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Pacific Asia Conference on Information Systems (PACIS)

December 2001

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Abstract

This paper investigates the multiple attribute decision making problem with preference information on alternatives, in which multiple decision makers give their preference information in three forms, i.e., preference orderings, utility values and fuzzy preference relation. A new approach is presented to make use of both the decision makers' social fuzzy preference relation on alternatives and decision matrix to form an optimization model. The optimization model can be used to determine the attribute weights and rank the alternatives. The approach provides a new way to reflect the decision makers' social preference information and the decision matrix. Finally, an example is used to illustrate the proposed approach.

Keywords: Multiple attribute decision making, Preference ordering, Utility value, Fuzzy preference relation, Optimization model, Alternative ranking

1. Introduction

In multiple attribute decision making (MADM) problems, decision makers often need to select the most desirable alternative or rank the alternatives that are associated with noncommensurate and conflicting attributes (Hwang and Yoon, 1981). MADM problems arise in many real-word situations (Chen and Hwang, 1992; Hwang and Yoon, 1981; Cook and Kress, 1994; Ma et al., 1999; Malakooti and Zhou, 1994). For example, in production planning problems, attributes such as production rate, quality, and cost of operations are considered in selecting the satisfactory plan. Although lots of research on MADM problems have been done (Chen and Hwang, 1992; Hwang and Yoon, 1981), the area of MADM problems is still open for new challenges (Cook and Kress, 1994; Ma et al., 1999; Malakooti and Zhou, 1994). One of the hotter researches is the use of fuzzy set theory to solve MADM problems when imprecise information is represented in fuzzy terms (Chen and Hwang, 1992; Chiclana et al., 1998; Kacprzyk, 1986; Kacprzyk and Fedrizzi, 1990; Tanino, 1984, 1990).
In MADM problems, decision makers' preference information is often used to rank alternatives or to select the most desirable one. However, due to their different culture and education backgrounds, the decision makers' judgements vary in form and depth. A decision maker may express his/her preference on attributes or alternatives in specific style or may not indicate his/her preference at all. Different decision makers may use different ways to express their preference. The approaches to solving the MADM problems with preference information can be classified into two categories (Hwang and Yoon, 1981): (1) the

approaches with preference information on attributes (Carrizosa et al., 1995; Li, 1999; Ma et al., 1999; Marmol, 1998) and (2) the approaches with preference information on alternatives (Chiclana et al., 1996, 1998; Hwang and Yoon, 1981; Malakooti and Zhou, 1994; Tanino, 1984, 1990).

This paper focuses on the second category, where the decision makers are able to give their preference information on alternatives. In this paper, the preference information on alternatives employs three forms: preference orderings, utility values and fuzzy preference relation (Chiclana et al., 1998). Different forms of preference information on alternatives need to be uniformed. Fuzzy preference relation on alternatives is a choice for the uniform form (Chiclana et al., 1998; Delgado et al., 1998). Preference orderings of the alternatives are usually used by decision makers and transformed into fuzzy preference relations on the alternatives (Chiclana et al., 1996, 1998; Tanino, 1984). Also utility values of the alternatives are always converted into fuzzy preference relations for ranking the alternatives (Chiclana et al., 1998; Nakamura, 1986). Of course, there are other ways of handling these forms of preference information. For example, starting with the utility values of alternatives given by multiple decision makers, in Yen and Bui (1999), a formulized heuristic for consensus seeking is proposed, i.e., the negotiable alternative identifier is used to locate a candidate for compromise and then to search a collective alternative.

Given the individual fuzzy preference relations on the alternatives, two types of approaches, i.e., the direct approach and the indirect approach, can be used to select the most desirable alternative (Kacprzyk, 1986). In the direct approach, selecting the most desirable alternative is directly based on the individual fuzzy preference relations on the alternatives from the decision makers. In the indirect approach, the multiple individual fuzzy preference relations are firstly aggregated into a social fuzzy preference relation on the alternatives. Then the selection process is conducted based on the aggregation result. In order to make our proposed approach more applicable, the social fuzzy preference relation on the alternatives is used to assess the attribute weights and ranking of the alternatives is desirable. Therefore the indirect approach is considered in this paper.

