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Rakesh Kawatra
Minnesota State University

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Lower Bounds for the Multiperiod Capacitated Minimal Spanning Tree with Node Outage Cost Design Problem

Rakesh Kawatra, MH 150, Minnesota State University, Mankato, MN 56002 (507) 389-5341

Abstract

The Multiperiod Capacitated Minimal Spanning Tree With Node Outage Costs (MCMSTWOC) Design problem consists of scheduling the installation of links in a communication network so as to connect a set of terminal nodes $S = [2,3,\dots,N]$ to a central node (node 1) with minimal present value of costs. The cost of the network is the sum of link layout cost and node outage costs. The link capacities limit the number of terminal nodes sharing a link. Node outage cost associated with each terminal node is the economic cost incurred by the network user whenever the terminal node is disabled due to failure of a link. In the network some of the terminal nodes are active at the beginning of the planning horizon while others are activated over time. The problem is formulated as an integer-programming problem. A Lagrangian relaxation method is used to find a lower bound for the optimal objective function value. Subgradient optimization method is used to find good lower bounds. This lower bound can be used to estimate the quality of the solution given by a heuristic.

Introduction

One of the common sub problems in the design of communication networks is to find a spanning tree to connect a set of geographically remote terminal sites (nodes) to a central site, which could be a host computer or a backbone node. The limited capacity of a link restricts the number of terminal nodes sharing that link. This is also known as the capacitated minimal spanning tree (CMST) problem. Several heuristic methods for solving different varieties of the CMST problem have been developed in the past. Some of these heuristics (Altinkemer and Gavish, 1988; Esau and Williams, 1966; Frank et al., 1971; Gavish 1983, 1985; Gavish and Altinkemer, 1990; Gouveia, 1995; Woolston and Albin, 1988) can solve single period CMST problems with equal importance given to all terminal nodes in a reasonable length of time. Heuristics have also been developed for solving the single period CMST problem when each terminal node in the network is assigned an outage cost (Dutta and Kawatra, 1994; Kawatra, Dutta, and Bricker, 1999). Outage cost associated with a terminal node is defined as the economic loss suffered by the user whenever the terminal node is disabled due to the failure of a link (Campbell and Pimentel, 1986). Another variant

of the CMST problem is when the terminal nodes are added to the network over time. This is known as the multiperiod CMST problem. A heuristic for solving the multiperiod CMST problem was developed very recently (Kawatra and Bricker, 2000). However, they assumed that all terminal nodes are equally important to the user and did not consider outage costs in their study. (Kawatra, 2000) presented a branch exchange heuristic to solve the multiperiod CMST problem when terminal nodes have outage costs associated with them. Their branch exchange heuristic is a greedy heuristic that is likely to find a local optimum, which may not be the best possible solution. Designers using this heuristic would like to have an estimate of the quality of the solution given by it.

One of the approaches used to find quality of solutions given by the heuristics is to find a lower bound of the optimal objective function value. In this paper, we suggest a Lagrangean relaxation method to find lower bound of the optimal objective function value of Multiperiod Capacitated Minimal Spanning Tree with Node Outage Cost problem. In section 2 we present an integer-programming formulation of the MCMSTWOC problem. Section 3 presents a Lagrangian relaxation method for finding a lower bound of the objective function value. The lower bound can be used to estimate the quality of the solution given by the branch exchange heuristic. Computational results in Section 4 demonstrate the performance of the Lagrangean relaxation method for several different network structures. Section 5 concludes the paper.

Mathematical Model of the Problem

We use the following notation in the model:
S: $[2,3,\dots,N]$ is the set of terminal sites;
Node 1: central site;
 λ : annual link failure rate;
 d_m : the time period at which node m becomes active;
 O_m^t : node outage cost associated with terminal node m in time period t;
P: the set of time periods $[1,2,\dots,T]$ in the planning horizon;
H: a limit on the maximum number of nodes in any subtree rooted at the central node;
 R_j : limit on the maximum number of nodes in any subtree rooted at node j;

C_{ijt} : the discounted cost of installing a link(i,j) in period t and maintaining it during periods t through T .

The following decision variables are defined:

X_{ijt} : a binary variable such that $X_{ijt} = 1$ indicates that link(i,j) is installed in time period t ; otherwise $X_{ijt} = 0$;

Y_{ijt} : a variable which specifies the traffic flow on the link(i,j) in time period t . This flow is equal to the number of paths connecting active terminal nodes in period t to the central node that include link(i,j);

L^m_{ijt} : a binary variable such that $L^m_{ijt} = 1$ indicates that link(i,j) is on the path from node m to the central node in time period t ; otherwise $L^m_{ijt} = 0$.

