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# Performance and robustness issues in the compensation of FOLPD processes with PI and PID controllers

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## *Abstract*

This paper will discuss the compensation of first order lag plus time delay (FOLPD) processes using PI and PID controllers whose parameters are specified using appropriate tuning rules. The gain margin, phase margin and maximum sensitivity of the compensated system as the ratio of time delay to time constant of the process varies, are used as ways of judging the performance and robustness of the system.

**Keywords:** PID, time delay, gain margin, phase margin, maximum sensitivity.

## **1. Introduction**

The ability of proportional integral (PI) and proportional integral derivative (PID) controllers to compensate most practical industrial processes has led to their wide acceptance in industrial applications. The requirement to choose either two or three controller parameters has meant that the use of tuning rules to determine these parameters is popular. The author is aware of 72 such tuning rules in the literature to specify the PI controller terms to compensate a FOLPD process (with another 24 tuning rules which are non-process specific, such as the ultimate cycle tuning rule of Ziegler and Nichols [1]), with 116 tuning rules defined to specify the PID controller parameters for FOLPD processes (though it is fair to point out that in the case of the PID controller, separate tuning rules have been defined for a number of PID controller structures). Typical tuning methods are based on using process reaction curve data (e.g. Ziegler and Nichols [1]), integral error criteria (e.g. Rovira *et al.* [2]), ultimate cycle methods (e.g. Ziegler and Nichols [1]), direct synthesis methods (e.g. Smith and Corripio [3]), gain and phase margin specifications (e.g. Hang *et al.* [4]) and internal model control strategies (e.g. Morari and Zafiriou [5]).

There has always been some difficulty in objectively comparing the performance and robustness of closed loop compensated systems whose controllers are determined by these tuning rules, as the time delay of the FOLPD process varies. Recently, however, Ho *et al.* [6], [7] determined performance and robustness by analytically calculating a good approximation for the gain and phase margins of the resulting compensated systems, and applying the technique to consider 11 tuning rules (PI controller) and 10 tuning rules (PID controller). The present paper will apply the method to determine the gain and phase margins of processes compensated by a wider range of PI and PID tuning rules, to facilitate direct comparisons of a wide variety of tuning rules. In addition, the paper will extend the method to determine analytically the maximum sensitivity of the compensated system, as an alternative performance

and robustness measure. Insight will be obtained into the range of time delay to time constant ratios over which it is sensible to apply various tuning rules, to compensate a FOLPD process.

The paper is organised as follows. Formulae for calculating analytically the performance and robustness metrics are outlined in Section 2. The performance and robustness of FOLPD processes compensated with a wide variety of PI and PID tuning rules are evaluated in Section 3. In Section 4, conclusions of the work are provided.

## 2. The analytical determination of performance and robustness metrics

### 2.1 The calculation of gain and phase margins

The calculations of the gain and phase margins of systems compensated by both PI and PID controllers are presented. Most of this information also appears in Ho *et al.* [6], [7].

#### 2.1.1 PI tuning formulae

The process and controller transfer functions are denoted by  $G_m(s)$  and  $G_c(s)$ , respectively, and the gain and phase margins as  $A_m$  and  $\phi_m$ , respectively. From the definition of gain and phase margin, the following sets of equations are obtained:

$$\phi_m = \arg[G_c(j\omega_g)G_m(j\omega_g)] + \pi \quad (1a) \quad A_m = \frac{1}{|G_c(j\omega_p)G_m(j\omega_p)|} \quad (1b)$$

where  $\omega_g$  and  $\omega_p$  are given by

$$|G_c(j\omega_g)G_m(j\omega_g)| = 1 \quad (2a) \quad \arg[G_c(j\omega_p)G_m(j\omega_p)] = -\pi \quad (2b)$$

The controller and process model are respectively given by

$$G_c(s) = K_c \left( 1 + \frac{1}{T_i s} \right) \quad (3a) \quad G_m(s) = \frac{K_m e^{-s\tau_m}}{1 + sT_m} \quad (3b)$$

