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# INCORPORATING KNOWLEDGE BUILDING IN REAL OPTIONS ANALYSIS OF TECHNOLOGY PROJECT INVESTMENT

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## **INCORPORATING KNOWLEDGE BUILDING IN REAL OPTIONS ANALYSIS OF TECHNOLOGY PROJECT INVESTMENT**

*Research-in-Progress* 

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## **Abstract**

*Real options theory has provided a useful framework for technology investment decision making. Researchers in this area have emphasized the importance of considering the option-like characteristics of IT investment projects. However, unlike financial options, investment in IT projects is typically irreversible: such investment cannot be recovered for other purposes without very costly rework. The objective of our work is to study the effects of knowledge building on the valuation of real options by using a continuous-time stochastic model. To our knowledge, this is the first model that formally builds a linkage between proactive learning and investment cost and examines the consequences of this linkage on the management of real options. Our main finding is that knowledge building can expedite the adoption of new technology and significantly enhance the value of technology options.* 

**Keywords:** Real options, Optimal control theory, Knowledge building, Analytical modeling, Black-Scholes model

## **Introduction**

The last three decades have seen significant and exciting growth in real options research – applying the financial option theory (Black and Scholes 1973; Merton 1973) to investment in real assets under uncertainty. Real options theory has provided a useful framework for corporate investment decision making. Information technology (IT) investment has been a fruitful area for real options applications given the rapid pace of change in the IT field over the past two decades.

Since the early works in this area (Clemons 1991; Dos Santos 1991; Kambil et al. 1993), researchers have stressed the importance of considering the option-like characteristics of IT investment projects. When the benefit of an underlying technology project is uncertain, a manager can defer the investment until the benefit exceeds the cost of implementing the project. In the case of point-of-sale (POS) debit card network (Benaroch and Kauffman 1999), there were uncertainties of customer acceptance and retailer adoption in 1987 at the time when Yankee 24 (an association of more than 200 member firms) was contemplating a move to provide POS service to its members. The Yankee's top management decided to wait to achieve the best timing for service deployment when the market acceptance of the POS technology became favorable or Yankee had better information about the investment return. Therefore, waiting to invest can create value – *good things come to those who wait*! In this aspect, IT investment projects are analogous to financial call options.

On the other hand, unlike financial options, investment in IT projects typically is *irreversible*: such investment cannot be recovered for other purposes without very costly rework. The effect of irreversibility is high when the investment cost is high, *ceteris paribus*. When both uncertainty and irreversibility are high, then it is worthwhile to valuate technology projects using real options framework (Dixit and Pindyck 1994). While the effect of technology uncertainty on the value and timing of investment have been well studied in the existing real options literature from the early works to the more recent works (Benaroch et al. 2010; Grenadier and Wang 2007), the issue of irreversibility has not been explored by the extant works which invariably assume a constant investment cost. Rather than waiting passively for the optimal investment time to arrive, a manager can alter such timing by changing the investment cost through proactive learning of technology to be adopted and therefore enhance the investment performance – *fortune favors the prepared mind*! Unlike other capital investments, technology investment involves a high percentage of expenditure for system development, in addition to hardware cost. Knowledge workers play a significant role in the system development process such as specifying project scope and generating business requirements, whether for the insourcing or outsourcing approach of system development. Therefore researchers have highlighted the importance of technical knowledge and skill within an organization in reducing the system development cost (Fichman 2004; Kogut and Kulatilaka 2001). In 2005, FBI had to scrap a US \$170 million software project due to poor planning, bad communication, and a lack of IT management and technical expertise (Goldstein 2005).

The objective of our work is to study the effects of knowledge building on the valuation of real options by using a continuous-time stochastic model. To our knowledge, this is the first model that formally builds a linkage between proactive learning and investment cost and studies the consequences of this linkage on the management of real options. Specifically, we examine the following research questions: how does proactive learning affect the value and exercise time of technology options? How should an organization put optimal effort into building technical knowledge over time? The rest of the paper is organized as follows. We first present the real option model which incorporates knowledge building effort. We then solve the model using optimal control theory (Sethi and Thompson 2000). Last section summarizes the paper and discusses future research direction.

