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# Exploratory Study on Qualitative Problem Solving: Probabilistic Information Processing

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# EXPLORATORY STUDY ON QUALITATIVE PROBLEM SOLVING: PROBABILISTIC INFORMATION PROCESSING

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## ABSTRACT

The experiment was designed in this study to compare the problem solving behaviors of subjects with various levels of proficiency in basic probability, using the Bayes' Theorem as a normative model for belief revision. The authors, through protocol analysis, investigated the nature of the cognitive biases observed in their probabilistic information processing.

We also explored the qualitative problem solving behaviors to verify whether such biases can be overcome with increased experience with quantitative normative model.

## 1. Introduction

A considerable amount of research from both laboratory and field experimentation indicates that humans in general have considerable difficulty in dealing with uncertainty, and are thereby poor and biased information processors [Keeney, 1982; Lichtenstein, Fischhoff & Phillips, 1976; Moskowitz & Bunn, 1986; Slovic, Fischhoff & Lichtenstein, 1977; Mowen & Mowen, 1986; Tversky & Kahneman, 1981].

In a series of papers, Tversky and Kahneman (1971, 1973, 1974, 1981) have presented detailed experimental evidence showing that individuals use certain heuristics in making judgements and decisions when under uncertainty. When adopted, these heuristics lead to certain predictable and consistent biases in individual judgements concerning the likelihood of uncertain events. It was also noted that the reliance on heuristics and the prevalence of biases are not restricted to novices. Experienced researchers are also prone to the same biases when they think intuitively.

Further, these kinds of consistent biases are not unique in assessment of subjective probability; they have also been reported as operating in the area of physics. DiSessa (1982) studied a group of elementary students learning to control a computer-implemented Newtonian object, and found that they revealed a uniform and detailed collection of naive strategies to move the object. These robust and naive strategies were contradictory to the Newtonian physics concepts taught in class. Adult subjects as well as a college student dealing with the same situation showed a large overlap with the set of strategies used by the elementary students, thereby indicating a marked lack of influence of classroom physics training on their naive physics.

This paper experimentally explores the nature of the biases observed in probabilistic information processing in the domain of basic probability. At the same time, an attempt is made to verify whether such biases can be overcome with increased experience with quantitative normative model. To this end, the experiment was designed to compare the problem solving behaviors of subjects with various levels of proficiency in basic probability, using the Bayes' Theorem as a normative model for probabilistic information processing or belief revision.

## 2. Experiment

The principal data for the study are the concurrent verbal protocols of the subjects. Protocol analysis is the process of translating the collection of the verbalization (i.e., verbal protocols) into more accessible and structured representations of a subject's cognitive processes (Ericsson & Simon, 1980, 1984; Newell & Simon, 1972).

### 2.1. Subjects

Twenty-nine subjects from the business school participated in the experiment: nine sophomores with high school probability background (group 1), ten juniors and MBA students who had taken a college probability course (group 2), and ten Ph.D. students who had taken a graduate-level probability course (group 3). None of them were currently taking any probability course.

### 2.2. Procedures

Subjects were tested individually by the researcher. When setting up the time for individual meeting for the experiment, subjects were told that they would be given basic probability problems to solve and the session would last an hour at most.

Two sets of problems were prepared: two problems in the first set and one problem in the second. There were all Bayesian problems. Problems of the first set, named Cat Burglar and Evaluation Report respectively, were different from the usual textbook or homework problems in that there were no specific numbers given in the problem statements. Instead, such phrases as "most of them", "good chances" and "equal amount of chances" were used to denote the magnitude of the likelihood. Efforts were made to make the context of the Cat Burglar problem different from those of the usual exercise problems. Also, the prior probabilities were not explicitly given in the Evaluation Report problem. The problem in the second set was a typical Bayes' Theorem problem that could be found in any college probability textbook (the texts of problems are shown in the Appendix).

Before the experiment, each subject was instructed about thinking aloud. The first set was administered first. After the subject's perusal of the problem statements for any ambiguity possible, the verbal protocols of problem solving were tape-recorded. A probability textbook, pencil and paper were available. The worksheets were used later for analysis with the tape-recorded verbal protocols. The first set of the problems was given first, followed by the second set. When all problems were tried, a brief interview was made with each subject to gather further information.

