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A Fuzzy-Logical Approach for Integrating Multi-Agent Estimators

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ABSTRACT

This paper proposes a novel approach for integrating estimations from multiple agents. The approach is based on the fuzzy set theory. However, compared to existing fuzzy logical methods that use fuzzy if-then rules, this method is based on solving an over-determined fuzzy equation system. The result is either a global inconsistency message or the consistent core of the equation system. We demonstrate the approach with data from an actual case study undertaken by a German automotive manufacturer.

Keywords

Fuzz Data, Fuzzy Logic, Estimation, Integration, Multi-Agent, Fuzzy Prediction, Forecasting, Consistency Check, Fuzzy Equation Systems

INTRODUCTION

Predictions, together with explanations, constitute some of the most important tasks of scientific activity. And, because most business decisions are made in consideration of future consequences, forecasts are of vital importance for business as well. Traditional techniques for predicting and forecasting, however, based on, for example, the fitting of trajectories to past data points, have produced mixed results. Here, we will present a competing method for producing long-range forecasts, based on the definition of overlapping contexts of the variables of interest. The technique is based on the fuzzy set theory.

We begin with a discussion of multi-agent estimations and the logic of scientific predictions. Then we present basic foundations of the fuzzy set theory and show how fuzzy data can be integrated into such estimations. This method—which we will call FuzzyCalc—enables us to calculate with over-determined fuzzy equation systems and to indicate inconsistencies. If no inconsistency is detected a reduction of the impreciseness is often possible. Finally, we illustrate the multi-agent estimation process with an example of long-range forecasting of car ownership.

MULTI-AGENT ESTIMATIONS

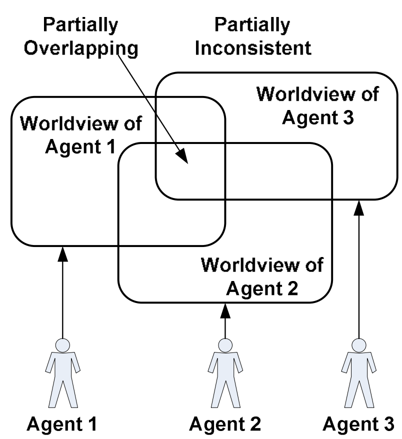

Figure 1. Integration of Partially Inconsistent Multi-Agent Estimation

The proposed approach assumes that multiple agents will estimate different parts of the reality (see Figure 1), and that their worldviews are also partially overlapping. We will focus on quantitative estimations and worldviews. Therefore the worldview or model is specified as an equation system. Agents will estimate one or more variables which are part of their worldview. These estimations are connected with other estimations through equations. Estimations are not precise but could be fuzzy sets and could be partially inconsistent. With this approach, we will attempt to 1) calculate the consistent core of all multi-agent estimations, or 2) or will give an inconsistency message. We assume that the agents have no incentives of opportunistically misrepresenting their estimations and therefore a collaborative environment of the agents is assumed.

One possible scenario could be to forecast the future environment for a group of agents. Later we will present an example in which a group of experts want to jointly estimate future car sales. Other examples are the distributed estimation of key performance indicators for a company or the estimation of costs, resources, and time required for a given project. The characteristic features common to these examples are 1) there is more than one agent, 2) the agents estimate one or more variables, and 3) the variables are connected through a shared model (i.e. equation system).

Related Work

Due to our focus on *cooperating* agents, many of the multi-agent techniques, such as multi-agent negotiation techniques or market-based multi-agent systems (Jennings, Faratin, Lomuscio, Parsons, Wooldridge, and Sierra, 2001), are not applicable.

For situations where agents are cooperating, but also competing, distributed constraint satisfaction techniques have often been used. Constraint satisfaction algorithms try to assign values for each variable so that each is consistent with all constraints, or it determined that no such assignment can be made (Shoham and Leyton-Brown, 2008). In our example, we are focused not on the precise assignment for each variable, but on the consistent core of all fuzzy estimations. Therefore constraint satisfaction techniques are not applicable. Fuzzy constraint-based methods generalize the notion of crisp constraints to fuzzy constraints (Ruttkay, 1994). However, fuzzy constraint satisfaction also tries to find a precise assignment for each variable and is therefore not applicable.

