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Abstract

Software products do not provide consumption benefit unless a hardware product is installed in advance. In the market with this software – hardware relationship, a software producer sometimes finds that it is profitable to make its product compatible with only a certain hardware product. This incompatibility decision is considered as a vertical foreclosure. The conditions under which vertical merger and foreclosure occur in equilibrium are analyzed. We find that the welfare reducing foreclosure arises in equilibrium even without the credible commitment of foreclosure decision.

Key Words: Compatibility Decision, Hardware-Software, Base–Supplemental Goods, Vertical Foreclosure

1. Introduction

Software products do not provide consumption benefits unless some other products are purchased in advance and used in conjunction. For example, consumers need to have hardware products. Some software products like operating system (OS) are also essential and prerequisite to other application software products. In this complementary relationship, we define 'base good' as one of the two complementary goods that must be purchased and installed prior to use application software products. Examples are hardware and OS. We also define 'supplemental good' as the other complementary good that provides consumption benefit when only used in conjunction with a base good. Any application software is a supplemental good. A base good may give consumption benefit without a specific supplemental good but a supplemental good always requires a base good.

An important property of many goods in the base-supplemental relationship is that the base good is typically much less substitutable than the supplemental good because of high switching costs. For instance, a consumer can pick Netscape Navigator or Internet Explorer as a default web-browser and switch from one to the other one easily. Changing computers from a PC (and Windows operating system) to a Mackintosh is, however, more problematic. We analyze a market with two complementary goods in 'base-supplemental' relation in this paper and the canonical example is hardware-software products.

The studies on the markets with two related products have focused on the monopoly reasons for vertical control. The issues in the studies are including the anti-competitive effects of vertical mergers, the possibilities of vertical foreclosure, and the welfare effect of vertical integration. While Chicago school argued that there is no monopoly reason for vertical

integration[1], many recent authors like Ordover, Saloner, and Salop (OSS) (1990), Choi and Yi(1997) showed that anti-competitive vertical merger or vertical foreclosure can occur as an equilibrium.

When they consider the vertically related industries, they focus on the upstream-downstream relationship, so the upstream firms produce intermediate goods and the downstream firms produce final goods with the upstream firms' intermediate goods. Therefore, no consumers' choices are directly involved in this vertical relationship. This paper, contrarily, considers a vertically related industry where the firms' products are sold directly to consumers. Therefore, consumers choose both components of the complementary goods.

Many works of Economides (Economides and Salop(1992), Economides(1994), and Economides(1997))are analyzing the market with two complementary goods. They are characterized by bundling purchase of two simple complementary goods, like bolt and nut. So, there is no consideration in purchasing timing or in difference of substitutability between base goods and supplemental goods in their models. Church and Gandal(1997) assume a timing structure where consumers purchase hardware first, then software. In this sense, this paper is following their setting. However, they do not capture the fact that hardware is less substitutable than software.

When a firm produces both hardware and software products, sometimes it makes its software products incompatible against the rival firms' hardware. This is called vertical foreclosure. For example, Nintendo and Sony do not have compatibility to each other. In this paper, we consider the possibility of vertical foreclosure and its welfare effect. We analyze the conditions that make vertical foreclosure be an equilibrium outcome. We find that there is more incentive for vertical foreclosure when the degree of substitutability is high and market share of foreclosing product is small. We also find that even without the credible commitment of foreclosure decision, the welfare reducing foreclosure may arise.

This paper is organized as follows. We describe the basic model in section 2. The equilibrium outcomes of the model are explained in section 3, and the welfare effect of integration and foreclosure are discussed in section 4. In section 5, we mention the possible extension of the model followed by conclusion and summary.

2. Model

2.1. Firms

Two firms (firm 1 and firm 2) are producing base goods (hardware) B_1 and B_2, and two firms (firm 3 and firm 4) are producing supplemental goods (software) S_1 and S_2 respectively. A base good and a supplemental good are, of course, complementary. Base goods firms and supplemental goods firms have constant marginal production cost, mc_B and mc_S, respectively. All fixed costs are sunk.

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1 see Tirole(1988) and Ordover, Saloner, and Salop(1990)

Each pair of firms are under price competition. The price of base goods B_1 and B_2 are denoted to be q_1 and q_2, and the supplemental goods S_1 and S_2 are denoted to be p_1 and p_2, respectively. We also denote the market share of B_1 to be ω and the market share of B_2 to be $1-\omega$.

We are assuming that the compatibility decisions are on supplemental good firms (firm 3 and firm 4). If either of two supplemental good firms makes its good compatible only with one of the base goods, then that behavior is considered as a foreclosure against the other base good. The compatibility structure will be one of the three cases; 1) full compatibility, 2) partial foreclosure, and 3) parallel foreclosure. We also assume that it is costless to change the compatibility structure and the ownership structure.

