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Heat Transfer Through The Boundary Layer On A Moving Cylindrical Fibre

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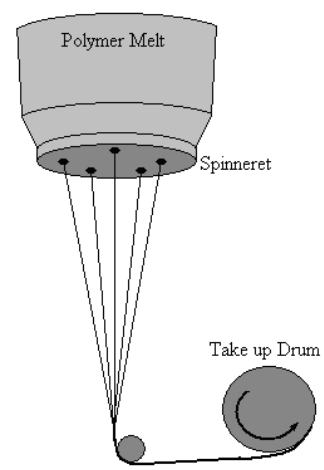
This paper applies boundary layer theory to the process of manufacturing polymer fibres known as the melt spinning process. The rate of heat loss of the fibre during this process, characterised by the local Nusselt number, is evaluated by means of a Pohlhausen integral method.

1 Introduction

The manufacturing of polymer fibres, by means of the melt spinning process, involves the extrusion of molten fibre through an orifice. The thin strands that are created are then passed through a cooling chamber before being wound onto a drum, as illustrated in Fig1. The rate of heat loss of the fibre during this process, characterised by the local Nusselt number, is of great importance in determining the overall properties of the fibre.

Fig. 1 The Melt Spinning Process



The model is set up as a boundary layer problem with the fibre treated as a continuous infinite cylinder passing through a fluid environment of infinite size. The model consists of both a thermal boundary layer and a momentum boundary layer and is solved by means of the Pohlhausen integral technique. Glauert and Lighthill[1] and Sakiadis[2] have examined the boundary layer on a stationary and moving cylinder with no temperature difference between the cylinder and the environment, respectively. Bourne and Elliston[3] have considered the case in which a cylinder is moving through still air with a temperature difference between both, thus forming a thermal boundary layer. In their paper, Bourne and Elliston[3] determine the local Nusselt number for Prandtl numbers less than one. This includes the case of a cylinder passing through air, which has a Prandtl number of 0.7. The aim of this paper is to add to the work of the aforementioned authors by looking at the case of a heated cylinder passing through fluids with Prandtl numbers in the range $1 \leq \sigma \leq 200$, which incorporates most liquids including water($\sigma = 7$) as well as more viscous liquids such as some light oils($100 \leq \sigma \leq 200$). The paper also compares the results with those of Crane[4], who has developed an exact solution for a variety of Prandtl numbers in this range.

2 Initial Observations

Bourne and Elliston[3] have used a Pohlhausen technique to determine the rate of heat transfer from a moving cylindrical fibre for Prandtl numbers in the range $0 \leq \sigma \leq 1$. This paper aims to build upon the work of Bourne and Elliston[3] by determining the Nusselt number for a range of Prandtl numbers greater than unity. The method used is similar to that of Bourne and Elliston, with the same assumptions made with respect to fibre velocity, temperature and radius. The most important difference to note is that for Prandtl numbers greater than unity the temperature boundary layer is smaller than the momentum boundary layer. Although the resulting differential equation is similar to that obtained by Bourne and Elliston, it is somewhat more difficult to obtain initial conditions for.

3 Formulation and Analysis

The boundary layer equations are

$$r\frac{\partial}{\partial x}(ru) + r\frac{\partial}{\partial r}(rv) = 0, \quad ur\frac{\partial u}{\partial x} + vr\frac{\partial u}{\partial r} = \nu\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r}), \quad ur\frac{\partial T}{\partial x} + vr\frac{\partial T}{\partial r} = \kappa\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r}) \tag{1}$$

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where (u, v) are the velocity components in the (x, r) direction, T is the temperature, ν is the kinematic viscosity and κ is the thermal diffusivity. The Pohlhausen method involves integrating the energy equation with respect to r across the thermal boundary layer to obtain the energy integral equation, given by $\frac{d}{dx}\int_0^{\infty}[u(r)(T-T_\infty)]dr = -\kappa a(\frac{\partial T}{\partial r})_{r=a}$, where T_∞ is the ambient temperature of the fluid. In order to solve this equation we must substitute in appropriate velocity and temperature profiles. The velocity and temperature profiles from Bourne and Elliston[3] are $u = U - \frac{U}{\alpha}\ln(\frac{r}{a})$ for $r \leq \delta = ae^{\alpha}$ and $\frac{T-T_\infty}{T_w-T_\infty} = 1 - \frac{1}{\beta}\ln(\frac{r}{a})$ for $r \leq \delta_T = ae^{\beta}$ where a is the fibre radius, T_w is the fibre surface temperature, α and β are dimensionless parameters in the momentum and thermal boundary layer profiles and δ and δ_T are the momentum and velocity boundary layer thickness respectively. Substituting these profiles into the energy integral equation and simplifying leads to the differential equation

