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PRICE CYCLES IN ONLINE ADVERTISING AUCTIONS

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Abstract

Paid placement in search engines has become one of the most successful and rapidly growing sectors of the online advertising industry. The innovative use of auctions to sell keyword-related advertisement positions is perhaps the most important factor driving the success of this market. There has been no systematic analysis, however, of the advertisers' strategies to bid for ranks in a dynamic environment, where each bidder's bid can be updated and observed by the competitors in real time. We capture this dynamic setting using a Markov process and identify the Markov perfect equilibrium. We find that in such a dynamic environment, bidders' bidding strategies follow a cyclical pattern (Edgeworth cycle) similar to that conjectured by Edgeworth (1925) in a totally different context. A new data set that contains a detailed bidding history of all advertisers for sample keywords in a leading search engine makes it possible for us to study the real-world behavior of bidders. We propose an empirical framework based on maximum likelihood estimation of latent Markov state switching to confirm the theory. We also discuss the theoretical and practical significance of finding such cycles in an online market place.

Keywords: Online advertising, search engine, keyword auction, price war, Markov perfect equilibrium, Edgeworth cycle, Markov switching regression

Introduction

Online advertising spending has steadily grown over the last few years, from 1999's $3.5 billion to 2004's $8.4 billion.[1] Among the different types of online advertising, sponsored search, where positions in a search engine's result page are sold to advertisers through auctions, has grown most rapidly and is widely credited for the revitalization of the search engine business.[2] Keyword search market will grow from $2.6 billion in 2004 to $5.5 billion in 2009, according to Jupiter Research.

In contrast to the traditional CPM (Cost-Per-1000-iMpression) model, the industry standard in these keyword auctions is performance-based payment models,[3] and different search engines adopt various auction mechanisms. For example, Yahoo!, the biggest online keyword auction broker, allocates positions to advertisers purely based on their bidding amount. Advertisers pay their own bid, as in a first price auction. Yahoo! now also allows bidders to choose a "proxy bidding" option, where each advertiser pays only one cent above the next highest bidder's bid, which effectively constitutes a second price auction. Google, the other industry leader, adopts a different mechanism, where the ad positions are determined by bidder's bids and click-through rates jointly.

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[1]Sources: Elliott and Scevak (2004); Internet Advertising Bureau (**http://www.iab.net/resources/ad_revenue.asp**).

[2]Source: Economist.com, "Searching for Relevance," July 17, 2003 (**http://www.economist.com/displaystory.cfm?story_id=1932434**).

[3]Also referred to as CPC (cost-per-click) or PPC (pay-per-click). The search engine gets paid only if an advertisement generates a click-through.

These keyword auctions have the following distinctive characteristics compared to the traditional auctions studied in the literature:

- It is a type of auction for multiple heterogeneous objects, where each bidder's valuations for these objects are commonly ranked; that is, all bidders value a higher position more than a lower one, but each bidder has private values for each of the positions.
- The advertisers are bidding for the "options" of future services. That is, the advertisers agree to pay the search engine *ex ante* for the click-throughs from the users. The actual payment is determined by the bid, other bidders' bids, and the frequency of clicks.
- These auctions are held in a dynamic environment in continuous time. That is, any bidder may join or exit the auction at any time, and both advertisers' bids and their rankings can be updated in real time.
- In contrast to traditional auctions, where bidders only know their competitors' valuation through a probability distribution function, in keyword auctions, bidders can learn the competitors' bids (thus can derive their valuations for each position if the learning period is sufficiently long), either from a public database,[4] or through repeated experiments in updating their own bids.

In practice, there does not seem to exist a straightforward strategy for advertisers to follow. In a survey conducted by Jupiter Research, it is estimated that "the overwhelming majority of marketers have misguided bidding strategies, if they have a strategy at all" (Stein 2004). This is surprising, given the vast amount of advertisers competing in such auctions.[5] The same survey also found that marketers without a strategy were less successful in their campaigns.

Then do most of the bidders really act irrationally in these auctions? What is the optimal bidding strategy for a rational advertiser? What kind of bidding price and rank outcome is implied by such an optimal strategy?

This paper tries to answer these questions. Instead of modeling a static game where each bidder can only bid once according to their expectations, we adopt a more realistic, dynamic approach, allowing bidders to update their bids as well as their rankings. Moreover, different from a classical sequential game, where each player's strategy is based on the whole history, our model incorporates a Markovian setting, where each bidder only reacts to the state variables that are directly payoff-relevant—as in a typical Markov process. In this model, bidders learn their competitors' valuations after a sufficiently long period of time. We identify the Markov Perfect Nash Equilibrium (MPE) bidding strategy, and find that such a bidding strategy does not produce a stable outcome in prices and ranks. Instead, a "price war" period is observed in which each bidder increases his bid by only the smallest increment above his/her competitor for a better position. A "cease fire" period follows when one bidder can no longer afford the costly price war and drops his bid to a lower level, then the other bidders drop their bids accordingly and a new round of price war begins.

