2002-01-01

Time delay estimation in signal processing applications: an overview

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Abstract: An extensive though scattered literature exists on the estimation of time delays in signal processing applications. However, it is possible to identify themes that are common to many of the available techniques. The intention of this paper is to provide a framework against which the literature may be viewed. In addition, original work on gradient based time delay estimation is described.

Keywords: Time delay, estimation, gradient algorithms.

1. Introduction

A time delay may be defined as the time interval between the start of an event at one point in a system and its resulting action at another point in the system. Delays, also known as transport lags, dead times or time lags, arise in physical, chemical, biological and economic systems, and in the process of measurement and computation. Delays also arise in signal processing, where a time delay is also known as a time difference of arrival between two signals; such a measurement arises in underwater tracking applications, biomedicine, geophysics, astronomy, acoustics, seismology and telecommunications. Quite often in these applications, the time delay is estimated in the absence of other process parameters.

Section 2 of this paper will briefly survey delay estimation methods, focusing on those that have appeared in leading signal processing theory and applications journals. Some analytical and simulation work on a gradient based delay estimation method is described in Section 3. Conclusions and references are subsequently provided.

2. Delay estimation methods

Gradient methods of time delay estimation are based on updating the delay by a vector that depends on information about the cost function to be minimised. The gradient algorithms normally involve expanding the cost function as a second order Taylor's expansion around the estimated delay. The choice of gradient algorithm for an application depends on the desired speed of tracking and the computational resources available. It is important that the error surface in the direction of the delay should be unimodal if a gradient algorithm is to be used successfully. However, the error surface is often multimodal. In these circumstances, strategies for locating global minima may involve multiple optimisation runs, each initiated at a different starting point with the starting points selected by sampling from a uniform distribution [1]. The global minimum is then the local minimum with the lowest cost function value among all the local minima identified. Gradient algorithms based on the Gauss-Newton, steepest descent and least mean squares (LMS) method have been defined [2-35].

Alternatively, the cross-correlation of two signals may be used to estimate the time delay between the two signals, as the time at which the cross-correlation term is maximised corresponds to the time delay estimate. Most cross-correlation methods, and variations of the method, are off-line in nature [23, 36-74], though some approaches have on-line potential [75]. Other authors use the technique to estimate time delays in MIMO environments or between multiple sensors and multiple targets [76-81]. Other authors also use power spectral density techniques to calculate the time delay [82-84]. The time delay may also be determined from a higher order spectral approach [85], though the use of these techniques are suited to a restrictive range of problems, in which noise signals on the input and output to the process cannot be effectively dealt with by pre-processing.

Finally, other off-line delay estimation algorithms have been defined; one example is discussed by Kenefic [86], in which the time delay between two sensors may be found by determining the maximum of the probability density function of the delay from a given prior distribution. Other off-line time delay estimation algorithms are also discussed [87-121]. On-line implementation of non-cross correlation based algorithms has also received attention [122-131]; Blackowiak and Rajan [128], for example, investigate the performance of a simulated annealing algorithm in the estimation of the amplitude scaling factors and the time delays of the separate arrivals in a signal composed of closely spaced arrivals with added noise. The method is particularly interesting as the cost function to be minimised has local minima that make the application of calculus based minimisation techniques (such as the gradient algorithms) difficult; the simulated annealing algorithm has the ability to slide through local minima.

3. Open loop gradient method for delay estimation

This paper will consider further the strategy proposed by Durbin [5], in which the process delay variation from the model delay is approximated by a rational polynomial, and a Gauss-Newton gradient descent algorithm is used to estimate the delayed model parameters. A previous paper (O'Dwyer and Ringwood [132]) has shown that the first order Taylor's series polynomial is the most appropriate choice of rational polynomial; this paper has also provided a proof of the convergence of the non-delay model parameters to the non-delay process parameters, when the process and model delays are equal, in the presence of uncorrelated noise. This proof is labelled Theorem 0 [133]. An outline proof of the convergence of the delay estimate will be provided in this paper; the full proof of the theorem (labelled Theorem 1), and associated simulation work, is available [134].

3.1 The delay as an integer multiple of the sample period

Theorem 1: For a first order discrete stable system of known non-delay parameters, the mean of the product of the errors (MPE) performance surface versus model delay index is unimodal, with a minimum value of the MPE occurring when the model delay index equals the process delay index, under the conditions indicated below. The delay index is the delay divided by the sample time.

