December 2004

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Yong-Seog Kim
Utah State University

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YongSeog Kim
Utah State University
Business Information Systems
Logan, UT 84341
ykim@b202.usu.edu

ABSTRACT

This paper presents an agent-based simulation approach to estimate the effects of auction parameters on the auction outcomes, and to find an optimal or, at least, close to an optimal infrastructure. In particular, this study intends to study how bidders' personalities and bidding strategies with other auction parameters affect the closing prices in two auction mechanisms: English and Yankee auctions. Experimental results show that the outcome of multiple English auctions is more favorable to auctioneers and sellers than that of a corresponding single Yankee auction. It is also shown that raising minimum bid increment or bid starting price positively affects the closing prices in both auction types. However, two auction systems respond differently to the changes in parameter values in terms of magnitude and robustness.

Keywords

Agent-based simulation, optimal auction design

INTRODUCTION

Online auction is one of the most successful mercantile processes in these days. For example, according to a self-report from eBay, on any given day, there are more than 12 million products across 18,000 categories and eBay members transacted \$14.87 billion in annualized gross merchandise sales in 2002. With the great success of online auction, there already exists a vast body of researchers from economics, finance, and e-commerce who studied the general theory of auction and attempted to test presented theories in real-world environments (McAfee and McMillan, 1987; Milgrom, 1989; Pekec and Rothkopf, 2003). The ultimate goal of their efforts is to understand and organize this emerging new mercantile process in an optimal way to maximize sellers' or bidders' welfare. For this purpose, mainly two different methodologies–laboratory and field experiments–have been used.

In laboratory experiments, real world phenomena are studied in a simplified and controlled setting for a specific research goal. While experimental results from controlled experiments are easily replicable (Kim, Barua and Whinston, 2002), it is not possible for researchers to have a completely controlled setting even in laboratory experiments. For example, researchers cannot control bidders' personalities, proportions of different types of bidders, distributions of bidders' personal valuations on auctioned items, new bidders' arrival, and so on. Further, results from laboratory experiments are often dependent on experimental settings and subjects (Lucking-Reiley, 1999). Many laboratory experiments also often involve an oversimplification of complex real worlds (Reinig et al., 1998). Researchers also use field experiments (e.g., hosting auctions on a real auction site) without worrying about control of extraneous variables as in laboratory experiments. However, field experiments are typically expensive (Kim et al., 2002), rarely used (Davis and Holt, 1993), and findings based on these data sets may reflect the mixed effects of several factors because key variables are not fully controlled. Finally, researchers often find it difficult to compare different auction mechanisms in laboratory and field experiments because subjects can "learn" and accumulate knowledges through experiments and use their knowledges in next experiments.

Researchers have also analyzed data sets collected from real auction sites with or without help of data collection agents (Roth02). However, these data sets are not appropriate for studying effects of individual auction parameter on outcomes of online auction because auction outcomes in real data sets reflect mixed effects of multiple parameters and it is almost impossible to extract individual contribution of each parameter from aggregated outcomes. For example, auction outcomes are dependent on auction specific parameters (e.g., auction type, auction duration, bid starting price, and minimum bid increment), bidder-related factors (e.g., number of bidders, bidders' personalities and private values, and proportions of different types of bidders), seller-related factors (e.g., sellers' reputation), environmental variables (e.g., popularity of auction sites), and item-related variables (e.g., used or new items, items with quick or slow depreciation). Therefore, not only is it extremely difficult to find many instances of auctions with the same values of parameters in the same environment but also it is often impossible to infer values of some parameters (mostly bidder-related parameters).

This study introduces a simulation-based auction optimization (SAO) model to better understand online auction and to find an optimal or, at least, close to an optimal infrastructure. In terms of research methodology, the SAO model is an extension of a simulation tool in (Bapna, Goes and Gupta, 2002; Bapna, Goes and Gupta, 2003) that generated an artificial bidding history based on bidding history in real data sets. The SAO model is also a generalized model of an agent-based simulation model in (Mizuta and Steiglitz, 2000) that studied the relationship between bidder types and auction outputs for a specific auction model. There are several advantages of using the SAO model: Using the SAO system, researchers can greatly reduce the amount of time for data collection, pre-processing, and validation. Further, the SAO system also makes it possible to compare different types of auction systems by generating the same pool of bidders that do not learn from previous experiments. Most of all, the SAO system provides researchers a great control over auction parameters and virtual online bidders, and hence allows them to separately analyze effects of multiple factors. Ultimately, the proposed system will help sellers make decisions about auction-related control variables and develop new auction mechanisms.

