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# Optimal Design of Contingency Pricing in IT-Intensive Commerce

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# OPTIMAL DESIGN OF CONTINGENCY PRICING IN IT-INTENSIVE COMMERCE

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## Abstract

*We propose the use of quality contingent prices, where a firm announces quality-price pairs for various levels of quality instead of a single price, as a mechanism for mitigating quality uncertainty. Contingency pricing is especially applicable to IT-intensive commerce where quality uncertainty is prevalent. The modern IT infrastructure allows easy capture, verification, and dissemination of performance and quality data essential for the implementation of contingency pricing framework. Under very broad conditions, we show that when the market underestimates firm performance, it is optimal to design a full-rebate contingent contract. The optimal quality threshold is set at the quality level that maximizes the gap between market and actual performance probabilities, and the optimal market size is independent of the quality threshold. When contingency pricing is optimal, it is sufficient to consider two-part contingent contracts: two-part contract performs as well as any multipart contract. Use of contingent prices include IT-intensive settings such as ASP service levels, Internet connectivity, and transaction execution in financial services.*

## 1 INTRODUCTION

Wide adoption of electronic commerce has exposed consumers increasingly to quality uncertainty. For digital goods, many of the phases of commerce, including search, selection, ordering, and delivery are conducted via remote interaction. Thus, consumers cannot observe quality until they consume the product or service. Even for many physical goods, consumers do not know product quality until delivery and use. This heightened quality uncertainty is especially prevalent in IT-intensive goods and services. The information technology (IT) sector has spawned several new firms with rapid introduction of new products. Both new firms and new products, along with the experience goods characteristics of IT-intensive goods and services, serve to increase quality uncertainty effects. Moreover, for IT services, consumers demand end-to-end services and quality levels, while providers control only some part of the supply-chain providing the service. Thus quality uncertainty manifests in many different IT-intensive commerce settings.

We propose the use of quality-contingent prices as a mechanism to mitigate the effects of quality uncertainty. In contingency pricing, the firm announces quality-price pairs for various levels of quality instead of a single price. Since quality is stochastic, any of the possible realizations of quality level may be achieved, and consumers pay depending upon the quality of the product or service received. Clearly, contingency pricing works well when quality is objectively verifiable and is unaffected by use, avoiding moral hazard. The contingency pricing framework also needs the availability of performance information: how well the firms deliver on quality. The modern IT infrastructure supports contingency pricing well in a setting where product quality is digitally captured, continuously metered, and hence can be verified and also aggregated for reporting performance statistics. Examples of such quality measures include download speeds, ISP uptime guarantees (Sprint), trade execution times (AmeriTrade), and ASP service level guarantees (see, for example, Kerstetter 2001).

Other mechanisms for dealing with quality uncertainty include money-back guarantees (Mann and Wissink 1988; Moorthy and Srinivasan 1995) warranties (Lutz and Padmanabhan 1998, Matthews and Moore 1987), and limited-time trials. Although contingency pricing shares some features with each of these, it differs significantly from them. In money-back guarantees, there is no notion of quality contingency. Consumers may return the product for any reason; hence transaction costs for the firm and consumer play a significant role, whereas with contingency pricing, products need not be returned—a simple monetary transaction completes the transaction. Warranties may be bundled with the product or sold unbundled as extended warranties, often by a different firm. They involve the return and repair of the product to original specification, upon which the warranty may restart, thereby involving the analysis of repeated gamble. Contingency pricing does not restore the product to original quality. For many IT-intensive goods and services, product delivery and consumption are simultaneous, leaving contingency pricing as an attractive solution for quality uncertainty. Although contingency pricing shares uncertainty and multiple levels of quality and prices with the adverse selection and price discrimination (Maskin and Riley1984; Mussa and Rosen 1978), unlike them, in contingency pricing, the combination of the stochastic quality levels and associated prices (along with associated probabilities) defines a single choice item offered to all users: users do not have the option to select a particular quality. Contingency pricing allows the product to be used before a claim of poor quality is made. For many IT services, especially online and real-time transaction services, usage is tightly coupled with purchase and delivery. Therefore, contingency pricing is especially suited to pricing of IT goods and services.

Bazerman and Gillespie (1999) argue that contingency pricing is highly relevant for business agreements when quality is uncertain, but find that the use of such pricing is limited by the difficulty in determining the optimal contract. In this paper, we develop the optimal design of contingent prices. Our earlier work motivates contingency pricing in detail and establishes some preliminary results on the usefulness of contingency pricing. In this paper, we describe the nature of contingency price contracts in a general framework. We address how and when contingency pricing should be used and how to set the optimal quality thresholds and prices.

