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# Dead-time compensators: performance and robustness issues

James Syder
*Technological University Dublin*

Thomas Heeg
*Technological University Dublin*

Aidan O'Dwyer
*Technological University Dublin*, aidan.odwyer@tudublin.ie



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# Dead-time compensators: performance and robustness issues

J. Syder, T. Heeg and A.O'Dwyer[1],
*School of Control Systems and Electrical Engineering,*
*Dublin Institute of Technology, Kevin St., Dublin 8, Ireland.*
Phone: +353-1-4024875, Fax: +353-1-4024992, E-mail: aodwyer@dit.ie

***Abstract*****:** This paper will compare a number of PID and predictive controller strategies to compensate processes modelled in first order lag plus time delay (FOLPD) form. The performance and robustness of the resulting compensated systems are evaluated analytically (where appropriate) and in simulation.

***Keywords*****:** PID, dead-time compensators, integral of squared error, stability plots, robustness plots.

## 1. INTRODUCTION

Processes with time delay (or dead-time) occur frequently in chemical, biological, mechanical and electronic systems. Many high order systems with a time delay can be approximated as an equivalent FOLPD model; one such procedure is defined by Ziegler and Nichols (1942). The transfer function representation of such a model is given as:

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m} \tag{1}$$

with $K_m$, $T_m$ and $\tau_m$ being the model gain, time constant and time delay, respectively.

The most common controller structure in process control applications is the proportional-integral-derivative (PID) controller and its variations (P, PI or PD structure). However, it has long been suggested that this controller structure is less appropriate for the control of processes with a dominant time delay. In a seminal contribution, Smith (1957) proposed a technique (subsequently labeled the Smith predictor) that facilitates the removal of the time delay term in the closed loop characteristic equation. This paper will compare the performance and robustness of the compensated system controlled using a number of PID and predictive control techniques, for the compensation of both delay dominant and non-delay dominant processes, whose parameters may vary. The following controller algorithms are considered:

1. Ideal PI controller: $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)$, $K_c$ = proportional gain, $T_i$ = integral time constant. The controller parameters are determined using the ultimate cycle method of Ziegler and Nichols (1942).
2. PID controller with filtered derivative: $G_c(s) = K_c\left(1 + \frac{1}{T_i s} + \frac{sT_d}{1 + sT_d/N}\right)$, $T_d$ = derivative time constant, N = 10. The controller parameters are determined using the ultimate cycle method of Ziegler and Nichols (1942).
3. Non-interacting PID controller - set-point weighting: $U(s) = K_c\left(b + \frac{1}{T_i s}\right)E(s) - \frac{K_c T_d s}{1 + sT_d/N}Y(s) + K_c(b-1)Y(s)$, U(s) = controller output, Y(s) = closed loop system output, E(s) = closed loop system input, R(s), minus Y(s), b = set-point weighting factor. The controller parameters are determined using the method of Hang *et al*. (1991).
4. PI compensator with filter: $G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(\frac{1 + sT_c + 0.5s^2 T_c^2}{[1 + sT_c/N]^2}\right)$. The controller parameters are determined using the method of Rad and Lo (1994).

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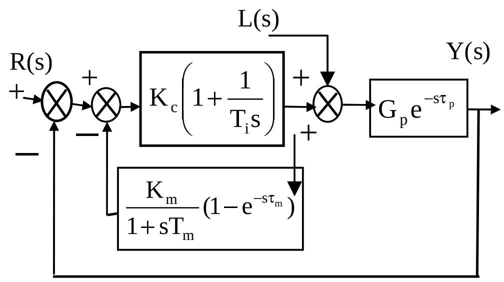
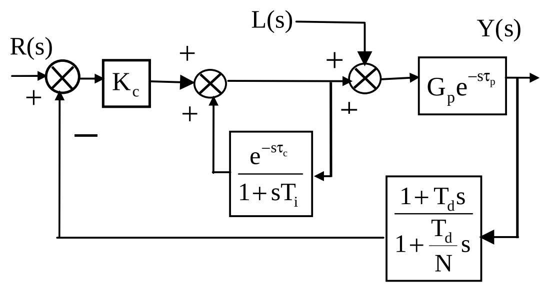
[1] Author to whom correspondence should be sent.

5. Smith predictor:




6. Dead-time compensator – Shinskey (1994):




The controller parameters for the Smith predictor may be determined using the method defined by Morari and Zafiriou (1989) or Hagglund (1992). Two dead-time compensators may be defined from the structure specified by Shinskey (1994); one compensator is defined when N = 1 ( $PI\tau$ controller) and the other when N = 10 ( $PID\tau$ controller). The open loop tuning rule (as specified by Shinskey (1994)) is used to determine the controller parameters. All of the tuning rules used will be described explicitly at the conference.