Fuzzy linear systems (Friedman, Ming, and Kandel, 1998) represent a system of linear equations with a crisp coefficient matrix and a fuzzy right-hand side column. Our approach is different because it can handle also non-linear equations. This is especially important because in the intended domains there are many non-linear relationships, including multiplication equations *(Revenue = Price * Volume)* or ratios *(Return on Equity = Income / Equity)*. Linear equation systems cannot model these relationships. Another difference of our approach, as compared to that of Friedman et al. (1998) or other fuzzy equation approaches (e.g. Buckley and Qu, 1990; De Andrés Sánchez, and Gómez, 2004; Wasowski, 1997) is that our approach allows over-determined equation systems. This is especially important for the integration of multi-agent estimations that are connected through non-linear equation systems.

Scientific Predictions

To understand the challenges and limitations of a multi-agent estimation method, we discuss in this section the scientific bases of prediction.

What is a scientific prediction? A deductive-nomological prediction according to the Hempel-Oppenheim model is a logical inference of a statement which describes a predicted event (explanandum) from a finite set of premises (explanans) (see Figure 2). The set of premises includes at least one general "law-like" statement (hypothesis), that expresses the nomological knowledge and at least one specific statement (antecedent) (Hempel and Oppenheim, 1948).

$$\begin{array}{llll} \text{Premisses} & H_1 \wedge H_2 \ldots H_k & \text{(Hypotheses)} \\ & A_1 \wedge A_2 \ldots A_l & \text{(Antecedents Conditions)} \\ \hline \text{Conclusion} & \text{E} & \text{(Explanandum)} \end{array}$$

Figure 2. Hempel-Oppenheim Model of the Deductive-Nomological Prediction

The logical deduction of the explanandum *E* from the conjunction *H* of the hypotheses $H_1 \wedge H_2 \ldots H_k$ and the conjunction *A* of the antecedent conditions $A_1 \wedge A_2 \ldots A_l$ — in other word the structure $(H, A) \rightarrow E$ — predicts the explanandum *E*.

For a forecast, at least one element of the premises must connect different time points or time spans. Such a time-spanning hypothesis could, for example, have the form $\forall x: Px_t \rightarrow Qx_{t+1}$, which means if an object *x* exists in *t* with the property *P* then this object has in *t+1* the property *Q* (see Figure 3).

$$\begin{array}{lcl} & \forall x(Px \rightarrow Qx) & \text{(Hypothesis)} \\ \text{Premises} & & \\ & Pa & \text{(Antecedents Condition)} \\ \hline \text{Conclusion} & \text{Qa} & \text{(Explanandum)} \end{array}$$

Figure 3. Forecasting Qa from the Event Pa and the Hypothesis $\forall x: Px_t \rightarrow Qx_{t+1}$

For predictions we can use not only currently confirmed natural laws, or economic or behavioral theories, but also indicator hypotheses or probabilistic relations. An indicator hypothesis, for example, might be "If the barometer reading is rapidly falling, then a storm is coming." However, indicator hypotheses are not suitable for bringing about different outcomes: intentionally manipulating the barometer reading so that it falls rapidly, will have no effect on the weather.

Structure laws and structure definitions include statements like "All chickens have two legs." Structure laws are falsifiable whereas structure definitions are tautological. This structure laws or definitions can be helpful for predictions. In the previous example, maybe we know the number of "legs" better than the number of "chickens".

Forecasting also faces some fundamental limits. Firstly, unpredictable events, or so-called "wild cards" can occur. "Wild cards" are low-probability, high-impact events–like an earthquake, for example. In the following we assume that no such structural breaks take place. These restrictions have to be included as additional hypotheses. Thereby the prediction becomes a conditional prediction.

There is a second reason why predictions fail: the formulation of the prediction leads to the start of actions that falsify the forecast ("self-defeating prophecy"). If, for example, an expert predicts that a lake will be ecologically dead within five years, this may lead to counter measures to save this lake. It would be unreasonable to blame the expert that his forecast has not come true, especially if the intent of making the prediction was to activate counter measures and therefore self-defeat the prediction. On the other hand, there exist also "self-fulfilling prophecies", which realize themselves or at least increase the chance of realization through the announcement of the forecast. Both phenomena are typical for social systems.

