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SLICING THE GORDIAN KNOT: A NOVEL MECHANISM FOR PROVIDING INNOVATION INCENTIVES FOR DIGITAL GOODS

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Abstract

Digital goods can be reproduced without cost. Thus a price of zero would be economically efficient in terms of eliminating deadweight loss. Unfortunately, zero revenues would also eliminate the economic incentives for creating such goods in the first place. We develop a novel mechanism which solves this dilemma by decoupling the price of digital goods from the payments to innovators while maintaining budget balance and incentive compatibility. Specifically, by selling digital goods via large bundles, the marginal price for consuming an additional good can be made zero for most consumers. Thus efficiency is enhanced. Meanwhile, we show how statistical sampling can be combined with tiered coupons to reveal the individual demands for each of the component goods in such a bundle. This makes it possible to provide accurate payments to creators which in turn spur further innovation. In our analysis of the proposed mechanism, we find that it can operate with an efficiency loss of less than 0.1 percent of the efficiency loss of the traditional price-based system. Innovation incentives in our mechanism are, of course, dramatically improved relative to the zero-price approach often favored by content consumers. The technology and legal framework for our proposed mechanism already exist and portions of it have already been implemented, although not in any coordinated fashion.

Keywords: Digital goods, bundling, innovation, incentives, mechanism design, information, online content

Introduction

Digital goods are different. Unlike other goods, perfect copies, indistinguishable from the original, can be created at almost zero cost. With the advent of the Internet, mobile telephony, satellite communications, broadband, and related technologies, these goods can be distributed to almost anyone in the world at nearly zero cost as well. Furthermore, when a person consumes a digital good, he doesn't reduce the stock for anyone else.

This should be a virtual nirvana. Yet, ironically, low cost digital goods are seen as a mortal threat to the livelihoods of many individuals, companies, and even whole industries. For example, digitized music has been blamed for an 8.9 percent decline in music CD sales in 2002 on top of a 6.4 percent decline in 2001. The availability of digital music is said to threaten the incentives for innovation and creativity itself in this industry. It has engendered a ferocious backlash, with thousands of lawsuits, fierce lobbying in Congress, major public relations campaigns, sophisticated digital rights management systems (DRMs), and lively debate all around.

Music is not the only industry affected. Software, news, stock quotes, magazine publishing, gaming, classified ads, phone directories, movies, telephony, postal services, radio broadcasting, and photography are just a few of the other industries also in

the midst of transformation. It's said to be difficult to predict the future, but a few predictions can be made with near certainty about the next decade: the costs of storing, processing, and transmitting digital information will drop by at least another 100-fold and virtually all commercial information will be digitized. While our colleagues in computer science, both in academia and industry, deserve much praise for this, it is incumbent upon information systems researchers to understand the business, social, and economic implications of these changes. Unfortunately, these implications have proven far less predictable.¹ What's more, we should go beyond prediction and seek to develop methods for maximizing the benefits from technological innovations while minimizing the costs.

Two schools of thought have dominated the debate on the economics of digital goods. One school stresses the benefits of the traditional market system. Clear property rights allow creators to exclude users from access to their creations. Users who wish to benefit from a creation must therefore pay the creator. This payment in turn assures that (1) the goods go to those individuals with the highest value for the good and (2) that the creator has incentives to continue to create valuable goods. This system has been pretty successful in modern market-based economies. To many people, it seems natural to apply the same principles to digital goods, typically via some combination of law (e.g., the Digital Millennium Copyright Act), technology (e.g., DRMs), and social education (e.g., the software industries ongoing anti-piracy public relations efforts).

Another school of thought thinks this approach is all wrong. "Information wants to be free," some argue. More formally, the point can be made that since digital goods can be produced at essentially zero marginal cost, the textbook economic principle of efficiency—"price equals marginal cost"—demands that price should never be greater than zero. After all, society as a whole is only made worse off if a user is excluded from access to a digital good which could have been provided without reducing the consumption of anyone else. While appealing, this approach begs the question of how to provide incentives for the goods creators. While some creators might continue to create for the shear joy of it, for indirect economic benefits such as enhancing their reputation or competency, or out of altruism,² economic systems and business models which rely solely on these motivations have historically not fared as well as those which provide more tangible rewards to innovators and creators.

Thus, the debate centers on who will be impaled on the two horns of the dilemma: should creators be deprived of the rewards from their creations or should users be deprived of goods which cost nothing to produce? Either approach is demonstrably suboptimal (see Lessig 2004). It would seem impossible to have both efficiency and innovation when it comes to digital goods. Improving one goal appears to be inextricably intertwined with hurting the other goal.

In this paper, we argue there is a third way. In particular, we develop and analyze a method for providing optimal incentives for innovation to the creators of digital goods. We show that it is possible to decouple the payments to the innovators from the charges to consumers while still maintaining budget balance. In this way, we can slice the Gordian knot and deliver strong incentives yet unhindered access to the goods for almost all interested consumers. In fact, we find that our system actually provides *better* incentives for innovation than the traditional price system, even when bolstered by powerful DRMs and new laws to enhance excludability and thus monopoly power.

We argue that it is misguided to try to force the old paradigm of excludability onto digital goods without modification. Ironically, DRMs and new laws are often used to strip digital goods of one of their most appealing, and economically beneficial, attributes: the ease of widespread use. At the same time, we take seriously the need to reward innovators financially if we wish to continue to encourage innovation and creativity.

The essence of our mechanism is to (1) aggregate a large number of relevant digital goods together and sell them as a bundle and then (2) allocate the revenues from this aggregation to each of the contributors to the bundle in proportion to the value they contribute, using statistical sampling and targeted coupons. We do this in a way which is fully budget-balancing and which provides accurate incentives for innovation with efficiency losses as small as 0.1 percent of the traditional price system. Furthermore, our mechanism provides better incentives for content creation than the traditional price based system where goods are sold individually and creators keep 100 percent of the revenues.

¹Although, as noted by Sorenson and Snis (2001), and by Lyman and Varian (2003) among others, we can predict with some confidence that there will be an increasing need for, and existence of, computer-supported codified knowledge and information, and the concomitant institutions for managing this information.

²These all seem to be factors in the success of open source software, for instance (see Crowston and Scozzi 2002).

Large digital collections are increasingly common as much Internet content moves from free to fee and as new forms of digital content, such as satellite radio, emerge. Consider XM radio, cable TV, AOL content, Rhapsody music, *Consumer Reports* reviews, JSTOR academic articles, and, last but not least, Microsoft Office software.

