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Recomended for You: The Impact of Profit Incentives on the Relevance of Online Recommendations

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RECOMMENDED FOR YOU: THE IMPACT OF PROFIT INCENTIVES ON THE RELEVANCE OF ONLINE RECOMMENDATIONS

Je vous le recommande : impact de l'intéressement aux bénéfices sur la pertinence des recommandations en ligne

Completed Research Paper

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Abstract

Recommender systems are commonly used by Internet firms to improve consumers' shopping experience and increase firm sales and profits. A large stream of work on recommender design has studied the problem of identifying the most relevant items to recommend to users. In parallel, recent empirical work has started to provide evidence that real-world recommenders contribute to increased sales and profitability for the firms. However, maximizing consumer welfare and firm profit are not the same. Given that recommenders impact sales and profits, a natural question is what is the impact of firm's profit incentives on recommender design? This paper studies optimal recommender design in a profit-maximizing framework to answer the question and identifies the conditions under which a profit-maximizing recommender recommends the item with highest margins and those under which it recommends the most relevant item. We further elaborate on the social cost of the mismatch between consumer and firm incentives.

Keywords: Recommender systems, incentive-centered design, Markov decision process, Internet commerce

Résumé

Les systèmes de recommandation en ligne sont communément employés par les entreprises pour accroître leurs ventes et leurs bénéfices. Ce papier étudie l'impact de l'intéressement aux bénéfices sur l'élaboration des recommandations et identifie sous quelles conditions un conseiller recommande le produit avec les plus fortes marges et celles où le conseiller recommande le produit le plus pertinent. Nous estimons ensuite le coût social d'inadéquation entre le consommateur et l'intéressement.

Introduction

The use of recommender systems has become widespread on the Internet. Recommender systems attempt to deliver personalized recommendations to their users, allowing them to discover new products and sort through large choice sets. With the explosion of products and information on the Internet, this functionality is becoming increasingly indispensable. As a result, several Internet firms including Amazon, Netflix, Yahoo, iTunes and others use recommenders extensively.

Designing an effective recommender system, however, is not a trivial task, and has attracted ample attention from computer science, information systems, and marketing research communities. A recommender system typically has to infer consumer's preference using limited information about the consumers and their purchase or rating histories. Various techniques have been proposed and analyzed in extant research, which can be classified into categories such as content-based, collaborative-filtering based, and hybrid approaches (Adomavicius and Tuzhilin 2005).

However, a gap exists between extant research on recommender systems and their use in real world. While increasing the accuracy of recommendation has been the focus of academic research, it is not clear that maximizing predictive accuracy is the eventual goal of all recommender systems. Recommender systems are deployed by firms whose incentives in deploying them may range from increasing customer loyalty to increasing profitability. Though recommending more relevant products to consumers increases the likelihood of purchase, firms may also consider many other important factors, chief among them is profit margin of the recommended item.

The case of Netflix (Shih et al. 2007) clearly demonstrates this point. Running a mail-delivered movie rental business, the firm provides a Web-based interface through which it recommends movies to consumers. The recommender system, not unlike others, strives to recommend the best fit to consumers. However, the system has a filter to avoid recommending new releases which are in high demand and have high carrying costs. Recommending them would perhaps increase the performance of the recommender system from the perspective of accuracy but lower profits. Clearly, from the perspective of profit maximization, the recommender system should balance accuracy and profitability. In the case of Netflix, a highly relevant movie which is also very popular, though possibly a great fit to a consumer, is also an expensive one. The firm could increase profit by recommending a less popular title which is a moderately good fit but is much cheaper to acquire and carry. E-commerce sites such as Amazon may call for a different strategy, where the popular items may also have lower unit costs due to volume discounts or lower holding costs, making them desirable for recommendation. Though the specific recommender policies may vary across firms, the notion of balancing the relevance and profitability of recommendations remain the same. This is one of the central tradeoffs that we study in this paper.

In addition to balancing relevance and margin, the firm also needs to concern itself with the trust that its consumers place in its recommendations when it interacts with consumers repeatedly over time. This trust translates into reputation of the recommender and sets up a tradeoff that is inter-temporal in nature. If the firm and a consumer only interact once, then the recommender may recommend products that are somewhat less relevant but have high margins, which more than compensate for low purchase probabilities. However, with repeated interactions, if the recommender system keeps recommending irrelevant products, consumers are less likely to pay attention to recommendations over time. Consequently, the recommender's reputation (or salience) will be diminished. Thus recommending high margin products that have low relevance may increase current profit at the cost of future profits.

To the best of our knowledge, the tradeoff between relevance and profit margin and that between short-term profitability and reputation have not been studied in existing research on recommender systems. Our study focuses on these two concerns. Through analytical modeling, we confirm that optimal recommendation policies must trade off product relevance and profit margin, and identify the conditions under which it is optimal to recommend one type of product or another. We show that the optimal recommendation policy under high recommender reputation is qualitatively different than that under low reputation. When reputation is high, only the balancing of relevance and profit margin needs to be considered, and the optimal recommendation typically maximizes current profit; but when reputation is low, in certain situations the optimal recommendation should be aimed at restoring reputation, even when doing so reduces profit temporarily. In general, an optimal recommendation policy calls for *harvesting* for profitability when reputation is high, and for *restoring* reputation when it is low. These results are first established in a two-product setting where the tradeoffs are more pronounced but are subsequently generalized to a multi-product setting.

The rest of the paper is organized as follows: in the next section we review existing literature; following that, we present the model that we use for our analysis; a two-product setting is then analyzed in detail, where our major results are established and discussed; the section following that show that the results also hold in generalized settings; finally, we conclude and discuss possible future research.

Literature Review

There are three streams of work highly relevant to our study. We briefly discuss these streams and position our work within the literature.

Recommender System Design: Designing effective recommender systems that can infer user preferences and recommend relevant items is a challenging task. A rich literature in several research fields including computer science, information systems, and marketing has studied the problem. Adomavicius and Tuzhilin (2005) provide a detailed survey of various techniques. These methods are classified into three main categories: content-based, collaborative filtering based, and hybrid approaches. Content-based approaches recommend items based on information about users and their past purchase behavior or usage patterns. These approaches work best when a significant amount of historical information about the users is available. Since this type of historical information is often sparse in reality, collaborative-filtering based approaches are often used to recommend items based on historical information of other users with similar preferences. Hybrid approaches combine both types of information in the recommendation process. Adomavicius and Kwon (2007) also provide a discussion on these methods. Breese et al. (1998) empirically evaluate several collaborative-filtering based approaches.

Recent research also attempts to leverage information in addition to that of users and items to increase the accuracy of recommendations. Ansari et al. (2000) use expert evaluations in addition to users' stated preferences and other user and item characteristics. Adomavicius et al. (2005) use contextual information together with user and item information to create a multi-dimensional rating estimation method. Sahoo et al. (2008) builds a multi-dimensional recommendation system to use information from multi-dimensional ratings using Yahoo movies as context.

While extensive research exists on the design of recommender systems, they mostly focus on improving the predictive accuracy of such systems, and little attention has been paid to the incentive of the firms that develop and deploy such systems. In creating these systems, firms often seek to improve profitability rather than just predictive accuracy. Although better predictive accuracy increases consumer purchase probability, many other factors are also taken into account by firms seeking to maximize profit. Bodapati (2008) is among the first bridge this gap. The author points out that even if the system recommends products that are most relevant, it may be of little value if those products would anyway be bought by consumers in the absence of recommendations. The study shows that, instead of recommending what is most likely to be purchased, the system should recommend products whose purchase probability can be most influenced by the recommendation.

Our work builds on the work by Bodapati (2008) and studies the relevance-profitability tradeoff in recommender design. This serves two purposes. First, it helps us understand how the profit incentives of firms impact system design and, more specifically, helps identify the conditions under which a policy that is compatible with firm's profit incentives is also the one that maximizes consumer welfare. Second, it helps inform design choices for firms that seek to balance relevance with profitability.