LITERATURE REVIEW: BIDDERS AND AUCTION TYPES

In this study, two types of bidders—risk-avoiders and risk-takers—are studied. The risk-takers are those who adopt a bidding strategy called “sniping”, last minute bidding to avoid unnecessary bidding wars against other bidders from the initial stage of an auction process. According to Roth and Ockenfels (2002), a significant portion of experienced bidders or experts uses a sniping strategy. By outbidding the highest bid at the last moment, risk-takers (or snipers) expect to outbid the current highest bidder without giving her a time to submit a higher bid. The other type of bidders is risk-avoiders who constantly increase their bidding prices by minimum bid increment (k) until the bidding price reaches their subjective upper bounds, their private values, whenever they are out of the winners’ list. The risk-takers and risk-avoiders are termed as Opportunists and Participators in (Bapna et al., 2003), respectively. In general, it is assumed that as more bidders adopt sniping strategy, the closing price becomes lower due to the less competition among bidders. This paper does not consider other types of bidders (e.g., Evaluators (Bapna et al., 2003) who bid only once at their private values and jump bidders (Easley and Tenorio, 1999; Issac, Salmon and Zillante, 2004) who bid higher than a minimum required bid) mainly because proxy bidding systems make it difficult to distinguish Participators who use proxy bidding system from Evaluators and preclude jump bids by automatically increasing bidding price by k.

Two different types of auctions—English and Yankee—are compared in this paper. English auction is one of the most popular auction models (Pinker et al., 2003). In English auction, only one item is available and a set of bidders compete against each other to obtain the item. Bidding in English auction starts at a minimum bidding price and the winning bid is increased by k. The bidder with the highest bidding price is determined as the winner but, in rare cases, payment arrangement can be also considered. Another very popular auction mechanism in the United States is Yankee auction. Both English and Yankee auction share some common characteristics. For example, bidding in both auctions starts at a minimum bidding price and the winning bid is increased by k. However, in Yankee auction, there are multiple items available and hence bidders should specify bid price and quantity of items that they want to purchase. Each bidder must bid on all items that she wants to buy at the same price in one bid. Ties in Yankee auction are broken in the order of bidding price (bidder with a higher bidding price wins), quantity (bidder who bids on more items wins), and bidding time (bidder who bids earlier wins). Since there are multiple items available in Yankee auction, multiple winning bidders are possible and they pay their own prices. Yankee auction can be considered a general case of English auction. However, bidders’ expectation and bidding strategies in two auctions often lead to the situation that the seller’s welfare in a Yankee auction is not necessarily the sum of the seller’s welfare from multiple English auctions. To measure the welfare or utility of sellers in both auction mechanisms, the closing price of the auction is used because it has a positive relationship with seller’s welfare.

RESEARCH MODEL AND EXPERIMENTAL SETTINGS

Simulation-based Auction Optimization (SAO) Model

The structure of a simulation-based auction optimization (SAO) model is shown in Figure 1. The SAO model consists of two components: auction environment and simulation components. In the auction environment component, various auction scenarios are randomly generated. Each generated auction scenario consists of various auction-related control variables including a proportion of risk-takers and risk-avoiders, a bid starting price, a minimum bid increment, and the number of bidders. Once a scenario is generated, it is passed into the simulation component in which a virtual auction is simulated with the given values of control variables and newly created bidders.

The simulation component is the core of SAO model. One of the main roles of the simulation component is to create virtual bidders (or bidding agents) that act like human bidders on real auction sites. Each agent is either a risk-taker or a risk-avoider, and has its own valuation about the item price. The private valuation of each bidder is drawn from a specific distribution (e.g., Gaussian, Uniform, or Exponential distribution). Agents are also different in terms of how many items they want and when they start bidding during the auction duration. Once agents are created, a virtual auction is simulated and agents compete to be a winner by following auction rules and their strategies through bidding process.