Bundling has been analyzed in some depth in the academic literature, including a cluster of articles specifically focusing on the bundling of digital information goods (e.g., Bakos and Brynjolfsson 1999, 2000 and the references therein). A key finding from the literature is that in equilibrium, very large bundles of information goods can provide content that is accessible to the vast majority of the consumers in the relevant market. It will not be profitable to exclude (via pricing) any consumers except those who simultaneously have unusually low valuations for virtually all of the goods in the bundle. Thus, bundling can dramatically increase economic efficiency in the allocation of information goods to consumers.

Given the prior literature on bundling information goods, our paper focuses on the second part of the mechanism, which involves designing a system for allocating revenues from such a bundle. This is necessary because by its very nature, bundling destroys the critical knowledge about how much each of the goods in the bundle is valued by consumers. Did I subscribe to XM radio for the classical music or for some other piece of content that was in the bundle? How much did I value each of these components? Unlike for unbundled goods, my purchase behavior for the bundle does not automatically reveal the answers to these questions. This creates a problem when it comes time to reward the creators and providers of the component goods. Surveys, usage data, and managerial "instinct" can all help allocate revenue to reward content creators, but none is likely to be anywhere as accurate as a true price-based system.

Our mechanism reintroduces prices, but only for a tiny fraction of consumers. For instance, in a large-scale implementation, only 1,000 consumers out of several million would face any prices for individual goods, typically via special coupons. This allows us to get accurate, unbiased assessments of value but because the vast majority of consumers do not face any non-zero price for individual goods, they incur virtually no inefficiency. Specifically, 99.9 percent of users have access to any given good as long as their value for that good is greater than zero and their values for all other goods in the bundle are not simultaneously unusually low.

The academic literature related to this part of our analysis is quite sparse. Some of the closest research is the work on a monopolist facing an unknown demand curve (e.g., Aghion et al. 1991) where it is shown that the seller can experiment by pricing to different buyers sequentially and updating the price accordingly. Some of the work on optimal market research is also relevant (e.g., Jain et al. 1995).

We are not aware of any systems that fully implement *both* parts of our mechanism, although bits and pieces are used in various industries and applications. For instance, as noted above, there are many examples of bundling for digital goods. Revenue allocation similar to our approach is more difficult to find. However, the American Society of Composers, Authors, and Publishers (ASCAP) does seek to monitor the consumption of its members' works and distribute its revenues to each creator in rough proportion to this consumption. However, they have no direct price data, and thus must work under the implicit assumption that all songs have equal value to each listener.

Thus, our paper both introduces a novel mechanism and rigorously analyzes it, finding that it is technically feasible and that it can dominate any of the approaches debated thus far. As noted by Fichman and Kemerer (1999) and by Iivari (1993), among others, feasibility and value are no guarantees of the success for an innovation. Barriers to diffusion and assimilation of this approach are likely to include overcoming knowledge barriers and some measure of organizational and institutional learning. Our analysis is meant to be a first step in addressing these obstacles. Notably, if this innovation succeeds, it should actually increase the pace of *future* innovations by improving incentives for the creation of useful digital goods. At a minimum, a broader discussion of this type of approach should change the terms of the existing debate about business models for digital goods.

The paper is organized as follows. The next section describes the basic assumptions and derives the asymptotic properties of massive bundling of information goods. The problem of revenue distribution in bundling is then introduced and the different types of solutions to this problem characterized. A mechanism to solve the revenue distribution problem is proposed and the convergence properties given. The traditional way of revenue distribution is shown not to provide a socially desirable innovation incentive, while our proposed mechanism can induce the correct innovation incentives. Practical issues of using the mechanism in the real world are discussed. The paper concludes with a brief summary and some implications.

A Basic Model of Bundling

Our goal here is to provide a theoretical framework to which we refer in later sections.

We consider a market with many providers of digital goods and many potential buyers. Digital goods are assumed to have a reproduction cost of zero.³ Therefore, any price greater than zero will be socially inefficient: some consumers (e.g., those with valuations less than the price but greater than zero) will be excluded from consuming the good even though it would be socially beneficial for them to have access to it. This is commonly called deadweight loss. In this section, we briefly review how bundling can radically eliminate this inefficiency, albeit at the cost of introducing a different problem involving incentives for innovation.

To be specific, suppose a monopolistic bundler connects the producers and the buyers by designing an optimal pricing and revenue distribution policy to maximize the bundler's profit. Each buyer has (at most) unit demand for any of the information goods. Suppose a buyer's valuations of the goods in the bundle are draws from a random variable *V* in the range normalized to [0,1], and that the random variable has a cumulative distribution function F(v), whose corresponding probability density function is f(v). In other words, a buyer's value for one good)e.g., a Britney Spears song) is independent of his value for an unrelated good (e.g., a news story about a local fire). At a price of *p*, the demand will be Q(p)=Prob(v>p)=1-F(p), yielding revenue of $\pi(p)=p[1-F(p)]$. This implies that the inverse demand curve is $P(q) = F^{-1}(1-q)$, and the bundler's problem is to solve

$$\pi^* = \max_{p} \{ p \cdot (1 - F(p)) \}$$
(1)

Taking first order condition, $\frac{\partial \pi}{\partial p} = (1 - F(p) - p \cdot \frac{\partial F(p)}{\partial p}) = 0$, we get

$$\frac{p^* \cdot f(p^*)}{1 - F(p^*)} = 1 \tag{2}$$

For the monopolistic bundler, it turns out that her profit maximizing decision is not hard. As shown in Bakos and Brynjolfsson (1999), the bundler's job is to find the optimal price for the sum of many random variables ($S_n = \sum_{i=1}^n v_i$). By the law of large numbers, it is easier to find an optimal price for the sum S_n than for individual goods v_i , because the coefficient of variation of S_n is decreasing as *n* becomes large.

In particular, it can be shown that for non-negative valuation, the expected value can be written as

$$E[X] = \int_0^\infty [1 - F_X(x)] dx$$
 (3)

Interestingly, this expression can be linked directly to the area under the demand curve. When price is v, demand is given by

Q(v) = 1 - F(v), so the area under the demand curve is just $\int_0^\infty Q(v) dv = E[V]$.