Substituting equations (3a) and (3b) into equations (1a) to (2b) gives

$$\phi_m = 0.5\pi + \tan^{-1} \omega_g T_i - \tan^{-1} \omega_g T_m - \omega_g \tau_m \quad (4a) \quad A_m = \frac{\omega_p T_i}{K_c K_m} \sqrt{\frac{\omega_p^2 T_m^2 + 1}{\omega_p^2 T_i^2 + 1}} \quad (4b)$$

From equation (2a),  $\omega_g$  may be determined analytically to be

$$\omega_g = \sqrt{\frac{T_i (K_c^2 K_m^2 - 1) + \sqrt{(K_c^2 K_m^2 - 1)^2 T_i^2 + 4K_c^2 K_m^2 T_m^2}}{2T_i T_m^2}} \quad (5)$$

From equation (2b),  $\omega_p$  is given by

$$0.5\pi + \tan^{-1} \omega_p T_i - \tan^{-1} \omega_p T_m - \omega_p \tau_m = 0 \quad (6)$$

An analytical solution of this equation is not possible. An approximate analytical solution may be obtained if the following approximation for the arctan function is made:

$$\tan^{-1} x \approx \frac{\pi}{4}x, \quad |x| < 1 \quad \text{and} \quad \tan^{-1} x \approx \frac{\pi}{2} - \frac{\pi}{4x}, \quad |x| > 1 \quad (7)$$

This is quite an accurate approximation, as is shown by Ho *et al.* [6] Looking at equation (6), four possibilities present themselves if the approximation in equation (7) is to be used; these possibilities are (i)  $\omega_p T_i > 1, \omega_p T_m > 1$  (ii)  $\omega_p T_i > 1, \omega_p T_m < 1$  (iii)  $\omega_p T_i < 1, \omega_p T_m > 1$  and (iv)  $\omega_p T_i < 1, \omega_p T_m < 1$  (Ho *et al.* [6] consider just the first of these possibilities). Table 1 shows the formula for  $\omega_p$  that may be determined analytically for each of these cases.

Table 1: Formulae for  $\omega_p$

$\omega_p T_i > 1, \omega_p T_m > 1$	$\omega_p T_i > 1, \omega_p T_m < 1$	$\omega_p T_i < 1, \omega_p T_m > 1$	$\omega_p T_i < 1, \omega_p T_m < 1$
$\omega_p = \frac{\pi \pm \sqrt{\pi^2 - 4\pi\tau_m \left( \frac{1}{T_i} - \frac{1}{T_m} \right)}}{4\tau_m}$	$\omega_p = \frac{\pi \pm \sqrt{\pi^2 - \frac{\pi}{T_i} \left( \frac{\pi}{4} T_m + \tau_m \right)}}{2 \left( \frac{\pi}{4} T_m + \tau_m \right)}$	$\omega_p = \sqrt{\frac{\pi/T_m}{4\tau_m - \pi T_i}}$	$\omega_p = \frac{2\pi}{4\tau_m + \pi(T_m - T_i)}$

Equations (4a) and (4b), together with equation (5) and the relevant equations from Table 1, may now be used to calculate the gain and phase margin of the compensated system, for each of the tuning rules, as a function of  $\tau_m/T_m$ .

### 2.1.2 PID tuning formulae

The controller is given by 
$$G_c(s) = K_c \left( 1 + \frac{1}{T_i s} \right) \left( \frac{1 + sT_d}{1 + s\alpha T_d} \right) \quad (8)$$

and the process model is given by equation (3b). Substituting equations (3b) and (8) into equations (1a) to (2b) gives

$$\phi_m = 0.5\pi + \tan^{-1} \omega_g T_i + \tan^{-1} \omega_g T_d - \tan^{-1} \omega_g T_m - \tan^{-1} \omega_g \alpha T_d - \omega_g \tau_m \quad (9)$$

and

$$A_m = \frac{\omega_p T_i}{K_c K_m} \sqrt{\frac{(1 + \omega_p^2 T_m^2)(1 + \omega_p^2 \alpha^2 T_d^2)}{(1 + \omega_p^2 T_i^2)(1 + \omega_p^2 T_d^2)}} \quad (10)$$