(a) The delay variation is approximated by a first order Taylor's series approximation.
(b) The noise is uncorrelated with the process input.
(c) The resolution on the process delay is assumed to be equal to one sample period.
(d) The error is calculated based on using a first order process model; the partial derivative of the error with respect to the delay variation is calculated based on using the first order Taylor's series approximation for the delay variation.
(e) The process delay index is greater than the model delay index, as the model delay index converges.
(f) The input signal to the process and the model allows the fulfilment of the necessary conditions for unimodality provided in the theorem.

Proof: The process difference equation, $y_2(n)$, based on using a first order process model, is ([134]):

$$y_2(n) = e^{-T_s/T} y_2(n-1) + K(1 - e^{-T_s/T})u(n - g_p - 1) + w(n) \quad (3.1.1)$$

with process time delay, $\tau_p = g_p T_s$, T_s = sample period, g_p = process delay index, u(n) = input and w(n) = noise. The model difference equation, assuming that the previous process output is used in its calculation and g_m = model delay index, is

$$y_{m3}(n) = e^{-T_s/T} y_2(n-1) + K(1 - e^{-T_s/T})u(n - g_m - 1) \quad (3.1.2)$$

Therefore, from equations (3.1.1) and (3.1.2),

$$e_3(n) = y_2(n) - y_{m3}(n) = K(1 - e^{-T_s/T})[u(n - g_p - 1) - u(n - g_m - 1)] + w(n) \quad (3.1.3)$$

The partial derivative of the error with respect to the delay variation may then be calculated by using a first order Taylor's series approximation for the delay variation. The corresponding model difference equation, assuming the previous process output is used in its calculation, is ([134])

$$y_{m2}(n) = e^{-T_s/T} y_2(n-1) - \frac{K(g_p - g_m)T_s}{T} u(n - g_m) - K(e^{-T_s/T} - 1 - \frac{(g_p - g_m)T_s}{T})u(n - g_m - 1) \quad (3.1.4)$$

Therefore, from equations (3.1.1) and (3.1.4),

$$e_2(n) = y_2(n) - y_{m2}(n) =$$

$$K(1 - e^{-T_s/T})u(n - g_p - 1) + \frac{K(g_p - g_m)T_s}{T} u(n - g_m) + K(e^{-T_s/T} - 1 - \frac{(g_p - g_m)T_s}{T})u(n - g_m - 1)] + w(n) \quad (3.1.5)$$

The corresponding partial derivative is

$$\frac{\partial e_2(n)}{\partial (g_p - g_m)} = \frac{KT_s}{T}\left[u(n - g_m) - u(n - g_m - 1)\right] \quad (3.1.6)$$

The update vector for updating the model delay, which depends on the product of $e_3(n)$ and $\partial e_2(n) / \partial (g_p - g_m)$, is then independent of g_p. The cost function that corresponds to this update vector is the MPE function; this function is defined as $E[e_2(n)e_3(n)]$ in this case. The MPE performance surface, $E[e_2(n)e_3(n)]$, may then be calculated to be ([134]):

$$2K^2(1 - e^{-T_s/T})^2[r_{uu}(0) - r_{uu}(g_p - g_m)] + K^2(1 - e^{-T_s/T})\frac{(g_p - g_m)T_s}{T}[r_{uu}(0) - r_{uu}(1) + r_{uu}(g_p - g_m + 1)]$$

$$- K^2(1 - e^{-T_s/T})\frac{(g_p - g_m)T_s}{T}[r_{uu}(g_p - g_m)] + r_{ww}(0) \quad (3.1.7),$$

$r_{uu}(n)$ and $r_{ww}(n)$ being the autocorrelation functions of $u(n)$ and $w(n)$ respectively. Therefore,

$$E[e_2(n)e_3(n)] = r_{ww}(0) \text{ for } g_m = g_p.$$

It may be shown by comparing the sizes of the individual terms in equation (3.1.7) that $E[e_2(n)e_3(n)] > r_{ww}(0)$ for $g_p > g_m$ only ([134]). Thus, the minimum value of $E[e_2(n)e_3(n)]$ occurs at $g_m = g_p$ (when g_m is restricted to be less than or equal to g_p) and the noise has no effect on the estimated process delay value. If $g_p > g_m$, then, from equation (3.1.7), the only situation that arises for which $E[e_2(n)e_3(n)] = r_{ww}(0)$ for

$g_m \neq g_p$ is when the input has a flat autocorrelation function, which corresponds to a constant level input. Thus, any input change is sufficient for correct process delay index estimation, provided that the required condition on g_m is fulfilled, if the delay index is estimated by determining the minimum of the MPE performance surface.