Following Bodpati (2008), our work also studies the design of recommender systems from firm's perspective. Rather than ask how to recommend what is most likely to be purchased, we investigate how to recommend products to best help the firm increase its profit. Our work is aimed at bridging the gap between the predictive accuracy of recommender systems and the firm's interest in designing recommendation policies that maximize profitability. Our work thus complements extant research on recommender system design and offers insight to practitioners.

Consumer Response to Recommendations: With the widespread use of recommender systems on the Internet, studies on consumer response to these systems have also been conducted. Using data collected from Amazon.com, Chen and Wu (2006) show that more recommendations are associated with higher sales. Senecal and Nantel (2004) also demonstrate through experiments that recommendations influence the choices made by consumers. Fleder and Hosanagar (2008) examine the impact of recommender systems on sales diversity, and show that though individual level diversity may increase, aggregate sales diversity can decrease at the same time.

Our work draws from this stream to model how consumers respond to recommendations. Recommendations have two main effects on consumer purchase decisions: awareness and salience (Fleder and Hosanagar 2008). Recommenders help address information asymmetry between consumers and firms. A consumer may not be aware of a product that is carried by the firm and the recommender can inform consumers about new products. This is the awareness effect of recommenders. Further, even if a consumer is aware of a product, a recommendation can increase the purchase probability of that product. This is the salience effect of recommenders. Experimental studies have validated the existence of such an effect (example, Senecal and Nantel 2004). The salience effect may be driven by several factors, including the ease of clicking on a recommended item, increased identification with the product due to the personalized nature of the recommendation, increased salience when comparing multiple products, persuasive effects akin to advertising, etc. Both awareness and salience effects increase the purchase probability of the recommended products. We model the increase in the purchase probability of recommended products but do not explicitly incorporate increased awareness on account of recommendations in the current study.

Asymmetric Information: A rich literature exists in economics to study the interaction between firms and consumers under asymmetric information. Kreps and Wilson (1982) show that in multi-period games, firms may seek to establish a reputation of being "tough" early on so as to benefit in later periods. Milgrom and Roberts (1982a) in a similar setting show that costly predation in early periods can be justified as it may deter future entry of competitors. Private information such as cost and quality can also be credibly signaled, through price and advertisement, to competitors and customers, as shown in Milgrom and Roberts (1982b) and Milgrom and Roberts (1986).

Our work is related to the asymmetric information literature, both because recommendation system conceptually rests on the existence of information asymmetry and, more importantly, because firms are concerned with the reputation of a recommendation system. When interacting with consumers, firms need to strike a balance between establishing a good reputation and benefiting from it. Tradeoffs are needed in such circumstances, as shown in Shapiro (1982) that when facing reputation concern, a firm will choose quality level lower than it would under perfect information. The main difference of our work is that while the economic literature generally focuses on characterizing equilibria of the games, our work focuses on the optimal design of recommendation systems. Furthermore, reputation concern is only one of the several factors that together influence the optimal policy, as will be discussed below.

Model

We study a firm's optimal recommendation policy when it sells products to consumers in a repeated interaction setting. The firm sells n products. Each product is characterized by its *profit margin* $M_i, i \in \{1..n\}$, and its *expected relevance* to the consumer $V_i, i \in \{1..n\}$. We assume that margin and expected relevance stay constant over time. To focus on the profitability implication of recommender policies, we assume that the firm knows the value of each product's V_i (existing research on recommender policies focuses on identifying V_i using information related to consumers and/or their purchase histories).

In each period, the consumer makes the decision to either purchase one of the products, or not to purchase in that period. Repeat purchases are allowed in our model and reflect recommendations in the context of purchase of consumables. In each period, the firm can recommend a product to the consumer. This recommendation is assumed to have a *salience effect* – the perceived relevance of the recommended product will temporarily receive a boost in that period. The magnitude of the boost, denote as δ_S, depends on the state S the consumer is in. The consumer's state reflects the *reputation* of the recommender to the consumer. Intuitively, if a recommender system consistently recommends the "right" products, then it would have a good reputation and the recommendations influence consumer choice to a greater extent than would be the case if it often recommends irrelevant products. In our model, we assume that the consumer can be in one of two states: H or L. The salience of the recommendation differs between the two states: $\delta_H > \delta_L > 0$.

The state a consumer is in is determined by the past performance of the recommender. We model it as follows: if a consumer is in state H in a period, and in that period she purchases the recommended product, then she remains in state H in the next period. If, however, she does not purchase the recommended product, then she will with probability p_1 transition to state L in the following period. If a consumer is in state L, and if in that period she does not purchase the recommended product, then she remains in state L in the next period. But if she purchases the recommended product, then she will with probability p_2 transition to state H in the following period. The transition probabilities p_1 and p_2 determine the persistence of reputation. The smaller these probabilities are, the more persistent is the reputation. The transition is depicted in Figure 1 below.

One important feature of this model setup is that a consumer's satisfaction with the recommender system is assumed to be reflected in her purchase behavior. This setup applies directly to search goods. Products are often classified into two categories in literature: search goods and experience goods. Consumers are usually able to assess the quality of search goods before purchase, while for experience goods consumers must use the product, after purchase, to assess the real quality. Since consumers can assess search goods thoroughly before purchase, it is reasonable to assume that the act of purchasing a product indicates that the consumer is satisfied. Our model is set up with search goods in mind, both because many recommendation systems in real world are designed for search goods and because of the analytical tractability it affords. Consumer's consumption experience needs to be accounted for to extend the model for experience goods.

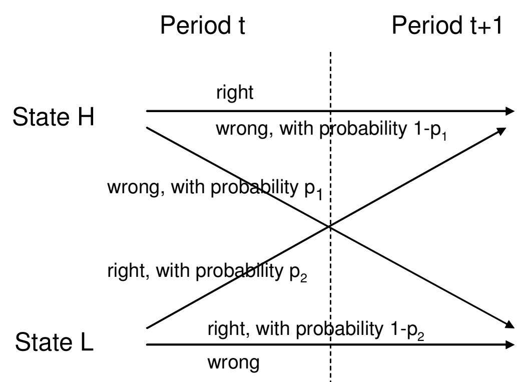

Figure 1: Consumer State Transition

In each period t, the consumer decides to buy one of the products, or decides not to buy at all. The utility of consumer purchasing product j in period t is expressed as:

$$U_{jt} = V_j + I_{jt}\delta_{s_t} + \varepsilon_{jt} \tag{1}$$

In the expression, $I_{jt} = 1$ if item j is recommended in the period, and 0 otherwise. The utility of the no-purchase option is normalized to zero. Let ε_{jt}, which represents a random shock to the utility that is observed by the consumer before making a purchase but not by the firm, follow a type I Extreme Value distribution. Then the purchase probability for item j in period t follows the Multinomial Logit specification and can be expressed in close-form:

$$\Pr(j,t \mid S) = \frac{e^{V_j + I_{jt}\delta_S}}{1+\sum_{l=1}^{n} e^{V_l + I_{lt}\delta_S}},\ j \in \{1..n\} \tag{2}$$

The firm chooses a recommendation policy to maximize profit. This profit maximization problem is a Markov Decision Problem (MDP), which is expressed as:

$$\max_R E[\Pi(R)] = E[\sum_{t=1}^{\infty} \beta^t M_{c_t(R)}] \tag{3}$$

In the expression, β is the discount factor and $c_t(R)$ is the item the consumer chooses to purchase in period t when the firm follows a recommendation policy R. The optimal item to recommend will likely depend on the state consumer is in. However, the firm faces two types of uncertainties on this. First, the firm might not know the consumer's initial state. And second, because a change in states is stochastic, the consumer's state in future periods may not be precisely known even if the initial state is known.

In order to simplify our analysis, we assume in the current model that $p_1 = p_2 = 1$. That is, if a wrong recommendation is made in state H, the consumer will immediately transition to state L, and vice versa. The assumption simplifies the analysis and enables us to highlight the relevance-margin tradeoff that is our main focus. We expect the central tradeoffs and the related insights to remain the same when these probabilities take less extreme values. With this assumption, firm learns consumer's state once the first period is passed. Since we study repeated interactions where a single period does not have dominant overall influence, we further assume that the firm knows the consumer's initial state. The discussion below focuses on the case where the initial state is H, The results would remain the same qualitatively if the initial state is L.