From equation (2a),  $\omega_g$  is given by the solution of

$$\omega_g^6 + a_1 \omega_g^4 + a_2 \omega_g^2 + a_3 = 0 \quad (11)$$

$$\text{with } a_1 = \frac{T_i^2 (T_m^2 + \alpha^2 T_d^2) - K_c^2 K_m^2 T_i^2 T_d^2}{T_i^2 T_m^2 \alpha^2 T_d^2}, \quad a_2 = \frac{T_i^2 - K_c^2 K_m^2 (T_i^2 + T_d^2)}{T_i^2 T_m^2 \alpha^2 T_d^2} \quad \text{and} \quad a_3 = -\frac{K_c^2 K_m^2}{T_i^2 T_m^2 \alpha^2 T_d^2}.$$

Following the procedure outlined by Ho *et al.* [7], the analytical solution of equation (11) is given as

$$\omega_g = \sqrt[3]{\sqrt{R + \sqrt{Q^3 + R^2}} + \sqrt[3]{R - \sqrt{Q^3 + R^2}} - \frac{a_1}{3}} \quad (12)$$

with  $Q = \frac{3a_2 - a_1^2}{9}$  and  $R = \frac{9a_1a_2 - 27a_3 - 2a_1^3}{54}$ .

From equation (2b),  $\omega_p$  is given by

$$0.5\pi + \tan^{-1} \omega_p T_d + \tan^{-1} \omega_p T_i - \tan^{-1} \omega_p T_m - \tan^{-1} \omega_p \alpha T_d - \omega_p \tau_m = 0 \quad (13)$$

As in Section 2.1.1, an analytical solution of this equation is not possible. An approximate analytical solution may be obtained if the approximation detailed in equation (7) is made. Looking at equation (13), twelve possibilities present themselves if the approximation in equation (7) is to be used; these possibilities are

- (1)  $\omega_p T_i > 1, \omega_p T_m > 1$     (2)  $\omega_p T_i > 1, \omega_p T_m < 1$     (3)  $\omega_p T_i < 1, \omega_p T_m > 1$     (4)  $\omega_p T_i < 1, \omega_p T_m < 1$   
(a)  $\omega_p T_d \leq 1, \omega_p \alpha T_d < 1$     (a)  $\omega_p T_d \leq 1, \omega_p \alpha T_d < 1$     (a)  $\omega_p T_d \leq 1, \omega_p \alpha T_d < 1$     (a)  $\omega_p T_d \leq 1, \omega_p \alpha T_d < 1$   
(b)  $\omega_p T_d > 1, \omega_p \alpha T_d < 1$     (b)  $\omega_p T_d > 1, \omega_p \alpha T_d < 1$     (b)  $\omega_p T_d > 1, \omega_p \alpha T_d < 1$     (b)  $\omega_p T_d > 1, \omega_p \alpha T_d < 1$   
(c)  $\omega_p T_d > 1, \omega_p \alpha T_d > 1$     (c)  $\omega_p T_d > 1, \omega_p \alpha T_d > 1$     (c)  $\omega_p T_d > 1, \omega_p \alpha T_d > 1$     (c)  $\omega_p T_d > 1, \omega_p \alpha T_d > 1$

It is interesting that Ho *et al.* [7] considers just the case where  $\omega_p T_i > 1$  and  $\omega_p T_m > 1$ . Table 2 shows the formula for  $\omega_p$  that may be determined for all of the cases above.

