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# A Solution Algorithm for the Supply Chain Network Equilibrium Model

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## Abstract

*A supply chain network equilibrium problem was recently formulated using the variational inequality approach and solved with the modified projection method. Assuming the vector function  $F$  that enters the variational inequality is strictly monotone and Lipschitz continuous (and that a solution exists). The realization of the modified projection algorithm for the supply chain network equilibrium model is guaranteed to converge; however, its algorithmic steps are not so interesting to the transportation community as the concept of traffic assignment is not used. In this paper, a new solution algorithm that iteratively solves a fixed-demand traffic assignment problem and makes comparisons among intermediate solutions is proposed. The proposed algorithm is well explained by a supernetwork representation, and in addition, loosely justified by a numerical example. The proposed solution algorithm can be extended to solve the time-dependent/dynamic supply chain network equilibrium model without difficulty.*

**Keywords:** supply chain network equilibrium, variational inequality, traffic assignment

## 1. Introduction

The supply chain network (SCN) equilibrium problem was recently formulated using the variational inequality approach (Nagurney et al., 2002, 2003). Such a mathematical model was first appeared in the literature and can be analytically solved. Despite its current “abstract” form, the SCN equilibrium model has great potential for further development. Interesting research direction includes dynamic/time-dependent extension, treatment of stochastic/fuzzy input data, incorporation of telecommunicating links and among others. In this paper, we only focus on the algorithmic issue.

For solving the SCN equilibrium model that uses the variational inequality (VI) approach, Nagurney et al. (2002) proposed the modified projection method. Under the assumption of the vector function  $F$  that enters the VI is strictly monotone and Lipschitz continuous (and that a solution exists), the realization of the modified projection algorithm for the SCN equilibrium model is guaranteed to converge. Indeed, the modified projection algorithm is not so intriguing to the transportation community as the commonly known traffic assignment concept is not adopted within the algorithmic procedure. In the following, a new solution algorithm will be proposed and elaborated. We first begin with the introduction of a supply chain supernetwork, consisting three tiers of the manufacturer, the retailer and the demand markets in Section 2. The proposed algorithm is described in Section 3 and validated with a numerical example in Section 4. Finally few remarks are given in Section 5.

## 2. The Supply Chain Network Equilibrium Problem

The SCN network represents economic sectors in tiers, each tier/sector contains a number of its members, and each member is denoted by a node. The nodes between the consecutive tiers are connected by links in which the products can flow from one sector to its successive sector. In Figure 1, three sectors: manufacturer, retailer and demand market are shown. The products are first produced by the  $m$  competitive manufactures (indexed by  $i$ ) and then shipped to the  $n$  retailers (indexed by  $j$ ) and finally, cleared at the  $o$  demand markets (indexed by  $k$ ). The quantities of the product produced, shipped, displayed, stored and transacted in the tiers are either explicitly or implicitly represented as the functions of the product price in the demand markets.