With the above assumption, a recommendation policy is a mapping from state to product: depending upon the consumer's state, the recommender recommends a certain product. With n products and two states, there are n^2 possible recommendation policies. Any policy is of the form: "recommend product j_1 in state H and j_2 in state L".

Let $\Pi_j(R), j \in \{H, L\}$ denote the overall expected discounted profit of following recommendation policy R when the initial state is j. Let $\pi_{tj}(R), j \in \{H, L\}$ denote the expected current period profit of following recommendation policy R when the current state is j. Let $P_{ij}(R), i, j \in \{H, L\}$ denote the probability of next period state being j when the current period state is i, when recommendation policy R is followed. We can then express the expected profit in the form of Bellman equations:

$$\Pi_H(R) = \pi_{tH}(R) + \beta[P_{HH}(R)\Pi_H(R) + P_{HL}(R)\Pi_L(R)] \tag{4}$$

$$\Pi_L(R) = \pi_{tL}(R) + \beta[P_{LH}(R)\Pi_H(R) + P_{LL}(R)\Pi_L(R)] \tag{5}$$

The system of equations of (4) and (5) can be solved to express the overall profit as functions of per-period profits and the transition probabilities, as stated below:

$$\Pi_H(R) = \frac{\pi_{tH}(R)(1 - \beta P_{LL}(R)) + \pi_{tL}(R)\beta P_{HL}(R)}{1 - \beta(P_{HH}(R) + P_{LL}(R)) + \beta^2(P_{HH}(R)P_{LL}(R) - P_{LH}(R)P_{HL}(R))} \tag{6}$$

$$\Pi_L(R) = \frac{\pi_{tL}(R)(1 - \beta P_{HH}(R)) + \pi_{tH}(R)\beta P_{LH}(R)}{1 - \beta(P_{HH}(R) + P_{LL}(R)) + \beta^2(P_{HH}(R)P_{LL}(R) - P_{LH}(R)P_{HL}(R))} \tag{7}$$

Because of the salience effect, recommending a product increases the chance that consumer purchases that product. Intuitively, the firm would like to recommend the products which have high margins, because selling higher margin products generates higher profits. However, when the high margin products are also of low relevance to the consumer, then the higher margin may or may not compensate the lower purchase probability. Furthermore, recommending low relevance products carries a higher risk of consumer becoming disappointed to the recommender system and transitioning into the low state, where the recommender system's influence is diminished. Therefore, in devising the optimal recommendation policy, the firm must balance the profitability concern and the reputation concern. Even when addressing profitability concern itself, the firm still needs to balance margin and relevance. These are the central tradeoffs that we analyze in this study.

The Two-Product Scenario

In this section, we first derive our results for a case in which the firm sells two products, i.e. $n = 2$. Focusing on this simplified setting enables us to clearly highlight the tradeoff the firm faces in the recommender policy design.

And as we demonstrate later, most of the results still hold in the general scenario where there are more than two products.

The two products are characterized by their relevance V_1 and V_2 as well as profit margins M_1 and M_2. If one product has both higher relevance and higher margin than the other, then the solution is obvious – always recommend the product with higher relevance and margin, regardless of the state. Therefore, we focus on the case in which the firm faces a tradeoff in recommendation, i.e. the item with higher relevance also has low margin, and vice versa. Without loss of generality, let $V_1 > V_2$, and thus $M_1 < M_2$.

With two products, there are four possible recommendation policies, as shown in Table 1 below.

Table 1: Recommendation Policies for Two-Product Scenario

Policy	Recommend in state H	Recommend in state L
$R1$	Product 1	Product 1
$R2$	Product 1	Product 2
$R3$	Product 2	Product 1
$R4$	Product 2	Product 2

As noted in the previous section, the profit of a recommendation policy is determined by the per-period profits and the transition probabilities. The per-period profits and transition probabilities corresponding to each recommender policy are listed in Table 2 below. Since $P_{HL} = 1 - P_{HH}$ and $P_{LH} = 1 - P_{LL}$, only P_{HH} and P_{LL} are presented. Note that both the current profits and transition probabilities are obtained from the purchase probabilities specified in (2).

Table 2: Per-Period Profits and Transition Probabilities

Policy	Per-Period Profits and Transition Probabilities
$R1$	$\pi_{tH} = \frac{e^{V_1+\delta_H}M_1 + e^{V_2}M_2}{e^{V_1+\delta_H} + e^{V_2} + 1}$, $\pi_{tL} = \frac{e^{V_1+\delta_L}M_1 + e^{V_2}M_2}{e^{V_1+\delta_L} + e^{V_2} + 1}$ $P_{HH} = \frac{e^{V_1+\delta_H}}{e^{V_1+\delta_H} + e^{V_2} + 1}$, $P_{LL} = \frac{e^{V_2} + 1}{e^{V_1+\delta_L} + e^{V_2} + 1}$
$R2$	$\pi_{tH} = \frac{e^{V_1+\delta_H}M_1 + e^{V_2}M_2}{e^{V_1+\delta_H} + e^{V_2} + 1}$, $\pi_{tL} = \frac{e^{V_1}M_1 + e^{V_2+\delta_L}M_2}{e^{V_1} + e^{V_2+\delta_L} + 1}$ $P_{HH} = \frac{e^{V_1+\delta_H}}{e^{V_1+\delta_H} + e^{V_2} + 1}$, $P_{LL} = \frac{e^{V_1} + 1}{e^{V_1} + e^{V_2+\delta_L} + 1}$
$R3$	$\pi_{tH} = \frac{e^{V_1}M_1 + e^{V_2+\delta_H}M_2}{e^{V_1} + e^{V_2+\delta_H} + 1}$, $\pi_{tL} = \frac{e^{V_1+\delta_L}M_1 + e^{V_2}M_2}{e^{V_1+\delta_L} + e^{V_2} + 1}$ $P_{HH} = \frac{e^{V_2+\delta_H}}{e^{V_1} + e^{V_2+\delta_H} + 1}$, $P_{LL} = \frac{e^{V_2} + 1}{e^{V_1+\delta_L} + e^{V_2} + 1}$
$R4$	$\pi_{tH} = \frac{e^{V_1}M_1 + e^{V_2+\delta_H}M_2}{e^{V_1} + e^{V_2+\delta_H} + 1}$, $\pi_{tL} = \frac{e^{V_1}M_1 + e^{V_2+\delta_L}M_2}{e^{V_1} + e^{V_2+\delta_L} + 1}$ $P_{HH} = \frac{e^{V_2+\delta_H}}{e^{V_1} + e^{V_2+\delta_H} + 1}$, $P_{LL} = \frac{e^{V_1} + 1}{e^{V_1} + e^{V_2+\delta_L} + 1}$

We first rewrite expression (6) in a more succinct way, where we use superscript to identify the policies:

$$\Pi^{R} = \frac{\pi_{tH}^{R}(1-\beta P_{LL}^{R}) + \pi_{tL}^{R}\beta P_{HL}^{R}}{1-\beta(P_{HH}^{R}+P_{LL}^{R}) + \beta^{2}(P_{HH}^{R}P_{LL}^{R} - P_{LH}^{R}P_{HL}^{R})} \tag{8}$$

Then for any two recommendation policies Ri and Rj:

$$\begin{aligned}\Pi^{Ri} - \Pi^{Rj} &= \frac{\pi_{tH}^{Ri}(1-\beta P_{LL}^{Ri}) + \pi_{tL}^{Ri}\beta P_{HL}^{Ri}}{1-\beta(P_{HH}^{Ri}+P_{LL}^{Ri}) + \beta^{2}(P_{HH}^{Ri}P_{LL}^{Ri} - P_{LH}^{Ri}P_{HL}^{Ri})} - \frac{\pi_{tH}^{Rj}(1-\beta P_{LL}^{Rj}) + \pi_{tL}^{Rj}\beta P_{HL}^{Rj}}{1-\beta(P_{HH}^{Rj}+P_{LL}^{Rj}) + \beta^{2}(P_{HH}^{Rj}P_{LL}^{Rj} - P_{LH}^{Rj}P_{HL}^{Rj})} \\ &= \frac{(\pi_{tH}^{Ri}(1-\beta P_{LL}^{Ri}) + \pi_{tL}^{Ri}\beta P_{HL}^{Ri})(1+\beta-\beta(P_{HH}^{Rj}+P_{LL}^{Rj})) - (\pi_{tH}^{Rj}(1-\beta P_{LL}^{Rj}) + \pi_{tL}^{Rj}\beta P_{HL}^{Rj})(1+\beta-\beta(P_{HH}^{Ri}+P_{LL}^{Ri}))}{(1-\beta)(1+\beta-\beta(P_{HH}^{Ri}+P_{LL}^{Ri}))(1+\beta-\beta(P_{HH}^{Rj}+P_{LL}^{Rj}))}\end{aligned} \tag{9}$$

