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The Identification and Control of Processes with Time Delays

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Abstract

The identification of the frequency response and process model parameters of systems with time delay using frequency domain techniques is evaluated in this paper. The performance of both Fourier Transform and Power Spectral Density techniques is considered. The benefits of employing modern model based design and delay compensation techniques, based on the identified process data, are assessed

Keywords: Frequency Domain Identification, Time Delay Systems, Optimal Control

1. Introduction

In recent years there has been much interest in the identification and control of systems with time delays. Although traditional self-tuning control schemes employ time domain techniques for process estimation, the use of frequency domain identification techniques has distinct advantages for time delay systems. The process time delay can readily be extracted from the process phase data and the non-delay process parameters computed. This paper compares the use of Fourier Transform and Power Spectral Density estimation methods in identifying the model parameters for a time delay process. Comparative results are presented to assess the performance of modern model based control methods with respect to those based on conventional controller tuning techniques.

The frequency domain identification techniques under consideration are presented in Section 2 of the paper. Section 3 discusses the analytical calculation of the parameters of a First Order Lag Plus Delay (FOLPD) model using the identified frequency response data. Techniques for model based delay compensation are also discussed. The classical and modern control laws considered are given in Section 4 and simulation results are presented in Section 5. The main conclusions are summarised in Section 6.

2. Process Identification using Frequency Domain Techniques

2.1 Discrete Time Fourier Transform

The process frequency response, $G(j\omega)$, can be identified recursively from the process input/output data ($u(t)/y(t)$), using the Discrete Time Fourier Transform (DTFT), as:

$$G(j\omega) = {}^{U(j\omega)}\!\big/_{Y(j\omega)} \qquad (1)$$

where: $$F(j\omega) = T\sum_{k=0}^{\infty} f(kT)e^{-j\omega kT}$$

2.2 Power Spectral Density

As an alternative to the DTFT approach to frequency domain identification, spectral analysis methods may be used. In the open-loop case the process frequency response is obtained as:

$$G(j\omega) = {S_{yu}(j\omega)}\Big/{S_u(j\omega)} \tag{2}$$

where:
$$S_{yu} = \int_{-\infty}^{\infty} R_{ry}(\tau)e^{-j\omega\tau}d\tau \tag{3}$$

$$S_u = \int_{-\infty}^{\infty} R_u(\tau)e^{-j\omega\tau}d\tau \tag{4}$$

with:
$$R_{yu}(\tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} y(t)u(t+\tau)dt \tag{5}$$

$$R_u(\tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} u(t)u(t+\tau)dt \tag{6}$$

3. Analytical Estimation of the Process Parameters and Delay Compensation

3.1 Process Parameters

Assuming a First Order Lag Plus Delay process model defined as:

$$\mathrm{G(s) = G_m e^{-st} = \frac{K}{T_c s+1}e^{-st}} \tag{7}$$

The model parameters can be estimated from the identified frequency response data as follows:

$$K = \frac{|G(j\omega_1)||G(j\omega_2)|\sqrt{\omega_2^2-\omega_1^2}}{\sqrt{|G(j\omega_2)|^2\omega_2^2-|G(j\omega_1)|^2\omega_1^2}} \tag{8}$$

$$T_c = \frac{1}{\omega}\sqrt{(\frac{K^2}{|G(j\omega)|^2}-1)} \tag{9}$$

$$\tau = \frac{1}{\omega}\left[\phi(j\omega) - \tan^{-1}(\omega T_c)\right] \tag{10}$$

Calculation of the parameters of an arbitrary order model is discussed in *O'Dwyer 1996.* To minimise the sensitivity of the process parameters estimates with respect to changes in the frequency domain data:

i. Select the lowest practicable frequency for ω_1 and the highest practicable frequency for ω_2. The frequencies ω_1 and ω_2 should be at least a decade apart.

ii. Calculate T_c from a magnitude in the range $0.25K<magnitude<0.75K$.

iii. Calculate τ using data corresponding to the magnitude of the response $<0.5K$.

3.2 Delay Compensation

Two methods are considered:

i) Smith Prediction

A Smith predictor structure takes the form shown in Figure 1 whereby the time delay is effectively removed from the control loop such that the design techniques described in section 4 are applied to the delay free process dynamics for design of the controller, C.

