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How Much Do Our Incoming First Year Students Know?: Diagnostic Testing in Mathematics at Third Level

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Abstract

A continuing cause for concern in higher education institutions is the poor core mathematical skills of incoming students. Many institutions now offer mathematics support services such as drop-in centres, online resources and short 'refresher courses' in an attempt to alleviate the problem. The majority of third level institutions in Ireland and internationally now make use of diagnostic testing of incoming first year students that both predict subsequent success and select groups for remediation. This project was developed to explore the issues around diagnostic testing and follow-up support for incoming students in the College of Sciences and Health. A large cohort of first year science students was tested and those who failed to achieve 50% on the test were offered support. This support came in the form of peer-assisted student led tutorials during which students had the opportunity to revise basic areas of mathematics. On comparison of the scores on the diagnostic test with the end of module results we have noticed a correlation between students who scored poorly on the diagnostic test and students who failed the Semester 1 mathematics module. The key recommendations arising from this study are; diagnostic testing provides useful information about the cohort as a whole and provides lecturers with information about gaps in the prior knowledge of the group allowing them to take particular care when introducing new topics, diagnostic testing helps to identify those students who are significantly weaker than the rest of the cohort and thus enables them to be targeted with support and attention. Furthermore, by carrying out diagnostic testing over an extended period of time, trends can be observed. This information

can then be used by Schools or Departments in an attempt to cope with diversity and ensuring that follow-up support is adequately provided.

Keywords: First Year Curriculum, Mathematics, Diagnostic Testing, Peer Learning

Introduction, Aims and Objectives

Over the past decade comprehensive concerns over student difficulties with mathematics, statistics and general numeracy have been expressed by many governments, employers and higher education providers. Abundant supplies of reports and articles have been produced to highlight these concerns (Hawkes & Savage, 2000; Savage, 2003; Smith, 2004). Furthermore, these issues are not exclusive to the UK and Ireland alone; reports of this kind are being produced worldwide. For example, an Australian article (McGillivray, 2008) studied the experiences of first-year undergraduate students and attempted to identify the weaknesses in mathematical skills and confidence that act as a barrier for success for many students. This gap between the level of preparedness either expected or required upon entry to third level and the mathematical capabilities acquired at school/college has become known as 'the mathematics problem' (Savage, 2003).

More recent studies into the changing nature of the mathematical skills which our undergraduates have acquired, have led to many third level institutions organising some form of mathematics support provision including the provision of drop-in centres, individual consultation and access to special provisions. The main aim is to aid students to overcome their difficulties with mathematics but to also help students with different backgrounds and challenges, such as mature students and students with disabilities, to get an introduction to the mathematical thinking required at third level (Gill, Mac An Bhaird, & Ni Fhloinn, 2010). Indeed, the Student Maths Learning Centre (SMLC) was established in DIT in 2006, with this purpose in mind.

Many third level institutions now use diagnostic testing in mathematics as a tool to assess their intake of students, in particular engineering students. Different third level institutions

adopt different types of diagnostic tests. An extensive study carried out in the UK in 2002 showed that many institutions use multiple-choice questions, either paper-based or computer-based and most tests will group questions together under a common heading such as algebra or calculus (LTSN., 2002). In Loughborough University, a novel diagnostic test is in use, incorporating a paired question method with the idea that both questions in a pair should test the same topic. Such a structure is believed to allow easy identification and subsequent follow up of topics where the student needs extra help (Lee & Robinson, 2004).

Diagnostic testing has helped to show that student performances have declined particularly in the areas of arithmetic and algebra (Atkinson, 2004; Gillard, Levi, & Wilson, 2010), as well as indicating an increase in the variability of results. In terms of practical significance to teaching and learning of mathematics at third level today, we are faced with the fact that within the profile of students entering mathematics lectures today, many students are hampered by a serious lack of fluency and reliability in numerical and algebraic manipulation and simplification (LMS, 1995a). Neither are the mathematical backgrounds of first year undergraduate students as strong as they were as recently as ten years ago (LMS, 1995a). Thus, this research shows us that the starting point of mathematics lecture materials for beginning undergraduate students needs to be adapted or 'in the absence of any change in starting point, a deterioration in the effectiveness of learning' will take place (Hunt & Lawson, 1996).