## 3. Protocol Analysis

### 3.1 Group 1

Nine subjects in this group were sophomores whose background in probability was the course taken in high school. Surprisingly, the group in general showed a very similar behavior in solving either quantitative or qualitative Bayesian problems.

In solving the Department Store problem, six subjects shared the same ad-hoc problem solving strategy of enumerative nature. They assumed there were 100 purchases, and put the probabilistic information given in the problem statements into a numerical example:

> "Let' assume there are 100 purchases, then there are 10 purchases returned and 90 purchases not returned ..."

Then, they used the semantic meaning of the conditional probability to answer the question and five of them successfully solved the problem:

> "... So, out of total 52 charged purchases, 7 purchases are returned ..."

The other three subjects, on the other hand, tried to solve the problem algebraically without resorting to such an ad-hoc example, but by following "a blind hunch." They revealed that they were not familiar with manipulating joint and conditional probabilities and could not decipher these relationships implied in the problem statements. Neither was the textbook helpful for these subjects:

> "I am looking for an example to follow (from the textbook) ... Oh, man! I don't see any. ... .1 time .7 ... not sure what it will give me .. I don't know .... I think I need to add them together because that is my condition ... I am following a blind hunch here .."

> " ... (referring to the formulas in the textbook) These are meaningless for me right now. I don't understand any of these."

The most noticeable feature of this group's problem solving of the Department Store problem was that none of the subjects used the mathematical notations for probability and, accordingly, there were no spontaneous mentioning of marginal, conditional or joint probability regardless of their strategies. A correct setting-up of an example showed that the subjects understood the context of the problem correctly, but there was no clear indication of whether they understood the concepts of joint and, especially, conditional probability. One of the subjects failed to produce a correct answer even though he had properly constructed the example for the problem.

As in the case of Department Store, there was made no mathematical notation or spontaneous mentioning of joint or conditional probability in solving Cat Burglar and Evaluation Report problems. Further, none of them recognized that both sets of problems were of the same characteristics--Bayesian Problems. Accordingly, each of them used a totally different problem-solving strategy from the one they later used to solve the Department Store problem. Instead of using ad-hoc example or algebra, seven subjects drew a conclusion simply following their "logic", "reasoning" or "common knowledge":

> "(The solutions) looked like more of an opinion even though there might be a correct answer. .. I was able to reason it through."
>
> "It just makes sense. .. very logical."
>
> "I tried to use my common knowledge."

Notably, six out of these seven subjects found Cat Burglar and Evaluation Report problems much easier to solve than Department Store: even though "there are no specific numbers to work with", "(The problems) were pretty logical if you straightened the things up in your mind" and "I am confident of my reasoning." In fact, they solved these problems much faster than Department Store and were very confident in their solutions although none of these six subjects had solved the problems correctly.

The last one of the above seven subjects, in contrast, thought Cat Burglar and Evaluation Report problems were a lot harder to solve than Department Store, since "the words problems didn't have something concrete to work with" and he "was confused about what were needed to solve them." So he tried to use a Venn diagram, which is not a good representation scheme for conditional probability. He ended up with two unrelated Venn diagrams representing the problem context incorrectly, and in consequence an incorrect solution.

These seven subjects who did not solve the problems correctly made a common mistake of ignoring the prior probability:

> " ... (for Hit-&-Run) 50% chances of getting caught and 50% of getting away with a crime ... Police have a pretty good chances to catch an Old-Faithful. Since we know this, then we can assume that it is more likely the burglar caught is an Old-Faithful."
>
> " ... Most candidates with favorable reports are successful later but half of unfavorable reports were successful ... so, it is quite fair .. It is more likely to receive a favorable report if it is a to-be-successful manager."

The other two subjects answered the problems correctly. They neither recognized the problems as Bayesian problems, nor adopted the same strategy as they used for Department Store. However, both of them drew diagrams to represent the problem context. One of them drew decision trees, assigning percentages to the branches in keeping with the problem statements and thereby successfully solved both problems.

The other subject made the same mistake of ignoring prior probability for the Cat Burglar. But later, when he realized the complexity of the Evaluation Report problem, he came up with an ingenious idea of representation to find the correct solution path.

