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Locating Base Stations for Mobile Servers

Tammy Drezner
California State University - Fullerton, tdrezner@fullerton.edu

Zvi Drezner
California State University - Fullerton, zdrezner@fullerton.edu

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Tammy Drezner
Mihaylo College of Business and Economics
California State University, Fullerton
Fullerton, CA 92834.
tdrezner@fullerton.edu

Zvi Drezner
Mihaylo College of Business and Economics
California State University, Fullerton
Fullerton, CA 92834.
zdrezner@fullerton.edu

ABSTRACT

In this paper we review location models, specifically covering location models, which are applicable to modeling the location of base stations for cellular phone users. The definition of cover is revisited, suggesting three new definitions of cover: two gradual cover models, and one sum cover model. A new objective for covering models, the variable radius model, is proposed. The probability that a connection can be made between the cellular phone and a base station at a given distance is defined. The objective is to minimize the probability that users cannot be serviced by at least one base station. These four new approaches are reviewed and once the research is completed four papers will be published in peer reviewed journals.

Keywords

Location, Mobile Servers, Covering Problems.

INTRODUCTION

Mobile commerce is a topic of great importance to businesses as well as to individual users. Use of mobile commerce has been increasing at an exponential rate in the past few years and is expected to continue to grow exponentially in the foreseeable future. Mobile technologies will become ubiquitous as devices taken for granted. Mobile services have diffused all over the globe. According to the International Telecommunication Union, by 2005 there were over 1.5 billion cell-phone users, about one-quarter of the world's population. Even in North America, which was lagging the adoption of mobile services compared to Europe and Japan, wireless services captured a significant market share. In the United States, there were almost 200 million wireless service subscribers by the end of 2005, and the penetration rates were expected to grow from around 65% to about 80% by 2009. Due to this surprising growth, mobile applications have required special attention from the research community, especially MIS scholars. For a review of mobile commerce the reader is referred to Mennecke and Strader (2003).

The proliferation of mobile servers will require the location of many base stations to provide the users reliable service. It is therefore essential to investigate the best approach to locating base stations. The goal is to provide good service to as many users as possible at a reasonable cost.

Locating base stations for mobile servers is an application of location theory. Location models deal with the location of one or more facilities in an area to satisfy customers' demand. A multitude of models were proposed for finding the location solution considering various conditions. For a review the reader is referred to two recent books by Drezner (1995) and Drezner and Hamacher (2002).

Covering problems are an important branch of location analysis (for reviews see Schilling, Vaidyanathan, and Barkhi 1993; Daskin, 1995; Current, Daskin, and Schilling, 2002; Plastria, 2002). Once the location of a facility is given, a certain rule determines whether a customer is considered covered by the facility or not. In the standard covering models, a customer is considered covered if he is within a given distance from a facility. In the set covering problem (suggested by ReVelle, Toregas, and Falkson, 1976), the objective is to cover all customers with the minimum number of facilities. In the maximal covering problem (suggested by Church and ReVelle, 1974), the objective is to cover the maximum number of customers with a given number of facilities.

REVISITING THE CONCEPT OF COVER

In traditional covering models cover is defined by a distance D. If a user is within a distance D from the facility, he is covered, and if he is located farther than distance D he is not covered. In modeling location of base stations for mobile cell phones (facilities) there is a distance at which the wireless network is effective. However, this distance is not fixed for all users because communication can be interrupted beyond a certain distance leading to dropped calls. Therefore, the proper modeling of the location of cellular base stations should be based on a probability that a customer is covered. This probability is a decreasing function of the distance. As in standard covering models we assume that cover depends on the distance alone.

Topographical characteristics of the cover area are not considered in the models. Further research is required to fine tune the models for such variations. The next three formulations: gradual cover; stochastic gradual cover; and sum cover; represent recent thinking by location theorists about the appropriate definition of cover.