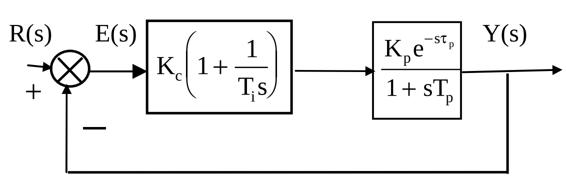
## 2. PERFORMANCE ISSUES

The performance of the compensated systems may be evaluated in a number of ways. The servo and regulator responses of compensated systems may be obtained in the time domain using MATLAB/SIMULINK; alternatively, performance indices such as the integral of absolute error (IAE) or integral of squared error (ISE) index may be determined in simulation. It is also interesting to evaluate how the ISE performance criterion, for example, varies with changes in the process parameters. More completely, an analytical calculation of the ISE or integral of time by squared error (ITSE) criteria for some compensated systems may be achieved using a method based on Parseval's theorem and contour integration (Marshall *et al.*, 1992). This method is summarised (for the calculation of the ISE criterion) as follows:

- E(s) of the system is calculated.
- It has to be proved that the system is asymptotically stable. A necessary, but not sufficient, condition for this is that the poles of E(s) lie in the open left half-plane.
- The ISE has the form: $ISE = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} E(s)E(-s)ds$. This may be determined using the method of residues.

The method is applicable when the transfer function for E(s) contains one time delay (corresponding to PI or PID compensation). If the transfer function for E(s) has more than one time delay (corresponding to predictive compensation), then the method may be used only when the time delays are commensurate (i.e. the delays are related by integer multiples). For such multiple delay problems, one possibility is to approximate the model time delay by a rational approximation (Marshall *et al.*, 1992). This issue will be discussed in more detail at the conference.

The ISE index for an ideal PI controller in series with a FOLPD process, in servo mode, is provided as a representative calculation.




(a) E(s) may be determined to be $\frac{T_i(1+T_p s)}{T_i s(1+T_p s)+K_c K_p(1+T_i s)e^{-s\tau_p}} = \frac{B(s)+D(s)e^{-s\tau_p}}{A(s)+C(s)e^{-s\tau_p}}$ (2)

(b) It may be calculated that

$$ISE = -\sum_k \operatorname{res}_{s=s_k}\left[\frac{T_i(1+T_p s)}{T_i s(1+T_p s)+K_c K_p(1+T_i s)e^{-s\tau_p}}\right]\left[\frac{T_i s(1-T_p{}^2 s^3)}{T_p{}^2 T_i{}^2 s^4 + T_i{}^2(K_p{}^2 K_c{}^2 - 1)s^2 - K_c{}^2 K_p{}^2}\right] \quad (3)$$

with $s_k$ determined from $T_p{}^2 T_i{}^2 s^4 + T_i{}^2(K_p{}^2 K_c{}^2 - 1)s^2 - K_c{}^2 K_p{}^2 = 0$.

Similar calculations are done to determine the ISE in regulator mode, and to calculate the ISE for both servo and regulator modes, for each of the PI and PID controlled systems considered. The MATHEMATICA package is used in some cases to determine the values of $s_k$, and to subsequently calculate the ISE criterion. These results will be presented at the conference.

## 3. ROBUSTNESS ISSUES

Robustness is the ability of a controller to maintain closed-loop stability in the face of variations in process parameters. One method of evaluating the robustness of a compensated system is by the determination of stability plots, which show the boundary of stability of a compensated system as the controller parameters change. These plots may be determined by simulation (using MATLAB/SIMULINK, for example) or may be approximately determined analytically. For a PI controlled FOLPD process, for example, marginal stability exists when

$$\left|G_c(j\omega)G_p(j\omega)\right| = \frac{K_p K_c}{T_i \omega}\sqrt{\frac{1+\omega^2 T_i{}^2}{1+\omega^2 T_p{}^2}} = 1 \quad (4)$$

and

$$\angle G_c(j\omega)G_p(j\omega) = \tan^{-1}\omega T_i - \omega\tau_p - \tan^{-1}\omega T_p - 0.5\pi = -\pi \quad (5)$$