In Chiclana et al. (1998), the three forms of preference information on alternatives are uniformed. Fuzzy majority method with fuzzy quantifier is used to aggregate the uniformed preference information and to select the most desirable alternative. However, the selection process is totally based on the decision makers' preference information on alternatives. In Miettinen and Salminen (1999), the weights of criteria are evaluated to exploit the outranking relations between the alternatives and to further make a certain alternative the best one. This paper presents a new approach to the MADM problem, where the decision makers can also give the three forms of preference information on alternatives. In the approach, the three forms of preference information on the alternatives are uniformed and aggregated into a social fuzzy preference relation. Based on the decision information of the alternatives, i.e., the decision matrix (see Section 2), an optimization model is constructed to assess attribute weights and thus to rank the alternatives so as to reflect the decision makers' social fuzzy preference relation. It is a new way of reflecting the decision makers' preference information based on the decision matrix. Comparison between the proposed approach and that in Chiclana et al. (1998) is also conducted to demonstrate the influence of the decision matrix on the ranking results in the proposed approach.

The organization of current paper is as follows: Section 2 presents the problem description. Section 3 proposes the new approach to the MADM problem, where the three forms of preference information on alternatives are uniformed, aggregated and used to assess attribute weights. In section 4, the numerical example in Chiclana et al. (1998) is used to illustrate the use of the proposed approach. Conclusion is given in section 5.

2. Problem description

This paper considers the MADM problem where three forms of preference information on alternatives are given by multiple decision makers, i.e., preference orderings, utility values and fuzzy preference relation. Following assumptions and notations are used to represent the MADM problem (Chiclana et al., 1998; Feng and Xu, 1999; Kacprzyk, 1986; Kacprzyk and Fedrizzi, 1990; Li, 1999; Nurmi, 1981; Orlovski, 1978; Tanino, 1984, 1990; Zadeh, 1983):

- the alternatives are known: let $S = \{S_1, S_2, \cdots, S_m\}$ denote a discrete set of $m\,(\geq 2)$ possible alternatives.
- the attributes are known: let $R = \{R_1, R_2, \cdots, R_n\}$ denote a set of $n\,(\geq 2)$ attributes.
- the weights of attributes are unknown: let $w = (w_1, w_2, \cdots, w_n)^T$ be the vector of weights, where $\sum_{j=1}^{n} w_j = 1$, $w_j \geq 0$, $j = 1, \cdots, n$, and w_j denotes the weight of attribute R_j.
- the decision matrix is known: let $A = [a_{ij}]_{m\times n}$ denote the decision matrix where a_{ij} (>0) is the consequence with a numerical value for alternative S_i with respect to attribute R_j, $i = 1, \cdots, m$, $j = 1, \cdots, n$.
- the decision makers involved are known: let E = (e_1, e_2, …, e_K) denote the set of decision makers ($K \geq 2$).

Different decision makers can express their preference on the candidate alternatives in different forms, i.e., preference orderings, utility values (vector) and fuzzy preference relation.

- preference orderings, or an ordered vector can be used by a decision maker e_k ($e_k \in E$) to express his/her preference on the alternatives: $O^k = (o^k(1), \cdots, o^k(m))$, where $o^k(\cdot)$ is a permutation function over the index set {1, …, m} and $o^k(i)$ represents the ranking position of alternative S_i, $i = 1, \cdots, m$. The alternatives are ordered from the best to the worst by the decision maker e_k.
- utility values or an utility vector can be used by a decision maker e_k ($e_k \in E$) to express his /her preference on the alternatives: $U^k = (u_1^k, \cdots, u_m^k)$, $u_i^k \in [0,1]$, $1 \leq i \leq m$, where u_i^k represents the utility evaluation given by the decision maker e_k to alternative S_i.
- fuzzy preference relation on the alternatives can be given by a decision maker. The decision maker's preference relation is described by a binary fuzzy relation P on S, where P is a mapping $S \times S \to [0, 1]$ and p_{ij} denotes the preference degree of alternative S_i over alternative S_j. We assume that P is reciprocal, by definition, (i) $p_{ij} + p_{ji} = 1$ and (ii) $p_{ii} = -$ (symbol '–' means that the decision maker does not need to give any preference information on alternative S_i), $\forall i, j$.
- a fuzzy linguistic quantifier Q can be represented by followings with a pair (a, b):