Popper (1965) noticed the following limit of long-range forecasts: while history will be influenced by ever-increasing human knowledge, scientific methods can't predict the increase of future knowledge. If we could, then we would know it already now and it would not be *future* knowledge anymore. Because the future actions are dependent on our future knowledge, no precise prediction of human history is possible. Popper's refutation of historicism does not, however, preclude all possible social forecasts. There is still the possibility of testing economic or social theories by predicting specific events under specific conditions.

FORECASTING WITH FUZZY DATA

Often companies expect that forecasts will result in very precise predictions. However, the unreliability of exact point forecasts has been shown repeatedly. As an example we refer to the Shell forecasts (see Shell, 1989)

Frequently, forecasting methods are based on the extrapolation of an optimal trajectory. The quality of the forecasting models can be assessed by the goodness of fit between the data points and the curve progression, measured e.g. with the sum of the squared errors.

Our method uses another paradigm: we identify spaces of possible future development, and constrain them through the evaluation of context information. During the forecasting process we search for context information of the predicted variable. A context is a set of statements concerning the concrete question in which at least fuzzy information is available.

The forecasting method based on our approach expresses the hypotheses as an equation system. Importantly, equations should not be considered the same as implications. In the former section, we used implications to describe hypotheses. The antecedent conditions are fuzzy numbers for variables, which are part of the equations.

For instance there may be a hypothesis, which states that a variable *X* will increase with the factor *f* (that means $X_{t+1} = X_t * f$). The fuzzy factor *f* and the equation would be the hypothesis, X_t the antecedent condition and X_{t+1} the explanandum.

With the help of the algorithm presented in the next section, we can calculate unknown fuzzy variables, check for inconsistencies, and reduce the fuzziness of the data. Thereby an improvement in the precision of the hypotheses (in our example the reduction of the fuzziness of *f*) can occur.

What advantages does the method have for forecasting? Firstly, the fuzziness of the forecast is made transparent for the decision maker. Secondly, there may be another hypothesis that can be used for predicting the variable. Thus, overlap and correction of separate partial views of multiple agents is made possible. Moreover, inconsistencies between the different views can be detected. This is also an important finding that can lead to the review and/or modification of the hypotheses or the antecedent conditions.

SOLVING OVER-DETERMINED FUZZY EQUATION SYSTEMS

Firstly we shall present some foundations of the fuzzy set theory that are needed for the further understanding of our approach. The method is based on the solution of over-determined fuzzy equation systems.

Model

In this approach a model for estimation consists of the structural relations of the variables and the data in combination with the error margins.

- **Structural relations of the variables.** The structural knowledge is specified with equations. (e.g. Total car expenditures in Europe in 2020 = Car sales in 2020 in Europe / Average price of a car in 2020 in Europe)
- **Data.** The source of the data could be measures, estimations, "gut feeling", or normative plans from multiple agents. (e.g. Car sales in 2020 in Europe = 20 Million units)
- **Error Margins.** We assume that the data is known only to a certain level of precision. This means the data has some absolute or relative error margins, e.g. "car sales in 2020 in Europe are 20 ± 4 million units". This statement can be interpreted as "car sales are around 20 million". In the following we use fuzzy sets to model this impreciseness.

Fuzzification

For the variables in the models we could have estimations from one or more agents, measures that are more or less accurate, or target ranges. In the following all variables will be represented with fuzzy sets.

The classical two-valued logic knows only the two logical values "true" and "false". This logic implies that, for example, the two-valued relationship "is element of ($\in$) and can only be interpreted in such a way that either an element *x* is part of a set *X* or it is not. Therefore for all *x* the truth value of ($x \in X$) is either "true" or "false".

The fuzzy logic replaces the two-valued predicate logic with a multi-valued logic with the interval [0,1] as the truth values. We can define a membership value $\mu(x)$ for each $x \in X$, which allows us to make statements about the membership of an element to a set. For instance we could define a fuzzy set "Car Sales is roughly 7 million units" with the membership function $\mu: X \rightarrow [0,1]$ as follows:

$$\mu_{(}x) = \begin{cases} 0 & x \leq 5 \vee x \geq 9 \\ \frac{1}{2}(x-5) \quad \text{if} & 5 < x < 7 \\ \frac{1}{2}(9-x) & 7 < x < 9 \end{cases}$$

Therefore a fuzzy set **A** for the basic set X with the membership function $\mu_{\mathbf{A}}(x)$ is defined through the pair of variates $(x, \mu_{\mathbf{A}}(x))$ with

$$A = \{(x, \mu_A(x)) \mid x \in X, \mu_A(x) \in \Re, \mu_A(x) \in [0,1]\}.$$

If there exists exactly one value with $\mu_A(x) = 1$, we call it *mean value* of **A**. Often we want to distinguish the elements of **A**, which belong to a set with at least a minimal level α. The α-cut consists of all $x \in X$ for which membership value is $\mu_A(x)$ is greater or equal to a threshold α (see Figure 4).