Table 2: Formulae for  $\omega_p$

	$\omega_p T_d \leq 1, \omega_p \alpha T_d < 1$	$\omega_p T_d > 1, \omega_p \alpha T_d < 1$	$\omega_p T_d > 1, \omega_p \alpha T_d > 1$
$\omega_p T_i > 1,$ $\omega_p T_m > 1$	$\frac{\pi \pm \sqrt{\pi^2 - \pi(4\tau_m - \pi T_d(1-\alpha))\left(\frac{1}{T_i} - \frac{1}{T_m}\right)}}{4\tau_m - \pi T_d(1-\alpha)}$	$\frac{2\pi \pm \sqrt{4\pi^2 - \pi(4\tau_m + \pi T_d \alpha)\left(\frac{1}{T_d} + \frac{1}{T_i} - \frac{1}{T_m}\right)}}{4\tau_m + \pi T_d \alpha}$	$\frac{\pi \pm \sqrt{\pi^2 - 4\pi\tau_m\left(\frac{1}{T_d} + \frac{1}{T_i} - \frac{1}{\alpha T_d} - \frac{1}{T_m}\right)}}{4\tau_m}$
$\omega_p T_i > 1,$ $\omega_p T_m < 1$	$\frac{2\pi \pm \sqrt{4\pi^2 + \frac{\pi}{T_i}(\pi T_d - \pi T_m - \pi \alpha T_d - 4\tau_m)}}{\pi T_m + 4\tau_m + \pi \alpha T_d - \pi T_d}$	$\frac{3\pi \pm \sqrt{9\pi^2 - \pi\left(\frac{1}{T_d} + \frac{1}{T_i}\right)(\pi T_m + \pi \alpha T_d + 4\tau_m)}}{\pi T_m + 4\tau_m + \pi \alpha T_d}$	$\frac{2\pi \pm \sqrt{4\pi^2 + \pi\left(\frac{1}{\alpha T_d} - \frac{1}{T_d} - \frac{1}{T_i}\right)(\pi T_m + 4\tau_m)}}{\pi T_m + 4\tau_m}$
$\omega_p T_i < 1,$ $\omega_p T_m > 1$	$\sqrt{\frac{\pi}{T_m(4\tau_m + \pi[\alpha T_d - T_d - T_i])}}$	$\frac{-\pi \pm \sqrt{\pi^2 - \pi\left(\frac{1}{T_m} - \frac{1}{T_d}\right)(\pi T_i - \pi \alpha T_d - 4\tau_m)}}{\pi T_i - 4\tau_m - \pi \alpha T_d}$	$\sqrt{\frac{\frac{\pi}{T_d} - \frac{\pi}{T_m} - \frac{\pi}{\alpha T_d}}{\pi T_i - 4\tau_m}}$
$\omega_p T_i < 1,$ $\omega_p T_m < 1$	$\frac{2\pi}{\pi(\alpha T_d - T_d) + \pi(T_m - T_i) + 4\tau_m}$	$\frac{2\pi \pm \sqrt{4\pi^2 - \frac{\pi}{T_d}(\pi T_m - \pi T_i + \pi \alpha T_d + 4\tau_m)}}{\pi T_m - \pi T_i + 4\tau_m - \pi \alpha T_d}$	$\frac{\pi \pm \sqrt{\pi^2 - \pi\left(\frac{1}{\alpha T_d} - \frac{1}{T_d}\right)(\pi T_i - \pi T_m - 4\tau_m)}}{4\tau_m - \pi T_m - \pi T_i}$

Equations (9) and (10), together with equation (12) and the relevant equations in Table 2, are now used to calculate the gain and phase margin of the compensated system, for each of the tuning rules, as a function of  $\tau_m/T_m$ .

## 2.2 The calculation of the maximum sensitivity

The maximum sensitivity is the shortest distance from the Nyquist curve to the (-1,0) point on the RI-Im axis. It is defined as follows:

$$M_E = \text{Max}_{\omega} \left| \frac{1}{1 + G_p(j\omega)G_c(j\omega)} \right| \quad (14)$$

For a FOLPD process model controlled by a PI controller,

$$|G_p(j\omega)G_c(j\omega)| = \frac{\sqrt{1 + \omega^2 T_i^2}}{\sqrt{1 + \omega^2 T_m^2}} \frac{K_c K_m}{\omega T_i} \quad (15)$$

and

$$\arg[G_p(j\omega)G_c(j\omega)] = -0.5\pi - \tan^{-1} \omega T_m + \tan^{-1} \omega T_i - \omega \tau_m \quad (16)$$