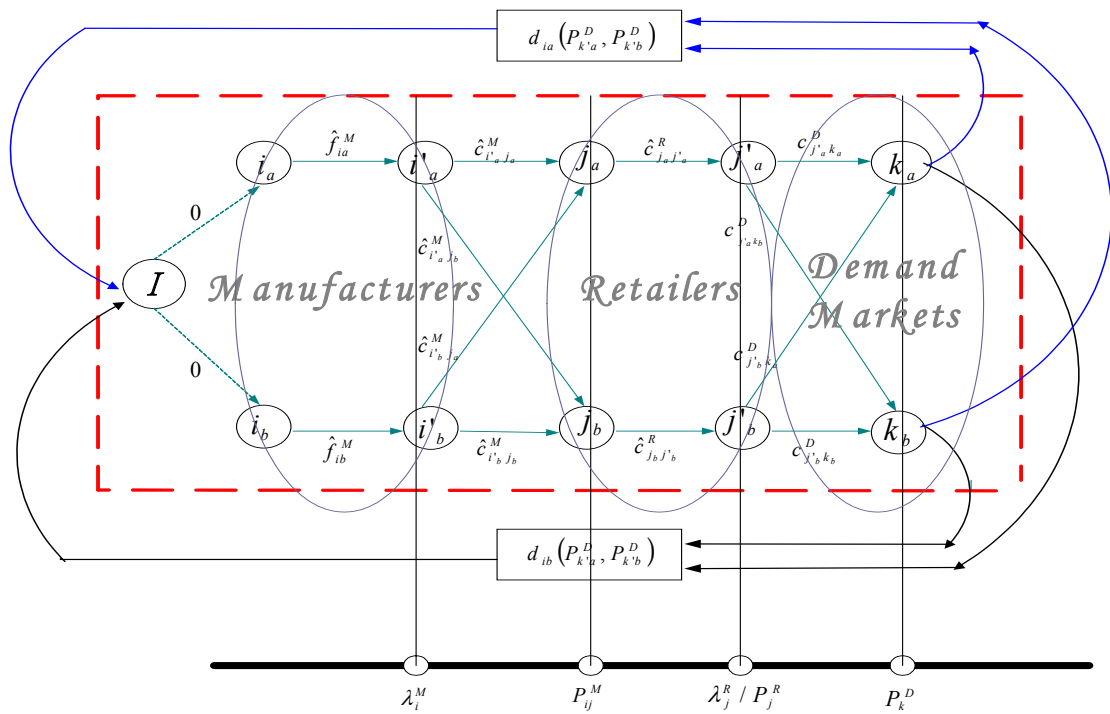


Figure 1: Supply Chain Network Representation

Let  $Q^1$ ,  $q_{ij}$  and  $Q^2$  denote product flows between manufacturer and retailer sector, the amount of the product transacted between manufacturer  $i$  and retailer  $j$ , product flows between retailers and demand markets. In each tier, cost incurred and revenue generated is different. A manufacturer is associated with the marginal production and marginal transaction costs and with the sales revenue. A retailer is associated with the marginal purchase and handling cost (such as display and storage cost) and with the sales revenue. A demand market is associated with the retailer's supply cost and the transaction cost in the market.

### 2.1 Equilibrium conditions

The equilibrium conditions for the SCN model state that product manufactured must be shipped to the successive retailer tier and then cleared in the demand markets. In equilibrium, all shipments between the tiers of network agencies will have to coincide. For the easier reference, the equilibrium conditions and the VI model for the SCN equilibrium problem that

appeared in Nagurney et al. (2002) is restated without detailed explanation.

### 2.1.1 Equilibrium conditions for the manufacturers

We assume that each manufacturer  $i$  is faced with a production cost  $f_i^{M*}(Q^{1*})$  and associated with a transaction cost  $c_{ij}^{M*}(q_{ij}^*)$  between manufacturer  $i$  and retailer  $j$ . Manufacturers obtain a price for the product (which is endogenous) and seek to determine their optimal production and shipment quantities, given the production costs as well as the transaction costs associated with conducting business with the different retailers. The equilibrium conditions for the manufacturers are stated as follows:

$$\hat{f}_i^{M*}(Q^{1*}) \begin{cases} = & \lambda_i^{M*}, q_i^* > 0 \\ \geq & \lambda_i^{M*}, q_i^* = 0 \end{cases} \quad \forall i \quad (1)$$

$$\lambda_i^{M*} + \hat{c}_{ij}^{M*}(q_{ij}^*) \begin{cases} = & P_{ij}^{M*}, q_{ij}^* > 0 \\ \geq & P_{ij}^{M*}, q_{ij}^* = 0 \end{cases} \quad \forall i, j \quad (2)$$

$$q_i^* = \sum_{j=1}^n q_{ij}^* \quad \forall i \quad (3)$$

Eqn (1) states that if a manufacturer  $i$  produces a positive amount of the product  $q_i^*$ , then the manufacturer's marginal production cost  $\hat{f}_i^{M*}(Q^{1*})$  should be equal to the minimum supply cost  $\lambda_i^{M*}$  (which is essentially the dual variable associated with the constraint 3 and is hence unrestricted in sign). Otherwise, the manufacturer's marginal production cost should be greater than or equal to the minimum supply cost. Eqn (2) indicates that if a manufacturer  $i$  ships to a retailer  $j$  a positive amount of the product  $q_{ij}^*$  (and the flow on that corresponding link will be positive) then the manufacturer's minimum supply cost  $\lambda_i^{M*}$  plus marginal transaction cost  $\hat{c}_{ij}^{M*}(q_{ij}^*)$  associated with that wholesaler is equal to the price  $P_{ij}^{M*}$  that the manufacturer  $i$  charges (and a wholesaler  $j$  is willing to pay) for the product. Otherwise, the manufacturer's minimum supply cost plus marginal transaction cost is greater than or equal to the price for the product. Eqn (3) expresses that the product  $q_i^*$  produced in the manufacturer  $i$  is equal to the sum of the quantities  $\sum_{j=1}^n q_{ij}^*$ .