2.2. Consumers

There are N consumers in the market, and N is normalized to 1. We assume that a consumer needs to have one unit of base good to consume supplemental goods. Consumers' preferences are identical. We denote the demand of S_1 with base B_1 and B_2 to be S_{11} and S_{21} respectively and the demand of S_2 with base B_1 and B_2 to be S_{12} and S_{22}.

If a consumer has base good Bi ($i=1,2$) and both S_1 and S_2 are compatible with Bi, then her utility function is

$$U(X, B_i, S_{i1}, S_{i2}) = X + h(B_i) + \alpha S_{i1} + \alpha S_{i2} - \frac{1}{2}(\beta S_{i1}{}^2 + \beta S_{i2}{}^2 + 2\gamma S_{i1} S_{i2}) \tag{1}$$

where X is the outside good and $h(B)$ is the stand-alone benefit of base good. B is defined over non-negative integer with $h(0)=0$, $h(1)>0$, and $h(1)\geq h(k)$ for all integer k. Note that we assume the utility function is separable in X, the outside good and the base good. We also assume that all coefficients are positive, and $\beta>\gamma$.[2] The budget constraint of consumers with base good B_i is:

$$X + q_i B_i + p_1 S_{i1} + p_2 S_{i2} = I, \qquad i = 1,2 \tag{2}$$

Maximization of (1) subject to (2) yields demand equations:

$$S_{i1} = a - bp_1 + cp_2, \qquad S_{i2} = a - bp_2 + cp_1, \qquad i = 1,2 \tag{3}$$

where $a \equiv \dfrac{\alpha}{\beta+\gamma}, b \equiv \dfrac{\beta}{\beta^2-\gamma^2}, c \equiv \dfrac{\gamma}{\beta^2-\gamma^2}$. At the consumer's optimal choice, the realized consumer's surplus with base good B_i^* is

$$CS_i = U(X^*, B^*, S_{i1}^*, S_{i2}^*) = I + h(B_i^*) - q_i B_i^* + \frac{1}{2}(\beta S_{i1}^{*2} + \beta S_{i2}^{*2} + 2\gamma S_{i1}^* S_{i2}^*) \tag{4}$$

Note that $h(B^*)=h(1)$ is a constant.

If a consumer already has a durable base good before the maximization, then her consumption benefit (CB) with that base good is

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2 This can be interpreted as "an increase in the price of all differentiated goods reduces the demand for each good." This restriction is quite common in oligopoly models.

$$CS_i = U(X^*, B^*, S_{i1}^*, S_{i2}^*) = I + h(B_i^*) + \frac{1}{2}(\beta S_{i1}^{*2} + \beta S_{i2}^{*2} + 2\gamma S_{i1}^* S_{i2}^*) \tag{5}$$

Now, if B_i and S_j are not compatible, then S_j does not give any consumption benefit to the consumers with B_i, so the utility function will not include the terms with incompatible good S_j. Therefore, when a consumer has B_i and only S_i is compatible with it, her utility function and budget constraint will be

$$\hat{U}(X, B_i, \hat{S}_{ii}) = X + h(B_i) + \alpha \hat{S}_{ii} - \frac{1}{2}\beta S_{ii}{}^2 \tag{1'}$$

$$X + q_i B_i + p_1 \hat{S}_{ii} = I, \qquad i = 1,2 \tag{2'}$$

We denote $\hat{S}_i$ to be the supplemental good S_i when S_j is not compatible with the base good B_i. The demand function and the realized consumer's surplus in this case are

$$\hat{S}_{ii} = d - ep_i, \qquad \hat{S}_{ij} = 0, \qquad i \neq j$$

(3')
where $d \equiv \alpha/\beta$, and $e \equiv 1/\beta$.[3]

$$CS_i = \hat{U}(X^*, B^*, \hat{S}_{ii}^*) = I + h(B_i^*) - q_i B_i^* + \frac{1}{2}\beta \hat{S}_{ii}^{*2} \tag{4'}$$

Again, if we drop the third term of the right hand side, then we have consumption benefit of a consumer who has a base good ex ante:

$$CS_i = \hat{U}(X^*, B^*, \hat{S}_{ii}^*) = I + h(B_i^*) + \frac{1}{2}\beta \hat{S}_{ii}^{*2} \tag{5'}$$

Since equation (3) or (3)' is the individual's demand function and consumers are identical, we can get the market demand function by simply multiplying (3) or (3)' by the number of consumers with that particular base good.

2.3. Setting of the game

We model the three-stage game. Before the beginning of the game, consumers already purchased and installed the base goods, and the market shares are exogenously given. We assume that the initial industrial structure is independent ownership with full compatibility. In the first stage, observing the initial value of ω (we denote it to be ω_0), firms decide the compatibility and ownership structures. In the second stage, firm 1 and firm 2 set the prices of base goods simultaneously, and consumers can switch their base goods. If a consumer chooses to switch it, she needs to purchase a new base good, but if she decides to stay with the previous base good, she pays nothing. Therefore, the price of the base good works as switching costs. In the final stage, firm 3 and firm 4 set the price of supplemental goods simultaneously, and consumers purchase them.