As shown by Bakos and Brynjolfsson (1999), in equilibrium, the profit maximizing price for a large bundle will be set low enough so that virtually all consumers interested in any of the goods in the bundle will choose to buy the whole bundle (even if they use only a small fraction of its components). For instance, most PC users buy Microsoft Office, even if they don't use all of its applications, or even all of the features of the applications that they do use. While there may be anti-competitive implications to this fact (see Bakos and Brynjolfsson 2000), such bundling does give the socially desirable result of dramatically reducing the deadweight loss because very few consumers are excluded from using any of the bundled goods in equilibrium. In essence, once consumers purchase the bundle, they can consumer any of the goods in the model at zero marginal cost. Thus, when the cost of (re)producing the goods is close to zero, bundling provides close-to-optimal allocation of goods to consumers (Bakos and Brynjolfsson 1999).

However these benefits come at a major cost. Bundling inherently destroys information about how each of the component goods is valued by consumers. Is the bundle selling because of the fresh sounds of a new artist or due to the lasting appeal of a traditional favorite? Without this information, it is impossible to allocate revenues to the providers of content in a way that accurately

³Digital goods typically satisfy this assumption easily. However, Bakos and Brynjolfsson (1999) show that the main results for bundling continue to hold even for small marginal costs.

encourages value creation. Selling goods individually would automatically solve this problem, but, as discussed above, individual sales create enormous inefficiencies because they exclude some users with positive value from access to the good.

Accordingly, the remainder of the paper studies the question of how to provide the correct rewards to content providers, and thereby give them financial incentives to create content.

The Revenue Distribution Problem

Bundling strategies help sellers to extract more consumer surplus. If one single seller can not provide enough numbers of information goods, it is worthwhile to have one content aggregator to negotiate with multiple sellers to offer a bundle of information goods from multiple sources.

The ideal revenue distribution mechanism would be one which somehow determined each good's demand curve, and distributed the revenue among the content providers in proportion to the social value of each good to all consumers. This value can be calculated by integrating the area below each good's demand curve. Various mechanisms used to derive demand curve proposed in the literature all fail here because bundle pricing does not automatically provide a way to observe the market's response to a price change of individual goods.

If the benefits created by each good cannot be observed or calculated, then a host of inefficiencies may result. First, the content providers may not have enough incentives to produce creative products, and consumers will eventually be harmed. Second, without a good signal of consumers' preference, content providers may not produce the content that best fit the consumers' taste. Third, in any effort to overcome these problems, the collection of content producers may force the potential bundler to adopt other strategies such as pay-per-view. However, such strategies reintroduce the deadweight loss problem discussed at the beginning of the previous section.

In the following subsections, we discuss the costs and benefits of several ways to distribute revenue to address this challenge, culminating with our proposed statistical couponing mechanism.

Payment Determined by Number of Downloads

In the context of digital information goods, it is natural to assume that the seller may be able to observe the number of times that each good is accessed. This gives us the following solution.

If one is willing to assume that the number of accesses signals popularity and popularity is a measure of value, we can infer the value by the number of accesses. Traditionally, this scheme is broadly used in the market of digital goods such as music, movies, TV shows, and software. For example, each episode of *Friends* had about 29 million viewers per week last year, which was far more than most other TV shows; as a consequence, each of the six stars was paid \$1.2 million per episode, which was far more than most other TV actors.

More formally, suppose we have *n* goods in the bundle, the price for the bundle is *B*. Also suppose there are *m* buyers of the bundle, each represented by j (j = 1, ..., m), then the total bundle revenue is $R = B \cdot m$. We assume the system can record the number of downloads of buyer *j* for good *i*: d_{ij} , then the provider of content *i* should be paid

$$revenue_i = R \cdot \sum_{j=1}^m \frac{d_{ij}}{\sum_{k=1}^n d_{kj}}.$$

This method is extremely easy to implement. In fact, the last equation implies that the bundler does not even have to keep record of all the downloads made by the *m* buyers; she can simply record d_i , the number of times good *i* has been downloaded.⁴

⁴Since
$$\sum_{j=1}^{m} \frac{d_{ij}}{\sum_{k=1}^{n} d_{kj}} = \frac{\sum_{j=1}^{m} d_{ij}}{\sum_{j=1}^{m} \sum_{k=1}^{n} d_{kj}} = \frac{d_i}{\sum_{k=1}^{n} \sum_{j=1}^{m} d_{kj}} = \frac{d_i}{\sum_{k=1}^{n} d_k}$$
, no *j* appears in the final term.

This method is powerful in the context when all the goods are approximately equal in value. If goods differ in value (bundling very cheap Joke-A-Day with more expensive Forrester Research Report), then pricing based on number of downloads is misleading. (The Joke-A-Day may be downloaded more times than the Forrester Research Report, but aggregate value of the latter may be much higher to consumers.)⁵

Payment Determined by Downloads Combined with a Stand-Alone Price

Number of downloads itself is not a good measure of consumer valuation in many cases. Assuming there also exists a stand-alone price for every information good in the bundle, and assuming these prices are all fair prices, we can then derive an improved mechanism to distribute the revenue.

Consider the market introduced in the previous subsection, suppose each item i (i = 1, ..., n) in the bundle also has a stand-alone price p_i .

Building on the equation from the previous subsection, an improved way to distribute the revenue is through the following formula:

$$revenue_i = R \cdot \sum_{j=1}^{m} \frac{p_i d_{ij}}{\sum_{k=1}^{n} p_k d_{kj}}$$
(4)

which says that the revenue to distribute to content provider *i* should be a proportion of the total revenue ($R = m \cdot B$), and the proportion is determined by the sum of each consumer's valuation of good *j*.

This method has the advantage of being more precise comparing to the previous solution. Indeed, if Joke-A-Day is sold separately, its price will probably be much lower than that of Forrester Research Report. The disadvantage of this method is that a fair and separate price may not always be readily available. If the distribution of revenue is set according to this method, and when bundling becomes a major source of revenue, there is room for content providers to misrepresent the stand-alone price. Furthermore, this approach implicitly assumes that the value from each good is proportional to the stand-alone price. However, this will only be true if the price paid by the marginal consumer of each good is proportional to the average price that would be paid by all consumers of that good, for all goods.⁶

Other Mechanisms

In his forthcoming book, William Fisher (2004) explores various solutions to the music piracy problem brought about by the new peer-to-peer technology. Specifically, he proposes to replace major proportions of the copyright and encryption-based models with a "governmentally administered reward system," and he correctly points out that what we really need is not the number of downloads, but the "frequency with which each recording is listened to or watched" (i.e., the real value to consumers). Fisher's proposal is similar to the Nielsen TV sampling approach, and he proposes to implement special devices to estimate the frequency of listening recording receives. He also suggests that the frequency should be multiplied by the duration of the works, and that consumer's intensity of enjoyment (obtained through a voting system) should be taken into consideration to make more precise estimates of the valuations.