The MCMSTWOC problem can be formulated as the following minimization problem:

$$Z_{IP} = \text{Min}_{X,L} \left\{ \sum_{i=2}^N \sum_{j=1}^N \sum_{t=1}^T C_{ijt} X_{ijt} + \sum_{m=2}^N \sum_{t=d_m}^T O^t_m \lambda \sum_{i=2}^N \sum_{j=1}^N L^m_{ijt} \right\} \quad (1)$$

subject to:

$$\sum_{j=1}^N \sum_{t \leq d_i} X_{ijt} = 1 \quad \forall i \in S \quad (2)$$

$$\sum_{j=1}^N L^m_{ijt} - \sum_{j=2}^N L^m_{jit} = \begin{cases} +1 & \text{if } i = m \quad \forall t \geq d_i \\ -1 & \text{if } i = 1 \quad \forall t \geq d_i \quad \forall i, m \in S \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

$$\sum_{m=2}^N L^m_{ijt} \leq R_j \sum_{\tau=1}^t X_{ij\tau} \quad \forall i \in S, j \in S \cup [1], t \in P \quad (4)$$

$$\sum_{i=2}^N \sum_{j=1}^N \sum_{t=1}^T X_{ijt} = (N-1) \quad (5)$$

$$L^m_{ijt} \in \{0,1\} \quad \forall i, m \in S, j \in S \cup [1], t \in P \quad (6)$$

$$X_{ijt} \in \{0,1\} \quad \forall i \in S, j \in S \cup [1], t \in P \quad (7)$$

where $R_j = \begin{cases} H-1 & \text{for } j=2,3,\dots,N \\ H & \text{for } j=1. \end{cases}$

Computation of Lower Bounds

In this study we use a Lagrangian relaxation approach to generate lower bounds for the MCMSTWOC problem. Lagrangian relaxation has been used very successfully to obtain tight lower bounds for a variety of integer-programming problems (Dutta and Kawatra, 1994; Kawatra and Bricker, 2000). For an application-oriented survey of Lagrangian relaxation, see Fisher, 1981. We add the following constraints to the integer-programming model (Z_{IP}) of the MCMSTWOC problem, which was presented earlier:

$$\sum_{j=1}^N Y_{ijt} - \sum_{j=2}^N Y_{jit} = \begin{cases} 1 & \text{if } t \geq d_i \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in S, t \in P \quad (8)$$

$$Y_{ijt} \leq (N-1) \sum_{\tau \leq t} X_{ij\tau} \quad \forall i \in S, j \in S \cup [1], t \in P \quad (9)$$

$$Y_{ijt} \geq 0 \quad \forall i \in S, j \in S \cup [1], t \in P \quad (10)$$

These constraints are redundant in Problem Z_{IP} but help in getting tighter lower bounds in its Lagrangian relaxation.

We form a relaxation of the MCMSTWOC problem by multiplying each constraint (4) by a nonnegative Lagrange multiplier θ_{ijt} and adding the product to the objective function. This results in the following relaxation of problem Z_{IP} :

$$M(\theta) = \text{Min}_{X,L} \left\{ \sum_{i=2}^N \sum_{j=1}^N \sum_{t=1}^T X_{ijt} (C_{ijt} - R_j (\sum_{\tau \geq t} \theta_{ij\tau})) + \sum_{m=2}^N \sum_{t=d_m}^T \sum_{i=2}^N \sum_{j=1}^N L^m_{ijt} (\lambda * O^t_m + \theta_{ijt}) \right\}$$

subject to (2),(3),(5)-(10),

where elements of θ are nonnegative. This can be separated as the following independent sub problems:

$$J(\theta) = \text{Min}_X \left\{ \sum_{i=2}^N \sum_{j=1}^N \sum_{t=1}^T X_{ijt} (C_{ijt} - R_j (\sum_{\tau \geq t} \theta_{ij\tau})) \right\} \text{ subject to}$$

(2),(5), (7) - (10),

and for $m = 2, \dots, N$ and $t = d_m, \dots, T$

$$Q^t_m(\theta) = \text{Min}_L \left\{ \sum_{i=2}^N \sum_{j=1}^N L^m_{ijt} (\lambda * O^t_m + \theta_{ijt}) \right\} \text{ subject to (3)}$$

and (6).