Gradual Cover Models

In the traditional cover problem there exists a distance *D* such that users within distance *D* from a facility are considered covered while users beyond that distance are not. An abrupt discontinuity in cover occurs at distance *D*. This does not simulate accurately actual reception of cell phones. For example, assume that the cover distance is *D*=5 miles. In traditional cover models, cover is 100% at distance of 4.99 miles while it is 0% at a distance of 5.01 miles. This discontinuity in cover is not realistic. The gradual cover problem in the plane was suggested in Drezner, Wesolowsky, and Drezner (2004) for modeling cover situations in a more accurate and realistic way. A similar formulation in a network environment was suggested by Berman and Krass (2002) and Berman, Krass, and Drezner (2003). Two distances *r* and *R*, ($r \le R$), are defined. If the user's distance to a base station does not exceed *r*, the user is fully covered, i.e., reception is excellent. If the distance to a base station exceeds *R*, the user is not covered at all, i.e., no reception is possible. Between the distances *r* and *R* there is a linear decline in the probability of reception. The cover functions applied in the standard and gradual cover models are depicted in Figure 1.

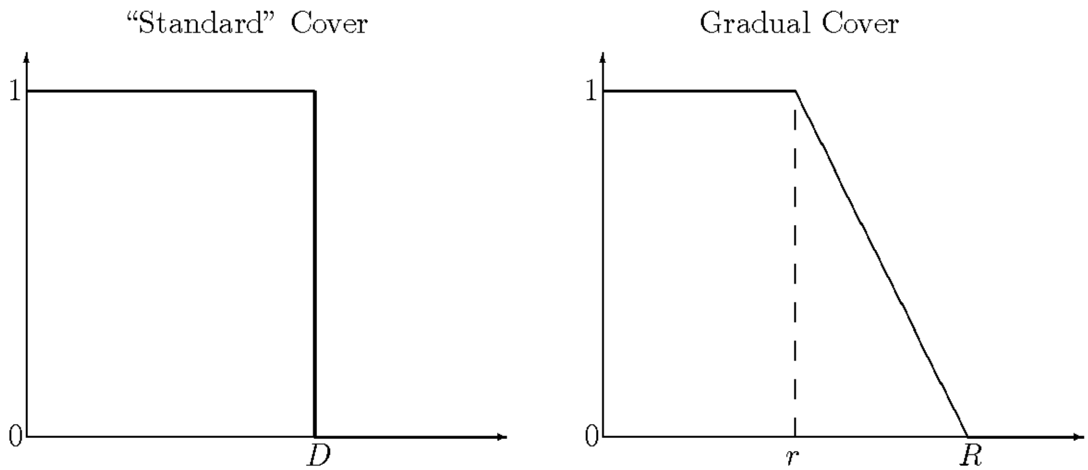

Figure 1. Standard and Gradual Cover

For example, if *r*=4 miles and *R*=6 miles (rather than *D*=5 miles), then up to a distance of 4 miles there is excellent reception. Cover declines gradually, reaches a 50% chance of getting reception at a distance of 5 miles, the probability continues to decline and reaches the point of no cover at 6 miles. At a distance of 6 miles or more no reception is possible. This cover function is a better depiction of actual behavior. The model reduces to the "standard" covering model when *R*=*r*.

Standard covering models can be solved by using this new definition of cover. In Drezner et al. (2004) an algorithm for finding the optimal solution for locating one base station is proposed. Algorithms for locating more than one base station are being developed

The Stochastic Gradual Cover Model

In this model (Drezner, Drezner, and Goldstein, 2008) the gradual cover model is taken a step further to make it yet closer to reality. It is assumed that the values of *r* and *R* are *not* fixed, rather they are assumed stochastic. The users residing or traveling at a certain location may have different values of *r* and *R* thus these values are random variables.

As in the gradual cover model, within a radius *r* from the base station, reception is excellent whereas farther than distance *R* there is no reception at all. Within a distance between *r* and *R* reception deteriorates and it can be assumed that only a portion of the users connect successfully. The values of *r* and *R* are fuzzy and should be modeled as following some stochastic distributions. Users may have different cellular phone models. Some models may provide stronger reception capabilities thus greater distances for perfect reception, and greater distances for which there is no reception at all.

The stochastic distributions of the distances *r* and *R* can be modeled in many ways (Drezner et al., 2008). In Figure 2, the probability of coverage as a function of the distance is depicted for the following modeling assumption: two random draws from a normal distribution of distances with a mean of *D* and standard deviation of σ are taken. The smaller of the two draws is *r* and the larger of the two draws is *R*. Observe the resemblance of this function to the Logit function used in many statistical analyses.

In Drezner et al. (2008) an algorithm was developed for optimally locating one base station in the plane. Algorithms for locating more than one base station are being developed.