## **Technology Investment Problem with Endogenous Investment Cost**

In this section, we formulate a technology investment model using a contingent claims approach. First developed in financial options theory (Black and Scholes 1973; Merton 1973), the contingent claims approach has been extensively used in real options applications (Dixit and Pindyck 1994). By taking appropriate long and short positions dynamically, we can construct a risk-free portfolio which consists of the technology investment option and a risky asset based on the underlying technology. In equilibrium, such portfolio earns a risk-free rate of return. In this way, the value of a technology investment option is determined. Table 1 defines the notation used in this paper.

#### **Table 1. Variable Notation and Definitions**





Let  $A(t)$  be the value of the risky asset at time  $t$ . It represents the present value of expected revenue generated from the technology project when put into operation. We assume that the asset pays no dividends, i.e., there is no revenue loss during waiting. In a later section, we will extend the model to the case of a dividend-paying asset. For now, assume that the asset evolves according to the following standard geometric Brownian motion:

$$
dA(t) = \mu \cdot A(t)dt + \sigma dz \tag{1}
$$

where  $\mu$  is the growth rate of the underlying asset value,  $\sigma$  the volatility of asset value, and *z* a standard Wiener process.

In order to reduce the investment/adoption cost of a new technology, an organization needs to spend effort in continuously gaining knowledge about the new technology by pilot studies, technical training, and R&D. Denote this effort at time *t* by  $u(t)$  and the cumulative level of knowledge by  $K(t)$ . We model the growth rate of  $K(t)$  as a function of effort  $u(t)$  in the following manner:

$$
\dot{K}(t) = dK(t) / dt = a \cdot u(t) - \delta \cdot K(t), \ K(t_0) = 0 \tag{2}
$$

In Equation (2), as an organization puts more effort  $u(t)$  into learning a technology, knowledge level  $K(t)$  grows faster. Constant *a* can be thought of as the effectiveness of learning for a particular organization. A higher *a* means that an organization is relatively more effective in learning. In the second term,  $\delta$  captures the decay effect of knowledge and represents the speed of change in technology. If an organization spends no learning effort  $(u(t)=0)$ , then knowledge about the new technology gradually becomes outdated as the technology evolves. A higher  $\delta$  leads to a faster decay of knowledge. Finally, we normalize the initial level of knowledge to be zero. Let function  $f(u(t))$  denote the corresponding cost rate of putting effort  $u(t)$  into knowledge accumulation. We assume a

commonly used quadratic form of effort cost function  $f(u) = b \cdot u^2$  (Sethi and Thompson 2000).

Let  $X(K(t))$  represent the investment cost of a new technology if an organization decides to invest in the technology at time *t* with *K*(t) being the corresponding knowledge level. There are many ways in which knowledge about new technology can reduce the implementation cost. For example, an organization with a good knowledge of new technology can make a better decision about evaluating and selecting a system vendor, creating a superior project plan of system implementation, and integrating new technology with existing systems more smoothly. Since more knowledge can reduce the investment cost, we have  $dX/dK < 0$ . Then an organization needs to choose the appropriate amount of knowledge building effort *u*(t) in order to maximize the value of technology adoption options. Let  $C(t, A(t), K(t), T)$  denote the present value of technology investment option for any given starting time *t* and option expiration time  $T$  ( $t < T$ ). By using the contingent claims approach, we can show that the option value satisfies the following equation (Details of proof can be found in the Appendix A):

$$
0 = \max_{u} \left[ C_t + rA \cdot C_A + C_K \cdot \dot{K} + \frac{1}{2} \sigma^2 A^2 C_{AA} - f(u) - r \cdot C \right], \quad C \big|_{t=T} = \max \left[ 0, A(T) - X(K(T)) \right] \tag{3}
$$

where *r* is the risk-free discounted rate, and  $C_x = \frac{\partial C}{\partial x}$ ,  $x = t, A, K$ . Equation (3) is an extension of Ito's Lemma (Hull 2008) with the inclusion of knowledge building *K*(*t*) as a new variable and an additional step of optimizing Equation (3) over knowledge building effort *u*(*t*).