The denominator in expression (9) has positive sign. Therefore, the difference between the profits of the two recommendation policies has the same sign as the numerator above:

$$\begin{aligned}&sign[\Pi^{Ri} - \Pi^{Rj}] \\ &= sign[\pi_{tH}^{Ri}(1-\beta P_{LL}^{Ri})(1+\beta-\beta(P_{HH}^{Rj}+P_{LL}^{Rj})) - \pi_{tH}^{Rj}(1-\beta P_{LL}^{Rj})(1+\beta-\beta(P_{HH}^{Ri}+P_{LL}^{Ri})) + \\ &\beta(\pi_{tL}^{Ri}(1-P_{HH}^{Ri})(1+\beta-\beta(P_{HH}^{Rj}+P_{LL}^{Rj})) - \pi_{tL}^{Rj}(1-P_{HH}^{Rj})(1+\beta-\beta(P_{HH}^{Ri}+P_{LL}^{Ri})))]\end{aligned} \tag{10}$$

State H:

First, we look at state H. Notice that $R1$ and $R3$ recommend the same product in state L, same for $R2$ and $R4$. Pair-wise comparisons of the profits reveal the following (we outline here the major steps in deriving the result, while detailed technical proof can be found in the Appendix):

$$\begin{aligned}&sign[\Pi^{R3} - \Pi^{R1}] = sign[\\ &M_2((e^{V_1+\delta_L} + e^{V_2} + 1)(e^{V_1+V_2+2\delta_H} + e^{V_2+\delta_H} - e^{V_1+V_2} - e^{V_2}) + \beta(e^{V_1}+1)e^{V_1+V_2}(e^{\delta_H} - e^{\delta_L}) + \beta(e^{V_2}+1)e^{V_1+V_2+\delta_H}(e^{\delta_L} - e^{\delta_H})) \\ &- M_1((e^{V_1+\delta_L} + e^{V_2} + 1)(e^{V_1+V_2+2\delta_H} + e^{V_1+\delta_H} - e^{V_1+V_2} - e^{V_1}) + \beta(e^{V_2}+1)(e^{V_1+V_2+\delta_H+\delta_L} - e^{2V_1+\delta_L} + e^{2V_1+\delta_H} - e^{V_1+V_2+2\delta_H}))]\end{aligned} \tag{11}$$

And

$$\begin{aligned}&sign[\Pi^{R4} - \Pi^{R2}] = sign[\\ &M_2((e^{V_1} + e^{V_2+\delta_L} + 1)(e^{V_1+V_2+2\delta_H} + e^{V_2+\delta_H} - e^{V_1+V_2} - e^{V_2}) + \beta(e^{V_1}+1)(e^{V_1+V_2+\delta_H+\delta_L} - e^{2V_2+\delta_L} - e^{V_1+V_2+2\delta_H} + e^{2V_2+\delta_H})) \\ &- M_1((e^{V_1} + e^{V_2+\delta_L} + 1)(e^{V_1+V_2+2\delta_H} + e^{V_1+\delta_H} - e^{V_1+V_2} - e^{V_1}) + \beta e^{V_1+V_2}(e^{\delta_L} - e^{\delta_H})((e^{V_1}+1)e^{\delta_H} - (e^{V_2}+1)))]\end{aligned} \tag{12}$$

It is easy to verify that in both expressions (11) and (12), the coefficient of M_2 is positive. This shows that when the margin of the high margin product is sufficiently high, $R3$ outperforms $R1$ in profit and $R4$ outperforms $R2$. Since both $R3$ and $R4$ recommend product 2 in state H, we know that the optimal recommendation policy should recommend the high margin item when that margin is sufficiently high. The result is stated formally below:

> *Proposition 1: When M_2 is sufficiently high, that is, when M_2/M_1 exceeds a threshold value TH, it is optimal to recommend the high margin product, product 2, in state H. The threshold value is:*

$$\begin{aligned}TH = \max\{&\frac{(e^{V_1+\delta_L} + e^{V_2} + 1)(e^{V_1+V_2+2\delta_H} + e^{V_1+\delta_H} - e^{V_1+V_2} - e^{V_1}) + \beta(e^{V_2}+1)(e^{V_1+V_2+\delta_H+\delta_L} - e^{2V_1+\delta_L} + e^{2V_1+\delta_H} - e^{V_1+V_2+2\delta_H})}{(e^{V_1+\delta_L} + e^{V_2} + 1)(e^{V_1+V_2+2\delta_H} + e^{V_2+\delta_H} - e^{V_1+V_2} - e^{V_2}) + \beta(e^{V_1}+1)e^{V_1+V_2}(e^{\delta_H} - e^{\delta_L}) + \beta(e^{V_2}+1)e^{V_1+V_2+\delta_H}(e^{\delta_L} - e^{\delta_H})}, \\ &\frac{(e^{V_1} + e^{V_2+\delta_L} + 1)(e^{V_1+V_2+2\delta_H} + e^{V_1+\delta_H} - e^{V_1+V_2} - e^{V_1}) + \beta e^{V_1+V_2}(e^{\delta_L} - e^{\delta_H})((e^{V_1}+1)e^{\delta_H} - (e^{V_2}+1))}{(e^{V_1} + e^{V_2+\delta_L} + 1)(e^{V_1+V_2+2\delta_H} + e^{V_2+\delta_H} - e^{V_1+V_2} - e^{V_2}) + \beta(e^{V_1}+1)(e^{V_1+V_2+\delta_H+\delta_L} - e^{2V_2+\delta_L} - e^{V_1+V_2+2\delta_H} + e^{2V_2+\delta_H})}\}\end{aligned}$$

The threshold value TH in the proposition is determined by the relevance of both products as well as the salience effects in the two states. It is easy to see that TH is always greater than 1, which is rather obvious. It can be shown analytically that this threshold increases as V_1 increases, and decreases as V_2 increases. This is also not surprising, since higher/lower difference in margin is needed to compensate for a wider/narrower gap in relevance. Also can be shown analytically is that the threshold decreases as δ_L increases[1]. A monotone relationship between the threshold and δ_H does not exist, however. Figure 2 below shows that with low V_1, TH decreases as δ_H increases, while with high V_1, TH initially increases as δ_H increases.

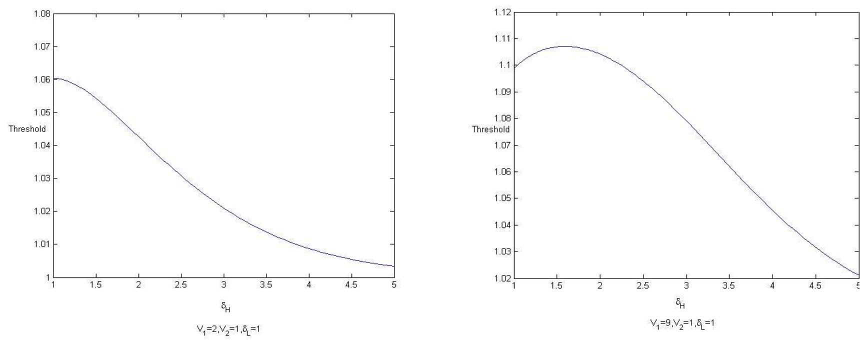

Figure 2: Change of Threshold in Response to Salience Effect

Further we find that as $M_2 \to M_1$, $R1$ outperforms $R3$ and $R2$ outperforms $R4$, so it is optimal to recommend product 1, the high relevance product, in this case. Similarly, as $V_2 \to V_1$, $R3$ outperforms $R1$ and $R4$ outperforms $R2$, so it is optimal to recommend the high margin product.