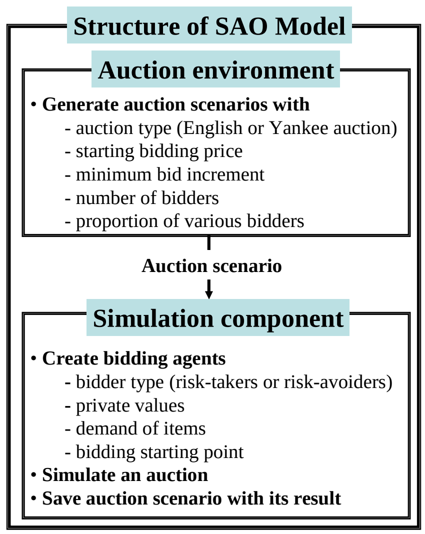

Figure 1. Structure of the SAO model

All bidding agents in simulated auctions behave like proxy bidding agents in real auction sites. Therefore, they increase their current bids by minimum bid increment when they are out of the winners' list. However, to add a realistic flavor, bidders do not always increase their bids even if other bidders outbid them. This is the case when some bidders want to sit back from and watch bidding processes rather than raising their bids right away. Note that all bidders are aware of their private values but they do not know other agents' private values. Note also that, in most open ascending auctions, other bidders' revealed bidding prices can affect the other bidders' private valuations on the auctioned item, and all agents want to wait few rounds of bidding so that they can estimate the true market value of the item from others' bids (Bulow and Klemperer, 1999). Therefore, in this paper, bidding orders are randomly determined during the auction duration. The chosen bidder may or may not place a new bid by comparing current winning bid and her private values. Further, all agents start bidding only after randomly assigned bidding points that is represented as a percentage of the auction duration. For example, while a risk-avoider will start bidding anytime between 0% and 99% of the auction duration, no risk-taker bids before the starting point of the last quarter of the auction duration. In multi-unit auctions like Yankee auctions, all bidders try to secure as many as items up to their demands at the same price. Given their preferences, all bidding agents in simulation follow the same optimal bidding strategy, which is summarized as follows:

- Step 1: Determine whether it is eligible to bid based on its pre-assigned bid starting time. If it is not eligible or it is eligible but it is already a winner, wait for next round of bidding. If it is eligible but its current bidding price plus a minimum bid increment (k) exceed its private value, do not bid any longer. Otherwise, go to Step 2 for English auction or go to Step 3 for Yankee auction.
- Step 2: In English auction, increase its bidding price by k. Wait for next round of bidding.
- Step 3: In Yankee auctions, determine whether it is necessary to raise current bid. Note that, in Yankee auctions, if multiple bidders bid at the same price, bidders who bid later but bid on more items have higher priority than those who

bid early but bid on fewer items. Therefore, if it can be a winner by bidding up to its demand at the current winning price, do not raise current bid. Otherwise, increase the current bid by *k*.

Experimental Settings

In this paper, the market value of the auctioned item is subjectively set to $1,000. Each randomly generated auction scenario contains information about the proportion of risk-takers (from 0% to 100% of bidders), bid starting price (from 10% to 50% of the item price), and minimum bid increment (from 2% to 10% of the item price). By representing auction-related parameter values as percentages of item price whenever possible, the findings from a specific simulation can be compatible with findings from simulations with different parameter values. The same rule applies to the private values of bidding agents. The private values of both risk-takers and risk-avoiders are drawn from a Normal distribution with a mean value (item price) and a standard deviation (item price × 0.25). The standard deviation is set to large enough to generate bidding agents with significantly different preferences. In preliminary experiments, experimental results from various distribution types were not significantly different when the variances of private values are the same. Further, the normality assumption is computationally convenient and closely approximates the observed distribution of bids on eBay (Bajari and Hortacsu, 2003). In particular, the presence of price-comparison agents creates a mass of consumer valuation at or around the prevailing market price (Bapna et al., 2002). Therefore, this paper presents results only from Normal distribution. The number of bidders in each auction scenario is negatively dependent on the bid starting price. When the bid starts at a lower price, it attracts more bidders who are constantly looking for a great deal. To reflect this negative relationship, the number of bidders is determined using a formula, λ/P_s, where P_s and λ represents a bid starting price as a percentage of the item price and a constant multiplier (λ=10), respectively. Based on the specified formula, when the starting bid is set to 10% and 20% of the item price in two scenarios, 100 bidders (=10/0.1) and 50 bidders (10/0.2) are created, respectively.