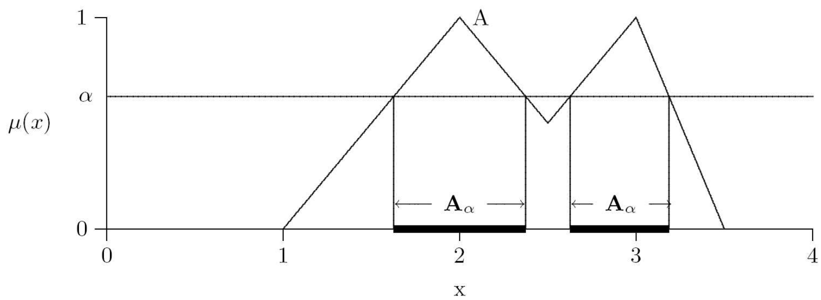

Figure 4: α-cut A_α for a Fuzzy Set A

The *support* of a fuzzy set is the set of all elements with a membership value > 0. A convex fuzzy set **A** is a *fuzzy number*, if the basic set X is ℝ and at least one value $x \in X$ exists for that $\mu_A(x) = 1$ is and $\mu_A(x)$ is at least piecewise continuous.

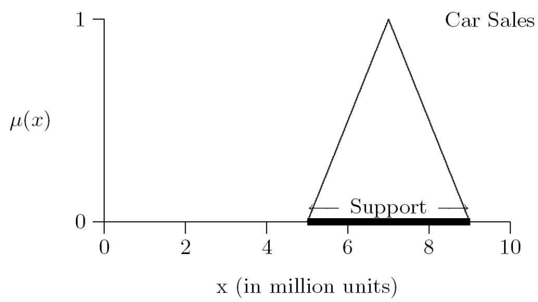

Figure 5: Number of Cars Sales in 2020 in Europe ("around 7 Million")

With the help of the operations t-norm and t-conorm we can define negation (¬), conjunction ($\wedge$), disjunction ($\vee$) and implication (⇒) for fuzzy sets (Zadeh, 1965). Interestingly we can use the extension principle of Zadeh (1965) to calculate with fuzzy numbers. Thereby mathematical concepts of normal numbers are transferred to fuzzy numbers. This enables us to define functions for fuzzy numbers. Through the extension principle we can infer calculation rules for every function but especially for the basic mathematical operations. Kruse (1993), pp.38 shows that the basic operators +, -, * and / can be realized by interval arithmetic for each α-cuts.

In the following, imprecise data in the model are specified as fuzzy numbers. Thereby the mean value corresponds to the most possible value for the variable according to an agent. If a measurement exist alternatively the means value corresponds to the measured value. The error margin can be specified with the support of the fuzzy number. With this information a simple triangle fuzzy number can be constructed. Figure 5 depicts the fuzzy number "car sales in 2020 in Europe are 7 ± 2 million units".

The Algorithm

Methods tasked with handling models with fuzzy data should be able to detect inconsistencies in the model and to calculate with fuzzy numbers semantically-correct forward and backward (Lenz and Müller, 2000).

The main idea of the algorithm is to overlap the different alternative values which are resulting due to the redundancy in the fuzzy equation system. The algorithm works as follows: First it calculates, for all variables in each equation, all the alternative values by solving the equation and using fuzzy arithmetic. Secondly the algorithm calculates for each variable the cut between all alternative values and the a priori value. If this cut is empty then the model is inconsistent. Otherwise the new value for the variable is set to the renormalized value of the cut. The algorithm will repeat until no further adjustment occurs or until an inconsistency is found.

```
FuzzyCalc-Algorithm
Input:  Fuzzy variables, Equations
Output: Adjusted Fuzzy variables or Inconstancy Alert
Repeat
    For all equations
        For all fuzzy variables in the current equation
            Solve current Equation for current fuzzy variable
            Calculate with fuzzy arithmetic alternative value for current fuzzy variable
        End For
    End For
    For all fuzzy variables
        Calculate cut between current fuzzy variable and all alternative values
        If cut is empty, then the equation system is inconsistent
        New value of current fuzzy variable is renormalized cut
    End For
Until no further adjustment or inconsistency alert
End FuzzyCalc
```