> *Proposition 2: When the margins of the two products are sufficiently close, that is, when M_2 / M_1 is below a threshold value TL, it is optimal to recommend the high relevance product, product 1, in state H. When the relevance of the two products are sufficiently close, it is optimal to recommend the high margin product, product 2, in state H. The threshold value is:*

$$TL = \min\{\frac{(e^{V_1+\delta_L} + e^{V_2} + 1)(e^{V_1+V_2+2\delta_H} + e^{V_1+\delta_H} - e^{V_1+V_2} - e^{V_1}) + \beta(e^{V_2}+1)(e^{V_1+V_2+\delta_H+\delta_L} - e^{2V_1+\delta_L} + e^{2V_1+\delta_H} - e^{V_1+V_2+2\delta_H})}{(e^{V_1+\delta_L} + e^{V_2} + 1)(e^{V_1+V_2+2\delta_H} + e^{V_2+\delta_H} - e^{V_1+V_2} - e^{V_2}) + \beta(e^{V_1}+1)e^{V_1+V_2}(e^{\delta_H} - e^{\delta_L}) + \beta(e^{V_2}+1)e^{V_1+V_2+\delta_H}(e^{\delta_L} - e^{\delta_H})},$$
$$\frac{(e^{V_1} + e^{V_2+\delta_L} + 1)(e^{V_1+V_2+2\delta_H} + e^{V_1+\delta_H} - e^{V_1+V_2} - e^{V_1}) + \beta e^{V_1+V_2}(e^{\delta_L} - e^{\delta_H})((e^{V_1}+1)e^{\delta_H} - (e^{V_2}+1))}{(e^{V_1} + e^{V_2+\delta_L} + 1)(e^{V_1+V_2+2\delta_H} + e^{V_2+\delta_H} - e^{V_1+V_2} - e^{V_2}) + \beta(e^{V_1}+1)(e^{V_1+V_2+\delta_H+\delta_L} - e^{2V_2+\delta_L} - e^{V_1+V_2+2\delta_H} + e^{2V_2+\delta_H})}\}$$

The recommendation policy needs to balance profitability and reputation concerns. Since the salience effect is high in state H, the profitability concern is more pronounced. As we have shown, when the margin of a product is sufficiently high compared to the other, it is optimal to recommend that high margin product. However, even out of profitability concern alone, the firm need not always recommend the high margin product. This is because the consumer has the option to not purchase either product. If the difference between the margins of the two products is small compared with the difference between their relevance, it may be optimal to recommend the high relevance product even out of profitability concern itself, since the firm does not make any profit if the consumer ends up not buying anything. When $M_2 \to M_1$, the optimal policy recommends the high relevance product in state H not out of reputation concerns but purely to maximize short-term profits.

State L:

Next, we look at the optimal product to recommend in state L. Though in state H the profitability concern is dominant, in state L the policy needs to also address the reputation concern: not only should the policy increase the short-term profit, it also needs to attempt to restore its reputation. These two concerns, however, may conflict with each other in certain cases.

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[1] The detailed technical proof on the directional change of TH in response to changes in V_1, V_2, and δ_L is available from authors upon request.

This conflict is illustrated in Figure 3. Suppose the consumer is in state L in the current period. In the diagram, the superscripts depict which product is recommended. The recommender needs to decide whether to recommend the high relevance product 1 or the high margin product 2, and it needs to take into account both the short-term and the future profit. From the perspective of short-term profit, when the margin of product 2 is sufficiently high, recommending it will bring higher expected current period profit, i.e. $\Pi_{tL}^2 > \Pi_{tL}^1$. From the perspective of future profit, since the salience effect in state H is higher than that in state L, we expect the profit to be higher in state H, i.e. $\Pi_H > \Pi_L$, and this difference will be higher when the salience effects in two states differ more significantly. Though recommending product 2 can generate higher current period profit, it also implies lower probability of transitioning to state H to generate future profit Π_H. Thus when the future is not heavily discounted and when the salience effects of the two states differ significantly, it may be beneficial to recommend the high relevance product to get a higher chance to go back to state H for higher future profits, even though doing so may reduce the current period profit.

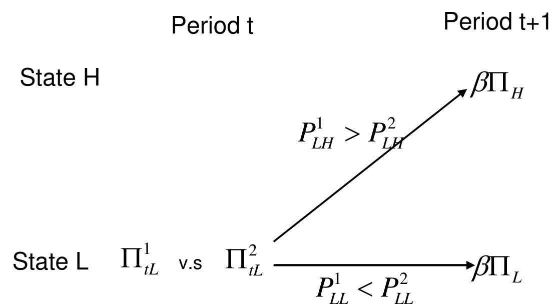

Figure 3: Tradeoff Between Reputation and Profitability Concerns

We now formally establish the result suggested by the above line of reasoning. Making pair-wise comparison between the profits under $R3$ and $R4$:

$$\begin{aligned} & sign[\Pi^{R3} - \Pi^{R4}] = sign[\\ & M_1((e^{V_1} + e^{V_2+\delta_H} + 1)(e^{V_1+V_2+2\delta_L} - e^{V_1+V_2} + e^{V_1+\delta_L} - e^{V_1}) + \beta e^{V_1+V_2+\delta_L}(e^{V_1} + 1)(e^{\delta_L} - e^{\delta_H}) + \beta e^{V_1+V_2}(e^{V_2} + 1)(e^{\delta_H} - e^{\delta_L})) \\ & - M_2((e^{V_1} + e^{V_2+\delta_H} + 1)(e^{V_1+V_2+2\delta_L} - e^{V_1+V_2} + e^{V_2+\delta_L} - e^{V_2}) + \beta e^{V_1+V_2+\delta_L}(e^{V_1} + 1)(e^{\delta_L} - e^{\delta_H}) + \beta e^{2V_2}(e^{V_1} + 1)(e^{\delta_H} - e^{\delta_L}))] \end{aligned} \tag{13}$$

The coefficient of M_2 in expression (13) can be expressed in the following way:

$$\gamma = e^{2V_1}(e^{V_2}(e^{2\delta_L} - 1) + \beta e^{V_2+\delta_L}(e^{\delta_L} - e^{\delta_H})) + o(e^{2V_1}) \tag{14}$$

As we have shown above, when M_2 is sufficiently high relative to M_1, either $R3$ or $R4$ is the optimal policy. In this situation, the difference in profit of these two policies is determined by the sign of coefficient of M_2, i.e. that of γ. This leads us to the following result:

Proposition 3: Suppose V_1 is sufficiently large compared with V_2 and M_2 is sufficiently high compared with M_1. If $e^{\delta_H} > \frac{1+\beta}{\beta} e^{\delta_L} - \frac{1}{\beta e^{\delta_L}}$, it is optimal to recommend the high relevance product, product 1, in state L. If $e^{\delta_H} < \frac{1+\beta}{\beta} e^{\delta_L} - \frac{1}{\beta e^{\delta_L}}$, then it is optimal to recommend the high margin product, product 2, in state L.

The difference between the two states can be highlighted when we contrast Proposition 1 with Proposition 3. Although the recommendation policies need to take both product margins and relevance into account in both states, in state H it does so mainly to increase the expected profit in that state, i.e., the near term profit. Therefore, as the product margin increases, the higher margin alone will eventually be sufficient to justify recommending the high margin product in state H despite its low relevance. In state L, however, the policy needs to address not only the profit in that state, but also the restoration of the reputation. If the high margin product is recommended, as its product margin increases, it increases the expect profit in that period. But no matter how high the margin is, recommending it does not increase the purchase probability, and thus does not help resolve the low salience of the recommender due to its low reputation. When there are significant differences in the recommender's salience between the two states, it is more beneficial to maximize the chance of returning to state H in the next period than to maximize the current profit. Thus the high relevance product should be recommended as highlighted in Proposition 3.

Additional findings for state L are stated in the proposition below, the proof of which is in the Appendix.