Learning in the 21st century is characterised by prompt change, an overabundance of sources of information, new technologies and new ways in which graduates study, live and work. In view of these changes, most third level institutions now recognise that they must help students to cope with complexity and adjustment (Hibberd, 2010). Indeed there is an

increased expectation from governments that third level institutions who offer mathematics or mathematics-related programmes should accustom their graduates to work in a wide range of sectors that require strong numerical skills (Higher Education Funding Council for England (HEFCE), 2005). Ireland has a unique situation in terms of the mathematical homogeneity of its third level students; most students enter via the route of the Leaving Certificate, on completion of 13 years of formal mathematics education. With regards to the transition from second level to third level mathematics, the current complaints of educators at Irish third level institutions, including DIT (Ni Fhloinn, 2006; Russell, 2005a), about the level of mathematical knowledge and skills of incoming students are much the same as those outlined 16 years ago by the London Mathematics Society in the report *Tackling the Mathematics Problem (LMS, 1995b):*

1. Students lack reliability and fluency in manipulating and simplifying numerical and algebraic problems;

2. There is a marked decline in students' analytical powers when faced with simple two-step or multi-step problems;

3. Many students no longer understand or appreciate that mathematics is a precise discipline in which exact, reliable calculation, logical exposition and proof play essential roles.

Inadequate provision of follow-up support is a shortcoming of the whole process of diagnostic testing. Indeed, in situations where students are simply given their result and advised to revise certain topics on their own and using their own initiative, there is little evidence that this happens. One of the most basic issues regarding the provision of mathematics support is that of funding. The majority of drop-in centres are staffed between 20-30 hours each week. The question is who pays for this service? On one side of the coin the budgetary arguments are strong: the cost of providing this service would be more than

covered by the fees of 10 first year students who drop out of their course because they are unable to cope with the mathematical modules. The problem arises when we endeavour to ascertain incontrovertibly that 10 students a year have been retained who would have been lost if the centre was not available. Furthermore, and perhaps even more disputable, is the fact that the 10 retained students are unlikely to be evenly spread across the institution. This problem has no easy solution and is often decided not by logical reasoning but instead by internal politics (Lawson, 2010).

In Ireland, recent papers about this topic have included examining the various ways of measuring the success of the support service offered by a mathematics support centre (Gill & O'Donoghue, 2007), looking at the pass-rates of at-risk students (Dowling & Nolan, 2006), analysing the role of student feedback in evaluating effectiveness and the impact of the mathematics support centre on first year grades(Mac An Bhaird & O'Shea, 2009). All of the reports concluded that mathematics support appeared to have an impact on the majority of students who regularly attended and also made a positive contribution to student retention (Gill, et al., 2010). More detailed evaluations still, have been carried out worldwide. In the UK researchers concluded that results of diagnostic tests and attendance at mathematics support centres were significant predictors of first year marks (Lee, Harrison, Pell, & Robinson, 2008). In the Netherlands a special mathematics course has been designed to help the transition from second to third level, the success of which has been documented by (Terlouw, de Goede, & Keinhuis, 2008). Australian research has concluded that students who avail of supports are almost twice as likely to complete the course (McGillivray, 2008). Mathematics support services were originally conceived as being services for "weaker" students i.e. students who were weak in terms of their preparedness (as opposed to weak in terms of their ability). In recent years, statistics have shown a change in the makeup of the

'clientele' of the drop-in centres with more and more very able students - often mature students whose mathematical background has not adequately prepared them for their course of study in higher education, using the drop-in centres as a valuable learning resource. Furthermore, these students only occasionally consult the duty staff, instead preferring to work in groups and make use of the non-staff resources (Lawson, 2010). Clearly the evaluation of the provision of mathematics support is an important but complex issue. The evaluation of services and the impact on students, their attendance and non-attendance certainly have very important implications.