Approximating $\tan^{-1} x$ as $0.25\pi x$, $|x| \le 1$ and $\tan^{-1} x$ as $0.5\pi - (0.25\pi / x)$, $|x| > 1$, $\omega$ may be determined from either

$$\left[\frac{4\tau_p T_i T_p}{\pi}\right]\omega^3 - \left[2T_i T_p\right]\omega^2 + \left[\frac{4\tau_p - \pi T_i + \pi T_p}{\pi}\right]\omega - 2 = 0 \quad (6)$$

or

$$\left[\tau_p(T_i - T_p) + 0.25\pi T_i T_p\right]\omega^2 - \left[\pi(T_i - T_p)\right]\omega + 0.25\pi = 0 \quad (7)$$

Thus knowing $T_p$ and $\tau_p$, a value for $T_i$ is chosen and $\omega$ is determined. $K_c$ may then be calculated from (4) i.e. $K_c = \frac{T_i \omega}{K_p}\sqrt{\frac{1+\omega^2 T_p{}^2}{1+\omega^2 T_i{}^2}}$.

The MATHEMATICA package may be used to solve for $\omega$ for each of the PI and PID controllers considered, and to draw the stability plot. A typical stability plot is shown in Figure 1; here, the process $G_p(s) = 2e^{-1.4s}/(1+0.7s)$ is compensated using an ideal PI controller.

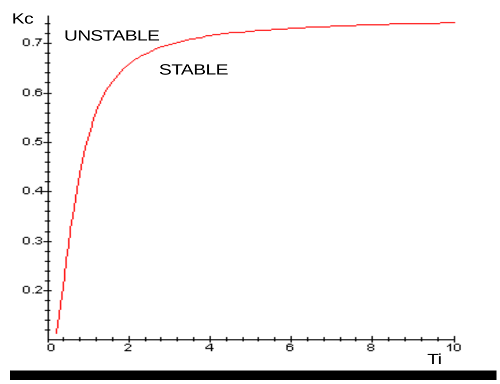



Figure 1: Typical stability plot

Three dimensional stability plots may be plotted when a process is compensated with a PID controller.

An alternative method of evaluating the robustness of the compensated systems is by the determination of robustness plots (as described by Shinskey (1990)), which show the boundary of stability of a compensated system as the process parameters change. These plots may be determined by simulation (using MATLAB/SIMULINK, for example) or may be calculated analytically. For a PI controlled FOLPD process, for example, marginal stability exists when $\left|G_c(j\omega)G_p(j\omega)\right| = 1$ (as above); $\omega$ may subsequently be determined from $\left[T_i{}^2 T_p{}^2\right]\omega^4 + \left[T_i{}^2 - K_c{}^2 K_p{}^2 T_i{}^2\right]\omega^2 - K_c{}^2 K_p{}^2 = 0$. This equation may be reduced to a quadratic equation, from which $\omega$ may be determined to be

$$\omega = \left[\frac{-(1-K_c{}^2 K_p{}^2)T_i + \left\{(1-K_c{}^2 K_p{}^2)^2 T_i{}^2 + 4K_c{}^2 K_p{}^2 T_p{}^2\right\}^{0.5}}{2T_i T_p{}^2}\right]^{0.5} \quad (8)$$

Therefore, knowing $K_c$ and $T_i$, and letting the time constant, $T_p$, be constant, a value of $K_p$ may be chosen and $\omega$ determined. $\tau_p$ may then be calculated by substituting $\angle G_c(j\omega)G_p(j\omega) = -\pi$, giving

$$\tau_p = \frac{1}{\omega}\left[0.5\pi + \tan^{-1}\left\{\frac{\omega(T_i - T_p)}{1+\omega^2 T_i T_p}\right\}\right] \tag{9}$$

The MATHEMATICA package may be used to solve for $\omega$ for each of the PI and PID controlled systems considered and to draw the robustness plot. A typical robustness plot is shown in Figure 2, when process $G_p(s) = 1.75e^{-12s}/(1+8s)$ is compensated using an ideal PI or PID controller, whose parameters are chosen according to the ultimate cycle tuning rules of Ziegler and Nichols (1942). The delay ratio equals the time delay divided by the time delay for which the controller was tuned. The gain ratio equals the gain divided by the gain for which the controller was tuned.