> *Proposition 4: When the margins of the two products are sufficiently close, it is optimal to recommend the high relevance product, product 1, in state L. When the margin of the high margin product is sufficiently high, as $\beta \to 0$, or as $V_2 \to V_1$, or as $\delta_L \to \delta_H$, it becomes optimal to recommend product 2.*

These results are not hard to understand. When the difference in margin is small compared with difference in relevance, recommending the high relevance product not only increases the current period profit, but also increases the chance of returning to state H. When M_2 is sufficiently high, recommending it increases the current period profit, so it is optimal to recommend it if the future is heavily discounted. When the salience effect in state L is close to that in state H, then the concern of returning to state H is not as overwhelming, so again maximizing current period profit is more important and product 2 should be recommended when M_2 is sufficiently high.

In summary, the optimal policy calls for *harvesting* in state H by recommending the high margin products. The policy focuses on short-term gains rather than long-term implications for the recommender's reputation. In contrast, the policy in state L could call for recommending the high relevance product to restore the reputation.

Welfare:

We next study the welfare implications of recommendation policies. We analyze both consumer welfare and social welfare which is the sum of consumer welfare and firm profit. To measure consumer welfare, we first assume that the consumer has the same discount factor β as the firm. We use only the expected product relevance, V_1 and V_2, to compute welfare, while ignoring the effect of salience and random shock. Recollect that the expected value of the random shock is zero. In addition, we treat the salience effect as a factor that only temporarily alters purchase probability but does not change the eventual utility derived from the product. Similar to expression (3), the consumer welfare can thus be expressed as:

$$CW(R) = E[\sum_{t=1}^{\infty} \beta^t V_{c_t(R)}] \tag{15}$$

Calculation of social welfare is also similar, where the sum of profit margin and relevance is used.

From a consumer's perspective, welfare is enhanced when her probability of purchasing the high relevance product is increased. Therefore, the consumer-welfare-maximizing recommendation policy will be the policy which always recommends the product with the highest relevance, $R1$ in this two-product scenario. Any deviation from this policy would increase the chance of purchasing the low relevance product and reduce consumer welfare.

The impact on social welfare is not as clear, since a recommendation policy may increase firm profit but simultaneously reduce consumer welfare. When the optimal policy for profit maximization also recommends high relevance product, there is no tradeoff and the social welfare would be higher. Instead, we focus on the cases where the profit maximizing policy calls for recommending the high margin product. As discussed above, this is more likely when M_2 is large relative to M_1.

When M_2 is large, similar to the discussion of firm profit, we perform pair-wise comparison of social welfare between $R1$ and $R3$. A result similar to expression (11) can be derived:

$$\begin{aligned} & sign[SW^{R3} - SW^{R1}] = sign[\\ & (V_2 + M_2)((e^{V_1+\delta_L} + e^{V_2} + 1)(e^{V_1+V_2+2\delta_H} + e^{V_2+\delta_H} - e^{V_1+V_2} - e^{V_2}) + \beta(e^{V_1} + 1)e^{V_1+V_2}(e^{\delta_H} - e^{\delta_L}) + \beta(e^{V_2} + 1)e^{V_1+V_2+\delta_H}(e^{\delta_L} - e^{\delta_H})) \\ & - (V_1 + M_1)((e^{V_1+\delta_L} + e^{V_2} + 1)(e^{V_1+V_2+2\delta_H} + e^{V_1+\delta_H} - e^{V_1+V_2} - e^{V_1}) + \beta(e^{V_2} + 1)(e^{V_1+V_2+\delta_H+\delta_L} - e^{2V_1+\delta_L} + e^{2V_1+\delta_H} - e^{V_1+V_2+2\delta_H}))] \end{aligned} \tag{16}$$

Recommending the high margin product may increase firm profit while reducing social welfare. To see this, denote the coefficient of $V_2 + M_2$ in the expression (16) as α_2 and that of $V_1 + M_1$ as α_1. To have the social welfare under $R3$ to be higher than under $R1$, M_2 needs to be higher than a threshold value:

$$\tilde{M}_2 = M_1 \frac{\alpha_1}{\alpha_2} + V_1 \frac{\alpha_1}{\alpha_2} - V_2 \tag{17}$$

According to expression (11), the threshold value above which firm profit is higher under $R3$ is:

$$\hat{M}_2 = M_1 \frac{\alpha_1}{\alpha_2} \tag{18}$$

It is easy to show that $\alpha_1 > \alpha_2 > 0$. And since $V_1 > V_2$, it follows that $\tilde{M}_2 > \hat{M}_2$. Thus when $\hat{M}_2 < M_2 < \tilde{M}_2$, firm profit is higher under $R3$ but social welfare is lower. If we hold other factors constant and increase M_2, then after a certain threshold value it is profit-enhancing to recommend it. At that threshold value, a marginal increase of M_2 has no first-order effect on firm profit, but recommending it reduces consumer welfare. Thus the social welfare will be lower. The above findings are summarized in the proposition below:

> *Proposition 5: When M_2 is sufficiently high, the profit maximizing policy that recommends the high margin product also increases social welfare, as firm profit gain outweighs consumer welfare loss. When M_2 is just above the profit-enhancing threshold, recommending the high margin product may increase firm profit but at the same time decrease social welfare.*

Figure 4 illustrates the difference in consumer welfare or social welfare between policies that recommend product 2 in a certain state and the policy that always recommends the high relevance product 1. The charts clearly show that recommending product 2 always reduces consumer welfare, while social welfare may either increase or decrease. As the margin of product 2 increases, social welfare implied by policies recommending product 2 also increases.

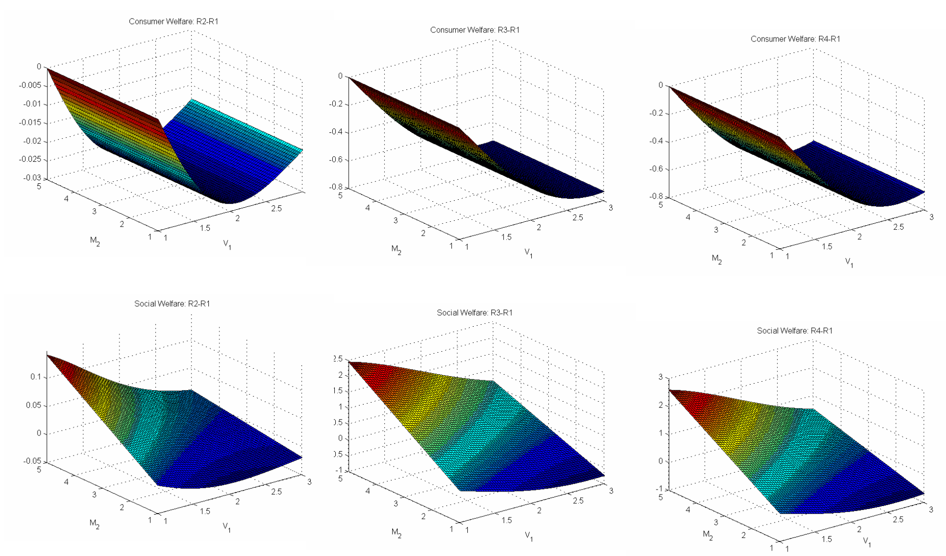

Figure 4: Consumer/Social Welfare Comparison. Ri v.s. R1

The *n*-Product Scenario

The two-product scenario highlights the tradeoff between profitability and reputation concerns in a clear way. When we turn to a general n-product setting, this tradeoff is not as easily illustrated. However, we note that the results are qualitatively similar in the n-product scenario as well. These are stated in the following two propositions, which can be proved in a similar fashion as in the two-product scenario:

Proposition 6: When there are $n \geq 3$ products and when consumer is in state H:

- *As the margin of one product, say product n, gets sufficiently high relative the other products, it becomes optimal to recommend that product.*
- *As $M_i \rightarrow M, \forall i$, so all products have almost the same margin, it is optimal to recommend the product with highest relevance.*
- *As $V_i \rightarrow V, \forall i$, so all products have almost the same relevance, it is optimal to recommend the product with the highest margin.*

Proposition 7: When there are $n \geq 3$ products and when consumer is in state L:

- *If one product, say product n, has sufficiently high margin M_n compared with other products, and another product, say product 1, has sufficiently high relevance compared with other products, then if $e^{\delta_H} > \frac{1+\beta}{\beta} e^{\delta_L} - \frac{1}{\beta e^{\delta_L}}$, it is not optimal to recommend product n, which has highest margin.*
- *As $M_i \rightarrow M, \forall i$, so all products have almost the same margin, it is optimal to recommend the product with the highest relevance.*
- *If one product, say product n, has sufficiently high margin M_n, then as $\beta \rightarrow 0$, or as $V_i \rightarrow V, \forall i$, or as $\delta_L \rightarrow \delta_H$, it is optimal to recommend item n.*

Proposition 6 is qualitatively similar to Propositions 1 and 2 and Proposition 7 is similar to Propositions 3 and 4. In this generalized setting, the recommender still needs to balance relevance and profit margin, both in terms of near-term profit and long-term reputation concerns. Because there are many products, an exact generalization of the result in proposition 3 is hard to characterize. Nonetheless, proposition 7 still shows that when future is not discounted too heavily and when the two states differ significantly, the optimal policy in state L should sacrifice immediate profits and not recommend the product with highest margin.