This proposal, if carried out, should be superior to the current practice taken by ASCAP (and BMI, SESAC, etc.) to compensate music producers, and it comes very near to our ideal of learning consumers' valuations and distribute money accordingly, but it also suffers from several inherent problems. First, unlike from Nielson TV sampling, people may use different devices to enjoy the same digital content. For example, a song can be played with an MP3 player in the car, a CD player in the home entertainment

⁵Another problem with this method is that it gives dishonest content providers a way to distort the values by manipulating the number of downloads of their own content. This has been a problem, for instance, with some advertising-supported content where prices are based on thousands of impressions recorded (Hu 2004).

⁶Barro and Romer (1987) explore how similar proportionalities can explain a number of pricing anomolies.

system, or a DVD drive on a computer. Second, and more critically, as shown in the public goods literature, votes are not reliable because individual hidden incentives may induce voters to misrepresent their true values. For instance, consumers might falsely claim to have an extremely high or low value for a good in an attempt to influence the voting. In essence, the Fisher approach still does not provide a reliable, incentive-compatible way to determine the true value of each good to consumers.⁷

Couponing Mechanism

As discussed in the previous section, the ideal way to provide correct incentives is to learn consumers' valuations for each good and make corresponding payments. Since bundling itself obscures consumers' valuations for individual goods, here we propose a mechanism to reveal the demand curve for each good by issuing targeted coupons to a small sample of consumers. For large populations, it is possible for the targeted sample to be large enough to be representative statistically while still being small enough to be fairly unimportant economically. Our mechanism is substantially different from the traditional use of coupons as a marketing method to price discriminate consumers. Instead, coupons in our mechanism are similar to the price experiments suggested in the optimal pricing literature.

Suppose the monopolistic bundler offers a bundle of information goods to a group of consumers. In order to derive the demand curve for one of the components, we choose $m \cdot n$ representative consumers and issue each of them a single coupon, where n is the number of price levels covering the range of the valuations, which we call *coupon levels* (one simple way to get these levels is to offer coupon values from $\frac{1}{n}\overline{V}$ to $\frac{n-1}{n}\overline{V}$ where \overline{V} is the upper bound of consumer valuations for this good), and m is the number of coupons to be offered for each of the price levels in total, which we call *sample points* (there will be m consumers who can receive a coupon with face value $\frac{i}{n}\overline{V}$, i = 1,...,n-1). While $m \cdot n$ should be large enough to make statistically valid inferences, it can nonetheless be a very small fraction (e.g., 1/1000 or less) of the total set of consumers buying the bundle.

If a consumer receives a coupon with face value \tilde{v} , then he can either choose to ignore the coupon and enjoy the complete bundle or choose to redeem the coupon and forfeit the right to use the indicated component. So upon observing a consumer's action, the bundler can learn whether that consumer's valuation is higher or lower than the face value of that particular coupon. Aggregating the *m* consumers' valuations will give the bundler a good estimate of the valuations at that price, summarizing the results for the *n* coupon levels, the bundler can plot a fairly accurate demand curve.⁸ The area under the resulting demand curve is the total social valuation for that particular good, and also the maximum revenue which that good can contribute to the bundle revenue. Using the same method for all the components, the bundler can learn the approximate social valuation and revenue potential of each of the goods in the bundle. She can then distribute the revenue among the content providers according to their contribution share to the total valuation. Let *R* be the total revenue from selling bundles and v_i be the social value of the component *i* in the bundle, content provider of *i* should be paid

$$revenue_i = R \frac{v_i}{\sum_{j=1}^N v_j}$$

where N is the total number of content providers.

⁸An alternative approach to revealing consumer demand would be to require the targeted consumer to pay an offer price (i.e., some price from $\frac{1}{v}\overline{V}$

⁷The public goods mechanism design literature seeks to provide a remedy to the voter misrepresentation problem. Specifically, the Vickrey-Clarke-Groves (VCG) mechanism can be shown to induce truth-telling by all participants. However, it has two fatal flaws. First, it is not budgetbalancing—significant inflows (or net penalties) are generally needed. Second, it is quite fragile. Each participant must believe that all other participants are truth-telling or he will not tell the truth himself. Accordingly, while VCG design is intriguing in theory, it is rarely, if ever, seen in practice.

to $\frac{n-1}{n}\overline{V}$ depending on his draw) in order obtain access to the selected good. Consumers who did not pay the relevant price would not have

access to that good. As with the coupons, only consumers with a value greater than the relevant offer prices would choose to consume the good, thereby revealing the demand curve.

This method compares favorably to the traditional price mechanism. The traditional price mechanism subjects 100 percent of consumers to the inefficiency of positive prices. However, only data from a small fraction of consumers are needed to get extremely accurate estimates of the value created and contributed by each good. The greater precision obtained by increasing the sample declines asymptotically to zero while the cost for subjecting each additional consumer to a positive price remains just as high for the last consumer sampled as for the first one. When balancing the costs and benefits, the optimal sample size is almost surely less than 100 percent. Secondly, the proposed couponing mechanism actually provides a *more* accurate estimate of the overall demand curve than any single-price traditional system. Because multiple different prices for coupons are offered, a much more accurate overall picture of demand can be obtained than simply revealing the demand at a single price, as conventional prices do. As discussed in the next section, this has large and important implications for dynamic efficiency and innovation incentives.

One can also compare our couponing mechanism with that of the well-known Vickrey-Clarke-Groves (VCG) mechanism. We find that our couponing mechanism does not give us exact valuations for each consumer unlike the VCG. However, in general, all approximate demand functions of the components will suffice, and by increasing the sample size, the accuracy can be made almost arbitrarily precise. Our couponing mechanism is superior to the VCG mechanism in several ways. (1) Truth-telling is a robust and strong equilibrium in the couponing mechanism, in the sense that each consumer simply compares his valuation with the coupon's face value; he is not required to assign correct beliefs on all other people's votes. (2) In the VCG, if one respondent misreports his value (due to irrationality or due to error), the consequence may be very severe for the rest of the people. Furthermore, coalitions of consumers can "game" the VCG to their advantage. However, in the couponing mechanism, the effects on others from a consumer's misreport are minimal. (3) The couponing mechanism is fully budget balancing, unlike the VCG. (4) The couponing mechanism is more intuitive than the VCG for real world problems.

The following proposition asserts that the couponing mechanism indeed gives us correct demand curve estimations in expectation.