The cyclical pattern of bids described here is similar to the Edgeworth cycle in the literature modeling duopolistic price competition, where two duopolists with identical marginal cost undercut each other's price in an alternating manner. This is in contrast to a standard Bertrand competition argument in which case each competitor sets the price equal to the marginal cost. The Edgeworth cycle was formally demonstrated with the concept of Markov perfect equilibrium in a class of sequential-move duopoly models by Maskin and Tirole (1988a, 1988b). Building on the Maskin and Tirole model, Noel (2003) identifies the existence of Edgeworth cycle in the retail gasoline market in 19 Canadian cities. Different from these models, our paper extends to model players with different types. In our model, bidders have different private valuations for the positions, but their valuations for these positions are ranked in the same way. Moreover, the competition between bidders drives the price up.[6] We are also able to derive some unique properties under this asymmetric player game setting.

There are various papers studying how search engines should auction off the advertisement positions. Feng et al. (2006) examine the performances of various ranking mechanisms used by different search engines using computational techniques, and find that, depending on the correlation between advertisers' willingness to pay and their relevance (quality), both Google's and Overture's ranking mechanisms can out-perform each other under specific conditions. Feng (2005), using an optimal mechanism design approach, studies how to optimally sell a set of objects where bidders valuations for these objects are ranked in the same order. It is shown that the order to fill the positions can be different under different market situations, and the optimal mechanism

[4]In Overture, a nice interface actually shows the bids submitted by all the bidders.

[5]A popular keyword like "flower" or "car" in Yahoo! attracts hundreds of advertisers to compete.

[6]In contrast to the typical Edgeworth cycle, where firms undercut prices.

includes a reserve price for each position, with the winners paying at least as high as the next highest bidder's bid. This framework can be applied to keyword auctions. Weber and Zheng (2005) model explicitly how advertisers' performance (quality) should impact the search engine's selling mechanism. They find that the optimal search engine design should put non-zero weight on advertisers' monetary payment, in addition to their advertisements' performance. They also show that there is no pure-strategy equilibrium in advertisers' bidding behavior. Liu et al. (2005) study a different weighting mechanism. Instead of weighting their performance and bidding price uniformly across all advertisers, they find that the optimal mechanism should give different weights to advertisements with different performance. Lim and Tang (2005) focus on the bidding behaviors in keyword auctions in a two-firm environment, where the bidding prices can only take three discrete values. They characterizes the conditions under which both advertisers will submit high or low bids. Borgers et al. (2005) study bidders' behavior in Yahoo!'s top three listings, assuming the click-through rate for the three positions are identical. They identify the existence of multiple equilibria in users' bidding strategies, and demonstrate this with Yahoo!'s auction data.

In all of these settings, however, bidders are only allowed to bid once according to their expectations about their competitors' valuation. Once the bids are submitted, they cannot be changed no matter which ranks the bidders are allocated. As mentioned in Salmon and Iachini (2003) and Zhan et al. (2005), such models often result in bidders' *ex post* loss: when bidders submit their bids, they do not know for sure which rank they will win, thus they bid for an "average position" and it is possible that they pay more than what they end up winning. Our paper overcomes this problem because bidders are allowed to update their bids if they end up paying more than necessary for a certain position (this is one reason for the existence of the price cycles in our model). To our knowledge, this paper is also the first empirical work to study the price cycles in this exciting auction market.

The paper proceeds as follows. We set up the model in the next section, where we show that the traditional approach in auction theory can shed little light in this market. Instead, we find that MPE is a good theoretical concept to describe the rational behavior of bidders. In equilibrium, the best strategy is to wage a price war and compete for the best position, and when a certain threshold value is reached, one of the bidders should drop his bid down to a lower level, and the other bidder will follow this drop. This model predicts cyclical behavior in bidding for a position in search engine placement auctions. We then describe a dataset characterizing the real world strategies played by the advertisers. We develop an empirical framework based on maximum likelihood estimation of latent Markov state switching, and report the results. Finally, we present our conclusions.

The Model

Setup

Consider n risk-neutral advertisers competing for n positions in a search engine's result page, each position has different expected click-through rate (CTR). Assume that CTR is solely determined by the position of the advertisement,[7] and the higher the position of an advertisement, the more valuable it is, thus $\tau_i \geq \tau_j \quad \forall i \leq j$.[8]

Denote θ_i as bidder i's per-click valuation associated with the search engine—the maximum amount he is willing to pay to receive one click from the ad. We take as player i's type.[9] In reality, θ_i is determined by two factors: the unit profit of the good and the conversion rate. The unit profit of the good is a function of the unit price, unit cost, and frequency of repeat purchase. The conversion rate is determined by the design of the website and the attractiveness of the product. Both factors are independent from the advertisement's position or the CTR.

We concentrate on studying Yahoo!'s ranking mechanism, in which advertisers' ranks are determined purely by the ranks of their bids b_i, and each bidder pays the amount of his own bids. Denote the reservation price (the minimum price that is required by the search engine to enter this market) as r, and the minimum increment in bid as ε.