Conclusion and Future Research

Extant research on recommender systems focuses mainly on improving recommendation accuracy but little attention has been paid to the impact of firm incentives on the choice of recommender policies. This study attempts to fill this gap. Our study highlights the main tradeoffs a firm faces in designing a profit enhancing recommendation policy. The first tradeoff is between the relevance of products to a consumer and the firm's margins from selling the product. While recommending high relevance products increases consumer's purchase probability, recommending high margin products increases the profit per purchase. Our result shows that the optimal policy must always balance these two factors independent of the consumer's state. We also identify the conditions under which it is optimal to recommend the product with the highest margins and those under which to recommend the one with the highest relevance.

The second tradeoff the recommender system faces is between increasing near-term profit versus maintaining reputation to increase future profit. While recommending a high margin product could increase immediate profit, this is likely to affect the salience of future recommendations. Here, we find that the consumer's state impacts the optimal policy. When a consumer trusts recommendations, the policy harvests that by focusing on near-term profits over long-term reputation. In contrast, when the recommender enjoys low reputation then the policy trades off both factors. In this scenario, our results also identify the conditions under which it is optimal to recommend high relevance products to regain reputation, even if doing so will reduce near-term profit. This reputation concern is pronounced when future is not heavily discounted and when the difference between high and low reputation is significant.

By incorporating profits into the firm's objective function, our work is a notable departure from most of the existing work which focus only on predictive accuracy. One of our key insights, that the firm should harvest for profitability when the system has high reputation, and restore reputation when the reputation is low, has significant managerial implications for firms that design and deploy such systems.

Designing recommender systems to maximize profitability is as challenging as designing one to maximize accuracy, and our work is a first step in this direction. Though the findings of this study apply to a wide range of situations, much work needs to be done. In a real world situation, the changes in state (i.e. changes in consumers' trust of recommenders) may not happen with certainty, and the firm may not know for sure the reputation of its recommendation system to a given consumer. A more sophisticated model, involving Bayesian inference, can be used to derive the optimal policy in the presence of such uncertainties. In addition, when consumers face high product search costs, the awareness effect of recommender systems becomes important. An enhanced model which takes both salience and awareness effects can further generalize our results. While analytical study of more sophisticated models can offer significant insight by revealing the underlying drivers of the issue, certain research questions on this topic also call for an empirical approach. With individual level purchase data combined with recommendation history, for example, we could estimate the magnitude of the salience effect in different states, as well as consumer's actual state-transition probabilities. With well designed experiment, we can also evaluate the performance in terms of profitability of alternative recommendation methods.

Understanding the profit implications of recommender systems is important to both firms and consumers. For firms, it informs design choice. For consumers, it helps them better understand the firm's incentive in recommending different products in different situations. For the social scientist, it helps us understand the social implications of potential misalignment in the incentives of firms and consumers. Additional research in this area will prove useful.

Appendix

Detailed Proof of Propositions 1 and 2: Based on expression (10) in the main text, we have the following:

$$
\begin{aligned}
& sign[\Pi^{R3} - \Pi^{R1}] \\
&= sign[\pi_{tH}^{R3}(1-\beta P_{LL}^{R3})(1+\beta-\beta(P_{HH}^{R1}+P_{LL}^{R1})) - \pi_{tH}^{R1}(1-\beta P_{LL}^{R1})(1+\beta-\beta(P_{HH}^{R3}+P_{LL}^{R3})) + \\
&\beta(\pi_{tL}^{R3}(1-P_{HH}^{R3})(1+\beta-\beta(P_{HH}^{R1}+P_{LL}^{R1})) - \pi_{tL}^{R1}(1-P_{HH}^{R1})(1+\beta-\beta(P_{HH}^{R3}+P_{LL}^{R3})))] \\
&= sign[(1-\beta P_{LL}^{R1})((\pi_{tH}^{R3}-\pi_{tH}^{R1})(1+\beta(1-P_{LL}^{R1})) - \beta(P_{HH}^{R1}\pi_{tH}^{R3} - P_{HH}^{R3}\pi_{tH}^{R1}) + \beta\pi_{tL}^{R1}(P_{HH}^{R1}-P_{HH}^{R3}))] \\
&= sign[(\pi_{tH}^{R3}-\pi_{tH}^{R1})(1+\beta(1-P_{LL}^{R1})) - \beta(P_{HH}^{R1}\pi_{tH}^{R3} - P_{HH}^{R3}\pi_{tH}^{R1}) + \beta\pi_{tL}^{R1}(P_{HH}^{R1}-P_{HH}^{R3})] \\
&= sign[(\frac{e^{V_1}M_1+e^{V_2+\delta_H}M_2}{e^{V_1}+e^{V_2+\delta_H}+1} - \frac{e^{V_1+\delta_H}M_1+e^{V_2}M_2}{e^{V_1+\delta_H}+e^{V_2}+1})(1+\beta(1-\frac{e^{V_2}+1}{e^{V_1+\delta_L}+e^{V_2}+1})) - \\
&\beta(\frac{e^{V_1}M_1+e^{V_2+\delta_H}M_2}{e^{V_1}+e^{V_2+\delta_H}+1}\frac{e^{V_1+\delta_H}}{e^{V_1+\delta_H}+e^{V_2}+1} - \frac{e^{V_1+\delta_H}M_1+e^{V_2}M_2}{e^{V_1+\delta_H}+e^{V_2}+1}\frac{e^{V_2+\delta_H}}{e^{V_1}+e^{V_2+\delta_H}+1}) + \\
&\beta\frac{e^{V_1+\delta_L}M_1+e^{V_2}M_2}{e^{V_1+\delta_L}+e^{V_2}+1}(\frac{e^{V_1+\delta_H}}{e^{V_1+\delta_H}+e^{V_2}+1} - \frac{e^{V_2+\delta_H}}{e^{V_1}+e^{V_2+\delta_H}+1})]
\end{aligned}
$$

And similarly

$$
\begin{aligned}
& sign[\Pi^{R4} - \Pi^{R2}] \\
&= sign[(\frac{e^{V_1}M_1+e^{V_2+\delta_H}M_2}{e^{V_1}+e^{V_2+\delta_H}+1} - \frac{e^{V_1+\delta_H}M_1+e^{V_2}M_2}{e^{V_1+\delta_H}+e^{V_2}+1})(1+\beta(1-\frac{e^{V_1}+1}{e^{V_1}+e^{V_2+\delta_L}+1})) - \\
&\beta(\frac{e^{V_1}M_1+e^{V_2+\delta_H}M_2}{e^{V_1}+e^{V_2+\delta_H}+1}\frac{e^{V_1+\delta_H}}{e^{V_1+\delta_H}+e^{V_2}+1} - \frac{e^{V_1+\delta_H}M_1+e^{V_2}M_2}{e^{V_1+\delta_H}+e^{V_2}+1}\frac{e^{V_2+\delta_H}}{e^{V_1}+e^{V_2+\delta_H}+1}) + \\
&\beta\frac{e^{V_1}M_1+e^{V_2+\delta_L}M_2}{e^{V_1}+e^{V_2+\delta_L}+1}(\frac{e^{V_1+\delta_H}}{e^{V_1+\delta_H}+e^{V_2}+1} - \frac{e^{V_2+\delta_H}}{e^{V_1}+e^{V_2+\delta_H}+1})]
\end{aligned}
$$

Simplifying these two leads to expressions (11) and (12) in the main text. The rest of the proof of proposition 1 is provided in the main text.