Proposition 1: For any one of the components in the bundle, given a large number of randomly chosen respondents and level of coupons, the above mechanism gives an empirical demand function $\hat{Q}(p) = 1 - \hat{F}_V(p)$ that arbitrarily approximates the true demand function: $Q(p) = 1 - F_V(p)$.

Proof: All proofs are in the appendix.

Proposition 1 gives an asymptotic result; we run simulations to see the effectiveness of this mechanism.

The use of our coupon mechanism gives us empirical estimates of the inverse demand curves for each of the distributions, and we define the error rates to be the percentage differences between the area under the empirical demand curve and the area under the true demand curve. Figure 1 shows the result of the coupon mechanism applied to the uniform distribution; other distributions yield similar figures. We see that error rate is declining with more coupon levels and with more sample points for each coupon value. It is remarkable that with just 20 coupon levels, the error rate is as low as 5 percent. Adding more sample points for each coupon level (for a total of 800 respondents) gives us an error rate of 15 percent, and sampling 80 consumers improves the error rate to be near 5 percent. From the error rate curves, we can also see that when sampling 20 consumers, adding coupon levels beyond 10 does not improve the precision significantly; also, when sampling 80 consumers, adding coupon levels beyond 15 does not improve the precision significantly; also, when sampling 80 consumers, adding coupon levels beyond 15 does not improve the precision significantly; also, when sampling 80 consumers, adding coupon levels beyond 15 does not improve the precision significantly. This observation tells us that we have to add coupon levels and sampling points simultaneously in order to achieve the best result estimating the social values of goods. Error rate converges toward 0 more quickly for fatter demand curves (the ones with a higher expected value). In our simulations, for some demand curves, with just 5 coupon levels and 20 sample points (for a mere 100 respondents), the coupon mechanism can give us an error rate below 0.1 percent. Thus, sampling just 100 consumers can provide almost as accurate an estimate of demand as sampling the entire population of consumers of the good, which could be in the millions.

The deadweight loss is proportionately smaller, too. Consumers who cash-in the coupons forgo access to the corresponding good, which creates a deadweight loss (unless the consumer's value was exactly zero). For such a consumer, this decision is analogous to facing a market price, with similar costs, benefits, and overall incentives. However, in contrast to the traditional price approach, the couponing mechanism only subjects a fraction of consumers to this decision, so only a fraction chose not to buy, and the total deadweight loss is only a fraction of what it used to be.

This mechanism can be used to solve the revenue distribution problem discussed in section 2, it turns out that there are some additional benefits related to innovation incentives of the content providers.

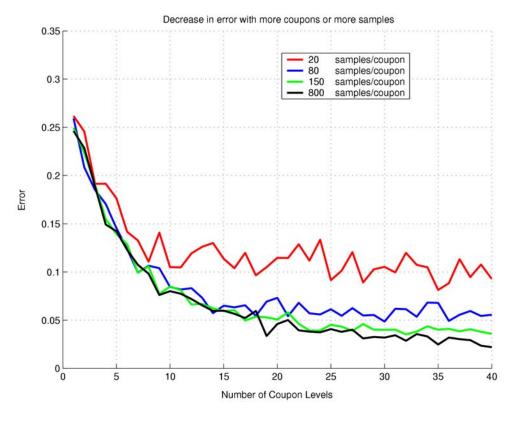


Figure 1. Simulation Results for the Couponing Mechanism

Innovation Incentives

In this section, we show that, contrary to common belief, the traditional price system based on excludability does not provide correct innovation incentives to producers. Thus, the proposed couponing mechanism can be a socially desirable way to promote innovation for digital goods.

Assume that the seller can invest in trying to create an innovation which improves consumers' valuations of her digital good. The investment can be in the form of improving product quality, functionality, or educating users to use the product more effectively. We now compare the innovation incentives of the seller under traditional pricing and under bundling combined with couponing.

Uniform Enhancement

Suppose the innovation can increase each consumer's valuation by δ , this is equivalent to moving the demand curve upward by δ .

Case 1 (Traditional Market): when the demand is shifted upward, the monopolistic seller can do some combination of two things: charge a higher price of $p^* + \delta$ or still charge the price p^* and sell to more people (the demand will be $q'=1-F(p^*-\delta)$ now). We next show, in lemma 1, that both strategies lead to the same expected profit for the seller.

Lemma 1: Marginally, the innovative monopolist seller can charge a higher price or enjoy an increased demand, and the two strategies are equivalent in terms of expected profit.

Lemma 1 says that in Figure 2, the area A and the area B should be equal. Essentially, by taking some effort, the seller can get expected marginal gain $A = B = \delta \cdot [1 - F(p^*)]$.

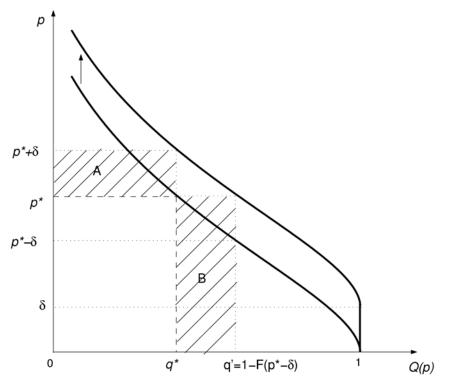


Figure 2. A Uniform Upward Shift of Demand Curve

Case 2 (Bundling with coupon mechanism): when the demand is shifted upward, the seller can get paid virtually the full amount of the extra valuation it created for the consumers. Let the original profit be $\pi = E[V] = \int_0^\infty [1 - F(v)] dv$, she can now earn $\pi' = \delta + \pi$.

Since p^* is the optimal price, and can never be zero, we have $1 - F(p^*) < 1$. So the marginal profit of innovation from bundling $\Delta \pi_T = \delta$ is strictly greater than the marginal profit of innovation from traditional market $\Delta \pi_T = \delta \cdot [1 - F(p^*)]$. Accordingly, we conclude that sellers' innovation incentive in the bundling scheme is higher than that in the traditional market. We can write this conclusion in the following proposition:

Proposition 2: If an innovation can increase consumers' valuations uniformly higher, the proposed mechanism gives the producer strictly greater incentives of innovation than does the traditional market mechanism.