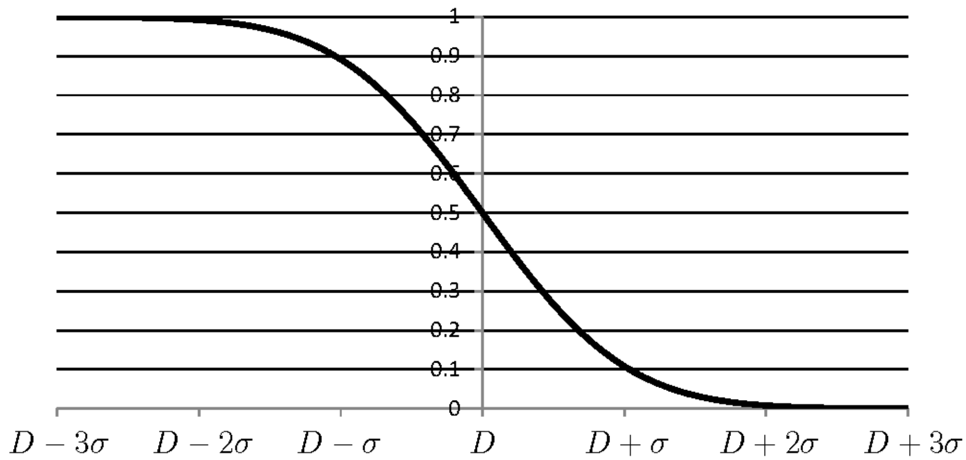

Figure 2. Stochastic Gradual Cover

Sum Covering Models

In pure sum cover models (Berman, Drezner, Krass, and Wesolowsky, 2008a), each facility emits a signal that declines with distance. The total signal received by users is the sum of all signals emitted by *p* facilities. A user is considered covered if the total signal exceeds a given threshold. For example, several light posts are constructed to illuminate a parking lot. Each light post covers the inside of a circle with sufficient light. However, it is possible that a point outside all circles (not covered by standard models) gets sufficient light from all light posts and thus is actually covered. See Figure 3 below for a depiction of two light posts. The thicker line surrounding the two circles encloses all points getting sufficient combined light from both light posts. The objective is to find the best locations of *p* facilities so that the number of covered users is maximized.

The cellular phone user emits a signal that decreases with the distance to the base station (facility). Conversely, the base station emits a signal that declines with the distance to the user. The probability that the base station can be utilized by the cellular phone is therefore a decreasing function of the distance. A user is considered covered if the probability that a base station can be utilized exceeds a certain threshold. Rather than defining a coverage distance, coverage is defined as the probability of coverage exceeding a threshold.

When locating base stations, a customer is considered covered if the probability he can utilize at least one base station exceeds 1-α (the reliability of the system). Alternatively, the user is considered covered if the probability he *cannot* connect is less than α. Let $p(d)$ be the probability that a connection with a base station at distance *d* is successful. Clearly, $p(d)$ is a decreasing function of the distance *d*. The decline can be modeled either as gradual cover or as stochastic gradual cover formulations described above, or any decay function such as an exponential decay of a power of the distance. Consider a user who is located at distances $d_1, d_2, \ldots, d_p$ from the *p* base stations. The probability that this user will not be able to connect is

$$\prod_{j=1}^{p}\left(1-p(d_j)\right)$$

The user is covered if and only if

$$\prod_{j=1}^{p}\left(1-p(d_j)\right) \le \alpha \quad or \quad \sum_{j=1}^{p}\log\left(1-p(d_j)\right) \le \log\alpha$$

Since all the values in this equation are negative, it can be written as

$$\sum_{j=1}^{p} -\log\left(1-p(d_j)\right) \ge -\log\alpha$$

which is a sum covering model with a threshold of $-\log\alpha$. Note that the decay function is $-\log\left(1-p(d_j)\right)$. The product of probabilities is thus converted to the sum of decay functions and thus the problem is converted to a sum cover model.

In Berman et al. (2008a) algorithms that find the optimal location for one or two facilities are developed. Heuristic algorithms are proposed for finding good solutions for the location of more than two base stations.