Diagnostic Testing was carried out for the first time using online submissions in the School of Mathematical Sciences in September 2011. Over previous years, the School of Civil And Building Engineering Services had developed a test and after discussions with personnel in this School and consultation with the research, it was decided to use the same test in our own School. This test was first implemented in 2006 and has been updated and improved each year. The test consists of a multiple-choice quiz on webcourses and is based on a large randomised question bank. Students are asked to answer 20 questions (ten paired questions) on basic topics such as algebra, fractions, indices, trigonometry, the equation of a line, logs, quadratic equations, simultaneous equations and basic differentiation and are given a time constraint of 90 minutes.

Students were given immediate results when they submitted their test and any student receiving less than 50% were advised to take part in a revision initiative. This initiative involved student-led tutorials over the first 10 weeks of Semester 1. Two 4th year Mathematical Sciences students facilitated the tutorials, and first year students revised basic mathematical concepts in each tutorial using specially prepared revision booklets (prepared

by Sigma Centre for Excellence in Mathematics and Statistics support). These revision booklets were developed with students such as those who participated in this study in mind, and so were not adapted by the author in any way. Students were then asked to retake the Diagnostic Test again during Weeks 11 and 12 of Semester 1.

In summary, the aim of this project was to identify students who were likely to struggle with mathematical concepts in first year by developing a diagnostic test, devising and implementing an appropriate feedback process and providing adequate follow-up mathematical support to these students in the form of student-assisted tutorials. The student groups were selected based on their participation in suitable modules taught at DIT, College of Science.

Summary of Main Findings

The following discussion provides a brief overview of the development of a mathematical diagnostic test for incoming first year students, the form of follow-up support provided to students identified as being significantly weaker, and summarises the results of student re-testing and student evaluation, including student comments. It also provides the authors reflections on the project, student reactions to the initiative, and indicates where improvements can be made in the future. Finally it outlines the key recommendations and conclusions from the research findings.

1) Diagnostic Test

Our initial challenge was in choosing what type of test to introduce – paper or computer-based. Using the research about best practise for mathematics diagnostic testing (LTSN., 2002), the decision made was that computer-based tests would best suit the needs of this

project. The main reasons for this decision were the time pressure involved in the correction of paper-based tests and also the relative ease of preparation and installation of the computer-based questions. Furthermore, if it had been decided to use paper-based tests, it would have been necessary for a capital spend to be made on an optical reader which would have added to an already overstretched budget within the School. One of the most advantageous benefits of using computer-based testing was the instant feedback that was given and answers were marked objectively against set criteria.

A second challenge related to the style of mathematical questions that were asked in our diagnostic assessment and their implementation. It was decided to make use of a multiple-choice style quiz where each question had four possible answers and negative marking applied. Each answer was sufficiently similar so as to ensure that the student understood how they arrived at an answer and negative marking ensured that students were discouraged from guessing. As outlined above, after consultation with colleagues in the School of Civil and Building Engineering, it was decided that a diagnostic test which they had developed and been using for some years would be appropriate for use in this initiative also. This particular quiz had been established online and so any technical glitches had already been corrected. Furthermore the developer of this quiz had improved and advanced it over the course of the five years of its' existence meaning that there was a large bank of possible questions that could be used for different topics, ensuring that students were much less likely to get the same question.

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The pilot groups chosen for this study were first year Honours Degree students on eight different programmes in the College of Science.

Course	Programme Code	Leaving Certificate Points*	
		Final	Midpoint
Mathematical Sciences	DT205/DT220	255	340
Physics Technology	DT222	305	365
Physics with Nanotechnology	DT227	325	375
Clinical Measurement	DT229	410	450
Physics with Bioengineering	DT235	320	390
Optometry	DT224	495	515
Computing	DT211	320	335
Computer Science	DT228	350	370

*In the Irish Leaving Certificate, six subjects are included for the purpose of calculating points. A maximum of 100 points can be attained in any one subject. The final point column shows the lowest points score achieved by an applicant who received an offer of a place on the course. The mid point is the points score of an applicant in the middle of a list of offerees placed in points score order.