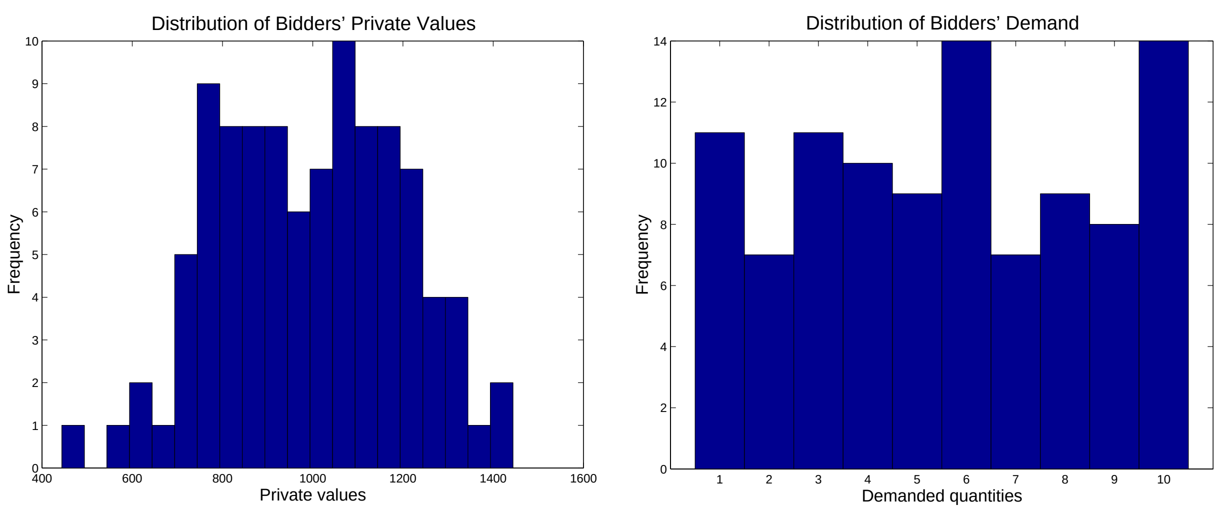

Figure 2. Distributions of bidders' private values and demands

Figure 2 shows the distribution of randomly generated 100 bidders' private values and their demands of an auction pro duct. Since the demand of each bidder is drawn randomly between one item and 10 items, demanded quantities are evenly distributed among bidders. On average, each bidder's private and demand quantity was $953.85 and 5.46 items. Although it is possible that few bidders can have negative private values, it is very unlikely to happen considering 95% confidence interval of a Normal distribution with given mean and standard deviation values. In case that there are few bidders with negative private values, they will not be able to participate for bidding process due to a high bid starting price. The auction duration in simulations is set to 50 bidding opportunities. Note that each time a bidder is randomly chosen is counted as one bidding opportunity and when 50 bidders are chosen, the auction ends. Overall, chosen parameter values in simulations are consistent with findings in (Easley and Tenorio, 1999) that collected data from 202 multi-unit auctions at Onsale.com and Ubid.com, and found that on average 10 units are offered and 52 bids are placed by 34 bidders.

EXPERIMENTAL RESULTS

Overall Comparison of English versus Yankee Auction

This section presents experimental results of comparing two representative auction systems, English and Yankee auction, in terms of seller's profit. In the following experiments, it is assumed that a seller has 10 items and she must decide which auction system is more profitable. The fair comparison is to compare an average of closing prices from 10 independent English auctions to an average of multiple closing prices from one Yankee auction. Note that for each simulation, a new group of bidders is generated. In order to account for the random difference in private values among bidders and provide reliable estimates, 50,000 English auctions and 5,000 Yankee auctions are simulated and results are summarized in Table 1.