## 2 CONTINGENCY PRICING

A contingency pricing structure (depicted in Figure 1) is defined when product quality $q$ is stochastic (it take on any value in the interval $[q_0, q_N]$) but is quantifiable and objectively verifiable. The firm announces a price vector $<<R_1,...,R_N>>$ and a vector of quality thresholds $<<q_0,q_1,...,q_N>>$, which defines an agreement wherein the buyer pays price $R_j$ if the quality level realization falls in the interval $[q_{j-1},q_j)$. Information about quality contingencies is available to consumers, and is represented as a cumulative distribution $F(q)$ representing the probability that the quality realized falls below $q$. Thus, $F(q_0)$=0 and $F(q_N)$=1. Consumers learn the function $F$ from independent third-party firms (such as BizRate or eTestingLabs) that monitor the firm's business performance and report historical data.

Likewise, the firm possesses private performance information represented by the probability distribution $G(q)$. It is possible that $F(q) \neq G(q)$ because of process or resource changes made by, and known only to, the firm. Note that if $F(q) > G(q)$ at some $q$, then the market underestimates the firm's likely performance at $q$. Consumers are heterogeneous in their valuation for the product and in their quality sensitivity as well. We index consumer types by $v$, so that $U(v,q)$ represents consumer type $v$'s valuation for quality level $q$. The consumer types and quality levels are ordered such that the first derivatives $U_v(v,q)$ and $U_q(v,q)$ are both positive; we also make the reasonable assumption that $U_{vq}(v,q) > 0$ (i.e., high-type consumers have a greater marginal valuation for increased quality).

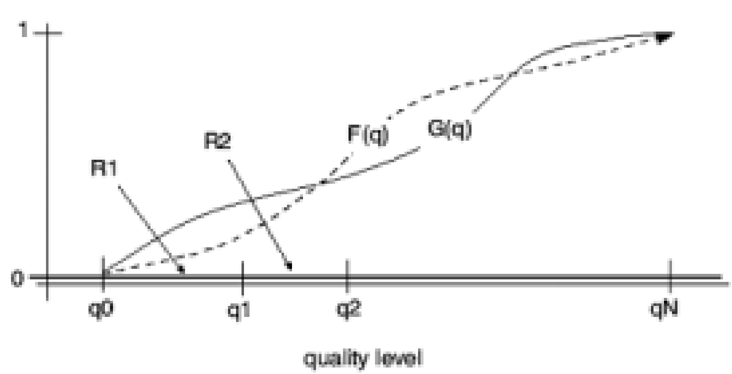


**Figure 1. Contingency Pricing Framework**

The dotted curve, $F(q)$, represents the market's estimated probability that quality will fall below level $q$. The curve $G(q)$ represents the firm's own belief based on private information

The firm incurs a positive marginal cost for delivering the product, and this cost may depend on the actual quality realized. For example, when inferior quality is produced, the firm may incur additional costs that include processing, goodwill loss, and other costs. Combining all of these costs over the probability distribution *G*, we denote the expected marginal costs to be $\hat{C}$. The firm's problem is to choose the quality thresholds and the corresponding prices so as to maximize total expected profits.

The firm's expected price from each sale is

$$\hat{R} = \sum_{j=1}^{N} R_j G((q_j) - G(q_j - 1))$$

Since consumers have access to only the public probability distribution, their price expectation is given by

$$\overline{R} = \sum_{j=1}^{N} R_j (F(q_j) - F(q_{j-1}))$$

Finally, the expected valuation of consumer type *v* is

$$U(v) = \int_{q_0}^{q_N} U(v,q)F'(q)dq$$

where $F'$ is the probability density function for the public probability distribution. Note that the expected valuation $U(v)$ is simply a function of consumer type *v* and is independent of how the firm defines the quality thresholds in its contingency pricing scheme. It may easily be verified that $U_v(v) > 0$.

Given the expected valuations and price expectations of consumers, the firm's contingency pricing scheme defines a marginal consumer $v_m$ (whose value and price expectations are equal) as one satisfying the condition $U(v_m) = \overline{R}$. The firm's profit expectation may therefore be written as

$$E[\pi] = (1 - v_m)(\hat{R} - \hat{C})$$

where we assume that *v* is uniformly distributed in the interval [0,1].