#### **Analytical Solutions**

To solve (3), we let  $C(t, A, K, T) = C^{BS}(t, A, X(K(T)), T) + C^{KN}(t, K, T)$  where  $C^{BS}(t, A, X(K(T)), T)$  is the solution to the following Black-Scholes equation:

$$
0 = C_t^{BS} + rA \cdot C_A^{BS} + \frac{1}{2} \sigma^2 A^2 C_{AA}^{BS} - rC^{BS}, \ C^{BS}\Big|_{t=T} = \max\big[0, A(T) - X(K(T))\big].\tag{4}
$$

Explicit expression of  $C^{BS}(t, A, X(K(T)), T)$  can be found in (Benaroch and Kauffman 1999; Hull 2008). In this study,  $C^{BS}(t, A, X(K(T)), T)$  represents the value of a call option on new technology adoption.

By utilizing Equations (3) and (4), we can derive the partial differential equation that  $C^{KN}(t, K, T)$  should satisfy:

$$
0 = C_t^{KN} + C_K^{KN} \cdot \dot{K} - f(u) - r \cdot C^{KN}, \ C^{KN} \Big|_{t=T} = 0
$$
 (5)

where  $K(t)$  is subject to knowledge growth equation (2). It is easy to verify that the following solution of  $C^{KN}(t, K, T)$  satisfies the differential equation (5) and its associated boundary condition:

$$
C^{KN}(t, K, T) = -\int_{t}^{T} e^{-r(x-t)} f(u) dx
$$
\n(6)

where the optimal form of  $u(t)$  has yet to be determined. The meaning of  $C^{KN}(t, K, T)$  is clear. It is the total discounted cost of spending effort  $u(t)$  in building knowledge  $K(t)$  from any time t to terminal time T.

However, it is not easy to determine  $u(t)$  from Equation (3) directly. Instead, we can recast the differential equation (3) as an equivalent optimal control problem for the starting time  $t_0$ :

$$
\max_{u} \left\{ C(t_0, A, K, T) = C^{BS}(t_0, A, X(K(T)), T) - \int_{t_0}^{T} e^{-r(x - t_0)} f(u) dx \right\}
$$
\n(7)

Equation (7) plays a very important role in this study. Through Equation (7), our paper contributes to real options theory by introducing the process of knowledge building in a technology investment project. Through the use of optimal control theory, such gradual build-up process is accomplished by continuously investing in  $u(t)$ . Specifically, by investing in knowledge-building effort  $u(t)$  continuously from the starting time  $t_0$  to the terminal time *T*, an organization can build a knowledge level  $K(T)$  which reduces the future adoption cost of a new technology project. As a result, the total value  $C(t_0, A, K, T)$  increases. Therefore, Equation (7) shows at the conceptual level how investing in knowledge-building effort  $u(t)$  can lead to an improvement in the overall value of a technology investment project. The term  $C^{BS}(t_0, A, X(K(T)), T)$  in (7) is called *salvage value* in optimal control theory. Once we obtain the solution to (7), then this solution also solves (5). This is what we are going to do next: find the optimal  $u(t)$  for  $t_0 < t < T$  and the associated  $K(t)$  to Equation (7).

To solve the problem formulated in (7), we first present the Hamiltonian of this problem:

$$
H[K, \lambda, u] = -e^{-r(t-t_0)} f(u) + \lambda \cdot (a \cdot u - \delta K)
$$
\n(8)

The adjoint variable  $\lambda(t)$  ( $t_0 < t < T$ ) satisfies the following equation:

$$
\dot{\lambda} = -\partial H / \partial K = \delta \cdot \lambda, \ \lambda(T) = \partial C^{BS} / \partial K(T). \tag{9}
$$

Adjoint variable  $\lambda(t)$  can be interpreted as the marginal value of knowledge K at time t; this marginal value should be positive since an increase in knowledge level K will cause the total system value  $C(t_0, A, K, T)$  to increase. In Appendix B we verify that  $\lambda(t)$  is indeed positive in the whole time duration.