$$Q(x) = \begin{cases} 0, & for\ \ x < a, \\ \dfrac{x-a}{b-a}, & for\ \ a \leq x \leq b, \\ 1, & for\ x > b, \end{cases} \qquad (1)$$

where *a, b, x* ∈ [0,1]. Different semantics correspond to different pairs of coefficients in (1), e.g., "at least half" corresponds to (0, 0.5), "most" corresponds to (0.3, 0.8), etc.
Since the attributes are generally incommensurate, the decision matrix A needs to be normalized so as to transform the various attribute values into comparable values. For the convenience of calculation and extension, the following two functions are used (Feng and Xu, 1999; Li, 1999):

$$b_{ij} = \frac{a_{ij} - a_j^{\min}}{a_j^{\max} - a_j^{\min}}, \qquad i = 1, \cdots, m, \ j = 1, \cdots, n, \qquad \text{for benefit criterion,} \tag{2}$$

$$b_{ij} = \frac{a_j^{\max} - a_{ij}}{a_j^{\max} - a_j^{\min}}, \qquad i = 1, \cdots, m, \ j = 1, \cdots, n, \qquad \text{for cost criterion,} \tag{3}$$

where $a_j^{\max}$ and $a_j^{\min}$ are given by

$$a_j^{\max} = \max\{a_{1j}, a_{2j}, \cdots, a_{mj}\}, \qquad j = 1, \cdots, n, \tag{4}$$

$$a_j^{\min} = \min\{a_{1j}, a_{2j}, \cdots, a_{mj}\}, \qquad j = 1, \cdots, n. \tag{5}$$

Then decision matrix $A = [a_{ij}]_{m \times n}$ can be transformed into a normalized one:

$$B = [b_{ij}]_{m \times n}. \tag{6}$$

The problem concerned is to rank the alternatives, based on the decision matrix A (or B) and the three forms of preference information on the alternatives given by the decision makers.
In the following section, a new approach to the MADM problem is proposed, where the three forms of preference information on alternatives are given by multiple decision makers. The approach is based on an optimization model which can be used to assess the attribute weights and then to rank the alternatives.

3. A new approach to the MADM problem

When multiple decision makers are involved in the decision process, using the indirect approach (Kacprzyk, 1986) to rank the alternatives, two phases are usually needed to attain the final solution (Chiclana et al., 1998): aggregation and exploitation. Aggregation is to combine opinions on the alternatives from different points of views; Exploitation is to rank the alternatives or to select the most desirable one based on the collective preference information on the alternatives. In this section, two forms of preference information on alternatives are firstly converted into the uniform fuzzy preference relation, then preference aggregation and approximation follow.

3.1 Preference uniformity

As discussed in Chiclana et al. (1998), a decision maker e_k ($e_k \in E$) can use preference orderings or an ordered vector $O^k = (o^k(1), \cdots, o^k(m))$ to express his/her preference on the

alternatives. The preference orderings can be transformed into fuzzy preference relation on alternatives S_i and S_j,

$$p_{ij}^k = \frac{1}{2}\left(1 + \frac{o^k(j)}{m-1} - \frac{o^k(i)}{m-1}\right), \qquad 1 \le i \ne j \le m, \tag{7}$$

where $o^k(j)$ is the ranking position of alternative S_j, as defined in section 2, $j = 1, \cdots, m$.