A simple example will help to clarify the basic idea of the algorithm. We have the variables P for Profit, R for Revenue and C for Costs. The variables are connected through the equation P = R – C. We have now fuzzy estimation from multiple agents for the variables: P = 15 ± 5, C = 35 ± 5 and R = 65 ± 10. The algorithm calculates the alternative values for each variable via fuzzy arithmetic. For example, P can also be calculated through R-C = 30 ± 15. Then the cut (that means overlap) between the a priori and the alternative value is calculated (the gray area in Figure 6 and in Figure 8). Then the fuzzy numbers are renormalized to a maximum value of 1 (see Figure 7). Then this procedure is repeated. However, after one iteration there is no further adjustment of the variables and the algorithm stops.

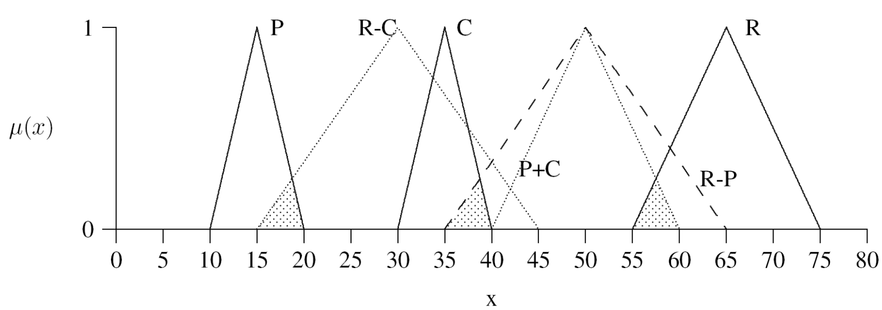

Figure 6: P, R, and C with alternative values and cut between alternative and a priori values

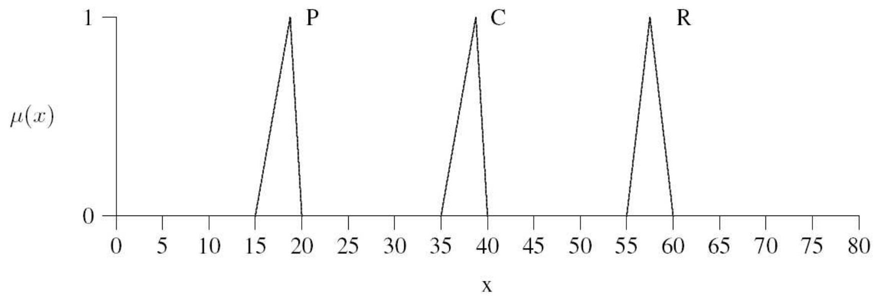

Figure 7: Renormalization of the cuts of P, R, and C

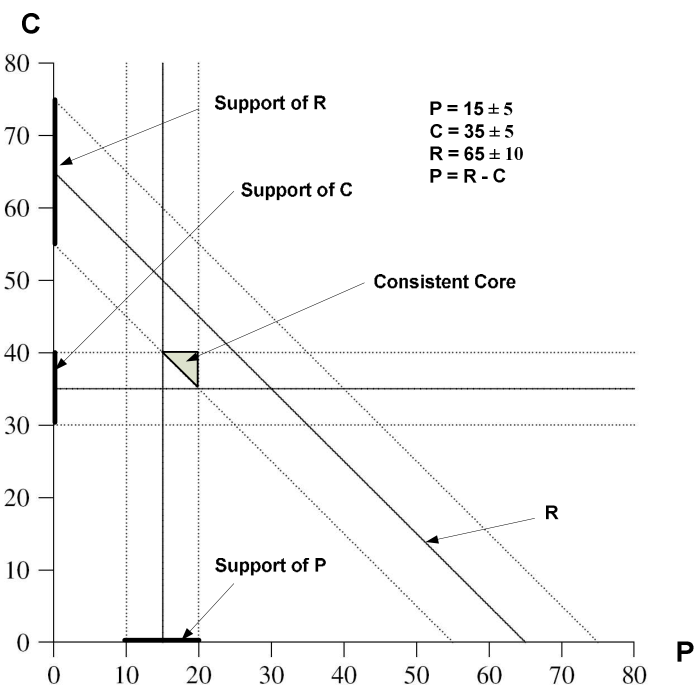

Figure 8: Equation *P* = *R* - *C* with partially inconsistent estimations and resulting consistent core