### 2.1.2 Equilibrium Conditions for the Retailers

Retailers must agree with the manufacturers as to the volume of shipments since they are faced with the handling cost associated with having the product in their retail outlet. In addition, they seek to maximize their profits with the price that the retailers are willing to pay for the product being exogenous. The equilibrium conditions for the retailers are stated as follows:

$$P_{ij}^{M*} + \hat{c}_j^{R*}(Q^{1*}) \begin{cases} = & \lambda_j^{R*}, q_{ij}^* > 0 \\ \geq & \lambda_j^{R*}, q_{ij}^* = 0 \end{cases} \quad \forall i, j \quad (4)$$

$$\lambda_j^{R*} \begin{cases} = & P_j^{R*} & , q_{jk}^* > 0 \\ \geq & P_j^{R*} & , q_{jk}^* = 0 \end{cases} \quad \forall j, k \quad (5)$$

$$\sum_{i=1}^m q_{ij}^* = \sum_{k=1}^o q_{jk}^* \quad \forall j \quad (6)$$

Eqn (4) states that a manufacturer  $i$  ships to a retailer  $j$  a positive amount of product  $q_{ij}^*$  (and the flow on that corresponding link will be positive) then the price that the manufacturer charges  $P_{ij}^{M*}$  (and a retailer  $j$  is willing to pay) plus minimum marginal transaction cost  $\hat{c}_j^{R*}(Q^{1*})$  associated with that retailer is equal to the retailer's supply cost  $\lambda_j^{R*}$  (which is essentially the dual variable associated with the constraint 6 and is hence unrestricted in sign) for the product. Otherwise, the price that the manufacturer charges plus the retailer's minimum marginal handling cost is larger than or equal to the retailer's supply cost for the product. Eqn (5) states that if a retailer  $j$  ships to demand market  $k$  a positive amount of the product  $q_{jk}^*$ , then the retailer supply cost  $\lambda_j^{R*}$  is equal to the price  $P_j^{R*}$  that the retailer charges. Otherwise, the retailer's supply cost is larger than or equal to the price. Eqn (6) expresses that for a retailer  $j$  the amount of the product purchased from manufacturer sector  $\sum_{i=1}^m q_{ij}^*$  is equal to the sum of the quantities  $\sum_{k=1}^o q_{jk}^*$  shipped to all retailers.

### 2.1.3 Equilibrium Conditions for Customers

Consumers determine their optimal consumption levels from the various retailers subject both to the prices charges for the product as well as the cost for conducting the transaction (which, of course, may include the cost of transportation associated with obtaining the product from the retailer). The equilibrium conditions for customers can be stated as follows:

$$P_j^{R*} + c_{jk}^{D*}(Q^{2*}) \begin{cases} = & P_k^{D*} & , q_{jk}^* > 0 \\ \geq & P_k^{D*} & , q_{jk}^* = 0 \end{cases} \quad \forall j, k \quad (7)$$

$$d_k(\mathbf{P}^{D*}) \begin{cases} = & \sum_{j=1}^n q_{jk}^* & , P_k^{D*} > 0 \\ \leq & \sum_{j=1}^n q_{jk}^* & , P_k^{D*} = 0 \end{cases} \quad \forall k \quad (8)$$

Eqn (7) states that if the customers at demand market  $k$  purchase from retailer  $j$  a positive amount of the product  $q_{jk}^*$ , then the price  $P_j^{R*}$  charged by the retailer for the product plus the transaction cost  $c_{jk}^{D*}(Q^{2*})$  is equal to the price  $P_k^{D*}$  that the consumers are willing to pay for the product. Otherwise, the price charged by the retailer for the product plus the transaction cost is equal to the price of the product at the demand market. Eqn (8) indicates that if the equilibrium price  $P_k^{D*}$  that the consumers are willing to pay for the product at the demand market  $k$  is positive, then the quantities  $\sum_{j=1}^n q_{jk}^*$  purchased of the product from the retailers will be precisely equal to the demand  $d_k(\mathbf{P}^{D*})$  for that product at the demand market. Otherwise, the demand is less than or equal to the total amount of commodities

available at the demand market. These conditions correspond to the well-known spatial price equilibrium conditions.