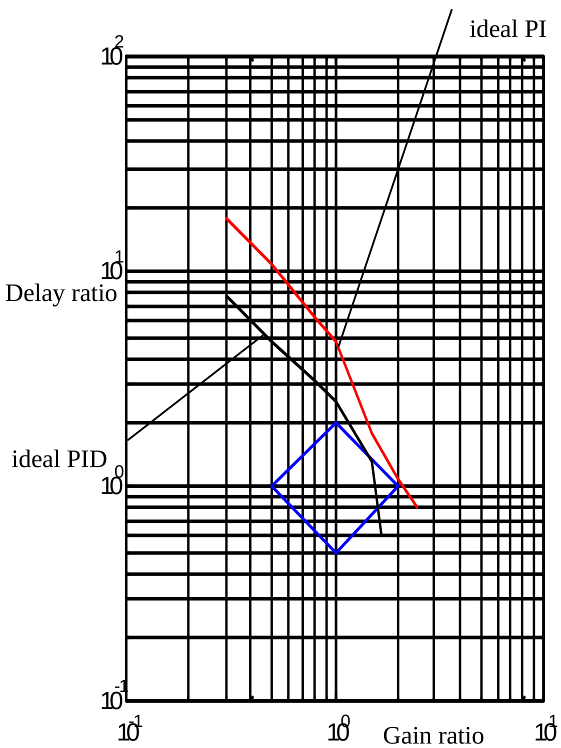



Figure 2: Typical robustness plot

The two lines in the figure represent the limit of stability for the PI and PID controller. To the right of the indicated lines (at higher gain ratios), the closed-loop system is unstable and to the left of them, the closed-loop system is stable. The upper-right side of the parallelogram represents the locus of all products of delay and gain ratio equalling 2.0; similarly, the lower-left side represents all products equalling 0.5. The other two sides represent all products of the two ratios equalling either 2.0 or 0.5. If the stability limit for a control loop stays outside of this window, that loop is considered to be robust (Shinskey, 1990). Of course, processes described by three parameters cannot be represented on this two dimensional surface. It is possible to produce a three dimensional plot with time constant ratio as the third dimension; simulation results have shown, however, that changes in time constant ratio have little effect on the nature of the robustness plots. Therefore, it is appropriate to let the time constant of the FOLPD model be constant.

## 4. RESULTS

Space considerations dictate that only representative simulation results may be provided. Three processes and their corresponding models are considered:

(A) $G_p e^{-s\tau_p} = G_m e^{-s\tau_m} = 2e^{-7s}/(1+0.7s)$ [i.e. $\tau_m/T_m = 10$ … strongly dominant delay]

(B) $G_p e^{-s\tau_p} = G_m e^{-s\tau_m} = 2e^{-1.4s}/(1+0.7s)$ [i.e. $\tau_m/T_m = 2$ … weakly dominant delay] and

(C) $G_p e^{-s\tau_p} = 2e^{-s}/(1+8.5s+22.5s^2+18s^3), G_m e^{-s\tau_m} = 1.82e^{-3.47s}/(1+7.68s)$ [i.e. $\tau_m/T_m = 0.45$ … dominant time constant]. This model for the process is determined from least squares fitting in the frequency domain (O'Dwyer, 1996).

Table 1 shows the ISE values determined in simulation using MATLAB/SIMULINK for a number of compensated systems.

Table 1: Determination of ISE criterion for some compensated systems

| | **ISE (servo mode)** | **ISE (regulator mode)** |
|---|---|---|

| **Process/Model** | A | B | C | A | B | C |
|---|---|---|---|---|---|---|
| Ideal PI – Ziegler and Nichols (1942) | 13.13 | 2.31 | 5.23 | 50.89 | 7.46 | 5.89 |
| PID – filtered D – Ziegler and Nichols (1942) | unstable | 1.66 | 3.35 | unstable | 4.14 | 2.26 |
| PID – setpoint weighting – Hang *et al.* (1991) | unstable | 1.81 | 4.71 | unstable | 3.63 | 2.25 |
| PI with filter – Rad and Lo (1994) | 8.37 | 1.90 | 4.08 | 31.47 | 5.61 | 3.34 |
| Smith predictor – Morari and Zafiriou (1989) | 7.50 | 1.90 | 3.55 | 28.02 | 5.78 | 3.75 |
| Smith predictor – Hagglund (1992) | 7.35 | 1.75 | 6.46 | 27.30 | 5.09 | 14.65 |
| PI$\tau$ controller – Shinskey (1994) | 7.37 | 1.65 | 6.17 | 27.39 | 4.17 | 2.79 |
| PID$\tau$ controller – Shinskey (1994) | 7.34 | 1.67 | 4.39 | 26.21 | 3.14 | 1.50 |

These results show that, as expected, the predictive controllers facilitate better performance than the PI/PID based controllers for processes with a strongly dominant delay. For processes with a less dominant delay (Process/Model B) the performance of some of the PID based controllers can be better that of the predictive controllers; the PID$\tau$ controller of Shinskey (1994) still allows very good performance in both servo and regulator mode. For processes with a non-dominant delay when modelled in FOLPD form, it is clear that the servo performance is best when the PID controller with filtered derivative is used, though the PID$\tau$ controller still facilitates the best regulator performance. Of course, the numerical value of the ISE index will change with the method of choosing the controller parameters. The results show, however, that it can be appropriate to use a PID controller to compensate dominant delay processes in some cases and that the PID$\tau$ controller may be indicated if good regulator performance is desired, for both dominant and non-dominant delay processes.