It can be shown that the algorithm has some appealing characteristics:

- The supports of the fuzzy data monotonically contract, or at least they will not expand. The reason is that the data in the model has redundancies and through the connections of the data the accuracy may increase.
- The size of a shift of a membership function is roughly proportional to its support.
- Crisp numbers are left unchanged.
- Membership functions are shifted into the intuitive correct direction and this is done coherently. For example, if two factors are "too small" and/or a product is "too large", then the factors are increased while the product is decreased.
- If the mean values of the adjusted fuzzy numbers are not sets, i.e. they are crispy, then the mean values confirm to all equations.
- Consistent data is left unchanged by the algorithm (invariance property).

- The algorithm is associative, i.e. the sequence of the variables and equations in the inner loop does not affect the solution, if it exists.

AUTOMOTIVE CASE STUDY

We will illustrate the method with an actual case study carried out by a German car company. All shown data are artificially altered to preserve confidentiality. First, a forecast of the car inventory in Germany at a specific time in the future (e.g. 2020) is developed. Conventional car projections estimate the parameter of a logistic or Gompertz function with the least-squares method by fitting a trajectory to the past data points. However, these points have in the past proven to be very unreliable and unable to correctly forecast the market size (Shell, 1989). The following approach describes the context of the stock of cars and calculates with fuzzy views. Thereby the a priori fuzziness of the forecast data often could be decreased. A forecast of the stock of cars at a given point in the future (e.g. for Germany) has to be embedded in a network model of the individual traffic.

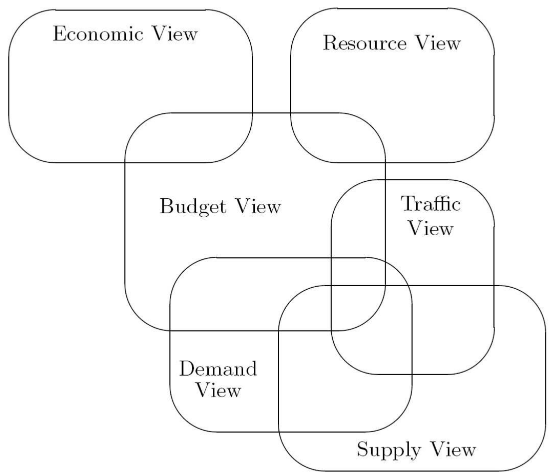

Figure 9: Overlapping Model Views for the Car Forecasting Model

We can distinguish between different views of the model: the demand view with the socio-demographic environment, the traffic system, the economic context, the budget view, the resource view, and the supply view. The different views are interdependent and are sketched in Figure 9.

We will limit our analysis to a small subset of all variables for illustrative purposes. This model is sketched in Figure 10. The graph represents the underlying equation system. Let us discuss how the equation *Avg. Journey Distance * Number of Journeys = Total Driven Distance* is visualized. Two directed links connect the two nodes of the factors with the node of the operator "*". Also one directed link connects the operator "*" with the product.