Also a decision maker e_k ($e_k \in E$) can use an utility vector $U^k = (u_1^k, \cdots, u_m^k)$ to express his/her preference on the alternatives. The utility vector can also be transformed into fuzzy preference relation on alternatives S_i and S_j as follows (Chiclana et al., 1998):

$$p_{ij}^k = \frac{(u_i^k)^2}{(u_i^k)^2 + (u_j^k)^2}, \qquad 1 \le i \ne j \le m, \tag{8}$$

where u_i^k is the utility evaluation given by the decision maker e_k to alternatives S_i, $i = 1, \cdots, m$.

3.2 Preference aggregation

Multiple decision makers are involved in the evaluation and selection process. After their preference information on the alternatives are transformed into uniform fuzzy preference relation, the next step is to aggregate these uniformed fuzzy preference relations into a social fuzzy preference relation. The social fuzzy preference relation can be formed by using the ordered weighted averaging (*OWA*) operator to aggregate the individual fuzzy preference relations (Yager, 1988, 1993, 1996, 1998). The *OWA* operator is an effective and common method to aggregate individual fuzzy preference information. An *OWA* operator of dimension *K* is a function *F* as follows,

$$F : [0,1]^K \rightarrow [0,1] \tag{9}$$

F is associated with a weight vector $V = [v_1, \cdots, v_K]$, $v_i \in [0,1]$ and $\sum_{i=1}^{K} v_i = 1$, and

$$F(p_{ij}^1, p_{ij}^2, \cdots, p_{ij}^K) = V \cdot C^T = \sum_{l=1}^{K} v_l c_l, \qquad 1 \le i \ne j \le m, \tag{10}$$

Where $C = [c_1, \cdots, c_K]$ and c_l is the *l th* largest value among the collection of $p_{ij}^1, p_{ij}^2, \cdots, p_{ij}^K$. $P^l = (p_{ij}^l)_{m \times m}$ is the matrix of the uniformed fuzzy preference relations on the alternatives from decision maker e_l, $l = 1, \cdots, K$. The weight vector *V* can be obtained by a proportional quantifier *Q* (Yager, 1988, 1993), i.e.,

$$v_l = Q(l/K) - Q((l-1)/K), \qquad l = 1, \cdots, K \tag{11}$$

Q can be a fuzzy linguistic quantifier with a pair(*a, b*) as defined in equation (1).

If $p_{ij}^1, p_{ij}^2, \cdots, p_{ij}^K$ are assigned importance $z_1, z_2, \ldots, z_K$ respectively, and t_l is the importance associated with c_l correspondingly ($l = 1, \cdots, K$), then formula (11) is changed into follows:

$$v_l = Q\left(\frac{\sum_{j=1}^{l} t_j}{\sum_{j=1}^{K} t_j}\right) - Q\left(\frac{\sum_{j=1}^{l-1} t_j}{\sum_{j=1}^{K} t_j}\right), \qquad l=1,\cdots, K. \tag{12}$$

In Chiclana et al. (1998), the fuzzy majority method with fuzzy linguistic quantifier "at least half" and "as many as possible" are used to find the social fuzzy preference relations. In Güngör and Arikan (2000), fuzzy preference relations on the alternatives are aggregated across the evaluation criteria by using the "simple additive weighting method" (Chen and Hwang, 1992; Hwang and Yoon, 1981). In the current paper, semantics "most", involved in the fuzzy linguistic quantifier with a pair (0.3,0.8), is used by the *OWA* to aggregate multiple individual preference relations.

3.3 Preference approximation

Using the "simple additive weighting method" (Chen and Hwang, 1992; Hwang and Yoon, 1981), the overall value of alternative S_i can be expressed by

$$d_i = \sum_{j=1}^{n} b_{ij} w_j, \qquad i=1,\cdots, m, \tag{13}$$

where d_i is an explicit function of the variables w_j (j =1,⋯, n). Based on the overall values, the ranking results of the alternatives can be obtained. The greater the overall value d_i is, the better the corresponding alternative S_i will be.