### 3.2. Group 2

The ten subjects in this group consisted of six juniors from business school and four MBA students. They had taken a college probability course before and were not taking one currently.

The most striking difference in contrast to Group 1 in solving Department Store problem is that they no longer adopted an ad-hoc problem space. Problem statements were translated into mathematical notations (even though two of the subjects could not properly do the transformation) and consequently were adopted mathematical problem spaces. Seven subjects adopted the AND/OR tree problem space, and other three subjects drew a joint probability matrix.

Two junior students were "weak" in probability: one of them tried to transform the problem statements into mathematical notations. However, he could not distinguish marginal or joint probability from conditional one and solve the problem. The other subject, not being much better than the first one, displayed totally erroneous problem-solving behavior.

The remaining four juniors produced the correct solutions. Two of them solved the problem algebraically in AND/OR problem space, showing the typical slow novice problem-solving behavior we saw in previous protocol analyses (the solution times were 13:48 and 16:05 respectively). The other two solved the problems efficiently by drawing a joint probability matrix with solution time of about six minutes. Four MBA students solved the problem correctly and efficiently in around four minutes. Three of them solved the problems algebraically and one used probability matrix.

Solution of Cat Burglar and Evaluation Report also seemed to have a correlation with the proficiency of quantitative skill of Bayes' rule. Three MBA students identified both sets of problems as Bayesian problems, and adopted a consistent problem space for both quantitative and qualitative problems to produce unbiased solutions. The fourth MBA student, who solved the Department Store problem as efficiently as the first three, gave up on the problem space he adopted for Department Store in solving Cat Burglar and Evaluation Report problems. He "tried to use common sense by deductive reasoning" only to neglect the effect of prior probabilities.

Of the six junior subjects, all committed the same mistake in the Cat Burglar and Evaluation Report problems. Four of them solved the Department Store successfully, but they failed to recognize the problems properly in solving Cat Burglar and Evaluation Report:

> "Words problems, they are typical problems that you solve logically as opposed to using numbers and formulas, ...typical day-to-day events going on in the world."

This kind of behavior was observed even when the subject had identified the problem as a Bayesian problem. One of those four advanced juniors identified the Cat Burglar and Evaluation Report problems as Bayesian problems without difficulty and even set up an equation accordingly for the Evaluation Report problem. However, when she was not sure how to evaluate the equation, she tried a Venn diagram which was not helpful in correctly evaluating the equation and then slipped into the prior-probability fallacy.

### 3.3 Group 3

Subjects in this group were ten Ph.D. students who had had advanced graduate-level probability and statistics courses one or two semesters before. As expected, all of the subjects had no difficulty in solving the Department Store problem correctly. Two subjects drew a probability tree and directly calculated the solution from the tree. All other subjects followed an algebraic derivation of Bayes' Theorem (AND/OR tree).

Eight out of ten subjects showed a consistent problem-solving behavior in solving the Cat Burglar and Evaluation Report problems as in Department Store:

> "Both (parts 1 and 2) are same problems ... Bayes' rule problems, one with specific numbers and the other without."

Nevertheless, when compared to that of Group 2, two ways of handling the non-numeric situation were noticeable. Some of them assigned variables such as p or q to unknown probabilities and assessed the resulting equation in terms of these variables for the conclusion.

The others, however, didn't bother with the specific numbers. They set up the equation for Bayes' Theorem and then tried to find how the values of each probability could be obtained from the problem statement.

An interesting observation was that a trend of using a heuristic in evaluating evidence still persisted. One subject of the two who used a heuristic also made the same mistake of disregarding prior probability in solving Evaluation Report. He first solved the Cat Burglar problem properly, but when he read the text of the Evaluation Report problem, the problem seemed to look like a trivial one ("Hey, it's too easy!") and solved the problem intuitively in less than two minutes so as to omit the prior probability in the evaluation. The last subject seemed to have made the same mistake of misinterpreting the Cat Burglar problem to make it trivial:

> "Given other robberies in the neighborhood, it's more likely an Old-faithful; Given few robberies in the neighborhood, more likely it is an Old-faithful."