However, if a gradient method is used to estimate g_p, then an additional restriction that the MPE function must be unimodal for $g_p > g_m$, with a minimum MPE value occurring at $g_m = g_p$, is imposed. The unimodality of the MPE function for $g_p > g_m$ may be proved by induction; an outline of the inductive proof (provided in full in [134]) is as follows:

It may be proved that the MPE function at $g_m = g_p - 1$ is greater than the MPE function at $g_m = g_p$ (using equation (3.1.7)), provided that

$$\left[2(1-e^{-T_s/T}) + \frac{(g_p - g_m)T_s}{T}\right] [r_{uu}(0) - r_{uu}(g_p - g_m)] > \frac{(g_p - g_m)T_s}{T} [r_{uu}(1) - r_{uu}(g_p - g_m + 1)] \quad (3.1.8)$$

It may also be proved that the MPE function at $g_m = g_p - n - 1$ is greater than the MPE function at $g_m = g_p - n$, provided that

$$2(1-e^{-T_s/T})[r_{uu}(g_p - g_m) - r_{uu}(g_p - g_m + 1)] + \frac{T_s}{T}[r_{uu}(0) - r_{uu}(1)]$$

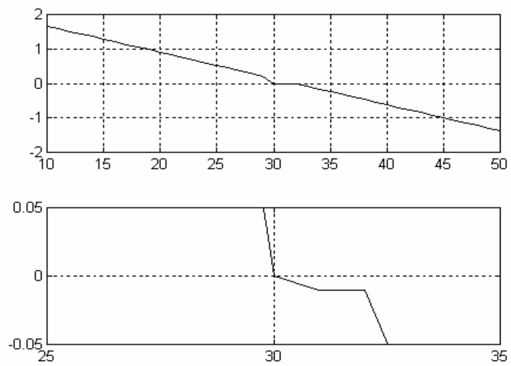
$$+\frac{T_s}{T}[(g_p - g_m)r_{uu}(g_p - g_m) - (2g_p - 2g_m + 1)r_{uu}(g_p - g_m + 1) + (g_p - g_m + 1)r_{uu}(g_p - g_m + 2)] > 0 \quad (3.1.9)$$

Both of the conditions in equations (3.1.8) and (3.1.9) are fulfilled by many excitation signals e.g. a white noise signal or a square wave signal ([134]). □

The behaviour of the MPE function (equation (3.1.7)) versus model delay index is confirmed by Figures 1 and 2, in representative simulation results. For these simulations, $K = 2.0$, $T = 0.7$ seconds and $g_p = 30$. The normalised MPE (= $MPE / r_{uu}(0)$) is plotted versus model delay index; $r_{ww}(0) = 0$. The plots show that the MPE surface is greater than $r_{ww}(0)$ for $g_p > g_m$ only, and that when the conditions in equations (3.1.8) and (3.1.9) are fulfilled, the MPE function is unimodal for $g_p > g_m$, with a minimum MPE value occurring at $g_m = g_p$.

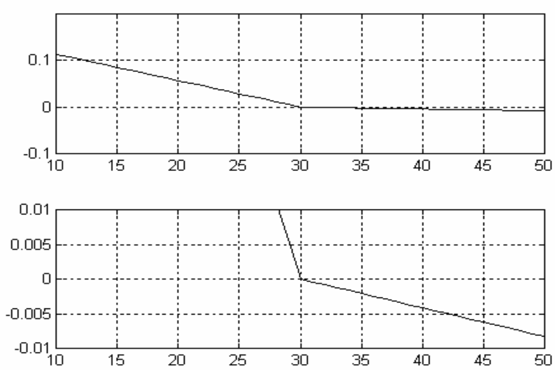
A representative simulation result corresponding to Theorem 1 is given in Figures 3 and 4. The starting values of the process and model delay index were both equalised; a step change was then made to the process delay index. The parameters K and T were put equal to 2.0 and 0.7 seconds, respectively (as above). The Levenberg-Marquardt gradient algorithm [135] was used to update the model delay index; the sample time is 0.1 seconds. Coloured noise, generated by low-pass filtering a white noise signal, was added. The model delay index was limited in variation to one sample period per iteration; such filtering was found to be desirable in simulation. Good convergence to the process delay index is seen for $g_p > g_m$. Other supplementary simulation results show no convergence to the process delay index when $g_p < g_m$. This verifies Theorem 1. The error, $e_3(n)$, in Figures 3b and 4b is non-zero due to the presence of the coloured noise.