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[7]*Ex ante*, each advertiser cannot do much about the writing of the advertisements so as to affect his or others' bidding behavior, thus we can safely make this assumption.

[8]This is a common assumption in the literature (Breese et al. 1998), and is frequently confirmed by industry reports.

[9]Harsanyi (1967) argues that all uncertainty faced by a player can be summarized as a single variable, called his type, and that the prior distribution over the vector of types is common to all the players. Vickrey (1961) also implicitly assumed that the joint distribution of values was commonly known to all bidders.

Main Model

In keyword auctions at Internet search engines, current information technologies not only enable bidders to observe their competitors' bidding prices (or infer their prices from some quick experiments by varying their own bids), but also allow them to update their bids in real time. To capture the dynamic feature of this game, consider the competition between *n* bidders that takes place in discrete time in infinitely many periods, with discount factor $\delta \epsilon (0,1)$. For simplicity, following Maskin and Tirole (1988a), we assume bidders adjust their bids in an alternating manner. In each period *t*, only one bidder is allowed to update his bid, which means each bidder commits to his bidding price for *n*-1 periods. This assumption is used to ensure that each bidder is committed to a particular bidding price in the short run. He can not change this price for a finite (can be brief) period, during which other bidders might act. This short-run commitment ensures that when other bidders respond to a particular bidder, this bidder will not already have changed his bid.

Most static auction models assume bidders make simultaneous bids without update. Some sequential-auction models (Jeitschko 1998) allow bidders to learn and bid multiple times, but usually in a "super game" framework, where bidders' strategies are based on the entire history. This assumption may not always be appropriate, however. For example, in reality, people may not have the ability to precisely calculate their strategies, especially when the history is long or the strategy is complicated. Moreover, it is natural to assume that "recent actions have a stronger bearing on current and future payoffs than those of the more distant past" (Maskin and Tirole 1988a). For this reason, we impose the Markov assumption: in each period, a bidder's strategy depends only on the variables that directly enter his payoff function (the bids set by other bidders in the last *n*-1 periods). Another advantage of adopting Markov strategies is for simplicity, as bidders' strategies depend on as little as possible while still being consistent with rationality.

We focus on the perfect equilibrium of this game, which means, starting from any period, the bidder who is about to act selects the bid that maximizes his inter-temporal profit given the subsequent strategies of other bidders and of his own. For simplicity, in this paper we establish the MPE in the setting of two bidders. We are interested in the stationary properties of this model, rendering the initial conditions irrelevant. This way, each bidder's action only depends on the other bidders' bids in the last *n*-1 periods, thus we can drop the time stamp *t* from the analysis. Let b_{-i} represents the state that bidder *i* faces when he is about to act.

Define $R = (R1, R2)$ as an MPE strategy profile, which represents the dynamic reaction functions forming the perfect equilibrium. *R* represents a Markov perfect equilibrium if and only if for each bidder i, *b*i = *R*i(*b*-i) maximizes bidder *i*'s inter-temporal profit at any time, given *b*-i and each bidder *i* bidding according to *R*i. Before we get into more details, the following assumptions are needed:

1. The click-though rate τ_i is constant across all periods. That is, the expected number of click-throughs a certain position can generate is the same in each period.

2. There is a minimum (reserve) price *r* to win any one of the positions. In this model, each participating bidder's value is greater than *r*.

3. If two or more bidders bid the same amount, they are allocated to two or more consecutive positions with the same probability.

4. The bidding space is discrete, that is, firms cannot set prices in units smaller than, for example, one cent. Let ε denote this smallest unit.

Bidder i's current period payoff function can be described as

$$\pi(b_i, b_{-i}) = \begin{cases} (\theta_i - b_i)\tau_1 & \text{if } b_i > b_{-i} \\ (\theta_i - b_i)\tau_2 & \text{if } b_i < b_{-i} \\ 0.5(\theta_i - b_i)(\tau_1 + \tau_2) & \text{if } b_i = b_{-i} \end{cases}$$

Following Maskin and Tirole (1988b), define a pair of value functions for advertiser 1 in this dynamic programming problem (advertiser 2's valuation function can be defined in the same way). Let

$$V_1(b_2) = \max_{b_1}[\pi(b_1, b_2) + \delta W_1(b_1)] \qquad (1)$$

represent bidder 1's valuation if (a) he is about to move, (b) the other bidders' current price is b_2, (c) both bidders play according to R = (R_1,R_2) thereafter. Where $W_1(b_1)$ is defined as

$$W_1(b_1) = E_{b_2}[\pi(b_1, b_2) + \delta V_1(b_2)] \qquad (2)$$

which represents bidder 1's valuation if (a) he played b_1 last period, and (b) both bidders play optimally according to R = (R_1,R_2) thereafter. Thus, R = (R_1,R_2) is MPE if $R_1(b_2) = b_1$ is the solution for equation (1), where the expectation in equation (2) is taken with respect to the distribution of $b_2$2, and the symmetric condition holds for advertiser 2. Solving for the equilibrium strategies, we have