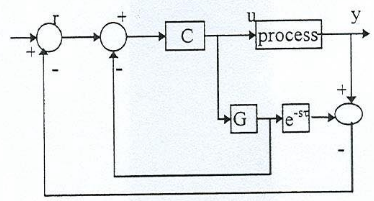


Figure 1: Smith Predictor Compensator

ii) Padé Approximation

The phase effect of a time delay on the process response can be approximated by an allpass network with a phase lag proportional to frequency (*Marshall 1979*). This delay approximation can be incorporated in the design of model based controller design methods such as the Linear Quadratic Gaussian method which is discussed in Section 4. The Padé approximants considered in this paper are summarised in Table 1.

First Order	*Second Order*	*Third Order*
$\frac{1 - sT/2}{1 + sT/2}$	$\frac{1 - sT/2 + (sT)^2/12}{1 + sT/2 + (sT)^2/12}$	$\frac{1 - sT/2 + (sT)^2/10 - (sT)^3/120}{1 + sT/2 + (sT)^2/10 + (sT)^3/120}$

Table 1: Padé Approximants for $e^{-s\tau}$

4. Controller Design

Two approaches to controller design are taken. The first directly uses the identified frequency response information to design a PID controller according to classical tuning rules. The second approach takes advantage of the availability of the process model parameters in the minimisation of an optimisation criterion (Linear Quadratic Gaussian). This latter approach has advantages for on-line controller updating in a self-tuning scenario.

4.1 PID Controller Design using Tuning Conventional Rules

The ultimate frequency can be found by iteration with respect to the evaluation frequency of the Fourier transform (*Ringwood and O'Dwyer 1994*) according to the equation:

$$\omega_{i+1} = \omega_i - \partial \frac{\omega_i - \omega_{i-1}}{\phi_i - \phi_{i-1}}(\pi + \phi_i) \tag{11}$$

with $0<\delta\leq 1$ (δ is considered as an uncertainty factor). This approach uses a straight line fit between two data points to extrapolate to the ultimate frequency. Classical ultimate cycle tuning rules such as Zeigler-Nichols can then be employed for controller design without the need for model parameter calculation.

4.2 LQG Controller Design

The non-delay portion of the plant is described by the polynomial transfer function in s, G(s) = B(s)/A(s). The low frequency process noise/ disturbance dynamics are assumed to be generated by a white Gaussian noise source coloured by the filter C(s)/A(s). The LQG cost function to be minimised is:

$$J = \int_0^{\infty}\{Q\Phi_{ee} + R\Phi_{uu}\}ds \tag{12}$$

where $\Phi_{xx}(s)$ is the power spectral density of a random function x(s). Φ_{ee} and Φ_{uu} denote the error and control spectral densities respectively and the cost function weightings Q(s), R(s) are the polynomial transfer functions:

$$Q(s) = (B^*_q(s)B_q(s))/(A^*_q(s)A_q(s)) \tag{13}$$
$$R(s) = (B^*_r(s)B_r(s))/(A^*_r(s)A_r(s)) \tag{14}$$

where the adjoint operator (*) in the Laplace domain is defined by: x*(s) = x(-s). The cost function weighting polynomials are designed to encapsulate the inevitable trade off between the output tracking error and control effort. The LQG controller which minimises the cost function (*Grimble 1985*) is:

$$C_0 = GA_r/HA_q \tag{15}$$

where the spectral factor D_c is the solution to:

$$D^*_cD_c = (BA_rB_q)^*BA_rB_q + (AB_rA_q)^*AB_rA_q \tag{16}$$

and the polynomials G,H and F are the minimal degree solutions with respect to F of the coupled Diophantine equations:

$$D^*_cG + FAA_q = (BA_rB_q)^*B_qC^* \tag{17}$$
$$D^*_cH - FBA_r = (AB_rA_q)^*B_rC^* \tag{18}$$

5. Results

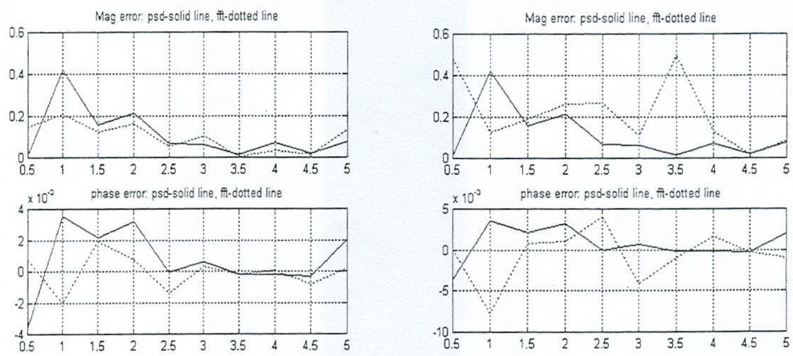
5.1 Simulation Model

A number of simulations were performed covering a range of process dynamics and time delays. A common rule of thumb is that PID controllers are suitable for control of a FOLPD process if $0.1 \leq \tau / T_c \leq 1.0$ (*O'Dwyer 1996a*). Hence illustrative results are considered in this paper for a FOLPD process with dynamics given by $G_m(s) = 2/(1+s)$ for the two cases:

Case 1: $\tau = 0.5$ seconds (small delay)
Case 2: $\tau = 3$ seconds (large delay)

5.2 Identification

Extensive identification tests were performed for both the Fourier Transform (FT) and Power Spectral Density (PSD) identification methods. Tests were carried out with and without noise, which was simulated using a random number generator (numbers from zero to one) multiplied by a scale factor and added to the process input signal. A comparison of the per cent magnitude estimation error and absolute phase estimation error for both the FT and PSD methods is shown in Figure 3 with a) no process noise and b) high process noise (scale factor 2). While it is more computationally intensive, simulation results consistently reveal the superiority of the PSD approach over the FT method with respect to insensitivity to even very high levels of noise.



a) identification without noise

b) Identification with noise (noise scale factor 2)

Figure 2: Frequency Domain Identification of $G = 2.e^{-0.5t}/(1 + s)$

Table 2 below summarises the model parameter estimates obtained (using the analytical estimation described in section two) for various levels of process noise (indicated by the noise scale factor used in the simulation). The results highlight the robustness of the PSD method with respect to process noise.

noise scale factor	FFT Identification			PSD Identification		
	K	T_c	τ	K	T_c	τ
0.1	1.9949	0.9952	0.5002	1.9998	0.9972	0.5005
0.2	1.9938	0.9943	0.5003	1.9998	0.9972	0.5005
0.5	1.9909	0.9939	0.5003	1.9998	0.9972	0.5005
1	1.9897	0.9939	0.5002	1.9998	0.9972	0.5005
2	1.9883	0.9960	0.5	1.9998	0.9972	0.5005
5	1.9620	0.9882	0.5005	1.9998	0.9972	0.5005
10	1.9686	1.0229	0.5008	1.9998	0.9972	0.5005

Table 2: Parameter Estimation Results

5.3 Control Results

Case 1: 0.5 second delay

Since the identification tests show the PSD estimation to be more robust in the presence of noise, the process model identified using this approach (Table 2) is used as the basis for controller and delay compensator design throughout this section. Hence, for the LQG design method presented in section 4:

$$A(s) = 0.9972s+1 \qquad B'(s) = 1.9998$$

The error and control weighting functions were selected as:

$$A_q(s) = s \qquad B_q(s) = 1 \qquad A_r(s) = 1 \qquad B_r(s) = 1.5$$

The above choice of error weight introduces integral action for steady state offset removal (since the polynomial $A_q(s)$ appears in the controller denominator). The control weighting function determines the relative importance which is placed on the penalisation of the control signal by the cost function. Simulation results are presented in Figure 3. Figure 3a compares the output regulation properties of a Zeigler Nichols tuned PID controller with that of an LQG controller designed with design parameters as listed above (see Table 3). As expected, in the case of a relatively small delay, the PID controller does stabilise the system, although the best response obtainable is rather oscillatory. The LQG controller (in this case designed without knowledge of the delay) provides a superior response from the points of view of reduction in oscillation and overshoot with no increase in controller order. Ease of controller design and redesign was also found to be an advantage of the LQG controller, a faster response is obtained simply by decreasing the value of the control weight (and vice versa).

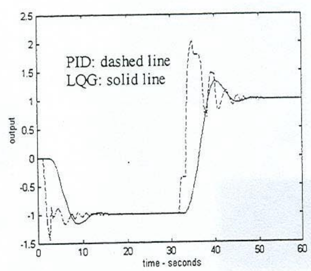
PID Controller	*LQG Controller*
$1.32\,[\,1 + 1/s + 0.25s]$	$\frac{s + 1.0028}{1.5s^2 + 2.8773s}$

Table 3: Controller design

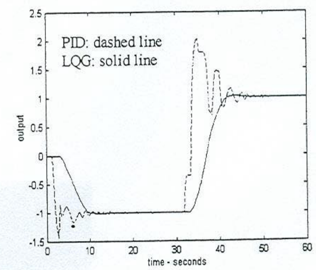
The effect of including a Padé approximation of the process delay in the LQG Controller design is investigated in Figures 3b and 3c.This is achieved by multiplying the polynomials A(s) and B(s) by the denominator and numerator polynomials respectively of the relevant order Padé approximation in Table 4 below. Figure 3b shows a significant improvement in regulator performance with respect to the conventional PID design by including a first order Padé approximation in the design. An increase in regulator performance with Padé order is demonstrated in Figure 3c, with corresponding increase in LQG controller order.