To illustrate the issue consider users uniformly distributed in a plane. The covered area represents the total number of users covered. Consider two base stations that are located at distance R from one another as in Figure 3. To illustrate the concept we apply a simple model. To simplify the illustration, we assume that the decay function is min{1, $1/d^2$}, rather than using the decay function $-\log\left(1-p(d_j)\right)$. This is equivalent to the assumption that the probability that a connection is successful is: $p(d)=1-e^{-1/d^2}$ and the probability $\alpha = 1/e$. If each base station is considered independently, users are covered within a radius of one from each base station.

In Figure 3 we depict the case R = 2.41 which is the distance providing the maximum coverage. The circles of radius one centered at each base station represent the area covered by the "standard" covering model. Each circle covers the users within a distance of one. The total area covered is thus 2π as long as the circles do not intersect (i.e., $R \ge 2$).

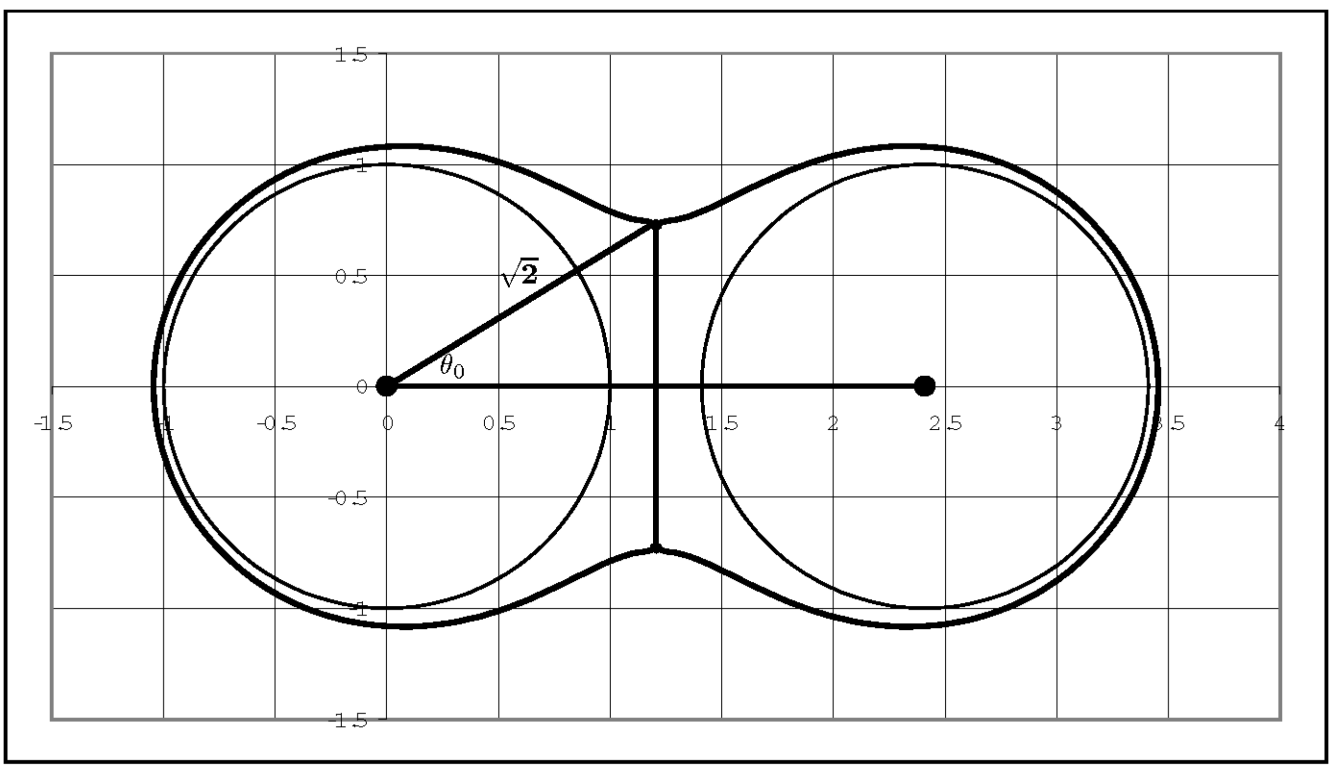

Figure 3. Coverage by two base stations

By the sum cover model, the area covered consists of all points X that satisfy

$$\frac{1}{d_1^2(X)}+\frac{1}{d_2^2(X)}\geq 1$$

where $d_1(X)$ and $d_2(X)$ are the distances between point X and the two base stations. This area receives from two base stations at least the signal intensity sufficient for coverage from one base station. The calculations detailed below lead to the area covered according to the sum cover model (which is bounded by the thicker line surrounding the two circles in Figure 3). This is the area covered by two cellular base stations, each covering a circle of radius one by classical covering models. It is clear that there is a significant gain in coverage when sum cover definition of cover is used.