## 3 TWO-PART CONTINGENT CONTRACT

To analyze the design of the optimal contingency pricing scheme, we begin by considering the optimal design of a two-part contingent price contract. Thus, $N = 2$, so that we have two prices and two quality intervals, i.e., one quality threshold. For simplicity in notation, we write $q_0 = 0$, $q_2 = 1$, and the quality threshold $q_1$ as just *q*. Similarly, we write the two prices as $R_2 = R$ (price when quality equals or exceeds *q*) and $R_1 = R - r$ where *r* can be considered the rebate for the inferior quality (below *q*). The probability that quality falls below *q* is $F(q)$ and the probability that high quality is realized is $1 - F(q)$.

The expected valuation, as before, is

$$U(v) = \int_{0}^{1} U(v,q)F'(q)dq$$

The price expectations of the consumer and firm are $\overline{R} = R - rF(q) = U(v_m)$ and $\hat{R} = R - rG(q) = \overline{R} + r(F(q) - G(q))$. The firm's profit function is

$$\begin{aligned}\pi &= (1 - v_m)(\hat{R} - \hat{C}) \\ &= (1 - v_m)(U(v_m) + r(F(q) - g(q)) - \hat{C})\end{aligned}$$

## 3.1 Exogenous Quality Threshold: When Is Contingency Pricing Optimal?

For the moment, we assume that the quality threshold $q$ is exogenously specified and the firm's objective is to set prices <<$R - r$,$R$>> to maximize profit.

Computing first derivatives with respect to the price variables $r$ and $R$, we see that

1. If the market perfectly estimates performance at $q$ (i.e., $F(q) = G(q)$), the first-order conditions yield many optima with $U(v_m{*}) = \overline{R} = \hat{R}$, where the optimal marginal consumer is given by the condition

$$v_m^* = \left\{ v_m : \frac{1}{1 - v_m} = \frac{U(v_m)}{U(v_m) - \hat{C}} \right\}$$

   We note that the condition yields a unique solution when $\frac{U^{'}(v_m)}{U(v_m)}$ is a decreasing function, satisfied by all concave functions and most convex utility functions.

2. When $F(q) = G(q)$ then the first-order conditions simplify to $v_m = 1$, i.e., $\pi = 0$, indicating that boundary conditions must be examined to determine the optimal solution.

Hence we see that for the case where the market is perfectly informed, there are many optimal contingent price contracts but all have the same marginal consumer, hence the same expected price. In other words, the firm can adjust the price variables $r$ and $R$ in infinite ways so long as $R - rF(q)$ equals $U_{v_m}^*$. Two solutions of special interest are $r = 0$ (i.e., the firm offers a standard single price with no rebate for inferior quality) and $r = R$ (the firm offers a full rebate for inferior quality). Since $r = 0$ is one optimal solution, we see

**Proposition 1** *Contingency pricing offers no economic advantage when the market is perfectly informed about performance.*

Although contingency pricing offers no economic advantage when the market is perfectly informed, consumers may prefer a contingent price contract because they will feel fairly compensated for any mis-performance.

For the case where $F(q) \neq G(q)$, the optimal pricing scheme must occur at one of the two boundaries $r = 0$ and $r = R$. Hence we can conclude

**Proposition 2** *When the firm has private information about its expected performance, it is never optimal to offer a contingency pricing scheme in which the firm offers less than a full-price rebate for inferior quality.*

The question remains whether the firm should ever employ contingency pricing, and we examine this below. We first provide the mathematical preliminaries for analyzing the two forms of pricing schemes.

| Standard single price $R^s$ (No Rebate) | Contingent prices $R^c, r^c = R^c$ (Full-price Rebate) |
|---|---|
| $\bar{R}^s = R^s = U(v_m)$ <br> $\hat{R}^s = R^s$ | $\bar{R}^c = R^c(1-F(q)) = U(v_m)$ <br> $\hat{R}^c = R^c(1-G(q)) = U(v_m)\frac{1-G(q)}{1-F(q)}$ |
| $\pi^s = \max_{v_m}(1-v_m)(U(v_m)-\hat{C})$ | $\pi^c = \max_{v_m}(1-v_m)\left(\left(\frac{1-G(q)}{1-F(q)}\right)U(v_m)-\hat{C}\right)$ |
| $\frac{\partial\pi^s}{\partial v_m} = -(U(v_m)-\hat{C}) + (1-v_m)U'(v_m)$ | $\frac{\partial\pi^c}{\partial v_m} = \left(\frac{1-G(q)}{1-F(q)}\right)\left(-\left(U(v_m)-\frac{1-F(q)}{1-G(q)}\hat{C}\right)+(1-v_m)U'(v_m)\right)$ |
| $v^s = \left\{v_m : \frac{1}{1-v} = \frac{U'(v)}{U(v)-\hat{C}}\right\}$ | $v^c = \left\{v_m : \frac{1}{1-v} = \frac{U'(v)}{U(v)-\frac{1-F(q)}{1-G(q)}\hat{C}}\right\}$ |