For a FOLPD process model controlled by a PID controller,

$$|G_p(j\omega)G_c(j\omega)| = \frac{\sqrt{(1 + \omega^2 T_i^2)(1 + \omega^2 T_d^2)}}{\sqrt{(1 + \omega^2 T_m^2)(1 + \alpha^2 \omega^2 T_d^2)}} \frac{K_c K_m}{\omega T_i} \quad (17)$$

and

$$\arg[G_p(j\omega)G_c(j\omega)] = -0.5\pi - \tan^{-1} \omega T_m - \tan^{-1} \omega \alpha T_d + \tan^{-1} \omega T_i + \tan^{-1} \omega T_d - \omega \tau_m \quad (18)$$

The maximum sensitivity may be calculated over an appropriate range of frequencies corresponding to phase lags of  $100^\circ$  to  $260^\circ$ .

### 3. Simulation results

Space considerations dictate that only representative simulation results may be provided. In these results, gain margin, phase margin and maximum sensitivity are calculated for processes compensated using a PI or PID controller following the tuning rules provided in Table 3.

Table 3: Tuning rules

Rule	PI tuning rules		PID tuning rules		
	$K_c$	$T_i$	$K_c$	$T_i$	$T_d$
Ziegler and Nichols [1] process reaction	$\frac{0.9T_m}{K_m \tau_m}$	$3.33\tau_m$	$[\frac{1.2T_m}{K_m \tau_m}, \frac{2T_m}{K_m \tau_m}]$	$2\tau_m$	$0.5\tau_m$
Regulator - IAE PI - Murrill [8], IAE PID - Kaya and Scheib [9]	$\frac{0.984}{K_m} \left(\frac{T_m}{\tau_m}\right)^{0.986}$	$\frac{T_m}{0.608} \left(\frac{\tau_m}{T_m}\right)^{0.707}$	$\frac{0.98089}{K_m} \left(\frac{T_m}{\tau_m}\right)^{0.76167}$	$\frac{T_m}{0.91032} \left(\frac{T_m}{\tau_m}\right)^{1.05221}$	$0.59974T_m \left(\frac{\tau_m}{T_m}\right)^{0.89819}$
Regulator - ISE PI - Murrill [8], ISE PID - Kaya and Scheib [9]	$\frac{1.305}{K_m} \left(\frac{T_m}{\tau_m}\right)^{0.959}$	$\frac{T_m}{0.492} \left(\frac{\tau_m}{T_m}\right)^{0.739}$	$\frac{1.11907}{K_m} \left(\frac{T_m}{\tau_m}\right)^{0.89711}$	$\frac{T_m}{0.7987} \left(\frac{T_m}{\tau_m}\right)^{0.9548}$	$0.54766T_m \left(\frac{\tau_m}{T_m}\right)^{0.87798}$
Regulator - ITAE PI - Murrill [8], ITAE PID - Kaya and Scheib [9]	$\frac{0.859}{K_m} \left(\frac{T_m}{\tau_m}\right)^{0.977}$	$\frac{T_m}{0.674} \left(\frac{\tau_m}{T_m}\right)^{0.680}$	$\frac{0.77902}{K_m} \left(\frac{T_m}{\tau_m}\right)^{1.06401}$	$\frac{T_m}{1.14311} \left(\frac{T_m}{\tau_m}\right)^{0.70949}$	$0.57137T_m \left(\frac{\tau_m}{T_m}\right)^{1.03826}$
Ziegler and Nichols [1] <sup>1</sup>	$0.45K_u$	$0.83T_u$	$[0.6K_u, K_u]$	$0.5T_u$	$0.125T_u$
Astrom and Hagglund [10]	$0.4698K_u$	$0.4373T_u$	$0.4698K_u$	$0.4546T_u$	$0.1136T_u$
Servo - Zhuang and Atherton [11]	$0.361K_u$	$0.083(1.935K_m K_u + 1)T_u$	$0.509K_u$	$0.051(3.302K_m K_u + 1)T_u$	$0.125T_u$

<sup>1</sup>  $K_u$  and  $T_u$  are the ultimate gain and ultimate period of the FOLPD model.