2.4. Three subgames

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[3] To compare the profits and consumer surplus in the next chapter, it is very helpful to express d and e in terms of a, b, and c as follows: $d = \dfrac{a(b+c)}{b}, e = \dfrac{b^2 - c^2}{b}$.

The equilibrium outcomes of this game can be derived by backward induction. So, we start with the final stage. Since the supplemental goods' prices are not affected by the ownership structure, we calculate the profits of firms under the three different compatibility structures.

2.4.1. Full compatibility

If firm 3 and firm 4 make their products compatible both with B_1 and B_2, then the objective functions of firm 3 and firm 4 will be

$$\pi_3 = p_1\,(\omega S_{11}+(1-\omega)S_{21})$$
$$\pi_4 = p_2\,(\omega S_{12}+(1-\omega)S_{22})$$

By solving the first order conditions, the optimal prices, profits and consumption benefits are obtained:

$$p_1^{full} = p_2^{full} = \frac{a}{2b-c}$$
$$\pi_1^{full} = \pi_2^{full} = \frac{a^2 b}{(2b-c)^2} \tag{6}$$
$$CB_1^{full} = CB_2^{full} = \frac{a^2 b^2}{(b-c)(2b-c)^2}$$

2.4.2. Partial foreclosure

When one of the two supplemental good firms (say firm 3) makes its good(S_1) be compatible with only one of the two base goods(say B_1), so forecloses against B_2, then the firms' objective functions will be

$$\pi_3 = p_1\,(\omega S_{11}+(1-\omega)\hat{S}_{21}) = p_1\omega S_{11}$$
$$\pi_4 = p_2\,(\omega S_{12}+(1-\omega)\hat{S}_{22})$$

The optimal prices and profits are

$$p_1^{part} = \frac{a(\omega c^2 + 2b^2 - c^2 + bc)}{b(3\omega c^2 + 4b^2 - 4c^2)}, \qquad p_2^{part} = \frac{a(2b+2c-\omega c)}{3\omega c^2 + 4b^2 - 4c^2)}$$

$$\pi_3^{part} = \frac{\omega a^2 (\omega c^2 + 2b^2 - c^2 + bc)^2}{b(3\omega c^2 + 4b^4 - 4c^2)^2}, \qquad \pi_4^{part} = \frac{a^2 (2b+2c-\omega c)^2 (b^2 - c^2 + \omega c^2)}{b(3\omega c^2 + 4b^2 - 4c^2)^2} \tag{7}$$

$$CB_1^{part} = a^2(8b^5+4b^4\omega c+4b^4 c+16b^3\omega c^2+b^3\omega^2 c^2\text{-}15c^2 b^3+5b^2\omega^2 c^3+2b^2\omega c^3\text{-}7b^2 c^3$$
$$\qquad {}^+9b\omega^2 c^4\text{-}16c^4\omega b+7c^4 b+3c^5\text{-}6c^5\omega+3c^5\omega^2) / 2b(b\text{-}c)(3\omega c^2+4b^2\text{-}4c^2)^2$$
$$CB_2^{part} = a^2(b+c)(2b^2+\omega bc+2\omega c^2\text{-}2c^2)^2 / b(b\text{-}c)(3\omega c^2+4b^2\text{-}4c^2)^2$$

Note that prices are higher under partial foreclosure than under full compatibility, i.e*., $p_2^{part}>$ $p_1^{part}>p^{full}$. This is because firm 4 becomes, in a sense, a monopolist for the consumers with B_2 after the foreclosure. Higher p_2 causes higher p_1 because prices are strategic complements

in the Bertrand game. Firm 4 will always be better off by the foreclosure of firm 3 against B_2, i.e., $\pi_4^{part} > \pi_4^{full}$, while we can not unambiguously determine whether or not firm 3 will be better. Finally, $CB_1^{part} > CB_2^{part}$, and this is because consumers with B_2 can not enjoy the product diversity of supplemental goods.