Note that when the market underestimates performance at $q$, then $F(q) > G(q)$ and $\frac{1-F(q)}{1-G(q)} < 1$. Conversely, when the market overestimates performance at $q$, then $\frac{1-F(q)}{1-G(q)} > 1$. Hence, examining the condition for the optimal marginal consumers in the two cases ($v^S$ and $v^C$, respectively) we see the following relationships.

1. When the market underestimates performance at $q$, then the denominator $U(v) = \frac{1-F(q)}{1-G(q)} < 1$ is greater than $U(v) - \hat{C}$, hence $v^S > v^C$. Therefore, the firm gains market share (which is $1 - v_m$) with contingency pricing since consumers see a lower expected price under contingency pricing $(\overline{R}^c < R^s)$.

   To see that contingent pricing increases profits in this case, consider the following contingent price contract $\ll 0, \frac{R^s}{1-F(q)} \gg$ such that the price expectation to consumers equals $R^S$, the optimal standard price. Since the price expectation is equal, the firm would obtain the same market share as with the optimal single price. On the other hand, the true expected margin is $R^s \frac{1-G(q)}{1-F(q)}$, which is greater than $R^S$, hence yielding greater profits. Hence the optimal contingency prices also yield greater profit than the optimal single price.

2. When the market overestimates performance at $q$, then it may be seen that $\pi^C < \pi^S$ hence the firm should offer the optimal single price.

To summarize,

**Proposition 3** *Given an exogenously specified quality threshold q, the firm will choose contingency pricing when the market underestimates its ability to delivery quality better than q. The optimal prices and marginal consumer are given by $v^C(q)$ satisfying the condition*

$$v^c(q) = \left\{ v_m : \frac{1}{1-v} - \frac{U'(v)}{U(v) - \frac{1-F(q)}{1-g(q)}\hat{C}} \right\}$$

*The firm obtains greater market share and profit compared with a single price strategy.*

## 3.2 What Is the Optimal Quality Threshold?

Now consider the general case where the firm can offer a two-part contingency pricing scheme, and it can *choose* the quality threshold $q$ on which to define contingent prices. We formulate the problem as

$$\pi^* = \max_q \pi(q)$$

where $(q) = \max_{R,r}(1 - v_m)(q))\left(\frac{1-G(q)}{1-F(q)}U(v_m(q)) - \hat{C}\right)$

In the previous section we have seen that, for any exogenous $q$, when $F(q) < G(q)$, the firm offers a standard single price. The firm employs contingency pricing if and only if $F(q) > G(q)$, and when this happens the optimal prices are set such that the marginal consumer $v^C(q)$ satisfies the condition

$$v^c(q) = \left\{ v_m : \frac{1}{1-v} = \frac{U'(v)}{U(v)} \right\} \tag{1}$$

### 3.2.1 Case: $\hat{C} = 0$

To develop the result, consider first the simple case where $\hat{C} = 0$. Then we see that for *all* $q$ such that $F(q) > G(q)$, the firm sets contingent prices such that the marginal consumer satisfies the condition

$$v^c(q) = \left\{ v_m : \frac{1}{1-v} = \frac{U'(v)}{U(v)} \right\}$$

which is identical for all $q$ (and which, incidentally, is also the same as that for the optimal standard single price). We denote the optimal marginal consumer by $v^C$, and note that the marginal consumer and market share are independent of the choice of quality threshold $q$. Hence we get

$$\pi(q) = (1 - v^c)U(v^c)\left(\frac{1-G(q)}{1-F(q)}\right)$$

from which it follows easily that the overall objective function $\pi^*$ is attained by choosing $q$ in order to maximize $\dfrac{1-G(q)}{1-F(q)}$.

### 3.2.2 Case: $\hat{C} \neq 0$

Now it is easy to generalize the result for the case where $\hat{C} \neq 0$. First, it is easily determined from Eq. 1 that $v^c(q)$ is minimized at the $q$ which maximizes the fraction $\frac{1-G(q)}{1-F(q)}$, and the firm obtains greatest market share when it designs a contingency pricing scheme around this quality threshold.