Targeted Innovation

We have assumed above that the innovation can uniformly increase consumers' valuations of all types. Now we look at the case that innovation can only affect a small subset of consumers' valuations. In particular, the innovation may be less significant so that only some consumers with valuation near some \tilde{v} are affected. For instance, a software developer could invest in adding features which would either (1) make satisfied users of its product even more satisfied, or (2) increase the value to consumers whose values were just below the market price, turning them into buyers, or (3) features which would increase the value of non-buyers but not enough to turn them into buyers. Suppose that the developer has a finite budget and can only pursue one of these three types of innovations. Even if innovations of type (1) or (3) might create more value, the traditional price system will only provide incentives for innovation (2). In contrast, combining bundling with couponing can provide balanced incentives for all three types of innovations, leading the developer to pursue any innovations whose expected benefits exceed expected costs.

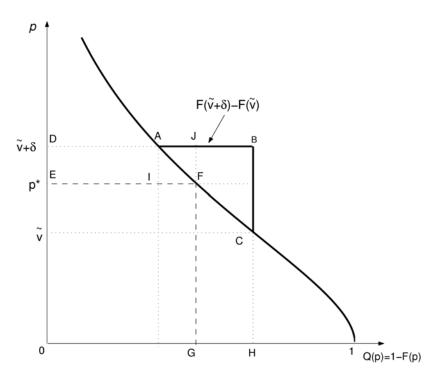


Figure 3. Social Benefit/Loss vis-a-vis Seller Innovation

More formally, the total potential social value of the good equals the area under the demand curve. If the seller makes a targeted innovation for some consumers with valuation \tilde{v} , the social gain of the innovation *ABC* is thus denoted by the area in Figure

3. When
$$\delta$$
 is small, $\Delta ABC \approx \frac{1}{2} \delta [F(\tilde{v} + \delta) - F(\tilde{v}) \approx \frac{1}{2} \delta^2 f(\tilde{v}).$

We shall need the following technical assumption to get a well-behaved demand curve.

Assumption: F(v) is twice continuously differentiable with F(0) = 1, F(1) = 1, f(v) > 0, $\forall v > 0$ and $\frac{1}{1 - F(v)}$ is strictly convex for $v \in (0,1)$.

This assumption is implied by log-concavity of 1 - F(v), which itself is implied by log-concavity of f(v). The intuitive meaning of the assumption that the random variable *V* has a log-concave density function is that it has a unique global maximum. Note that this assumption implies that the profit function $p \cdot [1 - F(p)]$ is concave.⁹

Given any innovation that increases some consumers' valuation by δ , there exists some, such that the seller is indifferent between carrying out the innovation and not carrying out the innovation. For the seller,

$$(\bar{v} + \delta)[1 - F(\bar{v})] = p^*[1 - F(p^*)]$$
⁽⁵⁾

⁹Log-concavity property is frequently assumed in the economics literature. See, for example, Laffont and Tirole (1988) in the context of games with incomplete information; Baron and Myerson (1982) in the context of theory of regulation; Myerson and Satterthwaite (1983), among others, in the context of auction theory. It is also well known that log-concavity of the density function implies the notions of IFR (increasing failure rate), and NBU (new better than used) in survival and reliability analysis literature (Barlow and Proschan 1975). Bagnoli and Bergstrom (1989) give a good review. In our context, log-concavity is sufficient to guarantee that solutions are unique and well-behaved.

and solving for \overline{v} , we have two values, \overline{v}_L and \overline{v}_H , such that $p^* \in (\overline{v}_L, \overline{v}_H)$ that satisfy (5).¹⁰ Also, $\forall v \notin (\overline{v}_L, \overline{v}_H)$, it must be that $(v + \delta)[1 - F(v)] < p^*[1 - F(p^*)]$, so the seller has no incentive at all to innovate for consumers with valuations outside the range $(\overline{v}_L, \overline{v}_H)$. This is very intuitive, in the traditional market, if the seller sells goods to consumers with valuation higher than \overline{v}_H , it makes no sense to increase their valuations further because that will only contribute to consumer surplus, and the seller will not be able to extract the added value. Similarly, for the potential consumers with lower valuations (lower than \overline{v}_L , to be precise), the seller will not take the effort to innovate because they will not be converted to consumers. For small δ , the range $(\overline{v}_L, \overline{v}_H)$ is very small, and even in this range, innovation may not be socially desirable.¹¹

Thus, in the traditional price mechanism, the seller has too little incentive to create innovations that mainly benefit consumers with very low or very high valuations. The lesson learned: if your valuation is substantially above or below the equilibrium price for a good, don't expect the good's provider to put significant effort into innovating to specifically address your needs.

In contrast, the couponing mechanism for bundling creates correct incentives for sellers. The seller will not discriminate against consumers with low or high valuations. To see this, suppose the seller takes some effort, and increases the valuation from \tilde{v} to $\tilde{v} + \delta$. Her expected reward is always the area *ABC*. This brings us to Proposition 3.

Proposition 3: If an innovation can increase only some consumers' valuations, the traditional price system does *not* provide correct incentives for the producer to innovate for people with relatively high or relatively low valuations. In contrast, the proposed mechanism always gives the producer a socially desirable level of incentives to innovate.

To see the socially wasteful incentive of innovation in the traditional price system, consider the case of the consumers' valuations near the optimal price. For example, if the seller takes an effort to innovate and increases the valuation for some consumers from

¹¹For consumers with valuation in the range $(\overline{v}_I, \overline{v}_H)$, one can look at three distinct cases:

(1) $v \in (\overline{v}_L, p^* - \delta)$. In this case, the seller would want to charge a price $p = v + \delta$ and earn profit $\pi = (v + \delta)[1 + F(v)]$. By lemma 1, $\pi > (\overline{v}_L + \delta)[1 - F(\overline{v}_L)] > p^*[1 - F(p^*)]$. So the seller prefers to lower the price from p^* to $p = v + \delta$, and earn a higher profit. The reduction in price has two socially desirable effects. First the consumer surplus is increased. For people with valuation in the range $(v + \delta, p^*)$, they are no longer excluded from accessing the good; and for people with valuation in the range $(p^*, +\infty)$, they can each enjoy an increase in consumer surplus of $\Delta CS = p^* - (v + \delta)$. Second, deadweight loss is reduced, the change in dead-weight-loss is $\Delta DWL = [F(p^*) - F(v + \delta)]\{v + \delta\}$. The reduction in deadweight loss is composed of two parts: first, for people with valuation in the range $(v + \delta, p^*)$, apart from the increase in consumer surplus, there is also reduction in DWL due to the fact that their demand is satisfied; second, for people with valuation in the range $(v, v + \delta)$, the innovation increases their valuation, and they are no longer excluded from purchasing the good.