2.4.3. Parallel foreclosure

If firm 4 also makes its product (S_2) be compatible only with B_2 under the partial foreclosure, then it will be changed into the parallel foreclosure. The objective functions will be

$$\pi_3 = p_1(\omega\hat{S}_{11} + (1-\omega)\hat{S}_{21}) = p_1\omega\hat{S}_{11}$$
$$\pi_4 = p_2(\omega\hat{S}_{12} + (1-\omega)\hat{S}_{22}) = p_2(1-\omega)\hat{S}_{22}$$

The optimal prices and profits are

$$p_1^{paral} = p_2^{paral} = \frac{a}{2(b-c)}$$
$$\pi_3^{paral} = \frac{\omega a^2(b+c)}{4b(b-c)}, \qquad \pi_4^{paral} = \frac{(1-\omega)a^2(b+c)}{4b(b-c)} \tag{8}$$
$$CB_1^{paral} = CB_2^{paral} = \frac{a^2(b+c)}{8b(b-c)}$$

Under parallel foreclosure, prices get even higher than under partial foreclosure, because firm 3 and firm 4 are like two independent monopolists. However, we cannot unambiguously say whether firm 3 and firm 4 are better off from partial foreclosure.

3. Equilibrium outcome

Now, we need to analyze the base good firms' pricing rule. Since B_1 and B_2 are homogenous, as long as both goods give the same consumption benefit, consumers have no incentive to switch their base goods. Therefore, if the compatibility structure is full compatibility or parallel foreclosure, nothing will happen in the second stage. Under partial compatibility, however, the consumption benefits from B_1 and B_2 are different ($CB_1 > CB_2$). If that difference is larger than the price of B_1, then consumers with base good B_2 purchase B_1 and switch their base goods.

The lowest possible price of B_1 is the marginal cost (mc_B). Therefore, we can consider two cases: 1) marginal cost is relatively low ($mc_B < CB_1^{part} - CB_2^{part}$), and 2) marginal cost is relatively high ($mc_B \geq CB_1^{part} - CB_2^{part}$).

3.1. High switching cost case

When $mc_B \geq CB_1^{part} - CB_2^{part}$, consumers always stay with their current base goods and base good firms don't get any additional profits in the second stage. This fact makes two notable points.

First of all, the initial market share ω_0 does not change till the end of the game. Second of all, it is meaningless to consider the joint profits of base good firm and supplemental firm good. Therefore, ownership structure is not a concern in this case. We analyze the equilibrium outcome of compatibility structure, which is decided by two supplemental good firms.

Firm 3 will make its product be compatible only with B_1, if $\pi_3^{part} > \pi_3^{full}$. Otherwise, it will make its good be compatible with both B_1 and B_2. From (6) and (7), we can compare the profits of two cases.

$$\pi_3^{part} > \pi_3^{full} \qquad iff$$

$$4\omega^2 c^5 b - \omega^2 c^6 - 4\omega^2 c^4 b^2 - 16\omega c^2 b^4 + 17\omega c^4 b^2 + \omega c^6 + 8\omega c^3 b^3 - 6\omega c^5 b - 16 c^4 b^2 + 32 c^2 b^4 - 16 b^6 > 0$$

To understand the economic meaning of this inequality, we divide above inequality by b^6. Then we have

$$4\omega^2 \eta^5 - \omega^2 \eta^6 - 4\omega^2 \eta^4 - 16\omega \eta^2 + 17\omega \eta^4 + \omega \eta^6 + 8\omega \eta^3 - 6\omega \eta^5 - 16 \eta^4 + 32 \eta^2 - 16 > 0 \qquad (9)$$

where $\eta \equiv c/b$. Note that η is the degree of substitutability of two supplemental goods, and since $b > c \geq 0$, η is between zero and one.($\eta \in [0,1)$) If η is equal to zero, two supplemental goods are independent, and if η is near 1, then two goods are close substitutes.

The inequality (9) cannot be determined unambiguously. It may be true with some combinations of ω and η. Since $\omega \in [0,1]$ and $\eta \in [0,1)$, we can indicate the combinations of ω and η to make (8) true in <figure 1>. In <figure 1>, the horizontal axis is ω and the vertical axis is η. There is a downward sloping curve which starts at *(0, 1)* and ends at around *(1, .9)*. The area above that curve indicates the combinations of ω and η to hold the inequality (8). In other words, if *(ω, η)* is in that area, firm 3 will choose to foreclose against B_2 and make its product be compatible only with B_1. If *(ω, η)* is below that curve, then firm 3 will make S_1 be compatible with both B_1 and B_2.[4]

We observe that the area for the partial foreclosure is with high η, the degree of substitutability. One can also see that area gets wider as ω, the market share of B_1 bigger. Therefore, we say that there are more incentives for partial foreclosure with higher degree of substitutability and with bigger market share of B_1.

What is the intuition behind this story? In other words, what is the source of the incentives for the foreclosure. If firm 3 forecloses against B_2, then it completely loses the customers with B_2. We call this **abandon effect**. However, once it forecloses, firm 4 becomes a monopolist for the consumers with B_2, and it has an incentive to raise the price of S_2. Since p_1 and p_2 are strategic complements, firm 3's best response to the firm 4's behavior is to raise p_1, but lower than p_2. (recall that $p_2^{part} > p_1^{part} > p^{full}$). If S_1 and S_2 are close substitutes, consumers with base good B_1

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4 In fact, we only need to look at the area for $\omega \in [1/2, 1]$. Because a supplemental firm , if it has choice, forecloses not against the base good with larger market share but against smaller market share.

are willing to buy more S_1 than S_2. We call this **substitution effect**. The latter effect will dominate the former effect when S_1 and S_2 are close substitutes or when the market share of B_1 is large.