Table 3 (continued)

Rule	PI tuning rules		PID tuning rules		
	$K_c$	$T_i$	$K_c$	$T_i$	$T_d$
Regulator - Zhuang and Atherton [11]	$\left( \frac{1.892K_m K_u + 0.244}{3.249K_m K_u + 2.097} \right) K_u$	$\left( \frac{0.706K_m K_u - 0.227}{0.7229K_m K_u + 1.2736} \right) K_u$	$\frac{4.434K_m K_u - 0.966}{5.12K_m K_u + 1.734} K_u$	$\frac{1.751K_m K_u - 0.612}{3.776K_m K_u + 1.388} T_u$	$0.144T_u$

The MATLAB package has been used in the simulations. It should be stated that  $\alpha$  in Equation (8) is taken as 0.1, with any tuning rules for the parallel form of the PID controller being converted into the PID controller structure used, following the procedure of Ho *et al.* [7]. Figures 1 to 6 show how gain margin, phase margin and maximum sensitivity vary as the ratio of time delay to time constant varies, if some PI tuning rules are used (Figures 1 to 3) and corresponding PID tuning rules are used (Figures 4 to 6). In these results, Z-N refers to the process reaction curve method of Ziegler and Nichols [1] with IAE reg, ISE reg and ITAE reg referring to the tuning rules for regulator applications that minimise the integral of absolute error criterion, the integral of squared error criterion and the integral of time multiplied by absolute error criterion, respectively, as defined by Murrill [8] for PI tuning rules and Kaya and Scheib [9] for PID tuning rules.

Figure 1: Gain margin

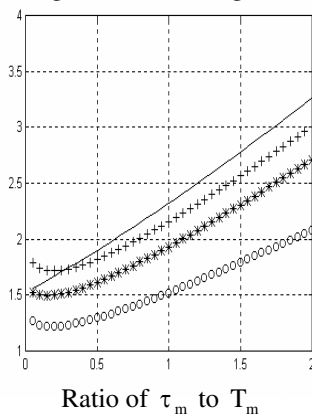


Figure 2: Phase margin

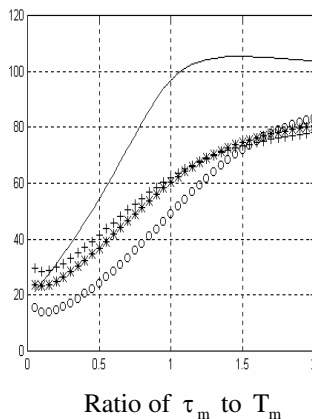


Figure 3: Maximum sensitivity

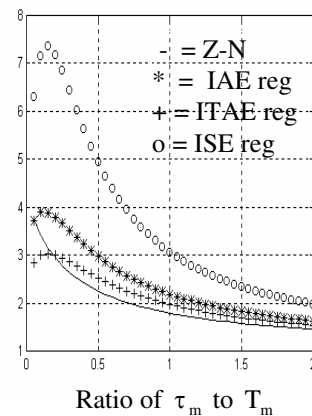


Figure 4: Gain margin

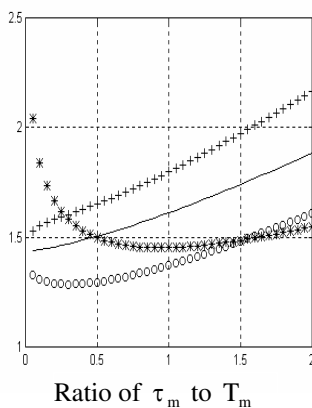


Figure 5: Phase margin

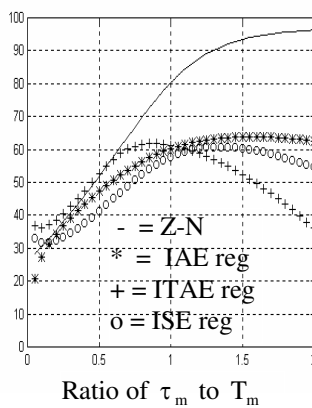
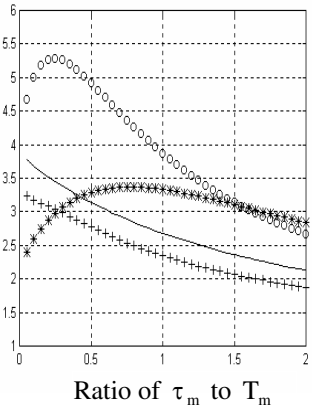