The Hamiltonian (8) is very useful in helping us obtain an analytical solution. Through adjoint variable  $\lambda(t)$ , it reduces a dynamic optimization problem (6) over time interval  $[t_0, T]$  to a series of static optimization problems. That is, we only need to maximize the Hamiltonian by choosing an appropriate  $u(t)$  at each instant *t*. We can do so since the two terms in the Hamiltonian have separately taken care of the direct and indirect contribution of knowledge building effort to the objective function (6). Equation (6) can now be solved analytically by using optimal control theory. The results are summarized in the following proposition:

PROPOSITION 1 (See Appendix B for proof): *The general form of the optimal solution u(t) to the problem formulated in (3) (and to the equivalent problem represented by (7)) is the following:* 

$$
u(t) = \frac{a}{2b} \cdot e^{(r+\delta)(t-T)} \cdot e^{r(T-t_0)} \frac{\partial C^{BS}}{\partial K(T)}, \ t_0 \le t \le T \ , \tag{10}
$$

*and the corresponding knowledge level is given by* 

$$
K(t) = a \cdot e^{-\delta(t - t_0)} \int_{t_0}^t e^{\delta(x - t_0)} u(x) dx, \ t_0 \le t \le T \ . \tag{11}
$$

A brief comment on the solutions (10) and (11) is in order here. At the first glance, it seems that (10) and (11) form a recursive relationship and become difficult to solve since the expression of  $u(t)$  contains  $K(T)$  and in turn  $K(T)$ contains  $u(t)$ . Actually, it is not so: simply plug  $u(t)$  from(10) into  $K(t)$  in (11); Set  $t=T$  in the newly obtained equation and solve for  $K(T)$  numerically since a closed-form solution of  $K(T)$  is impossible to obtain given the complexity of the expression  $C^{BS}$ . Plug  $K(T)$  into (10) and then  $u(t)$  is completely known. This method will be employed when we perform numerical studies later on.

As we can see from the expression of  $u(t)$ , more effort should be put into knowledge building as time  $t$  increases. This is due to knowledge decay and discount effects. For a given terminal time *T*, it is optimal to build more knowledge later to reduce the amount of knowledge becoming outdated since existing knowledge has a constant decay rate <sup>δ</sup> ,. Also after taking discount rate *r* into consideration, it is less costly to build the same amount of knowledge near the exercise time than at the beginning. These two effects jointly determine the optimal growth trend of knowledge building effort *u*(*t*).

#### **Further Numerical Studies**

In addition to increasing the value of technology option through  $K(T)$ , knowledge building effort can also affect the option exercise time *T*. Previous works (Benaroch and Kauffman 1999; Hull 2008) have considered how exogenous factors such as dividends and lost revenues can affect the option exercise time *T*. In our work, knowledge building is internal to an organization and an integral part of our real options model. As a result, effects of knowledge building and technology uncertainty jointly determine the optimal option exercise time *T*. We will use a numerical example to demonstrate the effect of knowledge building on both the value of options and the timing of options exercise. The Yankee's example (Benaroch and Kauffman 1999) introduced at the beginning of this paper serves as an ideal example to illustrate our points.

For this numerical example, we assume that knowledge level  $K(T)$  reduces technology investment cost in a squareroot form:  $X(K(T)) = X_0(1 - c \cdot \sqrt{K(T)})$ . Same parameter values as in (Benaroch and Kauffman 1999) are used: initial technical investment  $X_0 = 4 \cdot 10^5$ , expected revenue volatility  $\sigma = 0.5$  and risk-free rate  $r = 0.07$ . In addition, we choose the following reasonable, although hypothetical values that are related to knowledge building so

that realistic results and insights can be obtained:  $a = 1$ ,  $b = 1000$ ,  $\delta = 1$  and  $c = 0.3$ . Without loss of generality, we let  $t_0 = 0$ . The results are displayed in Table 2.