Proposition 1. *For a sufficiently fine grid (ε is sufficiently small), there exist threshold values* $\overline{b}_i$, $\underline{b}_{-i}$, *and* $\overline{b}_{-i}$, *and two probabilities σ(δ)ϵ[0,1] and μ(δ)ϵ[0,1], such that bidder i's optimal strategy can be expressed as*

$$b_i = R_i(b_{-i}) = \begin{cases} b_{-i} + \epsilon & & \text{if } r \le b_{-i} < \underline{b}_{-i} \\ b_{-i} + \epsilon & \text{with probability } \sigma(\delta) & \text{if } b_{-i} = \underline{b}_{-i} \\ \overline{b}_{-i} & \text{with probability } 1 - \sigma(\delta) & \\ \overline{b}_{-i} & & \text{if } \underline{b}_{-i} \le b_{-i} < \overline{b}_{-i} < \overline{b}_i \\ b_{-i} + \epsilon & \text{with probability } \mu(\delta) & \text{if } b_{-i} = \overline{b}_i \\ r & \text{with probability } 1 - \mu(\delta) & \\ r & & \text{if } b_{-i} > \overline{b}_i \end{cases} \qquad (3)$$

where r is the minimum required reserve *price, and* ε *is the smallest increment.*

All proofs are available in the technical appendix available at **http://nullvoid.mit.edu/keyword/**. Proposition 1 describes bidders' equilibrium bidding strategy, which is characterized by $\overline{b}_i$, $\underline{b}_{-i}$, $\overline{b}_{-i}$, *s, and m*. What equilibrium outcome does this strategy imply?

Corollary 1 *In general, the equilibrium bidding prices follow a cyclical pattern, with interchanging "price war" phases and "cease fire" phases.*

This cyclical price pattern can be easily inferred from bidders' equilibrium bidding strategies described in proposition 1. Beginning with the smallest possible price *r*, the bidders will wage a price war (outbidding each other by ε) until the price min{$\underline{b}_1$, $\underline{b}_2$} is reached. Without loss of generality, suppose $\theta_1 \ge \theta_2$, then bidder 1 will jump to bid $\overline{b}_2$, which is the highest bid that bidder 2 can afford to get rank 1. Now bidder 2 can no longer afford the costly competition, and will be better off remaining at the second position. He can choose any price lower than $\overline{b}_2$ and remain in the second position, but to minimize the cost, he has to bid *r*. Consequently bidder 1 should follow this drop to bid *r* + ε, where he remains at the first position and pays much less. Then a new round of price war begins, and so on. Note that on the equilibrium path, only the bidder with the higher valuation (bidder 1) has the option to jump up to bid $\overline{b}_2$ to force the price war phase to end early, and it is always bidder 2 who will first drop to bid *r* and restart the cycle.

Comparing to the Edgeworth cycle in Maskin and Tirole (1988b), the equilibrium in this paper has a different structure: there are two possible threshold values that could make the bidding price jump: $\overline{b}_i$ (jump down) and $\underline{b}_{-i}$ (jump up).

θ's Role in Price Cycles

First consider the upper bound bid of a bidder ($\bar{b}_i$). Intuitively, in the auction for ranking, the bidder who does not win the first position has a natural option with non-zero expected payoff: winning the second position ($\tau_2 > 0$). Taking this into consideration, bidders need not bid their full valuations for the first position. Thus,

Corollary 2 Bidder *i* won't pay more than his valuation per click for the top position, that is, $\bar{b}_i < \theta_i$.

This can be easily shown through Eq. (10) in the appendix.

The early-jumping behavior (jump bid at $\underline{b}_{-i}$) is a result of bidder's heterogeneity. Intuitively, when bidders have identical valuations, their gain from jump bidding is limited: first of all, they have to bid high enough (their own maximum bid for the first position) to stop the other bidder from competing in the price war, which is costly; second, given they both know the price at which their competitor will jump, and they both have the ability to stop their competitor from jumping, they both have incentive to jump bidding one period earlier than their competitors, which will eventually drive $\underline{b}_{-i}$ down to the extent that both are better off sharing the first position with their competitor in the price war. When two bidders' valuations are different, however, the lower-valued bidder (bidder 2) cannot afford to jump up to force the cycle to end. It is also straightforward that when the two advertisers' per-click valuations are sufficiently different ($\theta_1 >> \theta_2$), bidder 1 has incentive to jump bidding earlier, or even be willing to continue bidding $\bar{b}_2$ to dominate position 1. In summary,

Proposition 2. Bidder *i*'s maximum bid for position 1, $\bar{b}_i$, is increasing in θ_i.

Proposition 3. The price at which bidder *i* prefers to jump, $\underline{b}_{-i}$, is decreasing in θ_i.

The threshold $\underline{b}_{-i}$ can take two extreme values: when $\underline{b}_{-i} = \bar{b}_i - \varepsilon$, there is no jumping; when $\underline{b}_{-i} = r$, the jumping starts from the beginning—essentially no Edgeworth cycle pattern will be observed, since the bidder who jumps always occupies the first position. The following three corollaries discuss these cases.