Calculating the area in Figure 3

Since the results of the analysis are independent of translation and rotation, one base station is located at (0, 0), and the second base station is located at (0, R). Consider a point expressed by polar coordinates $X = (r, \theta)$ originated at (0, 0). The distance from the first base station is $d_1(X) = r$ and the distance from the second base station is by the cosine theorem: $d_2(X)=\sqrt{r^2+R^2-2rR\cos\theta}$. The point X is in covered if and only if:

$$\frac{1}{r^2}+\frac{1}{r^2+R^2-2rR\cos\theta}\geq 1$$

The boundary of the curve satisfies this equation as an equality. Algebraic manipulations lead to:

$$R^2-2rR\cos\theta=\frac{r^2(2-r^2)}{r^2-1} \quad (1)$$

If there exists a point X on the boundary fulfilling $d_1(X)=d_2(X)$, then $d_1(X)=d_2(X)=r=\sqrt{2}$. Therefore, for such a point

$$\cos\theta=\frac{R}{2\sqrt{2}}$$

If there is such a point on the perpendicular bisector between the two base stations (see Figure 3), then its angle θ satisfies this equation. It follows that the coverage area by the two base stations consists of disconnected two areas when $R>2\sqrt{2}$, the two areas are tangent to one another for $R=2\sqrt{2}$, and the area is connected when $R<2\sqrt{2}$.

In order to calculate the coverage area define $\theta_0=\arccos\frac{R}{2\sqrt{2}}$, and if $R>2\sqrt{2}$, then θ_0=0. We calculate one quarter of the area which lies above the x-axis and is left to the perpendicular bisector between the two base stations. The area consists of a right triangle whose sides are $\sqrt{2}\cos\theta_0$, $\sqrt{2}\sin\theta_0$, and $\sqrt{2}$. The area of this triangle is $\cos\theta_0\sin\theta_0$. Let the solution r of equation (1) for a given θ, which can be found by bisection on the range $(1,\sqrt{2})$, be $r(\theta)$. The rest of the area is

$$\int_{\theta_0}^{\pi}\frac{1}{2}r^2(\theta)d\theta$$

Therefore the total coverage area A is:

$$A=4\cos\theta_0\sin\theta_0+2\int_{\theta_0}^{\pi}r^2(\theta)d\theta$$

The integral can be calculated numerically by Simpson's rule. Calculating 300 points to an accuracy of 10^{-7} took a fraction of one second of computer time. The maximum coverage of 8.051 was obtained at R=2.41 and is used in generating the boundary of the cover area depicted in Figure 3. The coverage area is 28% more than 2π which is the cover area estimated by standard cover models for this configuration. For R=0, $A=2\pi$ and as $R\to\infty$, $A=2\pi$ as well both providing the same cover as standard covering models.

In Berman et al. (2008a) the problem of locating two cellular phone towers in Northern Orange County is solved. The locations found by the model are usable to cover the most population in Northern Orange County, California.

THE VARIABLE RADIUS MODEL

The variable radius suggests a new objective function for covering models. Berman, Drezner, Krass, and Wesolowsky (2008b) suggested this model in conjunction with traditional covering models. It can be generalized to incorporate any of the three new definitions of cover described above.

In this model it is assumed that the base station can be designed for providing different cover distances. Providing a larger cover distance requires more investment in constructing the base station and its transmitter. Each base station may have a different coverage area. One can build fewer base stations with a strong signal or more base stations with a weaker signal or a mix of signal strengths. In standard covering models the covering distance D is assumed to be an exogenous parameter, outside of the control of the decision-maker. However, the coverage radius of a base station can be one of the design parameters: a larger or smaller coverage radius can be achieved by increasing or decreasing the power of the transmitter.

The coverage radius as a function of the investment required for building a base station is known. In standard covering models the objective is either minimizing the number of base stations to cover all users or covering the maximum number of users with a given number of base stations. In the variable radius model, the objective is either the minimization of the budget required to cover all users or covering the maximum number of users with a given budget.