Let $Q = \left\{ argmax_q \left\{ \frac{1-G(q)}{1-F(q)} \right\} \right\}$. Suppose that the profit is maximized with a contingent contract $<< 0, R^j >>$ defined at some quality level $q_j \in Q$. We show that this leads to a contradiction.

Now consider some $q^* \in Q$, and define a contingent price contract $<< 0, R^j \frac{1-F(q_j)}{1-F(q^*)} >>$ at $q^*$. By construction, the price expectation to consumers $(\overline{R}^*)$ equals the price expectation for the $q_j$ solution, $(\overline{R}^j)$. Hence the market shares are the same under both solutions. By construction, $\frac{1-G(q^*)}{1-F(q^*)} \geq \frac{1-G(q_j)}{1-F(q_j)}$. Hence we see below that the $q^*$ solution delivers a greater true expected price

$$\hat{R}^* = \hat{R}^* \frac{1-G(q^*)}{1-F(q^*)} > \overline{R}^* \frac{1-G(q_j)}{1-F(q_j)} > \overline{R}^j \frac{1-G(q_j)}{1-F(q_j)} = \hat{R}^j$$

hence the q* solution also yields greater profit. This is a contradiction since we assumed that the $q_j$ solution is optimal. Since this argument holds for all $q_j \in Q$, it follows that the optimal solution is a two-part contract defined over any $q \in Q$. Therefore, we have proved the following result.

**Theorem 1** *It is optimal for the firm to employ contingency pricing if and only $F(q) > G(q)$ at some quality level q. The optimal two-part contingent price contract is defined over a quality threshold $q^*$ that maximizes the difference in beliefs about performance, computed as the ratio $\frac{1-G(q)}{1-F(q)}$. The optimal marginal consumer $v^c$ is given by Eq. 1, and this consumer has expected utility $U(v^c)$. The firm's optimal price (and rebate) are $R^c(q^*) = \frac{U(v^c)}{1-F(q^*)}$.*

# 4 MULTIPART CONTINGENCY PRICING: HOW MANY QUALITY THRESHOLDS?

We have analyzed two-part contingent contracts in the previous sections; now we address the question of multiple quality levels. If multipart contracts could be designed, how should the firm set the optimal quality levels and prices? Using the notation introduced earlier for multiple quality levels and prices, the firm chooses the price vector $<<R_1,...,R_N>>$ and a vector of quality thresholds $<<q_0,q_1,...,q_N>>$, to maximize its profits. Using the notation $\Delta F_i = F(q_i) - F(q_{i-1})$ and $\Delta G_i = G(q_i) - G(q_{i-1})$, we write $\overline{R} = \sum_{i=1}^{N} R_i \Delta F_i$ and $\hat{R} = \sum_{i=1}^{N} R_i \Delta G_i$. Note that $\hat{R} = \overline{R} + \sum_{i=1}^{N} \Delta G_i - \Delta F_i$.

Now, the profit function is given by

$$\pi = (1 - v_m)(\hat{R} - \hat{C})$$
$$= (1 - v_m)\left[ U(v_m) + \sum_{i=1}^{N} (\Delta G_i - \Delta F_i) - C \right]$$

First-order conditions for this problem indicate that the optima lie on a boundary. Each boundary, in turn, reduces to a specific two-part contingency pricing problem. The collection of these problems is the space of all two-part contingency prices. Hence the optimal two-part contingent contract is an optimal solution to the *N*-part problem.

**Theorem 2** *When quality is uncertain, and the quality level can take on any value in the interval* $[q_0, q_N]$*, then it is sufficient to consider only the space of two-part contingent price contracts in designing the optimal contingency pricing scheme. The firm's profits are maximized by a two-part pricing scheme designed around the quality threshold q* that maximizes the difference of beliefs expressed by the ratio* $\frac{1 - G(q)}{1 - F(q)}$.

Please contact authors for detailed proof. ■

## 5 CONCLUSIONS

In this paper, we propose the quality contingent prices as a mechanism for mitigating quality uncertainty effects. Under broad conditions, we show that when the market underestimates firm performance, it is optimal to design a full-rebate contingent contract. We show that the optimal quality threshold is set at the the quality level that maximizes the gap between actual and true performance probabilities. The optimal market size is independent of the quality threshold chosen. When contingency pricing is optimal, it is enough to consider two-part contingency pricing: a two-part contract performs as well as any multipart contract. These results provide useful recommendations to managers, especially since the information infrastructure and information content of the relevant quality metrics allow implementation of the contingency pricing framework. A significant contribution of our analysis of contingent pricing is its applicability in IT-related services and transactions.

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