	English auction	Yankee auction
Avg. of closing price	889.26	563.19
Std. dev. of closing price	236.51	217.73
t-test *p*-value		< 0.0001

Table 1. The average of closing prices from 50,000 English auctions and 5,000 Yankee auctions.

The average closing price from 50,000 English auctions is \$889.26 (±\$236.51), while the average price from 5,000 Yankee auctions is \$563.19 (±\$217.73). The simulation results show that multiple English auctions are more profitable than a corresponding Yankee auction. A careful investigation of two types of auction mechanism reveals why English auction returns a higher profit to a seller than Yankee auction. The fact that only one item is available in English auction and all bidders compete against each other indicates that any bidder who wants to win the only item should bid higher than the current winning price at least by minimum bid increment. Therefore, every new bid in English auction raises the current winning price during the auction process. In contrast, multiple items are available in Yankee auction and bidders can bid at the current winning price on the remaining items if there are more items left than their demand. Therefore, not all biddings in Yankee auction raise the closing price of auction. As a result, the closing price of Yankee auction can be significantly lower than that of multiple English auctions. This finding is also consistent with a traditional observation in economics, "more supplies lower a market price." However, this finding should be interpreted with caution because English auctions may incur additional costs including multiple set-up costs, inventory costs, and other opportunity costs related to depreciation of items. If all these costs were considered, it is possible that Yankee auction may return higher profit for sellers. Note also that this study is different from Hausche (1986) that compared simultaneous and sequential sales based on the same auction type and concluded that neither sale mechanism dominated the other sale.

Effects of Bidder Types on Closing Price

Different proportions of risk-takers and risk-avoiders can have significant impact on the outcomes of auctions. Figure 3 shows that the closing price in both auctions decreases as the proportion of risk-takers increases. This can be attributed to the fact that increasing number of risk-takers leads to less competition among bidding agents because of sniping strategy from risk-takers. However, the proportion of risk-takers seems to have slightly different effects on outcomes of English and Yankee auctions. For example, when all bidders are risk-avoiders, the difference in closing prices between English and Yankee auctions is \$365.38 (= \$1002.42 - \$637.04). However, when all bidders are risk-takers, the difference becomes \$269.91 (= \$743.15 - \$473.24). This indicates that Yankee auctions are more robust to sniping strategy because of possible multiple winners. In particular, closing prices in Yankee auctions do not decrease (with some fluctuations) when the proportion of risk-takers changes between 1% and 30% of bidders. However, the same change in proportions of risk-takers decreases the closing price in English auctions by maximum \$84 and its rate was accelerated as there are more risk-taking bidders. This finding is a counter-example to the revenue equivalence theorem (RET), which states that the expected revenue of the auctioneer will be the same under all four auction mechanisms: English, Dutch, first-price, and second-price auctions (Vickrey, 1961).