Building upon work already carried out in DIT (School of Civil And Building Engineering Services), the initiative was evaluated using a strategy devised to enhance the way in which the diagnostic test was implemented, and integrated, into programmes. Formative evaluation was also necessary to highlight areas where improvements could be made to the diagnostic test itself and its use within Science programmes. Both quantitative and qualitative research methods were implemented to ascertain the effectiveness of the diagnostic test, the follow-up revision tutorials and to determine where improvements can be made. The methods of data collection are diagnostic test results, attendance at revision tutorials, Leaving Certificate (LC) mathematics mark, diagnostic re-test mark, end of module maths mark and questionnaires.

Across all pilot groups, 329 students were eligible for consideration in the data. Some students had one or more pieces of data with missing values and so were excluded from some comparisons. Reasons for missing data include non-standard entry students, international students and a non-compulsory diagnostic test. 26% of students for which data was collected had taken Higher Level Mathematics at Leaving Certificate and 69% had taken Ordinary Level Mathematics at Leaving Certificate. The response rate for the diagnostic test was 47% (156 responses) giving an overall average mark of 52%.

Programme	No. Eligible Students	Average Mark	Response Rate	Less than 50%
DT205/DT220 – Mathematical Sciences	44	53%	84% (37 responses)	16/37 (43%)
DT222 – Physics Technology	15	58%	52% (8 responses)	4/8 (50%)
DT227 – Physics with Nanotechnology	16	57%	81% (13 students)	4/13 (31%)
DT229 – Clinical Measurement	30	58%	37% (11 students)	4/11 (36%)
DT235 – Physics with Bioengineering	9	50%	67% (6 students)	3/6 (50%)
DT224 –	30	61%	20% (6 students)	1/6 (17%)

Optometry				
DT211 – Computing	73	42%	34% (25 responses)	**16/25 (64%)**
DT228 – Computer Science	112	49%	45% (50 students)	**26/50 (52%)**

The following table is a breakdown of diagnostic test marks versus whether a student sat

Higher Level or Ordinary Level Mathematics at Leaving Certificate Level.
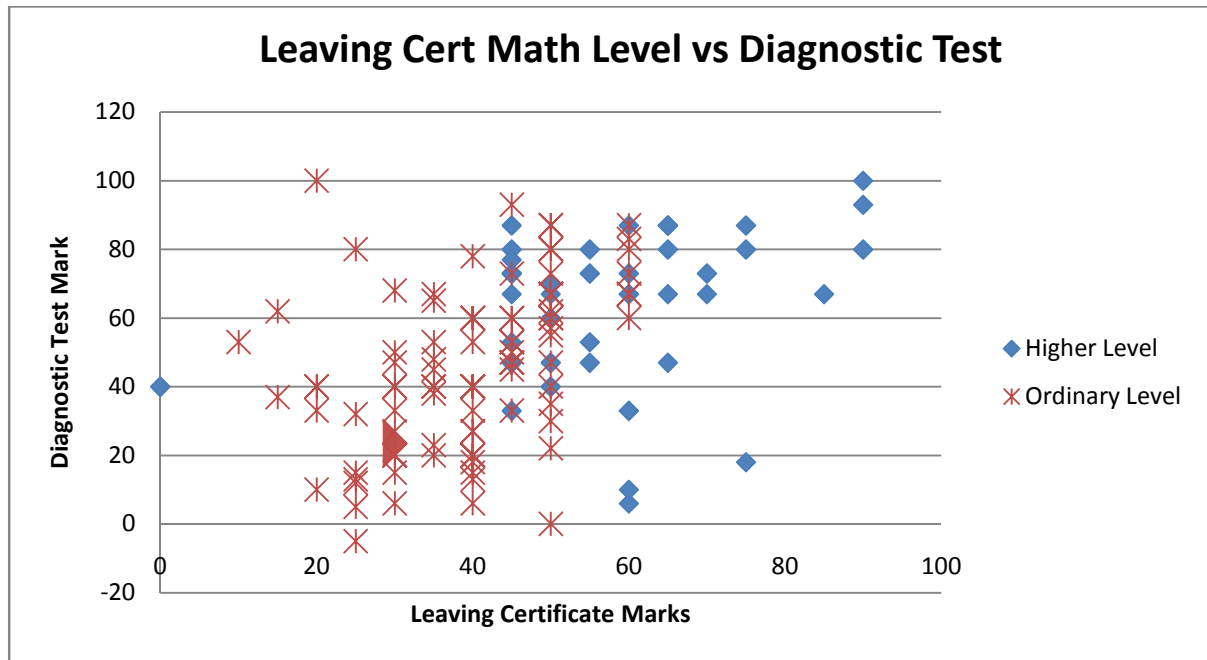
As can be seen from the diagram, students who had higher level mathematics were much