Now, under the partial foreclosure structure, we need to check whether firm 4 has an incentive to foreclose against B_1. This is called counter foreclosure. Firm 4 will do so if $\pi_4^{paral} > \pi_4^{part}$. By the same way as we did above, we get

$$\pi_4^{paral} > \pi_4^{part} \quad iff$$
$$-5\omega^2\eta^5 - 13\omega^2\eta^4 + 13\omega\eta^5 + 37\omega\eta^4 - 4\omega\eta^3 - 28\omega\eta^2 - 8\eta^5 - 24\eta^4 + 8\eta^3 + 40\eta^2 - 16 > 0 \quad (10)$$

The combinations of (ω,η) to hold this inequality are indicated in <figure 1>. They are the area above the upward sloping curve which starts at about $(0, 0.74)$ and ends at $(1, 0)$.

Again, as the degree of substitutability is higher and the market share of B_2, $(1-\omega)$ is bigger, counter foreclosure is more likely to occur. By foreclosing, firm 4 will lose all the sales to the consumers with base good B_1. However, that amount of sales is small when ω is small or when the degree of substitutability is high.

From the two inequalities (9) and (10), we can find the equilibrium compatibility structure under given combination of market share and degree of substitutability. If the combinations of (ω,η) are below the downward sloping curve, then the equilibrium compatibility structure will be full compatibility. If the combinations of (ω,η) are above the downward sloping curve but below the upward sloping curve, then partial foreclosure will be the equilibrium outcome. If combinations of (ω,η) are above both curves, then parallel foreclosure is the equilibrium outcome.

<Proposition 1> **The closer substitutes are the two supplemental goods, the more incentives does a firm have for foreclosure against one base good. The equilibrium compatibility structure depends on the given combinations of market share and degree of substitutability of two supplemental goods as indicated in <Figure 1>**

The downward sloping curve represents (9) and the upward sloping curve represents (10). The curves are drawn in GAUSS by numerically changing the combination value of (ω, η). No further proof is required in this proposition, but we can look at some specific values to see whether it works. For example, if $\omega=0$, then the left hand side of (9) becomes $-16(\eta^2-1)^2 < 0$. Therefore, the equilibrium structure is full compatibility. If $\omega=1$, then the left hand side of (9) becomes $2\eta^3+3\eta^2-4$, which is positive when η is between 0.9 and 1. In this manner, we construct the downward slopping curve in <Figure 1>

3.2. Low switching cost case

In the high switching cost case, π_1 and π_2 are always zero, because no consumers are buing the additional base good. However, in the low switching cost case, a consumer may purchase a new base good if the consumption benefits from the two base goods are different because a base good price may be lower than the consumption benefits difference. Recall two

consumption benefits are different only under partial compatibility. Therefore, *if* $mc_B < CB_1^{part} - CB_2^{part}$, firm 1 charges the price of B_1 to be slightly lower than the difference, and makes consumers with B_2 switch to B_1. In this case, $q_1^* = CB_1^{part} - CB_2^{part} - \varepsilon$, where ε is small enough positive number. Hence, $\pi_1^{part} = (1-\omega)(CB_1^{part} - CB_2^{part} - \varepsilon - mc_B)$ and $\pi_2^{part} = 0$, because $1-\omega$ portion of consumers purchase B_1 at price q_1^*.

However, firm 1 does not have the compatibility choice by itself, so firm 3 should be involved in that structural change. Firm 3 may agree to make S_1 be compatible only with B_1 if the joint profit of firm 1 and firm 3 under partial foreclosure is larger than firm 3's profit under full compatibility, even though its own profit under partial foreclosure is not larger. Therefore, by explicitly considering the ownership structure with compatibility structure, we may have a different equilibrium outcome. Firm 1 and firm 3 will integrate and foreclose against B_2, if

$$\pi_1^{part} + \pi_3^{part}(\omega=1) > \pi_3^{full}(\omega=\omega_0). \tag{11}$$

It is easy to see that there is stronger incentive in the low switching cost case than in the high switching cost case. In fact, the above inequality holds everywhere. This is *because* $\pi_3^{part}(\omega=1) = \pi_3^{full}(\omega=\omega_0)$ and $\pi_1^{part} > 0$. Hence, in low switching cost case, partial integration and foreclosure always occur.