In order to make information consistent, the overall values of the alternatives can be transformed into fuzzy preference relations on them. Thus, by using equation (13), $\overline{g}_{ik}$ can be defined as,

$$\overline{g}_{ik} = \frac{d_i}{d_i + d_k} = \frac{\sum_{j=1}^{n} b_{ij} w_j}{\sum_{j=1}^{n} (b_{ij} + b_{kj}) w_j}, \qquad 1 \le i \ne k \le m, \tag{14}$$

where the significance of $\overline{g}_{ik}$ is similar to that of g_{ik}. The difference between g_{ik} and $\overline{g}_{ik}$ is given by

$$f_{ik}(w) = g_{ik} - \overline{g}_{ik} = g_{ik} - \frac{\sum_{j=1}^{n} b_{ij} w_j}{\sum_{j=1}^{n} (b_{ij} + b_{kj}) w_j}, \qquad 1 \le i \ne k \le m. \tag{15}$$

Apparently, $f_{ik}(w)$ is an explicit function of w_j (j =1,⋯, n). To reflect the decision makers' social fuzzy preference relation based on the decision matrix, $\overline{g}_{ik}$ should approximate g_{ik} as far as possible by assessing the attribute weights w_j (j =1,⋯, n). So the following constrained optimization model can be constructed:

$$\textit{Minimize} \quad \sum_{i=1}^{m} \sum_{\substack{k=1 \\ k \ne i}}^{m} \left[\frac{\sum_{j=1}^{n} b_{ij} w_j}{\sum_{j=1}^{n} (b_{ij} + b_{kj}) w_j} - g_{ik}\right]^2 \tag{16a}$$

s.t.

$$\sum_{j=1}^{n} w_j = 1, \tag{16b}$$

$$w_j \geq 0, \qquad j = 1, \cdots, n. \tag{16c}$$

Model (16a)-(16c) can be easily solved by using the optimization toolbox for constrained optimization problems in Matlab (Redfern and Campbell, 1998). If equation (13) is substituted with the optimization solution to model (16a)-(16c), i.e., w_j^* (j =1,⋯, n), the overall values of the alternatives and then the ranking of them can be obtained respectively.

4. Illustrative example

Purchasing a house is a traditional MADM problem where the proposed approach can be used. A potential buyer intends to select a house from four alternatives (i.e. S_1, S_2, S_3 and S_4). The attributes considered include:

1) R_1: house price (\$10,000),
2) R_2: dwelling area (m^2),
3) R_3: distance between every house and the work locality (km),
4) R_4: natural environment (assessment value).

Among the four attributes, R_2 and R_4 are of benefit type, R_1 and R_3 are of cost type. The decision matrix with the four attributes (R_1, R_2, R_3 and R_4) and the four alternatives (S_1, S_2, S_3 and S_4) is presented as follows:

$$A = \begin{pmatrix} 3.0 & 100 & 10 & 7 \\ 2.2 & 70 & 12 & 9 \\ 2.5 & 80 & 8 & 5 \\ 1.8 & 50 & 20 & 11 \end{pmatrix},$$

which can be normalized into matrix B by using equations (2)-(5) as follows,

$$B = \begin{pmatrix} 0 & 1 & 5/6 & 1/3 \\ 2/3 & 2/5 & 2/3 & 2/3 \\ 5/12 & 3/5 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}.$$

Suppose six persons e_1, e_2, e_3, e_4, e_5, e_6 supply their opinions to help the buyer make a decision. They express their opinions in terms of ordered vector, utility vector and fuzzy preference relation as follows (Chiclana et al., 1998):

e_1: O^1={3, 1, 4, 2}, e_2: O^2={3, 2, 1, 4}, e_3: U^3={0.5, 0.7, 1, 0.1}, e_4: U^4={0.7, 0.9, 0.6, 0.3},

$$e_5:\ P^5=\begin{pmatrix} - & 0.1 & 0.6 & 0.7\\ 0.9 & - & 0.8 & 0.4\\ 0.4 & 0.2 & - & 0.9\\ 0.3 & 0.6 & 0.1 & - \end{pmatrix},\qquad e_6:\ P^6=\begin{pmatrix} - & 0.5 & 0.7 & 1\\ 0.5 & - & 0.8 & 0.6\\ 0.3 & 0.2 & - & 0.8\\ 0 & 0.4 & 0.2 & - \end{pmatrix}.$$