Further, in solving Evaluation Report problem, he made an unique mistake. He drew a probability tree for the problem as he later did for the Department Store problem and found out that the ratio of favorable and unfavorable reports (prior probability) was not given:

> "What is the ratio of favorable and unfavorable ...
> It's hard to tell ..."

Writing down "constant ratio" in the probability tree did not help him to evaluate the branches of the tree, and he ended up with an intuitive solution for the problem without taking account of the prior probability.

All other eight subjects, who solved both sets of problems correctly, showed consistent problem-solving behaviors, adopting the same problem space and strategy for both quantitative and qualitative problems.

## 4. Discussion

### 4.1. Bias in Probability Revision

The most striking feature found from the protocol analysis in Bayesian problem solving is that almost all naive or novice subjects revealed a uniform behavior in their evaluation of evidence when revising the probability: they tended to be insensitive to prior probability. This insensitivity to prior probability was observed to persist even for the experienced subjects as noticed by Kahneman and Tversky (1972).

Where does this bias come from and why is it persistent? The obvious source of this bias is the causal relationships between events intuitively assigned by the subjects. The evidence was abundantly found from the verbal protocols of naive subjects solving the Cat Burglar and Evaluation Report problems:

> "If the cat burglar is one of the "hit-&run", he is less likely to be caught in that area, compared to his counterparts who do burglaries in that area ..."
>
> "Police have a pretty good chance to catch "old-faithful" (while there is a 50/50 chance to catch "hit-&-run") ... Then we can assume that it is more likely that the burglar caught is an "old-faithful."
>
> " ... So, it is quite fair .. It is more likely to receive a favorable report if it is a to-be-successful manager."

This causal relationship is a very strong and straightforward relation between events. Thus, when the subjects implicitly assumed a causal relationship between events, the whole reasoning is simple and the logic behind the reasoning is robust and clear-cut.

When the naive subjects were asked about the confidence in their conclusions after the problem had been solved, most of them manifested that they were significantly more confident in their solutions for the word problems (where they used the logic) than in the solutions of the Department Store problem (where they used an ad-hoc example). Many of the subjects were more confident with the Evaluation Report problem than with the Cat Burglar problem, even though the prior probability was not given for the former problem. When asked for the reason, their usual answer was that the Evaluation Report problem was "more logical" or "more familiar" than the Cat Burglar one.

The logic based on the assumed causal relationship between two events, for example A and B, implies that P(A | B) is equal to P(B | A), which is one of the common misconceptions found in naive students in probability class. Hence, if a subject assumed a causal relationship between two events in solving a Bayesian problem such as the Cat Burglar or Evaluation Report problem, then there is no actual need to revise probability, and consequently, no need to consider prior probability to get a conclusion.

Since a false causal relationship is contradictory to the principles of probability, it is inevitably detected if one uses a probability theory or at least sets up a proper example in solving a Bayes' Problem. In fact, such a mistake of neglecting priors in revising probability was made neither by naive subjects who properly set up an ad-hoc example or diagram, nor by experienced subjects who did not solve the problem "intuitively."

The causal relationship is a very strong and logical relation, and the cause-and-effect phenomena are found everywhere in day-to-day routines. It is conceivable that a person learns about this relationship from everyday experience and reinforces the perceptive sense toward this causal relationship. It is quite often observed that people tend to assume a causal relationship between events, even if they are not related in that way. This tendency may be a plausible explanation for the fact that even an experienced researcher (or advanced students in our experiment) is prone to a biased judgement when he or she tries to think intuitively.

Two subjects mentioned in the previous section seemed to have a bias resulting from other than an assumed causal relationship. There was no definite clue about the sources of the biased judgement, but it seemed they were using some heuristics of which operations were based on the syntactic aspects of the problem.

One naive subject solved the Cat Burglar problem using the causality heuristic, thereby ignoring the prior probability. The reasoning for the Evaluation Report problem included the prior probability in its consideration, but rather ignored the specifics of the conditional probability. She easily identified the first event involved favorable and unfavorable reports, since they were mentioned in the problem consistently. But she seemed to be confused about the second event of successful and unsuccessful. They were not consistently annotated syntactically in the problem: successful candidates, unsuccessful manager or to-be-successful manager. Therefore, she seemed to focus on favorable and unfavorable aspects of the problem context, and anchored her judgement on the information about the prior probability.