From expressions (11) and (12), as $M_2 \to M_1$:

$$sign[\Pi^{R3} - \Pi^{R1}] = sign[M_1((e^{V_1+\delta_L} + e^{V_2} + 1)(e^{\delta_H} - 1)(e^{V_2} - e^{V_1}) + \beta e^{V_1}(e^{\delta_H} - e^{\delta_L})(e^{V_2} - e^{V_1})]$$

$$sign[\Pi^{R4} - \Pi^{R2}] = sign[M_1((e^{V_1} + e^{V_2+\delta_L} + 1)(e^{\delta_H} - 1)(e^{V_2} - e^{V_1}) + \beta e^{V_2}(e^{\delta_H} - e^{\delta_L})(e^{V_2} - e^{V_1})]$$

Since $V_2 < V_1$, we know $R3$ underperforms $R1$ and $R4$ underperforms $R2$ as $M_2 \to M_1$. Since both $R3$ and $R4$ recommend product 2 in state H, we know recommending product 2 will not be optimal. Thus it is optimal to recommend product 1.

Also, from expressions (11) and (12), as $V_2 \to V_1$:

$$sign[\Pi^{R3} - \Pi^{R1}]$$
$$= sign[(M_2 - M_1)((e^{V_1+\delta_L} + e^{V_1} + 1)(e^{2V_1+2\delta_H} + e^{V_1+\delta_H} - e^{2V_1} - e^{V_1}) + \beta(e^{V_1} + 1)(e^{2V_1+\delta_H+\delta_L} - e^{2V_1+\delta_L} + e^{2V_1+\delta_H} - e^{2V_1+2\delta_H}))]$$

$$sign[\Pi^{R4} - \Pi^{R2}]$$
$$= sign[(M_2 - M_1)((e^{V_1} + e^{V_1+\delta_L} + 1)(e^{2V_1+2\delta_H} + e^{V_1+\delta_H} - e^{2V_1} - e^{V_1}) + \beta e^{2V_1}(e^{\delta_L} - e^{\delta_H})(e^{V_1} + 1)(e^{\delta_H} - 1))]$$

Both of the above are expressions are positive, thus $R3$ outperforms $R1$ and $R4$ outperforms $R2$ as $V_2 \to V_1$. Similar to the case of $M_2 \to M_1$, we can conclude that it is optimal to recommend product 2 in this situation.

This completes the proof of proposition 1 and proposition 2.

Detailed Proof of Propositions 3 and 4: For $R3$ and $R4$, we know that $\pi_{tH}^{R3} = \pi_{tH}^{R4}$ and $P_{HH}^{R3} = P_{HH}^{R4}$. Thus we can simplify expression (10) to:

$$sign[\Pi^{R3} - \Pi^{R4}]$$
$$= sign[\beta(1 - \pi_{tH}^{R3})(\beta\pi_{tH}^{R3}(P_{LL}^{R4} - P_{LL}^{R3}) + (1 + \beta(1 - P_{HH}^{R3}))(\pi_{tL}^{R3} - \pi_{tH}^{R4}) - \beta(\pi_{tL}^{R3}P_{LL}^{R4} - \pi_{tL}^{R4}P_{LL}^{R3})]$$
$$= sign[\beta\frac{e^{V_1}M_1 + e^{V_2+\delta_H}M_2}{e^{V_1} + e^{V_2+\delta_H} + 1}(\frac{e^{V_1} + 1}{e^{V_1} + e^{V_2+\delta_L} + 1} - \frac{e^{V_2} + 1}{e^{V_1+\delta_L} + e^{V_2} + 1}) +$$
$$(1 + \beta\frac{e^{V_1} + 1}{e^{V_1} + e^{V_2+\delta_H} + 1})(\frac{e^{V_1+\delta_L}M_1 + e^{V_2}M_2}{e^{V_1+\delta_L} + e^{V_2} + 1} - \frac{e^{V_1}M_1 + e^{V_2+\delta_L}M_2}{e^{V_1} + e^{V_2+\delta_L} + 1})$$
$$- \beta(\frac{e^{V_1+\delta_L}M_1 + e^{V_2}M_2}{e^{V_1+\delta_L} + e^{V_2} + 1}\frac{e^{V_1} + 1}{e^{V_1} + e^{V_2+\delta_L} + 1} - \frac{e^{V_1}M_1 + e^{V_2+\delta_L}M_2}{e^{V_1} + e^{V_2+\delta_L} + 1}\frac{e^{V_2} + 1}{e^{V_1+\delta_L} + e^{V_2} + 1})]$$
$$= sign[M_1((e^{V_1} + e^{V_2+\delta_H} + 1)(e^{V_1+V_2+2\delta_L} - e^{V_1+V_2} + e^{V_1+\delta_L} - e^{V_1}) + \beta e^{V_1+V_2+\delta_L}(e^{V_1} + 1)(e^{\delta_L} - e^{\delta_H}) + \beta e^{V_1+V_2}(e^{V_2} + 1)(e^{\delta_H} - e^{\delta_L}))$$
$$- M_2((e^{V_1} + e^{V_2+\delta_H} + 1)(e^{V_1+V_2+2\delta_L} - e^{V_1+V_2} + e^{V_2+\delta_L} - e^{V_2}) + \beta e^{V_1+V_2+\delta_L}(e^{V_1} + 1)(e^{\delta_L} - e^{\delta_H}) + \beta e^{2V_2}(e^{V_1} + 1)(e^{\delta_H} - e^{\delta_L}))]$$

This is expression (13) in the main text. The rest of proof of proposition 3 is provided in the main text.

From expression (14), as $\beta \to 0$:

$$\gamma = e^{V_2}(e^{V_1} + e^{V_2+\delta_H} + 1)(e^{\delta_L} - 1) + \beta e^{2V_2}(e^{V_1} + 1)(e^{\delta_H} - e^{\delta_L}) +$$
$$e^{V_1+V_2}(e^{V_1} + e^{V_2+\delta_H} + 1)(e^{2\delta_L} - 1) + \beta e^{V_1+V_2+\delta_L}(e^{V_1} + 1)(e^{\delta_L} - e^{\delta_H})$$
$$\to e^{V_2}(e^{V_1} + e^{V_2+\delta_H} + 1)(e^{\delta_L} - 1) + e^{V_1+V_2}(e^{V_1} + e^{V_2+\delta_H} + 1)(e^{2\delta_L} - 1) > 0$$

Thus $R3$ underperforms $R4$ and it is optimal to recommend product 2 in state L.

Also from expression (14), as $V_2 \to V_1$:

$$\gamma \to (e^{V_1} + e^{V_1+\delta_H} + 1)(e^{2V_1+2\delta_L} - e^{2V_1} + e^{V_1+\delta_L} - e^{V_1}) + \beta e^{2V_1+\delta_L}(e^{V_1} + 1)(e^{\delta_L} - e^{\delta_H}) + \beta e^{2V_1}(e^{V_1} + 1)(e^{\delta_H} - e^{\delta_L}))$$
$$> \beta e^{V_1}(e^{2V_1+\delta_H+2\delta_L} - e^{2V_1+\delta_H+\delta_L}) > 0$$

Therefore, $R3$ underperforms $R4$ in this case and it is optimal to recommend product 2 in state L.

Finally, as $\delta_L \to \delta_H$, $\gamma \to e^{V_2}(e^{V_1} + e^{V_2+\delta_H} + 1)(e^{\delta_H} - 1) + e^{V_1+V_2}(e^{V_1} + e^{V_2+\delta_H} + 1)(e^{2\delta_H} - 1) > 0$

Therefore, $R3$ also underperforms $R4$ in this case and it is optimal to recommend product 2 in state L.

This completes the proof of propositions 3 and 4.

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