Corollary 3 *When $\theta_1 = \theta_2$, and δ is close to 1, bidder i's equilibrium strategy can be simplified as:*

$$b_i = R_i(b_{-i}) = \begin{cases} b_{-i} + \epsilon & & \text{if } r \le b_{-i} < \bar{b}_i \\ \left.\begin{array}{ll} b_{-i} + \epsilon & \text{with probability } \mu(\delta) \\ r & \text{with probability } 1 - \mu(\delta) \end{array}\right\} & & \text{if } b_{-i} = \bar{b}_i \\ r & & \text{if } \underline{b}_{-i} > \bar{b}_i \end{cases} \quad (4)$$

This degenerates to the Edgeworth cycle described in Maskin and Tirole (1988b), where there is no jump bidding. It follows naturally from Proposition 3. More specifically, δ needs to satisfy the following condition:

$$\begin{aligned} & (\theta_1 - \bar{b}_2)(1+\delta)\tau_1 + \delta^2 \bar{V}_1 \\ \le\ & (\theta_1 - \bar{b}_2 + 2\epsilon)(\tau_1 + \delta\tau_2) + \delta^2(\theta_1 - \bar{b}_2)(1+\delta)\tau_1 + \delta^4 \bar{V}_1 \end{aligned} \quad (5)$$

where $\bar{b}_2$ is specified in Eq. (11). This condition is easier to meet when $\delta \to 1$. Note that we only need to check the price $\bar{b}_2 - 2\varepsilon$ in the right hand side of Eq. (5), because if Eq. (5) is satisfied, Eq. (13) is satisfied for all the prices between (r, $\bar{b}_2 - 2\varepsilon$).

Corollary 4 If δ is close to 0, there is no jumping in the price war phase.

This is intuitive: when δ is close to 0, only the current period matters for the bidder. Thus bidder 1 should not incur a current period loss by jump-bidding.

> **Corollary 5** If the two advertisers' valuations are sufficiently different, the higher-valued bidder will dominate the first position. This way, no Edgeworth cycle pattern exists in equilibrium.

To illustrate, one sufficient condition for the absence of the price cycles can be: $(\theta_1-\theta_2)\tau_1 \geq (\theta_1 - r)\tau_2$. For example, consider $r = 0.05$, $\theta_1 = 30$ and $\theta_2 = 1$, with $\tau_1 = 1$ and $\tau_2 = 0.7$, then bidder 1 gets a larger current payoff by always bidding 1 and remaining in the first position than engaging in the price war with bidder 2, since: $\forall\, p \in [r, 1),\ (30 - p) * 0.7 < (30 - 1) * 1$.

τ's Role in Price Cycles

> **Proposition 4.** Bidder *i*'s maximum bid for position 1, $\overline{b}_i$, is decreasing in τ_2.

> **Corollary 6** If the CTRs of both positions are sufficiently close (τ_1 is close to τ_2), no Edgeworth cycle pattern will be observed.

Intuitively, the rankings do not matter when the two positions are worth the same for the bidders (they attract the same number of click-throughs). Both bidders have no incentive to compete for the first position. To illustrate, one sufficient condition for this to happen is when $(\theta_i - r - \epsilon)\tau_1 \leq (\theta_i - r)\tau_2$, or equivalently, $\frac{\tau_1}{\tau_2} \leq 1 + \frac{\epsilon}{\theta_i - r - \epsilon}$. The RHS of this condition is decreasing in θ_i. If both θ_1 and θ_2 satisfy this condition, then both bidders don't care about the first position, and they will both bid *r* and be allocated the first position with equal probabilities; if only θ_2 satisfies this condition, then bidder 1 and 2 will bid $r + \varepsilon$ and *r* respectively, and bidder 1 will dominate the first position. In either case, no Edgeworth cycle will be observed.

More than Two Bidders

When there are more than two bidders (say, three), intuitively the cyclical pattern in the bidding prices still preserves, but in a much more complex way. For example, one scenario can be, when bidder valuations are not significantly different, they may first start from the minimum price *r* and outbidding each other by ε. This way they occupy the first three positions in order, until the lowest valued bidder cannot afford the price war, drops to bid *r*, and gets the bottom position. On one hand, if the first two highest bidders' valuation is close, they will continue the pairwise competition for the first position, as we discussed in proposition 1, although with different threshold values. When the price war becomes too costly, the second highest bidder will drop to bid $r + \varepsilon$ and get the second position. Then the highest bidder follows the drop to bid $r + 2\varepsilon$ and gets the first position. Then a new price war phase begins. On the other hand, if the highest bidder's valuation is large enough, it is also possible that he dominates the first position, while the two other bidders compete for the second position. Overall, the price war can happen among all three bidders, or between two bidders. However, the formal demonstration of the bidding strategy with more than two bidders is beyond the scope of this paper.