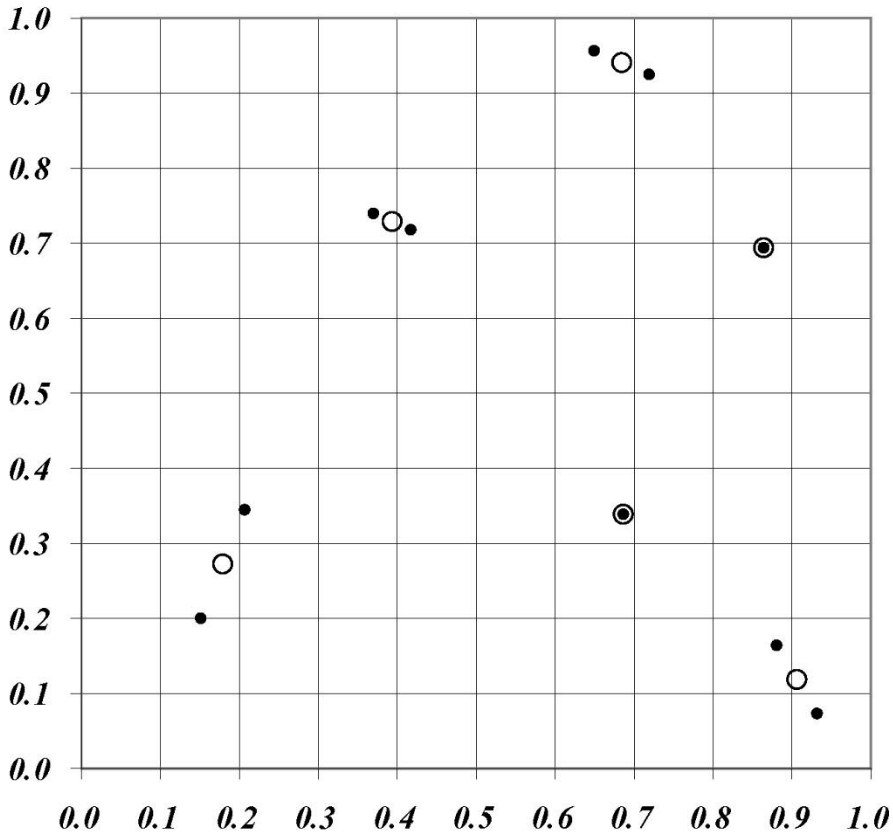

Figure 4. The solution to the ten user example

As an illustration consider a problem of 10 users depicted in Figure 4 as small black circles. These ten users can represent small villages in an area. Base stations need to be established to serve these ten villages. What is the most cost effective configuration of base stations to serve these villages? The cost of building a base station which covers a distance D is $0.01+D^2$. This means that there is a fixed cost of 0.01 for building a base station and a variable cost proportional to the square of the cover distance or proportional to the coverage area. If a base station with a very small cover radius is built at each village, the total cost is 0.1. The optimal solution is depicted in Figure 4. Base stations are depicted as rings. The solution suggests the establishment of six base stations. Four base stations cover two villages each and two base stations are dedicated to one village each. The total cost for this solution is 0.07486 which is a saving of 25.14% in construction cost. This solution is quite intuitive and could be obtained by inspection of the spatial distribution of the ten users. However, when the number of users is large, the solution may not be that easy to find. The solution may include base stations that cover three or more users each. Therefore, the problem is quite a contrived combinatorial problem that is not easy to solve. Berman et al. (2008b) proposed efficient approaches to solve such problems.

CONCLUSION

When locating cellular telephones base stations, it is common practice to apply location covering models. In standard cover models it is assumed that within a certain distance from the base station reception is perfect and beyond that distance there is no reception at all. Such models neither account for the overlap in cover when more than one base station is in the vicinity of a cellular telephone user nor do they consider deterioration in cover as distances increase. We reviewed several variations of standard covering models that consider these modeling improvements. These models are better suited than traditional approaches for the location of cellular telephone base stations. Three models suggest new definitions of cover: the gradual cover, the stochastic gradual cover, and the sum cover. Such models simulate the actual situation more accurately and thus provide superior locations for base stations. A fourth model, the variable radius model, provides the planner flexibility in determining the strength of the signal that is transmitted (and detected) by each individual base station. This formulation leads to covering more users with a lower construction cost.

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