more likely to get over 50% on the diagnostic test. 30% of higher level students failed to

achieve 50% on the diagnostic test whereas 56% of ordinary students failed to achieve over

50% on the same test.

Worryingly, given that the points requirements to all of the pilot courses are relatively high

(to very high in the case of Optometry) and that the basic mathematics requirement is an OB3

(Ordinary Level B3) for all Physics and an OC3 for all Computing programmes, almost half

of all respondents (47%) to the diagnostic test failed to achieve more than 50%. In particular,

high proportions of students on both computing programmes failed to achieve higher than 50%.

Furthermore, results from the diagnostic test highlighted particular topics about which students had misconceptions or areas where students had little or no prior knowledge on which to base their answers. Algebra and arithmetic were the two main 'problem areas' for these students and knowing this in advance of lectures allowed me to change my style of teaching and allow slightly more time allocation to these topics. Giving the students advance notice and materials to revise these topics and to go over the keywords which they would meet again in lectures, gave the weaker students in particular space to 'get their head around the basic concepts' and a direction in which to point themselves mathematically.

2) Follow-Up Support

Students who received less than 50% on the diagnostic test were advised to attend revision tutorials to help them to improve their understanding of basic mathematical concepts. These tutorials were held twice weekly for the first 10 weeks of Semester 1 and were facilitated by 2 fourth year mathematical science students. The average weekly student attendance was 11 students with the majority of attendees being mature students, particularly from the 2 computing programmes. Attendance was not compulsory nor did it account for any continuous assessment mark and for this reason, I feel that many students were not motivated enough to attend.

Each tutorial had a separate revision booklet prepared for it with each booklet covering a different basic mathematical topic e.g. Booklet 3 – Algebra, Booklet 5 – Factorising. Students were seated in small groups and were given a short introduction to the topic by the tutors.

Each group was then given time to work on and discuss some questions together and their solutions were checked by the tutors. If a question was consistently incorrectly answered, the tutors worked through this on the board. Booklets also contained extra questions and worked solutions for students to work through after each tutorial. The group work aspect of the tutorials allowed students to work together on problems and also to meet other learners in the same boat as themselves.

Student feedback about the initiative was very positive. More than 50% of original respondents (regardless of their mark on the diagnostic test) attended *some* revision tutorials.

Percentage Agreement	Response
87%	Revision Booklets were well-structured with good examples.
75%	Revision tutorials *had* improved their knowledge of basic mathematics.
88%	Revision tutorials had greatly improved their ***confidence*** with mathematics

3) Re-test

The diagnostic test was again made available during Weeks 11 and 12 of Semester 1 and students were reminded to attempt the test. 13% of students re-sat the test and again I feel that this figure was highly influenced by the non-compulsory nature of the initiative, coupled with a lack of motivation shown by many of the (especially weaker) students. The results however were more reassuring with 86% of respondents increasing their marks. The average increase in results was 62% which shows a significant improvement in understanding and competency in basic mathematical questions. These results also helped to improve students' confidence in their mathematical ability and keep them focused on the maths module.