Data

To examine the interesting equilibrium outcome of keyword auctions, and to learn how strategic the bidders in this market are, we obtained Overture's paid placement auctions data between June 2002 and June 2003, and chose one keyword for this study.[10] Each time someone submitted a new bid, the system would make a record on the bidder's ID, the date/time, and the bid value. A total of 1,613 records have been included in the sample dataset. There were a total of 49 bidders who submitted at least one bid during this period. Among these bidders, seven submitted more than 100 bids. These most-active bidders were primarily competing for the first position.

[10]All keywords display a similar pattern. Following the theoretical framework that studies competition for one keyword, we only consider the time series of one keyword here in our empirical analysis. The dataset, however, enables us to build much richer models to compare across keywords. This would be an interesting direction for future research.

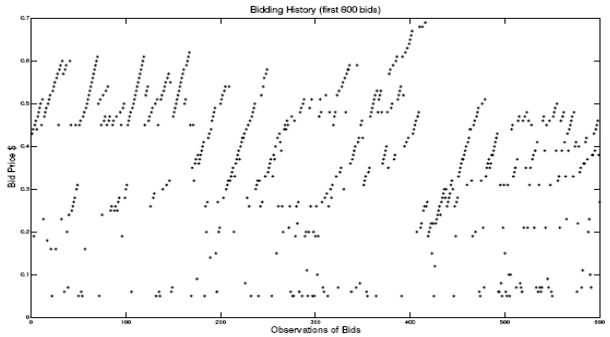

Figure 1. Bidding History

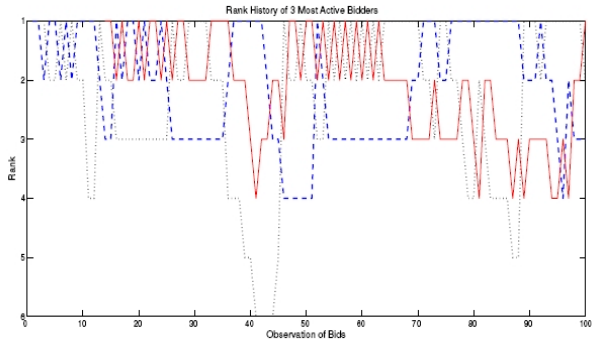

Figure 2. Rank History

Our empirical objectives are two fold. First, we would like to confirm the cyclical pattern of bids predicted in the theoretical model. A second objective of our study is to characterize the cycles so that we can learn something about the behavior of the bidders. Given the existing auction mechanism, it is very interesting to see how strategic the bidders are. This will also provide a lot of managerial insights for the practitioners (the publishers, the advertisers).

Figure 1 shows the bidding history of the first 600 bids. From the figure, we can easily identify the price cycles going through the price-war phase and the cease-fire phase.

From the raw data, we reconstruct the complete history of the bids and rankings.

Figure 2 gives the rank history of the first 100 bids submitted by the three most active bidders. Each instance of two lines crossing each other represents a change in ranking. Notably, most competition appears between people competing for the first rank.

A Markov Switching Model

Given the theoretical constructs of the Markov perfect equilibrium, the best empirical strategy is to use a Markov switching regression to characterize the cycles. Markov switching regression was proposed by Goldfeld and Quandt (1973) to characterize changes in the parameters of an autoregressive process. From the shocking cyclical trajectory of the bids shown in the figures in the previous section, it is very tempting for us to assign one of two states (W for price-war state; S for cease-fire state) for each observation of the bids and directly estimate the parameters with a discrete choice model. The use of the Markov switching regression, however, gives us a few preferable advantages over other potential empirical strategies. First, serial correlation can be incorporated into the model; the parameters of an autoregression are viewed as the outcome of a two-state first-order Markov process. Second, when the price trajectory is not as regular as displayed by our data, Markov switching regression can help us to identify the latent states.[11] This eliminates the need to subjectively assign dummy variable values for the states, making it less dependent on human interference. Third, from the estimation process, we can easily derive the Markov transition matrix, the parameter estimates can be directly used to validate our theory.

Formally, consider a two-state ergodic Markov chain as shown in Figure 3, with state space $S(S_t = \{w, s\}$, where $s_t = w$ represents the price war phase, $s_t = s$ represents the cease fire phase, and $t = 1, ..., T$. The process is a Markov chain with the stationary transition probability matrix $\Lambda = (\lambda_{ij})$, where

$$\lambda_{ij} = Pr(s_t = j | s_{t-1} = i), \qquad i, j \in \{w, s\}$$

[11]The smoothing technique we use in this paper was first proposed in Cosslett and Lee (1985). Hamilton (1989) further developed this technique. It is similar to a Kalman filter, which uses a time path to infer about an unobserved state variable. The nonlinear filter introduced by Hamilton draws inference about a discrete-valued unobserved state vector.

This gives us a total of four transition probabilities: λ_{ww}, λ_{ws}, λ_{ss} , and λ_{sw} , and we also have $\lambda_{ij} = 1 - \lambda_{ii}$, for $\epsilon\{w, s\}$ and $j \neq i$.