Now, we consider the possibility of counter foreclosure. If firm 4 merges with firm 2 and forecloses against B_1, then industrial structure will go back to symmetric structure. Since $CS_1^{paral} = CS_2^{paral}$, consumers would not change from the previous choice of the base goods and $q_1 = q_2 = \pi_1 = \pi_2 = 0$. Therefore, if firm 4's profit π_4^{paral} with initial market share ω_0 is greater than π_4^{part} with $\omega=1$, then counter merger and counter foreclosure will occur.

$$\pi_4^{paral}(\omega=\omega_0) > \pi_4^{part}(\omega=1) \quad iff \quad -\omega\eta^3 + 3\omega\eta^2 - 4\omega + \eta^3 - 3\eta^2 + 4\eta > 0 \tag{12}$$

This inequality is true for the area above the curve in <Figure 2>, which looks like a 45-degree line. Therefore, we can conclude that if (ω, η) is above the curve in <Figure 2>, then parallel integration and foreclosure is the equilibrium outcome, and if (ω, η) is below that curve, then partial integration and foreclosure is the equilibrium outcome. The intuition for the counter foreclosure is the same as in the high switching cost case.

<Proposition 2> If (ω, η) is above the curve in <Figure 2>, then parallel integration and foreclosure will be the equilibrium outcome and if (ω, η) is below that curve, then partial integration and foreclosure will be the equilibrium outcome. In the partial integration outcome, however, firm 1 will sweep all the consumers and the market share will be changed into $\omega=1$. The optimal price of B_1 in this case is $CS_1^{part} - CS_2^{part} - \bar{\varepsilon}$.

Proof: First, we need to show that at least partial foreclosure will always occur, i.e., $\pi_1^{part} + \pi_3^{part}(\omega=1) - \pi_3^{full}(\omega=\omega_0) > 0$. Since $\pi_1^{part} + \pi_3^{part}(\omega=1) - \pi_3^{full}(\omega=\omega_0) = \dfrac{a^2(1-\omega)(2b^2 + bc + \alpha c^2 - c^2)^2}{2b(4b^2 + 3\alpha c^2 - 4c^2)^2}$, and it is always positive, at least partial foreclosure

occurs. Now, parallel foreclosure arises in equilibrium when the inequality (12) holds. The downward sloping curve is drawn in GAUSS by numerically changing the combination value of (ω, η). When the combination of (ω, η) is in the area above this curve, then the inequality (12) is true and the equilibrium structure is parallel foreclosure. Otherwise, partial foreclosure is the equilibrium ownership structure. <**Q.E.D.**>

4. Welfare Effects

The social welfare is defined as the sum of the profits of firms and consumers' surplus. By comparing social welfare in three cases, we can conclude that $Welfare^{full} > Welfare^{part} > Welfare^{paral}$. It is interesting to note that firm 2 and firm 4 are not hurt by partial foreclosure at all. Firm 4 is even better off by it. All burdens of welfare loss are on consumers.

<Proposition 3> **Integration and foreclosure causes welfare loss to the society. Welfare is the best under full compatibility, and the worst under the parallel foreclosure. Partial foreclosure is between the two**.

proof : We need to show 1) $Welfare^{full} - Welfare^{part} > 0$, and 2) $Welfare^{part} - Welfare^{paral} > 0$.

1) $sign[Welfare^{full} - Welfare^{part}] = sign\,[20\eta^4(1-\eta)\omega^2 + 5\eta^6\,\omega^2 + (60\eta^2-44\eta^3-43\eta^4+46\eta^5 -15\eta^6)\omega + 48-6\eta +12\eta^6+16\eta^4-76\eta^2+40\eta^3-24\eta^5]$. Let the term in parenthesis of the right hand side be $\Theta(\omega,\eta)$. Since Θ is a quadratic function where ω is between 0 and 1, $\Theta(\omega,\eta)$ is positive if $\Theta(0,\eta)>0$, $\Theta(1,\eta)>0$, and $\Theta(\omega^m,\eta) >0$, where ω^m is the argument of $min\,\Theta(\omega,\eta)$. However, since ω^m is greater than 1 and we don't need to check the sign of $\Theta(\omega^m,\eta)$. We see that the sign $[\Theta(\omega,\eta)] = sign\,[-3\eta^3+9\eta^2-16\eta+12]$. The derivative of $-3\eta^3+9\eta^2-16\eta +12$ with respect to η is $-9\eta^2+18\eta -16$, which is always negative when $0<\eta < 1$. This means it is monotonically decreasing function of η between 0 and 1. Since $-3\eta^3+9\eta^2-16\eta +12 = 2$ when $\eta =1$, the sign of $-3\eta^3+9\eta^2-16\eta +12$ is positive. By the same procedure, we see that the *sign $[\Theta(1, \eta)] = sign\,[-2\eta^3+2\eta^2-5\eta +6]$* is positive. Therefore, $Welfare^{full} - Welfare^{part} > 0$.