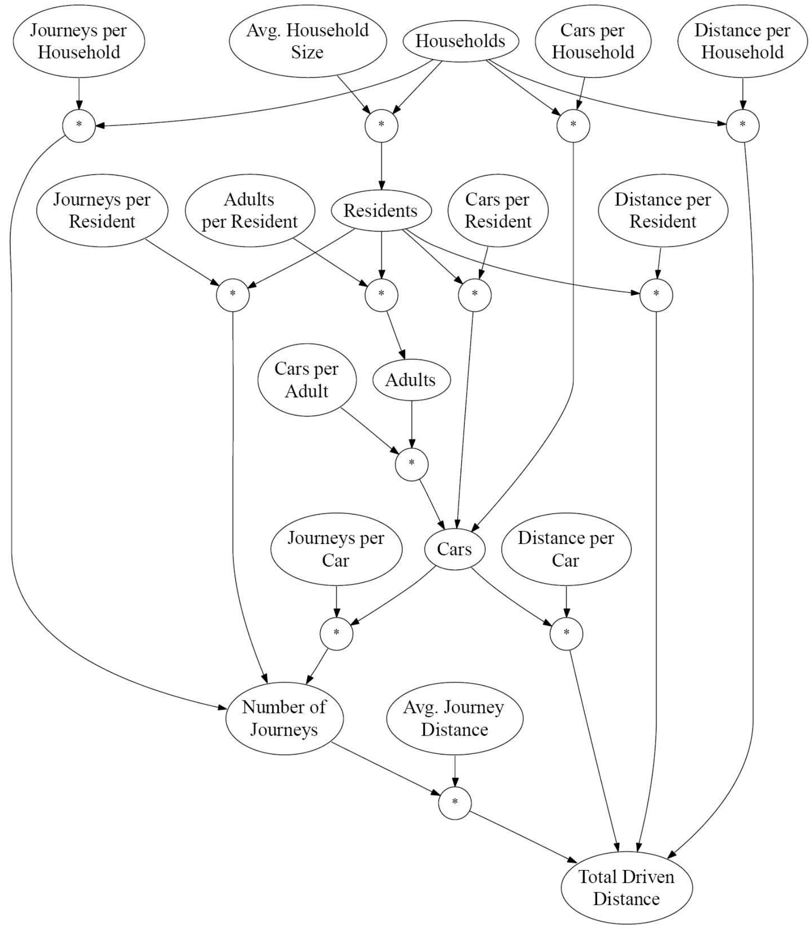

Figure 10: Part of Demographic and Demand View

For all these variables there could be estimations for a future point in time, of for span of time, e.g. for the year 2020. In the following, the data in Table 1 is fictitious and does not originate from real estimations of experts. The data is, however, sufficient for demonstrating the working mechanism of the approach. Thereby we show for one scenario the a priori estimations and the adjusted values calculated with our approach FuzzyCalc. An a priori value of e.g. $10 ± 2$ means that the value is a triangle fuzzy set with the mean value at 10 and a support in the interval [8, 12]. The result of this example shows that the fuzziness of the data could be decreased.

Variable	a priori Value	FuzzyCalc
Adults [Mio]	70 (+10; -30)	55.36 (+7.64; -12.5)
Residents [Mio]	65 ±5	68.13 (+1.87; -8.13)
Adults per Resident	0.6 ±0.2	0.81 (+0.09; -0.2)
Cars [Mio]	70 ±20	67.82 (+13.85; -7.82)
Cars per Adult	0.85 ±0.35	1.22 (+0.18; -0.26)
Households [Mio]	70 ±20	54.34 (+3.99; -4.34)
Cars per Household	1.3 ±0.1	1.25 (+0.15; -0.05)
Cars per Resident	0.8 ±0.5	1.01 (+0.35; -0.15)
Avg. Household Size	1.3 ±0.1	1.24 (+0.16; -0.04)
Number of Journeys [Mio]	?	40199.6 (+22800.4; -20199.6)
Avg. Journey Distance [km]	?	0.07 (+0.11; -0.04)
Total Driven Distance [Mio km]	?	2782.76 (+892.24; -682.76)
Journeys per Household	800 ±400	739.71 (+460.29; -339.71)
Journeys per Resident	500 ±200	590.04 (+309.96; -290.04)
Journeys per Car	600 ±300	592.73 (+307.27; -292.73)
Distance per Car [km]	40 ±5	40.67 (+4.33; -5.67)
Distance per Household [km]	50 ±30	51.45 (+22.05; -15.45)
Distance per Resident [km]	50 ±30	41.25 (+20; -11.25)

Table 1. Fuzzy Data before and after Adjustment

Already in this rather small model it is complicated for one expert to recognize inconsistencies and interdependencies between variables without a tool. If several agents with different experiences in different submodels want to merge their estimations, this consistency check becomes even more challenging. We assume that an expert can estimate one or more variables (a subset of all possible variables). One variable can be estimated by no expert (in Table 1 market as “?”), by one expert, or by more than one expert.