The ergodic probabilities for the ergodic chain is denoted π. This vector π is defined as the eigenvector of Λ associated with the unit eigenvalue; that is, the vector of ergodic probabilities π satisfies: $\Lambda\pi = \pi$. The eigenvector π is normalized so that its elements sum to unity: $1'\pi = 1$. For the two-state Markov chain studied here, we can easily derive

$$\pi = \begin{bmatrix} \lambda_w \\ \lambda_s \end{bmatrix} = \begin{bmatrix} (1-\lambda_{ss})/(2-\lambda_{ww}-\lambda_{ss}) \\ (1-\lambda_{ww})/(2-\lambda_{ww}-\lambda_{ss}) \end{bmatrix}$$

After defining the latent states, we can write the following model:

$$y_t = \begin{cases} \alpha_w + \mathbf{x}_{wt}\beta_w + \varepsilon_{wt} & if\ s_t = w \\ \alpha_s + \mathbf{x}_{st}\beta_s + \varepsilon_{st} & if\ s_t = s \end{cases}, \qquad t = 1, \ldots, T, \tag{6}$$

where is the bid submitted at time *t*, and $\mathbf{x}_{s_t t}$ $(s_t \in \{w, s\})$ is a vector of explanatory variables. The error terms $\varepsilon_{s_t t}$ are assumed to be independent of $\mathbf{X}_{\mathbf{s_t t}}$, also, we assume $\varepsilon_{wt} \sim N(0, \sigma_w^2)$ and $\varepsilon_{st} \sim N(0, \sigma_s^2)$. Notice that for each period, the regime variable (Markov state s_t) is unobservable.

Following Hamilton (1989), we develop a procedure to estimate model (6), and conduct the maximum likelihood estimation with EM algorithm (the detailed estimation procedure is relegated to the appendix and is available upon request).

Hamilton (1990) has shown that the maximum likelihood estimates for the transition probabilities can be calculated once we get the latent states

$$\hat{\lambda}_{ij} = \frac{\sum_{t=2}^{T} Prob(s_t = j, s_{t-1} = i | \mathbf{Y}_T; \hat{\theta})}{\sum_{t=2}^{T} Prob(s_{t-1} = i | \mathbf{Y}_T; \hat{\theta})}, \tag{7}$$

where $\boldsymbol{Y}_T$ is a vector containing all observations obtained through date *t*, and $\hat{\theta}$ denotes the full vector of maximum likelihood estimates. The meaning of this equation is that the estimated transition probability $\hat{\lambda}_{ij}$ is the number of times state *i* has been followed by state *j* divided by the number of times the process was in state *i*. We estimate the transition probabilities based on the smoothed probabilities.

Equipped with the estimates of the transition probabilities, we can derive very intuitive parameters to characterize the cycles. The expected duration of a typical price war (or cease fire) can be calculated with[12]

$$E(duration\ of\ phase\ i) = \sum_{k=1}^{\infty} k\lambda_{i,i}^{k-1}(1-\lambda_{i,i}) = \frac{1}{1-\lambda_{i,i}} \tag{8}$$

where $i \in \{w, s\}$ are the transition probabilities.

Estimation and Results

Following equation (6), we consider the next model:

[12]For details, please refer to Gallager (1996, Chapter 4).

$$y_{i,t} = \begin{cases} \beta_{w0} + \beta_{w1} y_{i,t-1} + \beta_{w2} r_{i,t} + \beta_{w3} r_{i,t-1} + \varepsilon_{wt} & if\ s_t = w \\ \beta_{s0} + \beta_{s1} y_{i,t-1} + \beta_{s2} r_{i,t} + \beta_{s3} r_{i,t-1} + \varepsilon_{st} & if\ s_t = s \end{cases}$$

where $y_{i,t}$ is the bid submitted by bidder i at time t, and $r_{i,t}$ is the achieved rank[13] after submission at time t.

The estimation procedure detailed in the last section will help us to calculate the latent state for each of the bids, along with the estimated probabilities, we can also get parameter estimates for $\beta_{\mathbf{w}} \equiv (\beta_{w0}, \beta_{w1}, \beta_{w2}, \beta_{w3})$, and $\beta_{\mathbf{s}} \equiv (\beta_{s0}, \beta_{s1}, \beta_{s2}, \beta_{s3})$.

The Markov switching regression based on Hamilton's algorithm turns out to be highly significant with the estimates reported in Table 1.

All parameters have expected signs. It is estimated that in regime 1, the bid is positively correlated with the previous bid when other things hold equal. The higher the previous rank is, the less likely he will be bidding higher, and the higher the new rank is, the higher he will be bidding. In regime 2, the higher the previous bid, the lower the new bid will be when other things hold equal. The higher the previous rank is, the more likely he will be bidding higher (the bidder in the second position is more likely to reduce the bid and enter the cease-fire phase), and the higher the new rank is, the higher he will be bidding (after all, each bidder is seeking the best position to introduce the cease-fire phase).

So far, we have validated the existence of the cycles, and the parameter estimates clearly show the distinctive properties of the two regimes. Our next objective is to characterize the cycles by applying the results of Markov chain theory.