Figure 1: Normalised MPE vs. time delay index g_m - white noise input



Model time delay index

Figure 2: Normalised MPE vs. time delay index, g_m - square wave input



Model time delay index

Figure 3a: Time Delay Index Estimate- white noise excitation

Figure 4a: Time Delay Index Estimate- square wave excitation

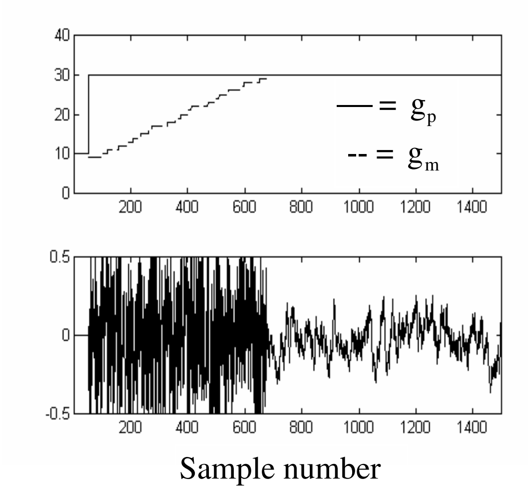


Figure 3b: Process minus Model output

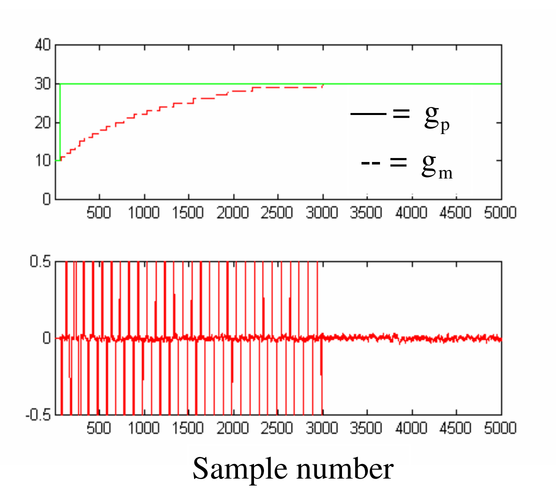


Figure 4b: Process minus Model output

3.2 The delay as a real multiple of the sample period

Theorem 1 dealt with the estimation of delays that are integer multiples of the sample period. For the estimation of delays that are real multiples of the sample period, the process difference equation is ([134]):

$$y_3(n) = e^{-T_s/T} y_3(n-1) + K(1-e^{g_b T_s/T})u(n-g_p) + K(e^{g_b T_s/T} - e^{-T_s/T})u(n-g_p-1) + w(n) \quad (3.2.1)$$

with g_b = process delay minus the process delay index. The corresponding model difference equation (assuming the previous process output is used in its calculation) is

$$y_{m4}(n) = e^{-T_s/T} y_3(n-1) + K(1-e^{g_a T_s/T})u(n-g_m) + K(e^{g_a T_s/T} - e^{-T_s/T})u(n-g_m-1) \quad (3.2.2)$$

with g_a = model delay minus model delay index. The model difference equation for calculating the partial derivative of the error with respect to the delay variation (and assuming that the previous process output is used in its calculation) is ([134]):

$$y_{m5}(n) = e^{-T_s/T} y_3(n-1) - \frac{K(g_p - g_m + g_b - g_a)T_s}{T} u(n-g_m) - K[e^{-T_s/T} - 1 - \frac{(g_p - g_m + g_b - g_a)T_s}{T}]u(n-g_m-1)$$

The MPE performance surface, $E[e_4(n)e_5(n)]$, may be obtained in a similar manner to the development outlined in equations (3.1.5) to (3.1.7), with $e_4(n) = y_3(n) - y_{m4}(n)$ and $e_5(n) = y_3(n) - y_{m5}(n)$. It may be shown that $E[e_4(n)e_5(n)] = r_{ww}(0)$ if $g_p = g_m$ and $g_b = g_a$ ([134]). Simulation results show that the MPE function versus model delay is multimodal when the delay is a real multiple of the sample period ([134]). The estimation of the real value of the process time delay, using the approach, is impossible using gradient methods.

4. Conclusions

This paper has surveyed a wide variety of methods for time delay estimation in signal processing applications. It is the hope of the author that the paper will provide a convenient reference for application work. The conclusions of the paper are that new design techniques have been accumulating, each claiming that it is the best suited for the application. In general, there is a lack of comparative analysis with other design techniques; associated with this is the lack of benchmark examples for testing the different methods. The main priority for future research should be a critical analysis of available design methods.

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