Test Mark vs Retest Mark
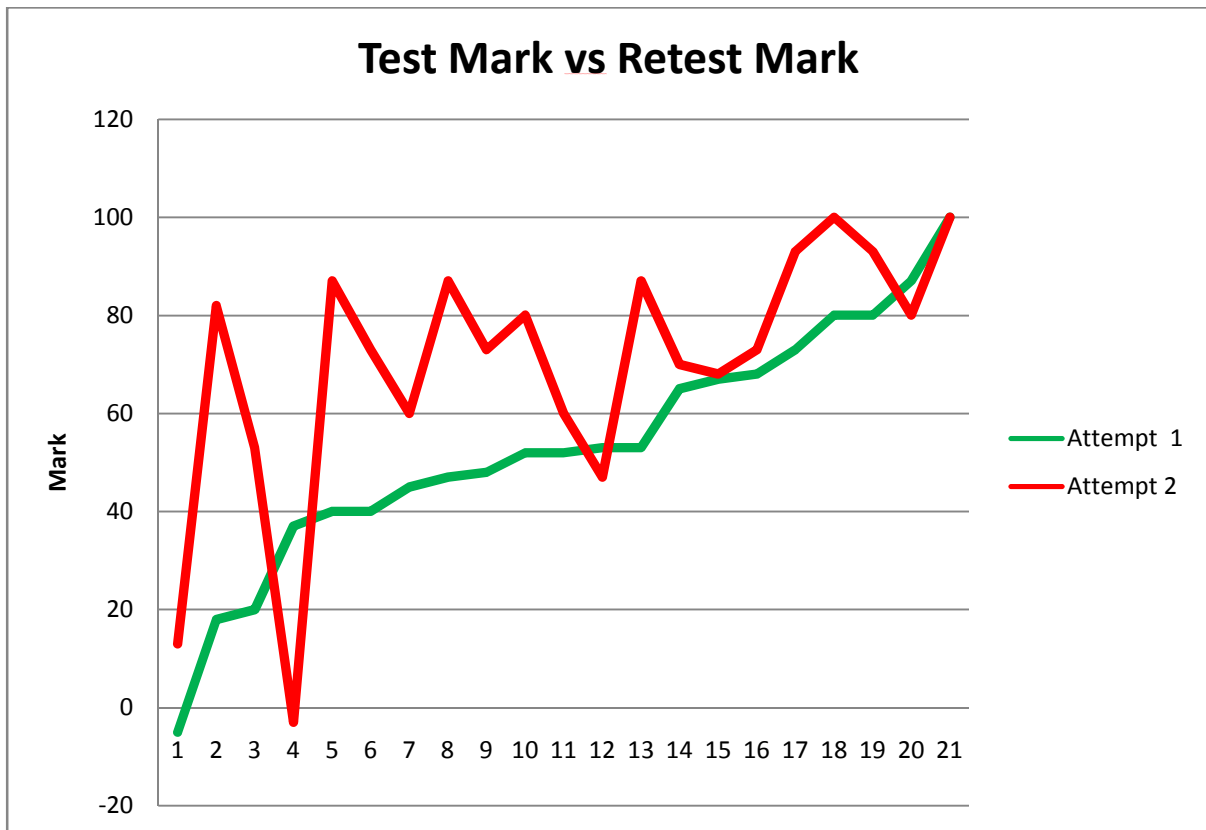
Correlation Analysis

To measure the strength of association (if any) between the diagnostic test mark and the end of module exam mark, we used the correlation coefficient. Any respondents who had either not taken the diagnostic test and/or had not taken the end of module exam mark were omitted from the analysis, leaving us with a sample size of 155. A non-compulsory submission of the diagnostic test was a major contributory factor to such omissions as well as students dropping out of their programme before the end of the semester.

Analysis of the data shows that there is a positive correlation between the diagnostic test mark and the end of module mark ($r = .390$) which is statistically significant($p = 0.000$). This tells us that those students who achieved a poor score on the diagnostic test tended to also obtain a poor end of module mark.

A scatter diagram of the data shows a linear relationship between the variables.