The first row of Table 2 shows the different possibilities of exercise time from 0.5 years to 4 years. The second row shows the corresponding present value *A* of the technology project. It increases first because the market is growing quickly and decreases later on because the loss of revenue dominates the gain from market growth. We show values of Black-Scholes options *BS COriginal* without knowledge-building in the third row. *BS COriginal* generally matches the trend

of the project value *A* but is not always so. When *T* goes from 2.5 to 3, the value of *A* decreases while  $C^{BS}$ increases. The reason is that options value is also determined by the exercise horizon: the longer the *T*, the higher the options value. The value of this deferral option reaches its maximum (\$152,955, as shown in bold) for a deferral of three years. For comparison, we show in the fourth row the net value of this technology project option  $C(0, A, K(T), T)$  obtained from Equation (7), where the benefit of knowledge building is taken into consideration. The maximum value of options ( $C = $245,598$ ) shown in bold occurs at  $T=2.5$ , six months earlier than the case without knowledge-building.

To provide some intuition for this interesting result, we first observe that in the absence of knowledge building, there are two opposing forces affecting the optimal timing of real options exercise. The first force is associated with asset uncertainty. A pure real options consideration would favor deferring a technology project in order to take the advantage of possible upswing in the future technology project value. The second force is related to revenue consideration. If an organization waits for too long, it could face an excessive loss of revenue and possibly market share. The timing of technology adoption is determined by balancing these opposing forces. When an organization puts effort into learning the new technology, it can lower the adoption cost and prefers to adopt earlier due to the knowledge decay effect. Therefore, the optimal option exercise time *T* tends to be reduced

$\boldsymbol{T}$ (Years)	0.5		1.5		2.5		3.5	
A	\$342.216	\$360,083	\$376,230	\$389,207	\$395,566	\$387,166	\$344.813	\$223,295
$C^{BS}_{Original}$	\$32,024	\$66,093	\$96,830	\$123,786	\$144,565	\$152,955	\$134,873	\$65,300
$\mathcal{C}$	\$139,187	\$183,066	\$211,354	\$232,435	\$245,598	\$243,748	\$208,840	\$102,693

**Table 2. Optimal Time and Value of Technology Option with Knowledge Building** 

Table 2 also shows the importance of enhancing the organizational knowledge level of new technology to be adopted within real options framework. From this table, we can see that option values have been significantly enhanced by knowledge building which increases option values in two ways. One is to reduce the option exercise cost  $X(K(t))$ . For the same realization of  $A(T)$  which is profitable to exercise without knowledge building, knowledge building can make the implementation of such project more profitable. The other is to increase the possibility of exercising technology options, i.e., the value  $Pr(A(T) - X(K(T)) > 0)$  increases with  $K(T)$ . Knowledge building makes it profitable to invest in new technology over a broader range of realizations of  $A(T)$ .

## **Conclusions and Future Research Directions**

In this work, we have extended real options framework of technology adoption by considering the effect of technological knowledge building that will benefit eventual technology adoption. We formulate this problem as a continuous-time stochastic model with knowledge building as an optimal control problem. An analytical solution is obtained and some interesting insights are gained through a numerical example.

This study makes several contributions to the management of new technology adoption. First, it calls for caution in adopting the well-accepted notion that the longer the exercise horizon, the higher the option value. One main finding of this study is that knowledge building can enable earlier technology adoption. A forward-looking organization can expedite the exercise of technology option by actively learning and building knowledge about the new technology to be adopted. Such early adoption of technology can greatly benefit organizations in scenarios where time-to-market

is critical. Also, knowledge building can lower the adoption cost, significantly increasing the option value. Our work demonstrates the need to incorporate knowledge in the technology investment decision.

Second, when planning the adoption of technology projects, top management should pay attention to more than the adoption timing which has been addressed by traditional real options theory. It should also devote resources throughout the planning horizon to knowledge building that can reduce technology adoption cost. Our analytical solution can provide some guidelines to project managers on resource allocation. As time goes on, project managers should devote more resources to activities related to knowledge building such as training and prototyping.