From the MLE procedure, we are able to use the smoothed probabilities to obtain the latent state of each bid. By examining the change of states, using equation (7), we can calculate the transition matrix as shown in Table 2.

Table 1. Markov Switching Estimates (t-statistic in Parentheses)

	Regime 1 (War)	**Regime 2 (Cease Fire)**
$\hat{\beta}_0$ (Constant)	0.16037 (7.16)	0.42510 (29.15)
$\hat{\beta}_1$ (Previous Bid)	0.81296 (43.17)	-0.15285 (-4.97)
$\hat{\beta}_2$ (New Rank)	-0.03223 (-53.32)	-0.01823 (-12.51)
$\hat{\beta}_3$ (Previous Rank)	0.02775 (30.24)	-0.02238 (-17.91)

R^2 = 0.91, Log-Likelihood = -1930.42

Table 2. Transition Matrix

λ_{ij}	w	S
w	0.9540	0.0460
s	0.7889	0.2111

[13]Note that a "high" rank has a "low" numerical value—the highest rank is associated with a rank r = 1.

Table 3. Summary of Change in Bids in the Two Phases			
$\Delta y_{i,t} \equiv y_{i,t} - y_{i,t-1}$	mean	std	median
War Phase	0.06	0.061	0.02
Cease fire Phase	-0.28	0.129	-0.21

We can also obtain the limiting unconditional probabilities.

$$\pi = \begin{bmatrix} \lambda_w \\ \lambda_s \end{bmatrix} = \begin{bmatrix} (1-\lambda_{ss})/(2-\lambda_{ww}-\lambda_{ss}) \\ (1-\lambda_{ww})/(2-\lambda_{ww}-\lambda_{ss}) \end{bmatrix} = \begin{bmatrix} 0.945 \\ 0.055 \end{bmatrix}$$

So in the long run, about 94.5 percent of states are in the price war phase, and about 5.5 percent of states are in the cease fire phase.

By (8), we also know

$$E(duration\ of\ regime\ w) = \frac{1}{1-\lambda_{ww}} = 21.74,$$

and

$$E(duration\ of\ regime\ s) = \frac{1}{1-\lambda_{ss}} = 1.27.$$

We infer that a typical cycle lasts about 23 periods.

Table 3 gives further evidence about the bidding behavior in the two phases. Using the smoothed probabilities calculated from the Markov switching regression, we confirm that the behavior is consistent with the story of two phases. In the war phase, a typical bidder bids 2 cents higher than his previous bid, and in the cease fire phase, a typical bidder reduces the bid about 20 cents down from his last bid.

Conclusion

Selling advertisements with keyword auctions is one of the most successful online business models and has attracted a lot of discussion. Much of the existing literature has focused on the strategy of the service providers. Little attention, however, has been given to the advertisers who rely critically on the market and aggregately contribute to the success of this business model. This paper provides a framework to study the dynamic bidding behavior in online keyword auctions, where bidders compete for the placement of their advertisements. Different from most of the existing papers in the literature, we allow for both the bids and the ranks to be updated dynamically.

We adopt a Markovian framework in studying bidders' bidding behavior, thus bidders only focus on the actions directly related to their payoff. This relaxes the strict assumption in a super game where each bidder acts according to the entire bidding history. The concept of Markov perfect equilibrium (MPE) has been used to describe the cyclical bidding behavior of bidders. Our model is sophisticated enough to depict the dynamic nature of the advertisers' decision process, and at the same time simple enough to be tractable. We follow closely Maskin and Tirole (1988a, 1988b) in deriving our results but our analysis is different from theirs in important ways. While Maskin and Tirole consider a dynamic Bertrand price-setting game with two identical firms producing homogeneous goods under constant costs, we are examining an auction game where firms competing for nonhomogeneous goods with indeterminate expected costs.

Our empirical analysis confirms that the price competition in the keyword auction market is consistent with the MPE. This result is contrary to the assertion that the majority of bidders do not follow a strategy. Our analysis suggests that bidders are following the equilibrium strategy in this market, and the competition has reached the steady state.

Edgeworth cycles are rarely observed in practice. This is actually not a surprise. A few necessary conditions have to be satisfied first.

1. Low cost in learning competitors' price changes in order to respond quickly

2. Low menu cost: the price tags should be easily changed

3. Relatively homogeneous competitors: no player is significantly different from the rest of players

4. There is no (or nonessential) branding effect

Indeed, the Canadian retail gasoline market studied by Noel (2003, 2004) is a rare example of an offline business satisfying these conditions. In the keyword auction market, these conditions are easily satisfied.

This work leaves a number of questions unexplored. First, we only looked at the bidding history of one keyword. It would be interesting to compare across keywords to see how different keywords can have different cyclical patterns. Second, it would be interesting to examine whether other equilibrium outcomes can be observed in this market; again, data from more keywords are needed. Third, a closer look at individual-level behavior of the bidders would reveal a lot of new information. Fourth, since Yahoo! enabled the feature of proxy bidding, many advertisers have started to use this feature; some others have also started using third-party tools to implement more complicated bidding rules. The impact of these new features should be further studied.

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