Scatter Diagram of Diagnostic Test mark vs End of Maths Module mark
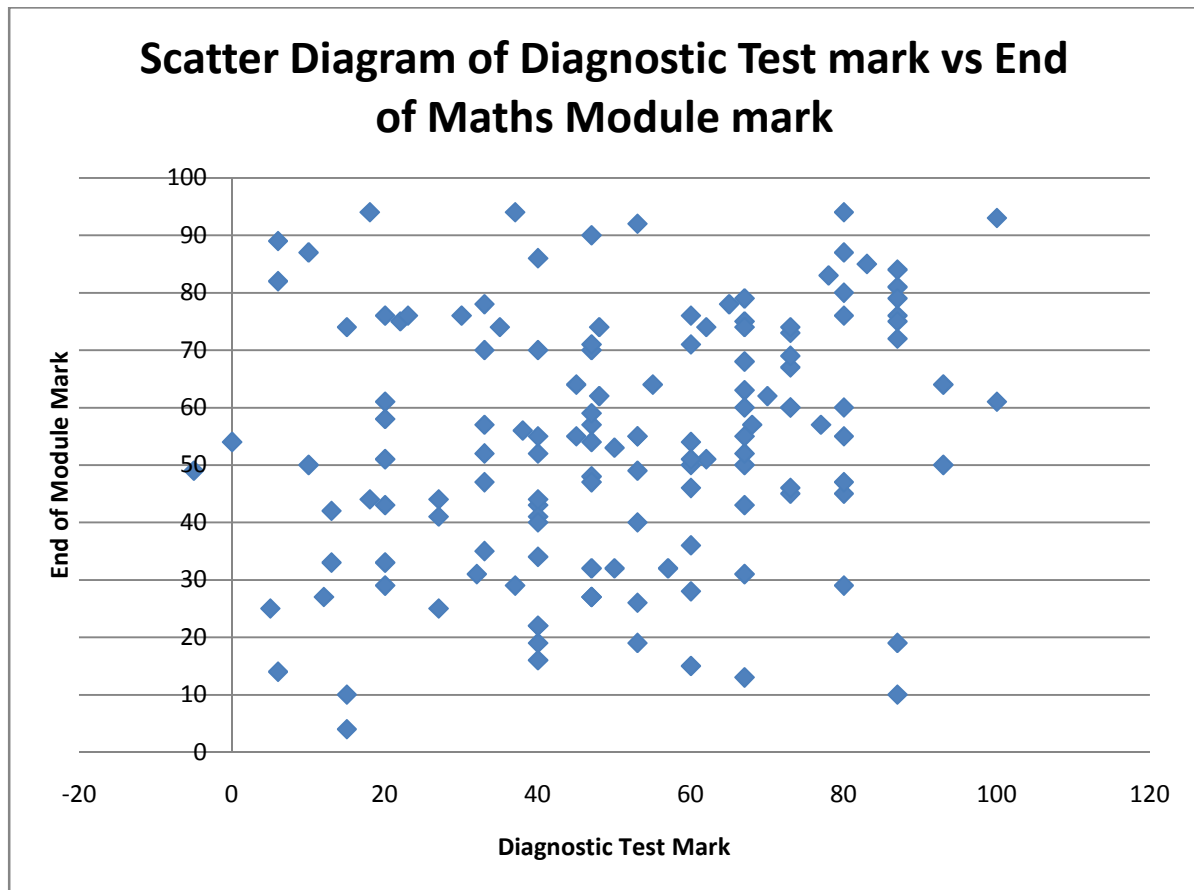
The regression equation is as follows:

$$End\ of\ Maths\ Module\ mark = 16.67 + 0.833(Diagnostic\ Test)$$

Conclusion

Diagnostic testing provided a positive approach to a situation. For our students it provided a constructive method which led to ongoing support and for myself as an academic interested in this research, it provided an indication of "what is needed" in terms of teaching and possible changes in the curriculum. There was a systematic improvement in the basic mathematical skills of students who participated in the initiative and this is evident from the results on the retest and also from student feedback. We hope that this initiative will become an integral part of mathematical education for first year students, as the number of students sitting the diagnostic test increases in future years.

To ensure a better initial response rate in future years, each group will be given a dedicated time slot in a computer lab during their induction schedule. This will help us to collect and analyse results in advance of the students beginning their first year mathematics modules and will allow us to discuss how and where students can avail of maths support early in the semester. Future work will investigate whether the presence of mathematics support is a factor which supports student retention. This will build upon work already carried out in the Faculty of Engineering in 2005 (Russell, 2005b).

Due to financial constraints, it is unlikely that the student-led revision tutorials will run again. In their place, an online resource will be developed over the coming years and made available to all students. This resource will be updated each week with a new tutorial. The revision booklets will again play a major role along with video links and dedicated quizzes for each topic.