Procedure for evaluating $J(\theta)$

The function $J(\theta)$ is evaluated by solving a spanning tree problem. For a given vector of Lagrange multipliers θ this problem can be accomplished very easily using Prim's algorithm (Prim, 1957).

Procedure for evaluating $Q^t_m(\mu)$

For each m and t , evaluation of $Q^t_m(\mu)$ requires solving a single commodity flow problem in which one unit of commodity m is to be shipped from node m to the central node during period t . Because the links are uncapacitated, the flow will be along the shortest path from node m to node 1, which can be found using Dijkstra's algorithm (Larson and Odoni, 1981) with $(\lambda * D^t_m + \theta_{ijt})$ as the cost of shipping one unit of commodity m from node i to node j .

We used the subgradient optimization method (Held, Wolfe, and Crowder, 1974) to compute the optimal Lagrangian multipliers.

Numerical Results

The effectiveness of the Lagrangean relaxation based heuristic was investigated by solving a randomly generated set of test problems with the number of nodes in the network varying from 20 to 60. The terminal nodes are uniformly distributed in a rectangle of dimension 500 by 1250 and the central node is either at the center or a corner of the rectangle. The entries for the node outage cost matrix were drawn from a uniform distribution over the interval [1,600]. A 10-year planning horizon was used in all problems. The fixed cost of installation of link(i,j) was chosen to be the Euclidean distance between points i and j. The time period d_i for activating each terminal node i was uniformly distributed between 1 and 6. The annual link failure rate, λ , was varied from 0.02 to 0.06. The link maintenance cost per period was assumed to be 6% of the fixed cost of installation. For discounting purposes a 5% annual interest rate was assumed. The limit on number of nodes in any subtree, H, was varied from 2 to 6 in increments of 2 for problems with 20 and 40 nodes in the network. For problems with 60 nodes in the network we varied the value of H from 4 to 8 in increments of 2.

For purposes of the subgradient optimization method, we used the solution value given by the branch exchange heuristic in [14] as the overestimate of the optimal objective function value. The Lagrange multipliers were initially set to 0. The stopping criterion in computation of the lower bounds was: stop if the total number of iterations exceeds 900 or if the objective function value changes by less than 0.8 in 30 successive iterations. The subgradient optimization method used for computing the lower bound on the objective function value was coded in Fortran 77 and run on a Vax-8550 computer. Computational results of the experiment are presented in Table 1.

The computational results presented in Table 1 show that the Lower bounds are within 27 percent of the upper bound given by the branch exchange heuristic. The table shows that the gap is smaller for smaller networks. We also observe that the gap decreases for smaller failure rate. It is possible that the heuristic solutions for smaller networks are closer to the optimal solution and the lower bounds are also tighter for smaller networks. Research is underway to improve the heuristic solutions as well obtain tighter lower bounds for networks with larger number of nodes.

Table 1. Experimental Results

No. of Nodes	K	Failure rate	Lower Bound	Heuristic solution	Gap (%)
20	2	0.02	10240	10649	4%
20	4	0.02	8138	8844	8%
20	6	0.02	7640	8251	7%
40	2	0.02	16605	19631	15%
40	4	0.02	12879	15394	16%
40	6	0.02	11739	14485	19%
60	4	0.02	16275	20024	19%
60	6	0.02	13835	17630	22%
60	8	0.02	12856	16758	23%
20	2	0.04	11139	11628	4%
20	4	0.04	8790	10219	14%
20	6	0.04	8282	9653	14%
40	2	0.04	19917	21617	8%
40	4	0.04	14879	17971	17%
40	6	0.04	13486	17517	23%
60	4	0.04	17969	22457	20%
60	6	0.04	15389	20114	23%
60	8	0.04	14542	19619	26%
20	2	0.06	11900	12565	5%
20	4	0.06	9538	11451	17%
20	6	0.06	9012	10897	17%
40	2	0.06	21297	23574	10%
40	4	0.06	16308	20231	19%
40	6	0.06	14927	19955	25%
60	4	0.06	19191	24463	22%
60	6	0.06	17092	22053	22%
60	8	0.06	15989	21952	27%

Conclusions

In this paper we presented a Lagrangean relaxation method to find lower bound of the optimal objective function value of the multiperiod capacitated minimal spanning tree with node outage cost problem. This lower bound can be used to estimate the quality of the solution given a heuristic method. Computational results for a variety of problems are reported. In our computational experiment, for all networks with up to 60 nodes, the lower bounds are within 27 percent of the optimal objective function value.

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