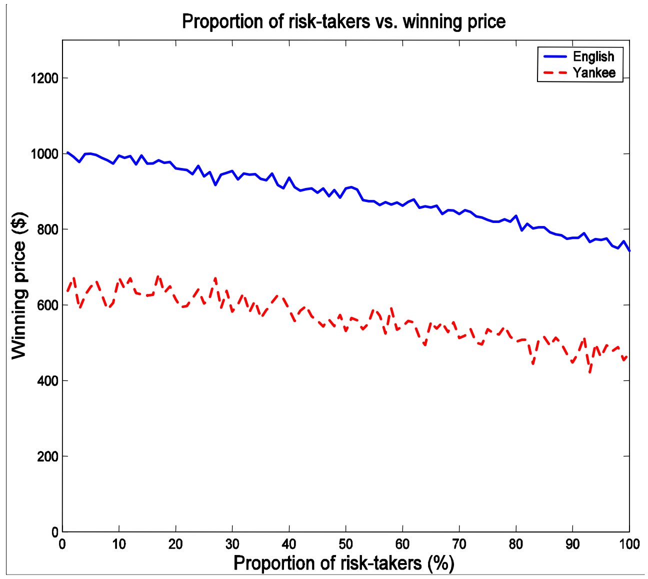

Figure 3. Results of auctions with various proportions of risk-takers

Effects of Bid Starting Price on Closing Price

Bid starting price (or minimum bid) is one of many parameters that sellers set to affect the outcomes of auctions. In general, if an auction starts at a higher starting price, it may lead to a higher closing price after predetermined number of bidding chances or times, all other things being equal. However, a higher bid starting price can negatively affect the closing price by reducing the number of participants. This is because participants in online auctions expect a minimum profit in advance and raising the minimum bid price reduces the expected profit of participants and, therefore, the number of bidders (Bajari and Hortacsu, 2000). Therefore, some sellers set the bid starting price very low to attract more bidders, hoping that more bidders make the current auction more competitive among bidders and result in a higher closing price. The net effect of bid starting price will be determined by the magnitude of negative and positive effects on the closing price.

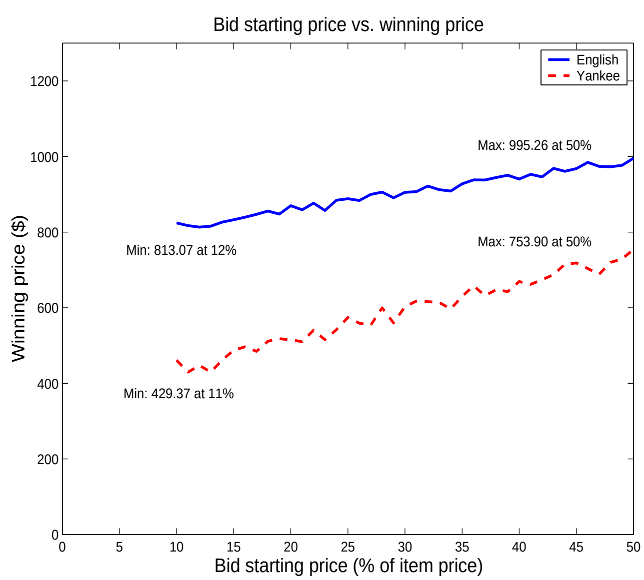

Figure 4. Results of auctions with various bid starting price

Figure 4 shows that raising minimum bid price increases the closing price in both types of online auctions. This is interesting because it implies that the positive effect of raising minimum bid price on the closing price is greater than its negative effect on the closing price. The author attributes this finding to the fact that whether or not a bid starting price is high is dependent on private values of bidders. Even when bids start from 50% of the item price, bidders who have high private values or even those whose private values are close to the item price may think that bids start at a very low price and they can purchase an item at a very low price. Therefore, these bidders will actively participate in bidding process and sellers may expect a high closing price. However, the slopes of two trend lines are significantly different. The positive relationship between the bid starting price and the closing price in Yankee auctions is stronger than that of English auction. For example, the difference between the lowest and the highest closing price in English auction was $182.19 (= $995.26 - $813.07), while the difference in Yankee auctions was $324.53 (= $753.90 - $429.37). This can be attributed to the fact that the closing price in Yankee auction is much lower than the average of private values of bidders and, therefore, there are many bidders who expect to purchase an item at a very cheap price from an auction even when it starts from 50% of item price. However, changing the bid starting price in English auction will not generate as much additional profit as in Yankee auction because the closing price of English auction is already close to the average of bidders' private values.

Effects of minimum bid increment on closing price

Figure 5 shows the effects of minimum bid increment (*k*) on closing prices in two auction systems. Although some studies have recognized the importance of *k* as an instrument to affect the outcomes of auctions, no study compare the relative effects of *k* in English and Yankee auctions. To illustrate the relationship between *k* and closing price, an average of closing prices of auctions with a specific *k* is computed. For example, the average closing price of English auctions at *k* = 0.02 is obtained by taking an average of closing prices of English auctions with the same value of *k* out of 50,000 simulated English auctions. In this paper, *k* varies from 2% ($20) to 10% ($100) of the item price ($1,000). Overall, Figure 5 shows that there is a positive relationship between *k* and the closing price in both English and Yankee auctions. Note that raising *k* can negatively affect the revenue of sellers by reducing the number of bidders who are eligible for next bids. However, over the range between 2% and 10% of the item, the positive effect seems to be dominant.