$$\frac{d\beta}{d\alpha}[e^{2\beta}(2\alpha\beta - 2\beta^2 + 2\beta - \alpha - 1) + \alpha + 1] + \beta\alpha^{-1}[e^{2\beta}(\beta - 1) + \beta + 1] = 2\sigma^{-1}\beta\alpha^{-1}[e^{2\alpha}(\alpha - 1) + \alpha + 1] \quad (2)$$

where $\sigma = \frac{\nu}{\kappa}$. The initial conditions required to solve equation (4) were found by substituting the power series expansion $\{\beta = a_1\alpha + a_2\alpha^2 + a_3\alpha^3....\}$ and the Maclaurin series expansions $\{e^{2\alpha} = 1 + 2\alpha + 2\alpha^2 + \frac{4}{3}\alpha^3 + \frac{2}{3}\alpha^4 + \frac{4}{15}\alpha^5 + ...\}$ and $\{e^{2\beta} = 1 + 2\beta + 2\beta^2 + \frac{4}{3}\beta^3 + \frac{2}{3}\beta^4 + \frac{4}{15}\beta^5 + ...\}$ into the equation and then solving the resulting cubic polynomial. The coefficients a_1,a_2 and a_3 are given by $a_1 = 1 - \left[\frac{\sigma-1}{\sigma}\right]^{\frac{1}{3}}\left[\left(1 + \frac{1}{9}\left[\frac{\sqrt{2\sigma-1}}{\sigma-1}\right]^2\right) - \sqrt{\frac{3}{4}}\left(\frac{2}{3}\left[\frac{\sqrt{2\sigma-1}}{\sigma-1}\right] - \frac{10}{81}\left[\frac{\sqrt{2\sigma-1}}{\sigma-1}\right]^3\right)\right]$, $a_2 = \frac{\sigma[a_1^5 - 2a_1^4] + a_1}{\sigma[6a_1^2 - 3a_1^3] - 1}$ and $a_3 = \frac{\sigma[9a_1^6 - 15a_1^5 + a_2(45a_1^4 - 80a_1^3) + a_2^2(60a_1^2 - 75a_1)] - 6a_1 + 10a_2}{\sigma[30a_1 - 5a_1^2 - 35a_1^3] - 20}$. Equation (4) was then solved using the 4^{th} order Runge-Kutta method in the range $0.15 \leq \alpha \leq 10$. The nusselt number was determined using the formula $Nu = \frac{2\pi}{\beta}$.

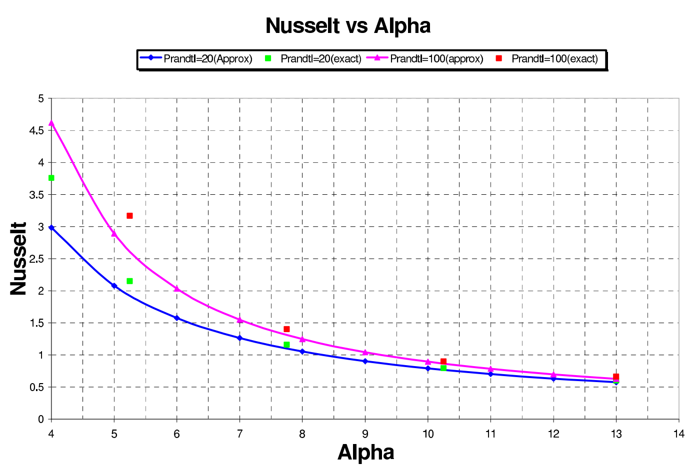
Table 1 Values of the Nusselt number for various values of α and σ

α	$\sigma = 1$	$\sigma = 7$	$\sigma = 20$	$\sigma = 50$	$\sigma = 100$	$\sigma = 200$
1	6.283185	15.4416	25.1932	39.00177	54.49424	76.43778
2	3.141593	6.273148	9.42148	13.81527	18.72783	25.64565
5	1.256637	1.730095	2.078529	2.486913	2.892411	3.417375
10	0.628319	0.73119	0.78999	0.847099	0.894926	0.947633

4 Comparison with Exact Solution

An exact solution to the above problem exists due to Crane[4] for various Prandtl numbers in the range $1 \leq \sigma \leq 100$. In Fig 2, the results from our Pohlhausen method are plotted against those of Crane[4]. From the graph we can see that there is good correlation between the approximate method used in this paper and the exact solution. The accuracy of the approximate method improves with distance from the orifice.

Fig. 2 Nusselt number vs Alpha



5 Conclusion

The results obtained from our analysis are in good agreement with those of Crane[4], as shown in Fig 2. The accuracy of the Pohlhausen method increases with increasing values of x, that is increased axial distance from the orifice. It is also important to note that for a Prandtl number equal to unity the results match up with those of Bourne and Elliston[3]. This paper has shown that the Pohlhausen method can be a powerful tool in predicting the rate of heat transfer from the surface of a cylindrical fibre moving through a stationary fluid at large distances from the orifice.

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