Acknowledgements

References

Atkinson, R. (2004). Don't just do it, Do it Right! Diagnostic Tests. *MSOR Connections, 4*, 35-37.

Dowling, D., & Nolan, B. (2006). *Measuring the effectiveness of a maths learning centre: The Dublin City University experience.* Paper presented at the CETL MSOR Conference 2006.

Gill, O., Mac An Bhaird, C., & Ni Fhloinn, E. (2010). The Origins, Development & Evaluation of Mathematics Support Services. *Irish Mathematical Society Bulletin, 66*, 51-63.

Gill, O., & O'Donoghue, J. (2007). Justifying the Existence of Mathematics Learning Support: Measuring the Effectiveness of a Mathematics Learning Centre. *Proceedings of the ALM, 14*.

Gillard, J., Levi, M., & Wilson, R. (2010). Diagnostic Testing at UK Universities: an e-mail survey. *Teaching Mathematics and its Applications, 29*, 69-75.

Hawkes, T., & Savage, M. (2000). *Measuring the Mathematics Problem.* London: Engineering Council.

Hibberd, S. (2010). Employability Skills: A Key Role for Mathematics. In C. M. Marr & M. J. Grove (Eds.), *Responding To The Mathematics Problem: The Implementation of Institutional Support Mechanisms*: HEA, MSOR, SIGMA (Centre for Excellence in mathematics and statistics support).

Higher Education Funding Council for England (HEFCE). (2005). *Strategically Important and Vulnerable Subjects*: London: HM Treasury.

Hunt, D., & Lawson, D. (1996). Trends in Mathematical competency of A-level Students on Entry to University. *Teaching Mathematics and its Applications, 15*(4), 167-173.

Lawson, D. (2010). *The Drop-In Centre Model of Mathematics Support*: Maths, Stats & OR Network.

Lee, S., Harrison, M., Pell, P., & Robinson, C. (2008). Predicting performance of first year engineering students and the importance of assessment tools therein. *Engineering Education, 3*, 44-51.

Lee, S., & Robinson, C. (2004). Diagnostic testing in mathematics: paired questions. *Teaching Mathematics and its Applications, 24*(4), 154-163.

LMS. (1995a). *Tackling the Mathematics Problem.* London: LMS, Institute for Mathematics and its Applications, Royal Statistical Society.

LMS. (1995b). *Tackling the Mathematics Problem.* London: Joint report of the LMS, Institute for Mathematics and its Applications and the Royal Statistical Society.

LTSN. (2002). *Diagnostic Testing for Mathematics*. UK: Learning and Teaching Support Network Maths Team Project.

Mac An Bhaird, C., & O'Shea, A. (2009). Is Mathematics Support worthwhile? An overview of the 3rd Irish Workshop on Mathematics Learning and Support Centres. *MSOR Connections, 9*(2), 52-59.

McGillivray, H. (2008). Learning Support in Mathematics and Statistics in Australian Universities: A guide for the university sector. Sydney: *Australian Teaching and Learning Council*.

Ni Fhloinn, E. (2006). *Maths Diagnostic Report. Internal Report*: Dublin Institute of Technology.

Russell, M. (2005a). *Academic Success, Failure and Withdrawal Among First Year Engineering Students: Was Poor Mathematical Knowledge a Critical Factor?* Dublin Institute of Technology.

Russell, M. (2005b). "Academic Success, Failure and Withdrawal Among First Year Engineering Students: was poor mathematical knowledge a critical factor?". *Level 3, 3*.

Savage, M. D. (2003). *Tackling the Maths Problem:Is it More Extensive than We Thought? Are Solutions in Sight?* Paper presented at the Mathematical Education of Engineers IV, IMA Conference.

Smith, A. (2004). *Making Mathematics Count: The report of Professor Adrian Smiths Inquiry into Post-14 Mathematics Education*. London.

Terlouw, C., de Goede, R., & Keinhuis, M. (2008). *The effectiveness of a special mathematics course for improving the transition from secondary education to higher professional education.* Paper presented at the 7th International conference of The London Scholarship of Teaching and Learning.