One Ph.D. student showed a unique heuristic when he solved the Evaluation Report problem. He drew a probability tree to solve the problem, but could not assign the specific probabilities for the first layer of the tree since the prior probability was not given. Further, he had the same problem with one of the branches in the second layer--no specific numbers were given for the conditional probability. Encountered with this difficulty, he simply ignored all incomplete information and drew his conclusion only based on the complete numerical information.

### 4.2. Categorization

Most naive subjects solved the quantitative and qualitative problems quite differently. When solving the Department Store problem, they set up an example that contained all the information given in the problem statement, and further obtained a correct answer from the example. This ad-hoc example was basically a joint probability matrix in a crude form, which is a very effective problem space in solving Bayes' problems. However, when trying to solve words problems, they gave up this effective representation and tried to use their "deductive reasoning." This phenomenon was same for those who solved the Department Store problem algebraically.

The major reason for this behavior is that they could not recognize the characteristics of the problems (i.e. Bayesian problem) or identify the similarity between them:

> "Department Store problem requires strict application of formulas according to numbers and certain rules ... The words problems are more of a matter of opinion .."
>
> "Working with numbers ... It's a matter of setting up an example.. (For words problems) it was pretty logical once you straighten things up in your mind."
>
> "It does not matter whether I solved the first part first or vice versa ..."
>
> "I didn't even think about it (the relationship between two parts). It didn't cross my mind at all."

This superficial categorization of problems was also witnessed from "weak" subjects in Group 2 who were not quite familiar with basic probability. If the superfluous categorization was not explicitly observed from other subjects, proficiency in basic probability principles or formulas did not automatically help them to solve the qualitative problems. One of them even identified both sets of problems as Bayesian problem and solved the Department Store problem accordingly without much difficulty. However, she solved the qualitative problems intuitively so as to disregard the prior probability in drawing a conclusion. Further, she said that she would have solved the problems in the same way as she had done even if the Department Store problem had been given before rather than after the Cat Burglar and Evaluation Report problems.

The separate application of intuitive and normative knowledge in solving quantitative and qualitative problems was also observed amongst Group 3, even though they identified the problems properly. One subject showed, when he solved the Department Store, that he could solve quantitative problems without difficulty, but made the same mistake about prior probability while solving the Cat Burglar and Evaluation Report problems. Another subject solved the Cat Burglar correctly in the same way as they later solved the Department Store. However, they used their intuitive logic to solve the Evaluation Report problem, since the problem was "very straightforward" and "simple."

As Chi, Feltovich & Glaser (1981) noted, the different categorizations evoke different kinds of knowledge associated with them, which consequently generates the different plan and behavior of problem solving. However, proper categorization alone in not enough to successfully complete the computation for a class of problems as we have seen in the performances of several subjects in Group 2 and 3. Quantitative refinements of knowledge must be integrated in appropriate ways with a qualitative belief or heuristic, and vice versa.

As one becomes proficient with a quantitative skill and gain more confidence in the applicability of the skill, the skill apparently receives more general consideration for its justification and its applicability is reinforced. The confidence in applicability and reinforcement of a quantitative skill lays a base for facilitated knowledge transfer to a qualitative one. This idea is consistent with the studies in human information processing where the successful application of a production rule increases the probability that the same rule will be applied in future situations [Lewis, 1978; Anderson, 1986].

### 4.3 Conclusion

One notable result of these analyses was that good quantitative problem solving skills are necessary for good qualitative problem solving skills. A number of students in the novice category were able to categorize the qualitative problems correctly as ones requiring the use of formal probability methods. However, they were unable to complete their analyses due to a lack of expertise in computing the relevant probabilities. One student after such a failure even resorted to using the incorrect approach based on intuitive logic. So a necessary but not sufficient condition for good qualitative problem solving skills is the ability to utilize quantitative problem solving techniques.

Another interesting outcome of these protocol analyses is that naive students already have some preconception of how probability knowledge should be represented. Interviews with these naive students revealed that almost all of the students have been informally exposed to probability in high school and favor a probability matrix type of knowledge representation for Bayes' Theorem/conditional probability problems.