Order	Padé Approximation (0.5 second Delay)	LQG Controller
First	$\frac{-0.2425s+0.9701}{0.2425s+0.9701}$	$\frac{0.2425s^2+1.21s+0.973}{0.364s^3+2.153s^2+3.764s}$
Second	$\frac{0.0202s^2-0.2425s+0.9699}{0.0202s^2+0.2425s+0.9699}$	$\frac{0.0202s^3+0.2627s^2+1.213s+0.9727}{0.0303s^4+0.422s^3+2.15s^2+3.763s}$
Third	$\frac{-10^{-3}s^3+0.0242s^2-0.2425s+0.9699}{10^{-3}s^3+0.0242s^2+0.2425s+0.9699}$	$\frac{10^{-3}s^4+0.025s^3+0.267s^2+1.2s+0.973}{10^{-3}s^5+0.039s^4+0.437s^3+2.152s^2+3.76s}$

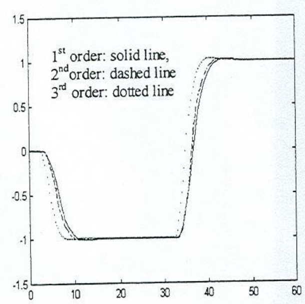
Table 4: Padé approximation of 0.5 second delay and LQG Design



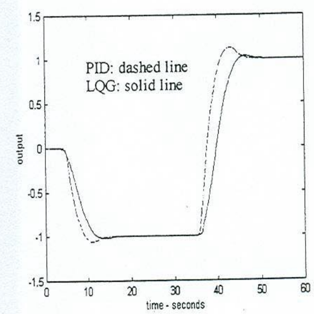
a) PID and LQG



b) PID and LQG with 1st order Pade



c) LQG design with pade approximation



d) PID and LQG with Smith Prediction

Figure 3: Output Regulation for Process with 0.5 Second Delay

The incorporation of a Smith Predictor structure in the feedback loop resulted in improved closed loop performance for both the PID and LQG design cases as demonstrated by Figure 3d.

Case 2: 3 second delay

For this relatively large delay, both the PID (Zeigler-Nichols tuned) and LQG controller with no delay compensation proved unable to stabilise the closed-loop system. The results for the Smith Predictor compensated loop are similar to those obtained in Figure 3d above. Hence only the results for the Padé approximation method in conjunction with LQG design are presented (Figure 4). A range of simulations show that this approach is effective for the control of even very high delay systems. As in the low delay case (Figure 3c) and as demonstrated in Figure 4, the output regulation performance is improved with increasing Padé order.

6. Conclusions

This paper has discussed approaches to the regulation of time delay systems based on the application of frequency domain identification methods and optimal control. The main results obtained reveal:

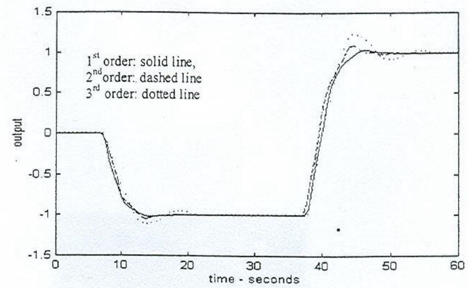


Figure 4: LQG Control for Process with 3 Second Delay (with padé approximations)

1. Both identification techniques are capable of providing accurate model parameter estimates (Figure 2). However, the PSD identification has distinct advantages were there is a high level of process noise, as may be expected in an industrial control environment.
2. Simulation results indicate benefits of the LQG approach over the PID method both in terms of ease and speed of design and closed-loop performance (Figures 3 and 4). PID performs adequately for low delay although output regulation is rather oscillatory. Improved performance can be obtained using the LQG approach which allows the control activity to be limited in the design to remove oscillations (which can be a feature of time delay systems). Subsequent improvement in LQG performance is reported by including a Pade approximation of the estimated process delay in the design.
3. The use of a smith predictor structure was also investigated and good regulation was achieved due to the high accuracy of the process identification employed.
4. For higher delay systems, the PID approach was not capable of stabilising the system while the LQG controller can provide good regulation properties if a pade approximation of the delay is included in the controller optimisation.

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