The variation of the ISE criterion as time delay changes has also been considered; one representative simulation result is provided in Figures 3 and 4. Process/Model B above is used and the time delay is varied from 0 to 2.8 seconds.

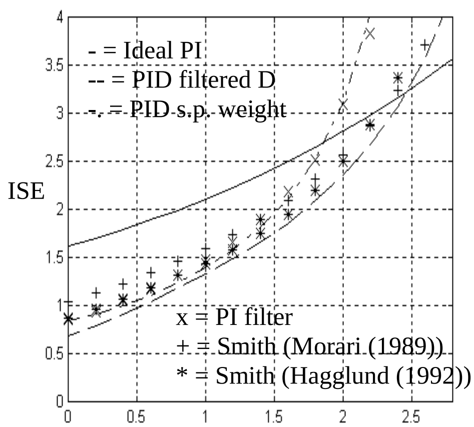



Figure 3: ISE (servo mode) vs. time delay

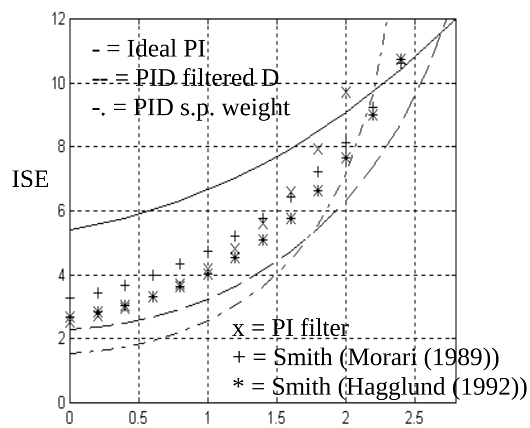
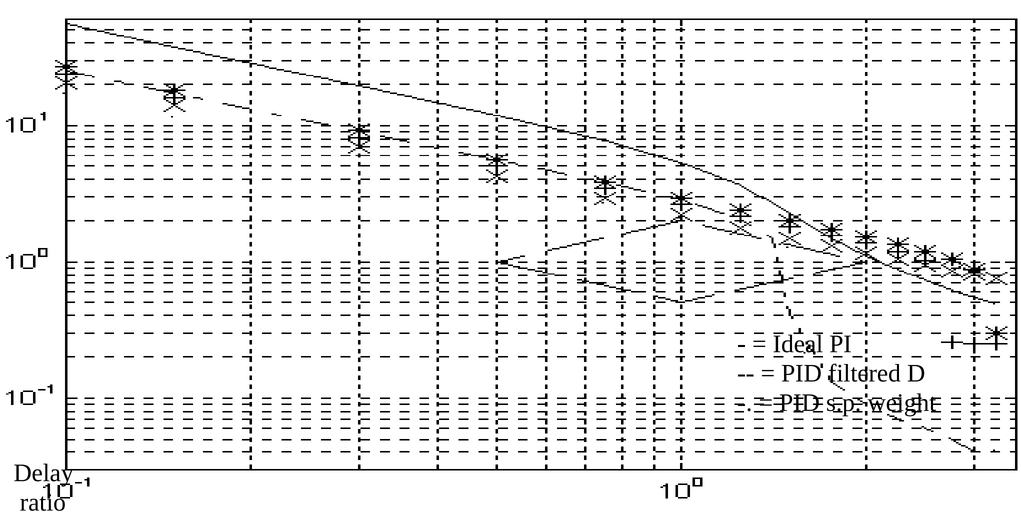



Figure 4: ISE (regulator mode) vs. time delay



As expected, the tuned PI controller tends to facilitate the greatest degree of robustness.

## 5. CONCLUSIONS

This paper has compared predictive and PID compensation strategies, using appropriate tuning rules, and has analysed methods to evaluate performance and robustness of these strategies, when applied to the control of FOLPD process models. Performance may be evaluated in simulation or, where possible, by analytically calculating the ISE or ITSE criterion. Robustness may be usefully evaluated using stability plots, as described by the authors, or by using the robustness plots described by Shinskey (1990). The performance and robustness design requirements, as well as factors such as the ratio of time delay to time constant, will determine the most appropriate compensator strategy to use. Further work will concentrate on the analytical determination of the performance indices for systems compensated using predictive controllers.

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