(2) $v \in (p^*, \overline{v}_H)$. In this case, the seller innovates for people with valuation just higher than the optimal price. By lemma 1, we know it is worthwhile for her to increase the price to $v + \delta$; however, there are two socially undesirable effects associated with this. First, for consumers originally having a valuation above $v + \delta$; however, there are two socially undesirable effects associated with this. First, for consumers originally having a valuation above $v + \delta$; however, there are two socially undesirable effects associated with this. First, for consumers originally having a valuation above $v + \delta$; however, there are two socially undesirable effects associated with this. First, for consumers $(v, v + \delta)$, although their valuation is increased due to the innovation, they no longer enjoy a surplus now. Second, for people with valuation in the range (p^*, v) , they can no longer afford to buy the good now, so there is an increase in deadweight loss.

(3) $v \in (p^* - \delta, p^*)$. This case has mixed effects. On one hand, there are socially desirable effects as in 1, and on the other hand, there are also socially undesirable effects as in 2. For people with valuation higher than $v + \delta$, they suffer a reduction in consumer surplus by $\Delta CS = v + \delta - p^*$. In Figure 3, the loss is indicated by the area *ADEI*. For people with valuation in the range $(p^*, v + \delta)$, due to innovation, they have a higher valuation now, but due to the increased price, they no longer enjoy a surplus (the area *AIF*). For people with valuation in the range (v, p^*) , their valuation is increased to $v = \delta$, but again, the seller gleans all the surplus due to innovation. A socially desirable side effect is that the deadweight-loss is reduced because this group of people is able to use the product now. The area *FGHC* indicates the social gain from reduced deadweight loss. In total, consumer surplus is hurt by the area *ADEF*, deadweight loss is reduced by the area *FGHC*, and the seller enjoys the extra value created by innovation indicated by area *ABC*.

¹⁰This is a direct consequence of the assumption of log-concave density function. Here we omit a formal proof of the existence and uniqueness of \overline{v}_L and \overline{v}_H , which can be derived by fixed point theorem.

 p^* to $p^* + \delta$, then her gain is $\delta[1 - F(p^*)]$. The ratio of her gain over her contribution is *incentive_ratio_{Traditional}* = $\delta[1 - F(p^*)]/[\frac{1}{2}\delta^2 f(p^*)] = 2\frac{1 - F(p^*)}{\delta f(p^*)} = \frac{2p^*}{\delta}$, and $\lim_{\delta \to 0}$ *incentive_ratio_{Traditional}* = ∞ . For the case of the proposed

mechanism, the *incentive* _*ratio*_{Bundling} = $\frac{1}{2}\delta^2 f(\tilde{v})/(\frac{1}{2}\delta^2 f(\tilde{v})) = 1$, which is fair. So we have the following proposition:

Proposition 4: The traditional market gives the producer too high an incentive to innovate where it is most harmful to the social welfare, no incentive elsewhere; the proposed mechanism induces the producer to make socially desirable innovation efforts.

Discussion

This paper contributes to establishing a more efficient approach to create, distribute, and consume digital goods. The theoretical foundation proposed here is just the first step toward this goal; in order to build viable business models, we need to address some practical issues to be discussed below.

In this paper, couponing has been analyzed solely as a mechansism for revealing existing demand, not for influencing it. Of course, in practice, couponing may also be viewed as a form of advertising that increases demand. If it increases more for some kinds of goods, and not for other kinds, then the estimates of values may be biased in a non-uniform fashion. There is a related, more conspicuous problem: due to the heterogeneity in people's tastes, some goods are surely downloaded less than some others (consider a Forrester report: maybe only tens out of millions of consumers would want to download it), if we do not offer enough sampling points, there will be a bigger error in estimating demand for these less popular goods. It turns out that both issues can be easily addressed by a practice we call *passive couponing*. Under a passive couponing regime, only those who downloaded a good will be offered a coupon for that good. After downloading, the consumer learns all the product characteristics, so the informative role of couponing as advertising is no longer valid. For goods downloaded by the majority of people, we can choose a small fraction out of them to offer coupons, and for goods downloaded only by a few, we may offer coupons to most or all of them. In either case, subsequent access to that good, or similar goods, can be restricted for consumers who prefer to redeem the coupon instead. By discriminating coupons offered to different types of goods, we can get a better overall estimate of the specific demands.¹²

In previous sections, we avoided the issue of duration of contracts. It is likely to be unnecessary to permanently block from access to a good for consumers who redeem the corresponding coupon. Temporary blockage will generally suffice. We can put this question into the context of subscription-based business models. Suppose the bundle is to be paid by month (e.g., \$20 per month), then for time-critical information goods (e.g., news, stock quotes, etc.), we can offer the coupons by month, too (e.g., "Take this \$1 coupon and sacrifice CNN news for the next month"). For those less time-critical information goods (e.g., music, software updates, etc.), we can offer the coupons by longer periods (e.g., "Take this \$10 coupon and give up downloading Madonna for the whole next year"). The mechanism proposed here may not work as well with information goods whose value is time-invariant (e.g., Symphony No. 9 by Beethoven). Depending on the nature of the DRM system used, once someone downloads a copy of the work, there may be no point in offering coupons because the consumer might not need to download any more copies in the future.

Conclusion

Revolutionary technologies often engender innovations in business organization. The digitization of information is no exception. We seek to advance the debate on how best to allocate digital goods and reward their creators by introducing a novel mechanism and analyzing its implications. Our approach eliminates the marginal cost of consuming digital information goods for the vast majority of consumers via massive bundling. For very large aggregations, this preserves most of the static efficiency which could be achieved with a zero price policy. However, in the long run, the more important issue is how to create incentives for ongoing

¹²What if a good is only downloaded by one consumer? First of all, in this case, if this good is not important in the bundle, the bundler can exclude it in the future. Second, the bundler can offer this consumer a different coupon in each period with the face value determined by a random draw. Within some periods of sampling, the bundler can still extract the true value, the math works exactly the same as in the proof of proposition 1. It can also be easily shown that there is no incentive for the consumer to misreport his value in each period.

innovation. Indeed, our living standards, and those of future generations, depend far more on continuing innovation than on simply dividing up the existing set of digital goods. In this area, the proposed mechanism shows particular promise. We find that our approach can provide substantially better incentives for innovation than even the traditional monopoly price system bolstered by artificial excludability (e.g., via DRMs, laws, etc.). In particular, the traditional price system, in which each good is sold for a specific price with the proceeds going to the monopolist creator, focuses virtually on incentives for innovations that help this narrow group of consumers. Rents transferred to the creator from such innovations exceed the social benefits. In contrast, our approach, using statistical sampling and couponing, can provide incentives which are nearly optimal for every type of innovation.