To make the preference information uniform, the transformation functions in (7) and (8) are used, and the results are as follows (Chiclana et al., 1998):

$$P^1=\begin{pmatrix} - & \frac{1}{6} & \frac{2}{3} & \frac{1}{3}\\ \frac{5}{6} & - & 1 & \frac{2}{3}\\ \frac{1}{3} & 0 & - & \frac{1}{6}\\ \frac{2}{3} & \frac{1}{3} & \frac{5}{6} & - \end{pmatrix},\qquad P^2=\begin{pmatrix} - & \frac{1}{3} & \frac{1}{6} & \frac{2}{3}\\ \frac{2}{3} & - & \frac{1}{3} & \frac{5}{6}\\ \frac{5}{6} & \frac{2}{3} & - & 1\\ \frac{1}{3} & \frac{1}{6} & 0 & - \end{pmatrix},$$

$$P^3=\begin{pmatrix} - & \frac{25}{74} & 0.2 & \frac{25}{26}\\ \frac{49}{74} & - & \frac{49}{149} & 0.98\\ 0.8 & \frac{100}{149} & - & \frac{100}{101}\\ \frac{1}{26} & 0.02 & \frac{100}{101} & - \end{pmatrix},\qquad P^4=\begin{pmatrix} - & \frac{49}{130} & \frac{49}{85} & \frac{49}{58}\\ \frac{81}{130} & - & \frac{81}{117} & 0.9\\ \frac{36}{85} & \frac{36}{117} & - & 0.8\\ \frac{9}{58} & 0.1 & 0.2 & - \end{pmatrix}.$$

The *OWA* operator with fuzzy linguistic quantifier "most" is used to aggregate the six persons' opinions, with the corresponding weight vector being $(0, 1/15, 1/3, 1/3, 4/15, 0)^T$. That is, by (10), the social fuzzy preference relation from these persons is obtained,

$$G=\begin{pmatrix} - & 0.2933 & 0.4899 & 0.7568\\ 0.6647 & - & 0.6385 & 0.7200\\ 0.4167 & 0.2670 & - & 0.8460\\ 0.1842 & 0.2200 & 0.2156 & - \end{pmatrix}.$$

With respect to *G*, by using the optimization toolbox for constrained optimization problem in Matlab (Redfern and Compbell, 1998) to solve model (16a)-(16c), the optimal weight vector of the attributes can be obtained. Therefore the overall values of the four alternatives and their rankings would also be obtained respectively. Results are showed in table 1.

To demonstrate the difference between the proposed approach and that in Chiclana et al. (1998) in ranking the alternatives, a ranking procedure (Chen, 2001; Hsu and Chen, 1997. See appendix A) is used by calculating the quantifier guided dominance degree (*QGDD*) and the quantifier guided non-dominance degree (*QGNDD*) (Chiclana et al., 1998) for each alternative repetitively. That is, starting from the social fuzzy preference relation matrix *G*, using the *OWA* operator with fuzzy linguistic quantifier of "most", to calculate the *QGDD* and *QGNDD* for each alternative and select the best alternative at each iterative step. The result is showed in table 2.

From table 2, it can be seen that there is a difference between the ranking results of the proposed approach and that in Chiclana et al. (1998). It is clear that, by using the approach in Chiclana et al. (1998), the ranking result of the alternatives is only influenced by the social fuzzy preference information from the decision makers. In other words, the approach in Chiclana et al. (1998) is based on the decision makers' preference information. The proposed

approach produces a different ranking result by introducing the optimization model to make use of both the decision matrix and the decision makers' preference information.