More work can be done in several directions. In this paper, we assume that knowledge building effort has a linear effect on knowledge growth. We can relax this assumption to generalize the model and get more robust results. Another interesting consideration would be to allow knowledge building to affect the asset value growth process and vice versa. Then we can gain more insights into how the interaction between these two growth processes affects option values. It is also important to collect empirical evidence that demonstrates the application of this model to a real technology adoption project where knowledge building is an integral component of the adoption process. Last but the least, we can extend the model to a situation that involves competition of technology adoption and examine the effect of competition on knowledge building process.

#### Appendix

#### **Appendix A Derivation of Equation (3)**

By expanding  $C(t, A, K, T)$  and keeping terms with order  $o(dt)$ , we have

$$
dC(t, A, K, T) = C_t dt + C_A dA + \frac{1}{2} C_{AA} (dA)^2 + C_K \dot{K} dt
$$

$$
= (C_t + \mu A \cdot C_A + \frac{1}{2} \sigma^2 A^2 C_{AA} + C_K \dot{K}) dt + C_A \sigma A \cdot dz
$$

where we have substituted *dA* using Equation (1) and  $(dA)^2 \approx \sigma^2 A^2 \cdot dt$ .

Consider a portfolio with owning this technology option and shorting  $n_A (= C_A)$  shares of underlying technology (it could mean the shares of a firm that has this technology). The value  $n_A$  remains constant within each time interval *dt* but could be adjusted at the beginning of each subsequent *dt*. During each interval *dt*, the change of portfolio value  $\Pi$  (=  $C - n_A \cdot A$ ) is due to the capital gain of this portfolio minus the expense of knowledge building effort  $(f(u)dt)$  by using the optimal amount of *u* 

$$
d\Pi = \max_{u} \left[ dC - n_A dA - f(u)dt \right]
$$
  
= 
$$
\max_{u} \left[ (C_t + \mu A \cdot C_A + \frac{1}{2} \sigma^2 A^2 C_{AA}) dt + C_A \sigma A \cdot dz - C_A (\mu A \cdot dt + \sigma A \cdot dz) - f(u) dt \right]
$$
  
= 
$$
\max_{u} \left[ C_t + \frac{1}{2} \sigma^2 A^2 C_{AA} + C_K \dot{K} - f(u) \right] dt
$$

which is riskless since it does not contain any stochastic term involving *dz*. Then, this portfolio should earn risk-free return rate *r*, i.e.,  $d\Pi = r \cdot \Pi \cdot dt = r(C - C_A A)dt$ . Therefore, we have:

$$
\max_{u} \left[ C_t + rA \cdot C_A + \frac{1}{2} \sigma^2 A^2 C_{AA} + C_K \dot{K} - f(u) - r \cdot C \right] = 0 \tag{A.1}
$$

which is Equation (3) with the boundary condition  $C(t = T, A, K, T) = \max[0, A(T) - X(K(T))]$ .

#### **Appendix B Proof of Proposition 1**

We can solve (8) for  $\lambda(t)$  and get

$$
\lambda(t) = e^{\delta(t-T)} \lambda(T) > 0, t_0 \le t \le T
$$
\n(A.2)

since  $\lambda(T) = \partial C^{BS} / \partial K(T)$  which can be shown to be positive by using the expression of  $C^{BS}$ .

The optimal knowledge building effort can be determined from  $H<sub>u</sub> = 0$  with *H* given by (7):

$$
u(t) = a \cdot \lambda(t) e^{r(t-t_0)} / (2b) = \frac{a}{2b} \cdot e^{(r+\delta)(t-T)} \cdot e^{r(T-t_0)} \partial C^{BS} / \partial K(T)
$$
 (A.3)

where we have used the expression of  $\lambda(t)$ .

It is easy to verify that the following solution of  $K(t)$  satisfies Equation (2) with initial condition  $K(t_0) = 0$ :

$$
K(t) = a \cdot e^{-\delta(t - t_0)} \int_{t_0}^t e^{\delta(x - t_0)} u(x) dx, \ t_0 \le t \le T
$$
 (A.4)

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