2) $sign\,[Welfare^{part} - Welfare^{paral}] = sign\,[20\eta^4(1-\eta)\omega^2 + (53\eta^5-51\eta^4-44\eta^3+60\eta^2)\omega - 36\eta^5+28\eta^4+52\eta^3-76\eta^2-16\eta +48]$. Let the term in the parenthesis of the right hand side be $\Xi(\omega, \eta)$. Since Ξ is a quadratic function of ω which is between 0 and 1, $\Xi(\omega,\eta)$ is positive if $(0,\eta)>0$, $(1,\eta)>0$, and $(\omega^m,\eta)>0$, where ω^m is the argument of $min\,(\omega,\eta)$. However, since ω^m is greater than 1, we do not need to check the sign of $\Xi(\omega^m, \eta)$. It is easy to see that sign *[$\Xi(0, \eta)] = sign\,[9\eta^2-16\eta +12]$* >0. *Sign $[\Xi(1, \eta)]$* is equal to *sign $[-3\eta^3+9\eta^2-16\eta +12]$*, which we already show the positive sign in 1). *Therefore, $Welfare^{part} - Welfare^{paral} > 0$. (Q.E.D.)*

In our setting, we find two remarkable points that depart from OSS (1990) and other vertical foreclosure literatures. First, contrary to other 'anticompetitive foreclosure' results, partial foreclosure (and integration) does not hurt the rival firm's profit even though it reduces social welfare. Second, even without the ability of commitment, the welfare reducing foreclosure may arise. We need to look at the details of the second point.

We've showed that partial foreclosure always occurs in the low switching cost case. When

firm 1 and firm 3 announce the merger and foreclosure decision in the first stage, then consumers with base good B_2 will purchase base good B_1 in the second stage. However, suppose some consumers with B_2 do not switch to B_1 in the second stage for some reason. Then, in the final stage, firm 3 may have an incentive to renege on its foreclosure commitment and supply S_1 to these consumers. Knowing this, consumers may not purchase and switch to B_1.

This commitment problem has been a big issue of vertical foreclosure literatures since OSS(1990).[5] They show that anticompetitive vertical integration can arise in equilibrium if the vertically integrated firm can commit not to sell its input to the unintegrated downstream firm. It has been argued that OSS's result breaks down if the vertically integrated firm cannot make the credible commitment because the integrated firm has a strong incentive to renege on its price commitment and undercut the unintegrated rival's price.

If we assume that the firms cannot commit to foreclose against the unintegrated rival firm, then our equilibrium ownership structure in Figure 2.2 is no longer valid. However, if the initial market share and the degree of substitutability are in the partial foreclosure area of the Figure 2.1, then without the ability of commitment, firm 3 forecloses against B_2 anyway. If the initial market share and the degree of substitutability are not in this area, then there is no incentive for firm 3 to foreclose. Therefore, we can say that even without credible commitment, welfare reducing foreclosure may happen if the market share of foreclosing product is small and the degree of substitutability is high.

5. Conclusion

We consider a market with two complementary goods, a 'base good'(hardware) and a 'supplemental' good (software). With the assumption that there are two base good firms and two supplemental good firms, we analyze the conditions that make vertical integration and incompatibility decision equilibrium outcomes. Incompatibility decision can be considered as a vertical foreclosure. We find that there are more incentives for partial foreclosure when the degree of substitutability of two competing supplemental goods (software products) is high and market share of foreclosing product is small. We also find that even without a credible commitment of foreclosure decision, the welfare reducing foreclosure may arise.

Our results may not be sensitive to the restriction on a consumer's initial purchase of a base good. This needs to be examined in a dynamic game that allows a certain portion of consumers repurchase base goods when their durabilities wear out. In this case, we need to consider the firm's present value of future profits. The equilibrium ownership structures may be different. This remains as an area for future study

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[5] If we do not want to worry about the commitment problem, then as Church and Gandal (1997) do, we can assume that the compatibility choice is not reversible because it is too costly, or that a firm has an incentive to build its 'reputation' over time.