Figure 5. Results of auctions with various bid increment (*k*)

The most contrasting difference between English and Yankee auctions is the fact that raising *k* increases the closing price in English auctions at a decreasing rate, while it steadily increases the closing price in Yankee auctions. In particular, three intervals with different slopes are visually identified in English auction case: steep (2%-4%), intermediate (4%-7%), and flat range (7%-10%). Note that the closing price ($1,000) at the intersection point (k = 0.07) between intermediate and flat intervals is about the item price and an average of private values of all bidding agents. Raising k higher than 7% in English auctions will not significantly raise the closing price because not many bidding agents are able to raise their bids higher than their private values. It is not very difficult to imagine that as k increases beyond a certain point, the winning price will eventually decrease. For example, when k is set to $2,000 (i.e., 200% of item price), there will be very few bidders who have higher private values than $2,200 (=$2,000 (k) + $200 (bid starting price)), and, hence, the winning price will be lower. This finding is consistent with finding from another simulation-based study (Bapna et al., 2003), claiming that on average smaller bid increments could lead to higher auction revenue to sellers than actually used bid increments. However, sellers can significantly benefit from raising k until its value is less than or equal to 7%. In contrast, in Yankee auction, even though sellers set k to a value higher than 7%, it still raises the closing price mainly because the closing price is far lower than private values of bidding agents.

In previous section, it is shown that an average of closing prices from multiple English auctions is higher than that of a corresponding Yankee auction. However, repeating multiple English auctions take much longer than one Yankee auction and, therefore, some items (e.g., perishable goods or items with a rapid depreciation) are not appropriate for English auctions. The next question is then how sellers can sell items quickly in Yankee auction while expecting the same or, at least, close to the closing price from multiple English auctions. One possible solution is to adjust k in Yankee auction and one solution can be easily found by using information in Figure 5. Assume that Alice as a seller wants to find an optimal value of k in Yankee auction that guarantees a closing price ($600) that she can enjoy from multiple English auction with k = 0.02. In Figure 5, P1 is the intersection point between a vertical line at k = 0.02 and the closing price curve of English auction, while P2 is a parallel point of P1 on y-axis. To find an optimal k, Alice draws an extended line of $\overline{P1P2}$ that intersects at P3 with the closing price line of Yankee auctions. The corresponding value of k to P3 is around 0.057. Therefore, if Alice sets $k \approx 0.06$ in Yankee auction, she may expect the same closing price that she would expect from English auctions with k = 0.02. Likewise, if Alice sets $k \approx 0.083$ in Yankee auction, she may expect the same profit that she would expect from English auctions with k = 0.03. As a rule of thumb, about three times higher value of k in Yankee auctions than in English auctions is recommended.

CONCLUSION

This paper presents an agent-based research model that analyzes various auction systems efficiently when traditional methods are not appropriate. Experimental results and findings by using this tool can help sellers to organize auctions in an optimal way by configuring control variables. The presented model can be also useful for researcher to test auction related theories and to develop new auction mechanisms. Several practical recommendations for auctioneers or sellers are summarized as follows:

- Multiple English auctions are more profitable than a single Yankee auction provided that items are durable and sellers do not need to sell items quickly.
- Raising a minimum bid increment (k) to a certain level will increase closing prices in both English and Yankee auctions. However, raising k will increase the closing price at a decreasing rate in English auctions. While it increases the closing price at a constant rate in Yankee auctions.
- In case that sellers need to sell multiple items quickly, use Yankee auction. In particular, it is recommended that sellers set k to high in Yankee auction to ensure the high closing price. Based on experimental results, sellers may set k in Yankee auction as three times high as in English auction.
- Setting a bid starting price at a very low price draws more bidders but does not help sellers make more money. In contrast, raising a minimum bid price to a certain level results in higher closing price because there are still active bidders with high private values and strong desire to buy items.

There are a few future directions related to the current work. One direction is to test the presented model more rigorously through various environmental settings. Another major direction for a future research is to generalize the presented system to study more complex and advanced auction systems. For example, in combinatorial auctions, there are multiple items of several different products. Therefore, bidders should consider not only needs of goods but also synergistic benefits of combining different goods when they make a new bid. Developing generalized form of the presented simulation model for combinatorial auctions will be challenging but are well worthy.

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