Table 1. Calculation results of the proposed approach

The social fuzzy preference relation	Attribute weight vector	Overall values of the alternatives	Ranking of the alternatives
$\begin{pmatrix} - & 0.2933 & 0.4899 & 0.7568 \\ 0.6647 & - & 0.6385 & 0.7200 \\ 0.4167 & 0.2670 & - & 0.8460 \\ 0.1842 & 0.2200 & 0.2156 & - \end{pmatrix}$	$w_1^* = (0, 0, 0.7546, 0.2454)^T$	d_1=0.7107, d_2=0.6667, d_3=0.7546, d_4=0.2454.	$S_3 \succ S_1 \succ S_2 \succ S_4$

Table 2. Results of the approach in Chiclana et al. (1998) by using the ranking procedure.

The social fuzzy preference relation	*QGDD* and *QGNDD* of the alternatives					Ranking of the Alternatives
		S_1	S_2	S_3	S_4	
$\begin{pmatrix} - & 0.2933 & 0.4899 & 0.7568 \\ 0.6647 & - & 0.6385 & 0.7200 \\ 0.4167 & 0.2670 & - & 0.8460 \\ 0.1842 & 0.2200 & 0.2156 & - \end{pmatrix}$	*QGDD*	0.4553	0.6614	0.4054	0.2075	$S_2 \succ S_1 \succ S_3 \succ S_4$
	QGNDD	0.9010	1	0.8521	0.4168	$S_2 \succ S_1 \succ S_3 \succ S_4$

5. Summary

This paper proposes a new approach to solve the MADM problem with three forms of preference information on alternatives. The approach is based on an optimization model which can be used to assess the attribute weights and then to rank the alternatives. In this approach, the different forms of preference information given by multiple decision makers are transformed into uniform fuzzy preference relations and aggregated. To reflect the aggregated preference information of the decision makers, the attribute weights are assessed by using the optimization model (16a)-(16c) based on the decision matrix. Then ranking of the alternatives is obtained. The proposed approach provides an extension for the study in Chiclana et al. (1998), which is totally based on the decision makers' preference information on alternatives. Instead of the fuzzy majority method for alternative selection in Chiclana et al. (1998), the proposed approach reflects decision makers' social preference information based on the decision matrix. The illustration example demonstrates that the proposed approach produces more stable solution to the MADM problem than that in Chiclana et al. (1998) by making use of the information in the decision matrix with the optimization model for ranking the alternatives. This paper is not without limitation. To correspond with Chiclana et al. (1998), only three forms of preference information on alternatives, i.e., preference orderings, utility values and fuzzy preference relation, are used by the decision makers. In a forthcoming paper, additional three forms of preference information on

alternatives, e.g., linguistic term vector (Güngör and Arikan, 2000; Herrera and Herrera-Viedma, 2000) will also be considered to provide the flexibility for decision makers to express their preference on the alternatives.

Acknowledgement
This research is partly supported by the Competitive Earmarked Research Grant (CERG) of Hong Kong (Project No. 9040375 and Research Grant Council of Hong Kong and NSFC joint research scheme (Project No. 9050137).

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Appendix A

Given a fuzzy preference relation matrix *G*, a ranking procedure (Chen, 2001; Hsu and Chen, 1997) can be used to rank the alternatives by calculating the QGDD and QGNDD (Chiclana et al., 1998) repetitively:

(1). Set *T*=0 and suppose the set of the alternatives is $\Omega = \{S_1, S_2, \ldots, S_m\}$.

(2). Select the alternative with the highest nondominated degree, say S_h, $\mu^{ND}(S_h) = \max_i \{\mu^{ND}(S_i)\}$. Set the ranking for S_h as $r(S_h) = T + 1$.

(3). Delete the alternative S_h from Ω, i.e., $\Omega = \Omega \setminus S_h$. The corresponding row and column of S_h are also deleted from the fuzzy preference relation matrix *G*.

(4). Recalculate the nondominated degree for each alternative S_i, $S_i \in \Omega$. If $\Omega = \phi$, then stop. Otherwise, set *T*=*T*+1, and return to step (2).