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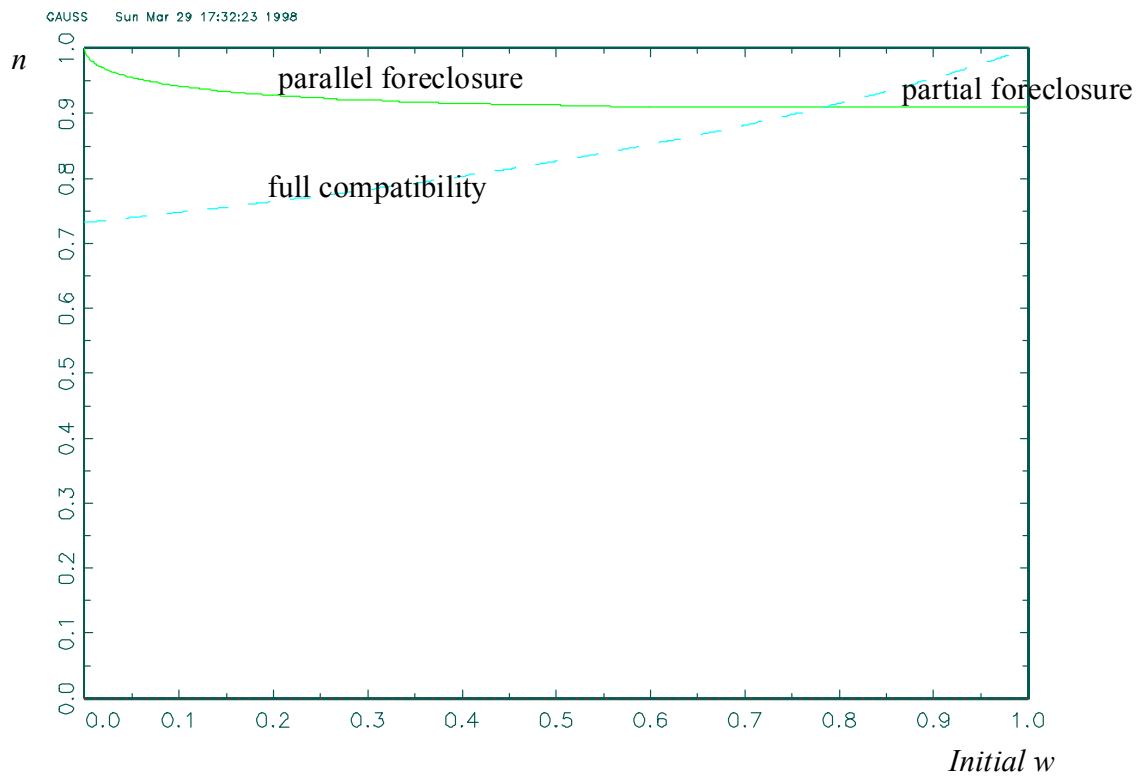
<Figure 1> Equilibrium Outcome under High Switching Cost
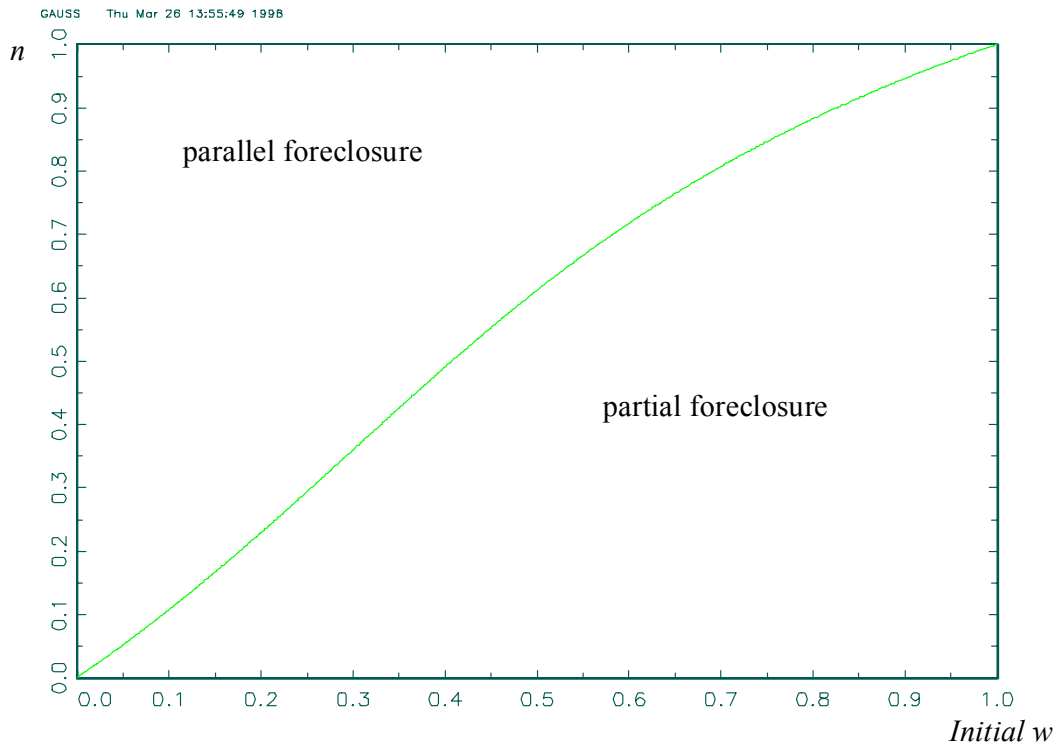
<Figure 2> Equilibrium Outcome under Low Switching Cost

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