## 2.2 Model Formulation

Note that the SCN equilibrium problem embeds asymmetric link interactions, which may be exemplified by the complete competition of the manufacturers with different sizes. Therefore, it is appropriate to formulate the SCN equilibrium problem as a VI model. For a description of the VI model, see Nagurney (1993) and Chen (1999). The following theorem defines the SCN equilibrium problem.

**Theorem 1:** The SCN equilibrium problem is equivalent to finding an equilibrium solution

$X^* = \{Q^{1*}, Q^{2*}, \lambda^*, P_k^{D*}\} \in \Omega$  to the following VI model.

$$\begin{aligned}
& \sum_{i=1}^m [\hat{f}_i^{M*}(Q^{1*})] \times [q_i - q_i^*] + \sum_{i=1}^m \sum_{j=1}^n [\hat{c}_{ij}^{M*}(q_{ij}^*) + \hat{f}_j^{R*}(Q^{1*}) - \lambda_j^{R*}] \times [q_{ij} - q_{ij}^*] \\
& + \sum_{j=1}^n \sum_{k=1}^o [c_{jk}^{R*}(Q^{2*}) + \lambda_j^{R*} - P_k^{D*}] \times [q_{jk} - q_{jk}^*] \\
& + \sum_{j=1}^n \left[ \sum_{i=1}^m q_{ij}^* - \sum_{k=1}^o q_{jk}^* \right] [\lambda_j^R - \lambda_j^{R*}] \quad \forall X \in \Omega \\
& + \sum_{k=1}^o \left[ \sum_{j=1}^n q_{jk}^* - d_k(P^{D*}) \right] \times [P_k^D - P_k^{D*}] \geq 0
\end{aligned} \tag{9}$$

where  $\Omega$  denotes the feasible region which is delineated by the constraints (10)~(16). Flow conservation constraints:

$$q_i = \sum_{j=1}^n q_{ij} \quad \forall i \tag{10}$$

$$\sum_{i=1}^m q_{ij} = \sum_{k=1}^o q_{jk} \quad \forall j \tag{11}$$

$$d_j(P^D) = \sum_{k=1}^o q_{jk} \quad \forall j \tag{12}$$

Nonnegative constraints:

$$q_i \geq 0 \quad \forall i \tag{13}$$

$$q_{ij} \geq 0 \quad \forall i, j \tag{14}$$

$$q_{jk} \geq 0 \quad \forall j, k \tag{15}$$

$$P_k^D > 0 \quad \forall k \tag{16}$$

Constraints (10)~(12) have been interpreted in Section 2.1. Constraints (13)~(16) require that productions, transactions and demand prices respectively be nonnegative. For the equivalency proof between the SCN model (9) and its corresponding equilibrium conditions (1)~(8), the

reader may refer to Nagurney et al. (2002) for details.

### 3. Solution Algorithm

We propose a two-loop solution algorithm in this section. The outer loop estimates the demands of the product and with this estimated demands, the inner loop performs a fixed demand traffic assignment. The product flows and prices that obtained are again used to update the product demands. This procedure is repeated until the convergence criterion is satisfied. The algorithmic steps are stated as follows:

**Step 0: Initialization.**

Step 0.1: Set initial product prices to zero,  $\{p_k^D\}^0 = \{0\}$  and compute their corresponding demands as the upper bound  $\{\bar{d}_k^{Max}(\mathbf{P}^{D*})\}$ . Also set zero as their lower bound  $\{\bar{d}_k^{Min}(\mathbf{P}^{D*}) = 0\}$  and the pre-specified increment  $\{\Delta_k\}^0$ . Set  $p=0$ .

**Step 1: Traffic Assignment.** (by the Frank-Wolfe method)

Step 1.0: Let  $\{\bar{d}_k(\mathbf{P}^{D*})\}^p = \{\bar{d}_k^{Max}(\mathbf{P}^{D*})\}$ . For each pair  $(I,k)$  (where  $I$  represents super-origin in the supernetwork), perform all-or-nothing assignment (the shortest path from super-origin  $I$  to demand market  $k$ ) based on  $\hat{f}_i^M(0)$ ,  $\hat{c}_{ij}^M(0)$ ,  $\hat{c}_j^R(0)$ , and  $c_{jk}^D(0)$ . This yields  $(Q^1)^0$ ,  $\{q_j\}^0$ ,  $(Q^2)^0$ , set  $l=0$ .

Step 1.1: Update. Set  $\hat{f}_i^M(Q^1)^l$ ,  $\hat{c}_{ij}^M(q_j)^l$ ,  $\hat{c}_j^R(Q^1)^l$ , and  $c_{jk}^D(Q^2)^l$ .

