermore, we need to check whether or not all payments w_i made by the users are non-negative, which is not given by the above equations. A simple computation shows us, that $w_3 \geq 0$ is equivalent to $v_3 \geq \frac{v_1v_2}{v_1+v_2}$, $w_2 \geq 0$ is equivalent to $v_2 \geq \sqrt{v_1w_3}$ and $w_1 \geq 0$ is equivalent to $v_1 \geq w_2+w_3$. Of course all of these conditions may be expressed by the valuations v_i, $i=1,\dots,3$, but results in much more complex inequations. This leads us directly to the conclusion that there is no unique Nash equilibrium for three users, since the realization of $v_3 = \frac{1}{3}\frac{v_1v_2}{v_1+v_2}$ yields to the negative bid $w_3 < 0$, which is not feasible. Finally, it is straightforward, that for any positive value of w_3 we obtain $w_{2-} \leq 0$, hence we may discard the second solution and focus on w_{1+} and w_{2+} though we may not speak of a unique equilibrium but rather of a feasible equilibrium.

Proof of Proposition 8. In order to be able to apply the mechanism, we have to determine the user who submitted the highest bid. Without loss of generalization we may assume an ascending order for the payment vector, thus being $w_3 = w_{max}$ the highest bid. Similar to the discussion of the Proportional Share allocation mechanism we have the following equation system

$$w_1 = \frac{3w_3(v_2 - 2w_3)}{v_2},\ w_2 = \frac{3w_3(v_1 - 2w_3)}{v_1} \text{ and}$$

$$v_3 = \frac{6w_3^3}{3w_3(w_1 + w_2) - 4w_1w_2},$$

which leads to two possible solutions of w_3:

$$w_{3\pm} = \frac{1}{48v_3}\left(9v_3(v_1 + v_2) - v_1v_2 \pm \sqrt{81v_3^2\left(v_1^2 + v_2^2 - \frac{9}{14}v_1v_2\right) - 18v_1v_2v_3(v_1 + v_2) + v_1^2v_2^2}\right).$$

It is obvious that in order to obtain a unique Nash equilibrium, the sum contained in the root has to be zero, hence we need to demand a further condition on one of the valuations. Choosing that valuation to be v_3 it has to be that $81v_3^2\left(v_1^2 + v_2^2 - \frac{9}{14}v_1v_2\right) - 18v_1v_2v_3(v_1 + v_2) + v_1^2v_2^2 = 0$ is valid for all valuations $v_1$and v_2 resulting in a direct dependency of v_3 from the other two valuations. We receive two possible solutions for $v_{3\pm} = \frac{1}{3}\frac{v_1v_2(3(v_1+v_2)\pm 4\sqrt{2}\sqrt{v_1v_2})}{9(v_1^2+v_2^2)-14v_1v_2}$ and therefore forcing v_3 to be on a cone-like structure.

As it is seen, the second solution gives us, even for very high valuations of the remaining two users, a relatively low value of the third valuation, though we assumed the third user to be the highest bidder. Since this scenario appears to be an extreme unlikely case, we may discard the second solution and focus on the first. Therefore we have a unique solution for v_3 and thus for w_3, both depending only on the valuations of the first two, lower-bidding users. The question is, if that is the one and only restriction even though it is a very tough condition and may rarely be achieved in real-world applications. Unfortunately the answer is that we need even more restrictions, this time on the realizations of the valuations of the lower-bidding users to ensure $0 \leq w_i \leq w_{max}$ for $i = 1, 2$, since otherwise we would have a contradiction to our primary assumptions. A simple computation shows us that the first inequality holds, if $2v_1 \geq v_2$ and $2v_2 \geq v_1$, while the second inequality holds if $\frac{8}{9}v_1 \leq v_2 \leq \frac{9}{8}v_1$. Since we need both inequalities to hold and the second condition already implies the first one, we may focus thereon, which leaves only a narrow margin for the possible choices of $v_1$and v_2. At this point we have to remark, that for any given valuations of the lower-bidding users, v_3 will take on a value which is less than any of this valuations, though we already have chosen the highest possible value accordingly to our calculations. The maximum for v_3 will be reached exactly if $v_2 = \frac{9}{8}v_1$or vice versa, since the lower-bidding users are symmetric, i.e. it does not matter who bids or valuates the good more than the other one, as long as they do not bid higher than the third user.