The protocol analyses provide some indirect support to the notion that procedures which have been successfully used in the past will be more likely to be used in future problem-solving situations (Gick & Holyoak, 1980, 1983). Novice subjects were quite familiar and confident about using the intuitive logic approach. However, some of them were not especially proficient with the Bayes' Theorem problem-solving procedure and thus probably were not confident with its use. In contrast, the expert subjects were quite proficient in using Bayes' Theorem and were probably very confident about its use.

There is one possible explanation for success of the expert students in categorizing the qualitative problems as one requiring the use of Bayes' Theorem. With their high level of confidence in the use of Bayes' Theorem these students were likely to invoke such an approach if it appeared to be useful. In contrast, the novice students may have tended to avoid the use of Bayes' Theorem since they had a low level of confidence in using the approach successfully. These novice students may have gravitated towards the use of an intuitive logic approach where they had a high level of confidence in its use.

4.4 Future Directions

The discussion about biases and categorization suggests that there is an apparent pattern of problem-solving skill evolution in dealing with Bayesian problems.

To fully understand the evolution of qualitative problem solving skills, two lines of research are especially of interest. The first one is the schema theory (Bobrow & Normal, 1975; Minsky, 1975; Thorndyke & Hayes-Roth, 1979; Rist, 1989), and the second the analogical problem-solving (Carbonell, 1983, 1986; Chi, Feltovich & Glaser, 1981; Chi, Glaser & Rees, 1982; Clement, 1988; Gick & Holyoak, 1980, 1983).

If these two lines of research have started with the different perspective from each other, they are closely related and complementary in explaining the acquisition of expertise. Thorndyke and Hayes-Roth (1979) define a schema as a configuration of concepts and association among the concepts that are repeatedly invoked to encode unique stimulus events. A schema as an abstraction induced from past experience, hence, afford the basis for facilitated analogy transfer by making salient the causal aspects of the situation that should trigger a particular plan of action (Gick & Holyoak, 1983).

The research next in order is to conduct an in-depth extensive analysis on the exploratory study described here in the light of the schema-based analogy transfer in problem solving. This would provide us with richer and more complete information about the evolution of qualitative problem-solving knowledge.

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Appendix

CAT BURGLAR

Recently in the metropolitan area of Chicago, there has been an outbreak of cat burglaries. The police know from the past experience that there are basically two types of cat burglars: "hit-and-run" and "old-faithful." The "hit-and-run" type burglars constantly change their target area for a crime and usually are not responsible for more than one robbery in one particular neighborhood. The counterparts of them are commonly referred to as "old-faithful"s because their radius of activity involves one specific neighborhood for a long period of time during their careers.

From the investigation of apprehended criminals, the police learned that most of the recent break-ins are of the "hit-and-run" type. The police also know they have a really good chance in capturing the burglar when a crime is committed by one of the "old-faithful"s. However, busting "hit-and-run" artists is more or less a matter of luck, and there is an equal amount of chance for them to get away with the crime.

When a burglar is caught, is it more likely, or less likely, that this burglar is responsible for recent robberies in the neighborhood other than the one he/she is apprehended to (i.e. old-faithful)?

EVALUATION REPORT

When hiring a manager, the COMPANY first starts the preliminary screening process by collecting information about the candidates' skills and previous experiences. Only those with a favorable preliminary report are given further consideration by the stockholders.

The stockholders of the COMPANY came to notice that the ratio of favorable and unfavorable preliminary reports has been consistent for the last five years, even though they have had different pool of candidates for each evaluation. Hence, they became concerned about the biasness of the process, and conduct a follow-up survey about the performances of all ex-candidates, hired or not, who had gone through this process during the last five years.

The result shows that most of the candidates who had received a favorable report from the preliminary evaluation later proved themselves as successful managers. However, the preliminary evaluation did not seem to do a good job for those who had gotten an unfavorable report: there were as many successful cases as

unsuccessful ones.
Is it more likely, or less likely, that a to-be-successful candidate would receive a favorable report than an unfavorable report from the COMPANY's preliminary screening process?

DEPARTMENT STORE

A store has found from past experience that the probability that a customer will return a purchase is 0.1. Further, 70% of all purchases that are returned are charged, and 50% of all purchases not returned are charged.
Find the probability that, if a purchase is charged, it will be returned.