Step 1.2: Direction finding. Perform all-or-nothing assignment based on the updated information obtained in Step 1.1. This yields a set of auxiliary flows  $(G^1)^l$ ,  $\{g_j\}^l$ ,  $(G^2)^l$ . Set  $d_{ij}^l = \left( (g_{ij}^M)^l - (q_{ij}^M)^l, (g_{jk}^D)^l - (q_{jk}^D)^l \right)$ .

Step 1.3: Line search.

Solve the linear combination optimization problem (by substituting the values of the decision variables at iteration  $l$  by those at iteration  $l+1$ ) for step size  $\alpha$ :

Step 1.4: Update of product flows. Update the link flows by the following formulas:

$$(q_{ij}^M)^{l+1} = (q_{ij}^M)^l + \alpha d_{ij}^l \quad (17)$$

$$(q_{jk}^D)^{l+1} = (q_{jk}^D)^l + \alpha d_{jk}^l \quad (18)$$

$$(q_j^R)^{l+1} = \sum_{i=1}^m (q_{ij}^M)^{l+1} \quad \forall j \quad (19)$$

Step 1.5: Convergence check.

If the convergence criterion is satisfied, set  $\{P_k^D\}$  as the shortest path distance from super-origin  $I$  to demand market  $k$ , and continue. Otherwise, let  $l=l+1$ , go to step 1.1.

**Step 2: Convergence check.**

Compute  $\{d_k(\mathbf{P}^{D*})^{p+1}\}$ . If  $\max_k \left\{ \frac{|d_k(\mathbf{P}^{D*})^{p+1} - d_k(\mathbf{P}^{D*})^p|}{d_k(\mathbf{P}^{D*})^p} \right\} \leq \varepsilon$ , the convergence criterion

is satisfied, stop. Otherwise, do the following:

Step 2.1: Comparison of product demands.

Case 1: If  $d_k(\mathbf{P}^{D*})^p - d_k(\mathbf{P}^{D*})^{p+1} > \varepsilon$ , set  $\Delta_k^{p+1} = \Delta_k^p$ .

Case 2: If  $d_k(\mathbf{P}^{D*})^p - d_k(\mathbf{P}^{D*})^{p+1} < -\varepsilon$ , set  $\Delta_k^{p+1} = \Delta_k^p / 2$ .

Case 3: If  $|d_k(\mathbf{P}^{D*})^p - d_k(\mathbf{P}^{D*})^{p+1}| < \varepsilon$ , set  $\Delta_k^{p+1} = 0$

Step 2.2: Update of product demands.

Set  $d_k^{\max}(\mathbf{P}^{D*}) = d_k^{\max}(\mathbf{P}^{D*}) - \Delta_k^p$ . Let  $p=p+1$ , go to step 1.

#### 4. Numerical Example

To demonstrate, a numerical example modified from Nagurney et al. (2002) is solved for the SCN model. As shown in Figure 1, the test network consists three sectors, i.e., manufacturer, retailer and demand market respectively. Each sector contains two agencies. Cost functions associated with the three tiers of the test network are given in Table 1.

Table 1: Cost Functions Associated with the Three Tiers of the Test Network

Cost Function	Production/Display Cost	Supply Cost	Transaction Cost	Price
$M$	* $f_i^M(Q^1) = 2.5q_i^2 + q_1q_2 + 2q_i$	$\lambda_i^M$	* $c_{ij}^M(q_{ij}) = 0.5q_{ij}^2 + 3.5q_{ij}$	$P_{ij}^M$
$R$	* $f_j^R(q_j) = 0.5(q_j)^2$	$\lambda_j^R$	—	$P_j^R$
$D$	—	—	$c_{jk}^D(Q^2) = q_{jk} + 5$	$P_k^D$

Remarks: “M” denotes manufacturers, “R” retailers, and “D” demand markets; “\*” denotes derivative information is required.

The demands can be expressed as functions of the relevant prices. Two demand functions for two agencies, are assumed in Table 2.

Table 2: Demand Functions  $d_k(\mathbf{P}^D)$

$d_{.1}(\mathbf{P}^D) = -2P_1^D - 1.5P_2^D + 1000$
-----
$d_{.2}(\mathbf{P}^D) = -2P_2^D - 1.5P_1^D + 1000$

The test results of link costs and product flows are summarized in Tables 3 and 4.