In summary the mechanism we introduce

- potentially has orders of magnitude less inefficiency than the traditional price system
- is budget balancing, requiring no external inflows of money
- works with existing technology and the existing legal framework
- requires no coercion and can be completely voluntary for all parties, since it is fully incentive compatible
- · doesn't assume that innovators will continue to innovate even without financial rewards
- can be implemented and run in real-time
- is scalable to very large numbers of goods and consumers (in fact, works better for larger numbers)

Our approach also has weaknesses and challenges. Compared to giving away all digital goods for free, our approach will exclude a small number of consumers and create some inefficiency as a result. More importantly, our approach does require the creation of new business institutions or models, which is never easy. Specifically, an entity is needed to manage the statistical sampling and couponing, analyze the resulting data, and allocate payments to the content owners accordingly. Near misses for this type of entity already exist. For instance, ASCAP does much the same thing already for broadcast music, but without accurate price information. Nielsen and similar organizations provide usage information, but again without accurate price information. There are organizations which regularly collect and distribute large sums of money to member companies based on various algorithms. The Federal Deposit Insurance Corporation which does this for banks is one example. Some cooperatives are also run this way. Last but perhaps not least, the government regularly makes these types of transactions. However, it should be stressed that our mechanism does not require any government role since all of the participants (consumers, content creators, bundlers) have incentives to participate completely voluntarily and it adheres to the existing legal framework. This stands in contrasts to the proposal by Fisher (2004) or the varied proposals to change copyright or other laws.

By offering this new framework and analysis, with a new set of opportunities and challenges, we hope to lay the foundation for future research on the critical question of providing incentives for innovation in the creation of digital content and implementing mechanisms to deliver that content to consumers efficiently.

We expect that the next 10 years will witness a scale of organizational innovation for creating and distributing digital goods surpassing even the remarkable pace of the last 10 years. New coordination mechanisms, such as the innovation incentive approach described and analyzed in this paper, will flourish. With a proactive attitude toward technology-enabled organizational innovation, we believe that academia can speed this process by framing the issues, and by providing tools, "cookbooks," and analyses.

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Appendix. Proofs

Proof of Proposition 1: We prove proposition 1 in two steps. First, we show that for each price level, the mechanism offers a consistent estimate of the true demand at that level. Second, we show given enough price levels, the demand curve can be arbitrarily closely approximated.

For one particular component, the seller first chooses the number n of coupon levels, then, for each coupon level, sends m coupons to m randomly chosen consumers. For a coupon with face value \tilde{v} for the component, the respondent will take it only if he has a valuation lower than \tilde{v} . The probability of the coupon getting accepted is $prob(V \leq \tilde{v}) = F_v(\tilde{v})$. We now define indicator variables Y_1, \ldots, Y_m where Y_i is 1 if the coupon with face value \tilde{v} is accepted by the i^{th} consumer, 0 otherwise. We have $Y_k = \begin{cases} 1 & X_k \leq \tilde{v} \\ 0 & X_k > \tilde{v} \end{cases}$, where $k = 2, \ldots, m$. Note that $prob(Y_k = 1) = prob(X \leq \tilde{v}) = F_v(\tilde{v})$, and $prob(Y_k = 0) = prob(X > \tilde{v}) = 1 - F_v(\tilde{v})$. For all the *m* people to whom we sent coupon \tilde{v} , we know the number of the experiments telling us what percentage of people accept the coupon \tilde{v} . We can show the expected value of the empirical cdf is the true unknown cdf. $E[\hat{F}(\tilde{v})] = E[\frac{a_m}{m}] = \frac{m \cdot E[Y]}{m} = E[Y] = 0 \cdot prob(Y = 0) + 1 \cdot prob(Y = 1) = F_v(\tilde{v})$. That completes

the step 1.

Next consider the interval between any neighboring coupon's value levels. For explanatory purposes, we now assume that the seller sets equi-distance intervals on the value range [0,1], that is, the coupon values are $0, \frac{1}{n}, ..., \frac{n-1}{n}$. Our result does not rely on this assumption; it holds as long as the distances are all weakly shrinking when adding more coupon levels.

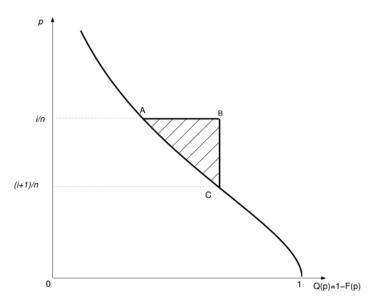


Figure A1. The Upper Bound of Error in Estimating Demand

For neighboring coupon levels $\frac{i}{n}$ and $\frac{i+1}{n}$, the seller may estimate points A and C from step 1. She can simply connect the estimated points to approximate the demand curve between the two points. Since the demand curve is monotonically decreasing from 1 to 0, when estimating the area below the demand curve, the triangle ABC is the upper bound for the error. The area of ABC is $\Delta ABC = \frac{1}{2}(\frac{i+1}{n} - \frac{i}{n})[\hat{F}(\frac{i}{n}) - \hat{F}(\frac{i+1}{n})]$. We know $\hat{F}(\frac{i}{n}) - \hat{F}(\frac{i+1}{n}) \le 1$, and given the assumption that $F_V(x)$ is continuously differentiable. We have $\lim_{n\to\infty} [\hat{F}(\frac{i}{n}) - \hat{F}(\frac{i+1}{n})] = 0$, so we have

$$n$$
 n n

$$\lim_{n \to \infty} \Delta ABC = \frac{1}{2} (\lim_{n \to \infty} \frac{1}{n}) \cdot [\lim_{n \to \infty} (F(\frac{1}{n}) - F(\frac{1}{n}))] = 0$$

which says that when n is large enough, the error in estimation will converge to 0.

Proof of Lemma 1: We need to compare $\pi_p = (p^* + \delta)[1 - F(p^*)]$ and $\pi_q = p^*[1 - F(p^* - \delta)]$, and show that as $\delta \to 0$, they are equal. Equivalently we need to show: $\lim_{\delta \to 0} (p^* + \delta)[1 - F(p^*)] = \lim_{\delta \to 0} p^*[1 - F(p^* - \delta)]$, which is

Q.E.D.

$$\lim_{\delta \to 0} \frac{F(p^*) - F(p^* - \delta)}{\delta} = \frac{1 - F(p^*)}{p^*} \Leftrightarrow f(p^*) = \frac{1 - F(p^*)}{p^*}, \text{ which is true due to equation (2).} \qquad Q.E.D$$