Our approach checks the consistency of every variable with every other variable because every variable is directly or indirectly connected through an equation. Through the redundancies of the views a reduction of the support of the fuzzy variables is often possible. The algorithm calculates the consistent core of the possibility space of the variables, if possible. If the model represents the reality correctly and the true values of the variables are inside the support of the a priori fuzzy sets, then the true values are also in the support of the adjusted fuzzy sets.

The model graph in Figure 10 describes the connection of the variables at *one* future point in time, (for stocks) or for *one* time span (for flows). For these estimations we could question agents, use "fuzzy laws" or rules of thumb, use plausible extrapolation of the data, or create a scenario. When we elicit the expert knowledge, we could, for example, ask a family expert to identify, for each possible value of the *avg. household size in the year 2020*, a membership level in the interval [0,1].

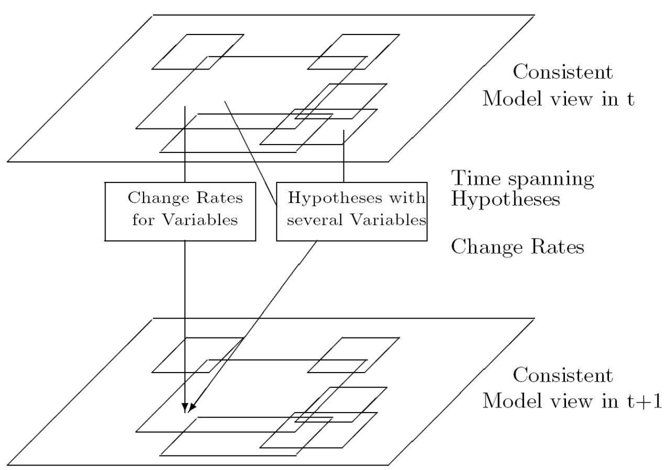

Figure 11: Dynamic Connection of the overlapping Model View of Figure 9

The resulting fuzzy number shows the expert's subjective assessment of the possibility of the different realizations. According to our definition this would not be a forecast but a prophecy because no hypothesis or law is stated. However, the expert could come up with this estimation due to theories they have not mentioned to us. Rules of thumb, experiences and conjectures can all play a role for the estimations, which could possibly be operationalized with the help of fuzzy data.

With "fuzzy laws" or theories we could incorporate the hypotheses in the model. Thereby we could add further equations, such that, for example, the avg. household size in *t* is the avg. household size in *t-1* multiplied by a change rate *f*. The factor *f* could be directly estimated as a fuzzy number or calculated from other equations e.g. from the settlement structure, or from economic growth. The method allows an overdetermination of a variable (see Figure 11).

Another alternative is the assumption that a variable will not change very much. In such case, we would set the factor *f* to "approximatively 1", which means a "fuzzy one". We could also create scenarios: we could build two or more forecasts with different values for a variable or a set of variables. With the presented method, the consistency of the resulting scenario is ensured.

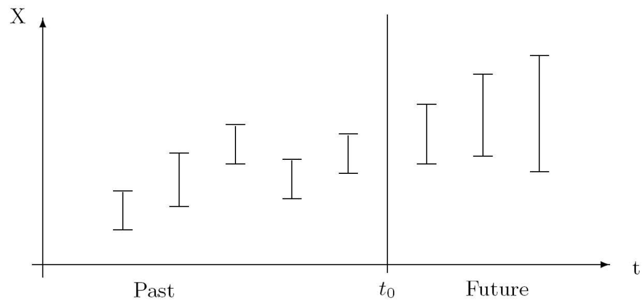

Figure 12: Information about a Variable X with Accuracy Level

If we forecast a variable *X* over a time span, we would get a result like in Figure 12. Note that in the figure also the past and current values of the variable are not exactly known. The farther we look into the future, the fuzzier our forecast becomes. The accuracy of the forecast is thereby better understood by the decision maker. All forecasting methods that promise exact point estimations for future variables should therefore be taken with caution.

CONCLUSION

We have presented a novel method for integrating multi-agent estimations. This approach is based on fuzzy set theory. It integrates the different estimations by solving an over-determined fuzzy equation system. We demonstrate the approach with data from a case study in the automotive sector.

Further research is needed to specify the multi-agent estimation process. The approach should also be compared with probabilistic methods like Markov Chain Monte Carlo simulations. Also the practical applicability of the method has to be researched.

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