Table 3: Link Costs

Cost Function	Production/Display Cost	Supply Cost	Transaction Cost	Price
$M$	$\hat{f}_1^M(Q^1) = 5q_1 + q_2 + 2 = 201.296$ $\hat{f}_2^M(Q^1) = 5q_2 + q_1 + 2 = 201.296$		* $\hat{c}_{ij}^M(q_{ij}) = q_{ij} + 3.5 = 20.108$	$P_{ij}^M = 221.404$
$R$	* $\hat{f}_j^R(q_j) = q_j = 33.216$	$\lambda_j^R$	—	$P_j^R = 254.616$
$D$	—	—	$c_{jk}^D(Q^2) = q_{jk} + 5 = 21.608$	$P_k^D = 276.224$

Remarks: “M” denotes manufacturers, “R” retailers, and “D” demand markets



Test 4: Product Flows

$Q^1 = \{q_{ij}\}$	$q_{11}$	16.608	$q_1 = 33.216$
	$q_{12}$	16.608	
	$q_{21}$	16.608	$q_2 = 33.216$
	$q_{22}$	16.608	
$Q^2 = \{q_{jk}\}$	$q_{11}$	16.608	$d_1 = 33.216$
	$q_{21}$	16.608	
	$q_{12}$	16.608	$d_2 = 33.216$
	$q_{22}$	16.608	
$d_1(\mathbf{P}^D) = -2P_1^D - 1.5P_2^D + 1000 = 33.216$			
$d_2(\mathbf{P}^D) = -2P_2^D - 1.5P_1^D + 1000 = 33.216$			

#### 4. Conclusion and Suggestion

This paper proposed a trial-and-error solution algorithm for solving the SCN model using the VI approach and validated with a numerical example that was taken from Nagurney et al' (2002). The proposed algorithm is very intriguing in that the well-known traffic assignment procedure can be iteratively applied to a supernetwork representation.

It is known that under the assumption of perfect competition, the product price will usually affect the amount of the manufacturers' productions, which will in turn affect the product price in the demand market. This recursive procedure is expected to approach equilibrium conditions. However, this is not the case with our numerical example and therefore the alternative solution algorithm that adopts the trial-and-error technique is employed. For insights, the characteristics of demand functions need to be elaborated in the future and in addition, the computational efficiency of the proposed algorithm be further tested/improved with larger and asymmetric networks.

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## Notations

$c_{ij}^M$  : transaction cost function for shipment from manufacturer  $i$  to retailer  $j$

$\hat{c}_{ij}^M$  : marginal transaction cost function for shipment from manufacturer  $i$  to retailer  $j$

$c_{jk}^R$  : transaction cost function for shipment from retailer  $j$  to demand market  $k$

$\hat{c}_{jk}^R$  : marginal transaction cost function for shipment from retailer  $j$  to demand market  $k$

$f_i^M$  : production cost function associated with manufacturer  $i$

$\hat{f}_i^M$  : marginal production cost associated with manufacturer  $i$ , i.e.,  $\frac{\partial f_i^M}{\partial q_i} = \hat{f}_i^M$

$\hat{f}_j^R$  : marginal display cost associated with retailer  $j$ , i.e.,  $\frac{\partial f_j^R}{\partial q_j} = \hat{f}_j^R$

$i$ : manufacturer designation,  $i \in I$ ; set  $I = \{1, 2, \dots, i, \dots, m\}$

$j$ : retailer designation,  $j \in J$ ; set  $J = \{1, 2, \dots, j, \dots, n\}$

$k$ : demand market designation,  $k \in K$ ; set  $K = \{1, 2, \dots, k, \dots, o\}$

$P_{ij}^{M*}$  : equilibrium price that the manufacturer  $i$  charges to the retailer  $j$

$P_{jk}^{R*}$  : equilibrium price that the retailer  $j$  charges to the demand market  $k$

$P_k^{D*}$  : equilibrium price in the demand market  $k$

$q_i$  : productions from manufacturer  $i$

$q_{ij}$  : the amount of transactions between manufacturer  $i$  and retailer  $j$

$q_{jk}$  : the amount of transactions between retailer  $j$  and demand market  $k$

$T$ : time period designation,  $t \in T$ ; set  $T = \{1, 2, \dots, t, \dots, T\}$

$\lambda_i^M$  : minimum supply cost for manufacturer  $i$

$\lambda_j^R$  : minimum supply cost for retailer  $j$

$\lambda_k^R$  : minimum supply cost for demand market  $k$