Using the results above, we may compute a solution under the given conditions on the valuations v_i, $i = 1, \ldots, 3$. Since the components of the resulting payment vector w are relatively complex, we just state here the final result of the utility functions: For any given valuations v_i, $i = 1, 2$, of the lower-bidding users their utility functions equal zero, i.e. $u_i = 0$ for $i = 1, 2$ while the utility of the higher-bidding user is besides being relatively low, non-negative for $\frac{8}{9}v_1 \leq v_2 \leq \frac{9}{8}v_1$.

Finally, omitting the restriction of a unique Nash equilibrium we may analyze the case of two equilibria. As is seen in Figure 1 a unique equilibrium is achieved for v_3 being on the cone like structure. For v_3 not holding that requirement we can distinguish between two cases: First, v_3 being inside the structure and second, v_3 taking a value outside the cone. For the first case, it is easily seen, that we have no Nash equilibrium at all, because we have to deal with a square root of a non-positive value. In the second case, we may discard the possibility of v_3 taking a value below the structure because we assumed the third user to be the highest bidder and thus this choice seems odd. So we proceed with the following condition

$$v_3 > \frac{1}{3}\frac{v_1 v_2(3(v_1+v_2)+4\sqrt{2}\sqrt{v_1 v_2})}{9(v_1^2+v_2^2)-14 v_1 v_2}$$

which gives rise to two Nash equilibria depending on the choice of $w_{3\pm}$. To ensure a feasible solution, i.e. $0 \leq w_i \leq w_{max}$ for $i = 1, 2$, a subsequent analysis shows that it has to hold that $2w_{3\pm} \leq v_i \leq 3w_{3\pm}$ for $i = 1, 2$ resulting in a zero utility for these users as well. At this point we may remark, that $w_{3\pm}$ is ascending and limited for all choices of v_3 and converges so that we have

$$\lim_{v_3 \to \infty} w_{3\pm} = \frac{3(v_1+v_2) \pm \sqrt{9(v_1^2+v_2^2)-14 v_1 v_2}}{16}.$$

We see, that the above condition on the valuations of the lower-bidding users is based upon the value of $w_{3\pm}$. Therefore, instead of being able to check the conditions just on the given valuations of the three users, we have to compute $w_{3\pm}$ first, followed by a decision, which solution of w_3 to use. Having in mind the conditions on v_i, $i = 1, 2$, gives rise to the consideration choosing strategically w_{3+} and discard w_{3-} though one cannot eliminate the possibility of a feasible equilibrium with the last realization.

Proof of Corollary 10. We just have to show that the conditions on the valuations of the users which are being implied by the unique Nash equilibrium satisfy also the conditions necessary for a feasible equilibrium for the Proportional Share mechanism. This is straightforward. We may fix without loss of generalization the valuation of the first user v_1. Then v_2 has to satisfy $\frac{8}{9}v_1 \leq v_2 \leq \frac{9}{8}v_1$. A simple analysis shows us that v_3, depending on the other valuations, is ascending and thus we choose the lowest possible value for v_3 with $v_2 = \frac{8}{9}v_1$. This yields $v_3 = v_2$ and thus the first condition $v_3 \geq \frac{v_1 v_2}{v_1+v_2}$ for Proportional Share holds. To check the second condition $v_2 \geq \sqrt{v_1 w_3}$ we confirm that w_3 is ascending as well and with v_1 fixed and the above choice of $v_2 = \frac{9}{8}v_1$ this results in $w_3 = \frac{3}{8}v_1$and the second condition holds as well. Since w_2 depending on w_3 is also ascending we continue with the above choice of w_3 and have $w_2 = \frac{9}{32}v_1$ which makes the third condition hold for all possible realizations of the valuations of the users implied by the conditions of the unique Nash equilibrium by the Sanghavi-Hajek mechanism thus concluding our proof.