Figure 6: Maximum sensitivity



It is clear that the gain margin is consistently less when the PID rather than the PI tuning rules are considered, for all ratios of time delay to time constant taken; the difference between the phase margins is less clear cut. This suggests that these PID tuning rules should provide a greater degree of performance than the corresponding PI tuning rules, but may be less robust. Comparing the individual tuning rules, it is striking that the ISE based tuning rules

have the smallest gain margin and have also a small phase margin, suggesting that this is a less robust tuning strategy. The results in Figures 3 and 6 confirm these comments.

Figures 7 to 12 show how the quantities vary for the other tuning rules in Table 3; Z-N ult refers to the ultimate cycle method of Ziegler and Nichols [1], A-H refers to the method specified by Astrom and Hagglund [10] with Z-A ser and Z-A reg referring to the optimal servo and regulator methods, respectively, defined by Zhuang and Atherton [11]. Figures 7 to 9 are associated with the use of the relevant PI tuning rules, with Figures 10 to 12 associated with the use of the PID tuning rules. The results speak for themselves.

Figure 7: Gain margin

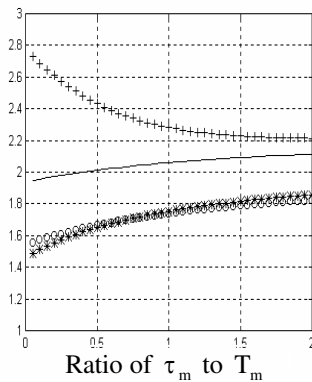


Figure 8: Phase margin

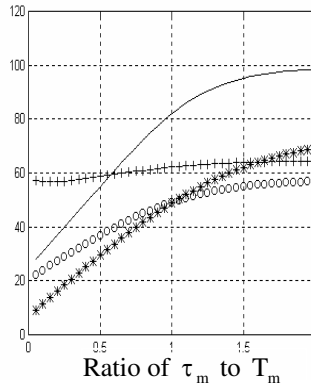


Figure 9: Maximum sensitivity

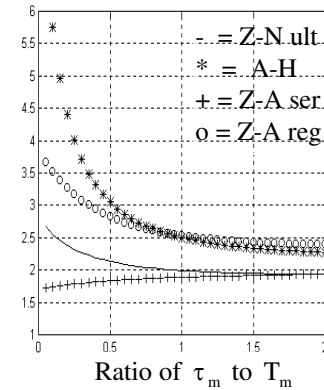


Figure 10: Gain margin

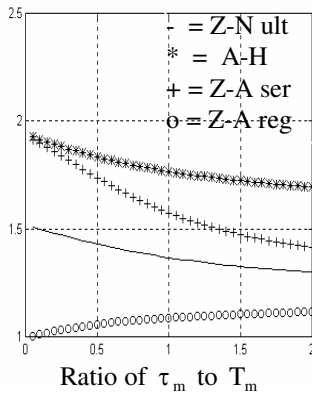


Figure 11: Phase margin

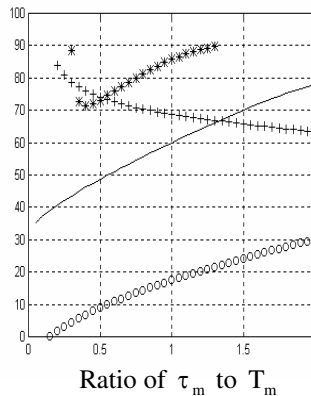
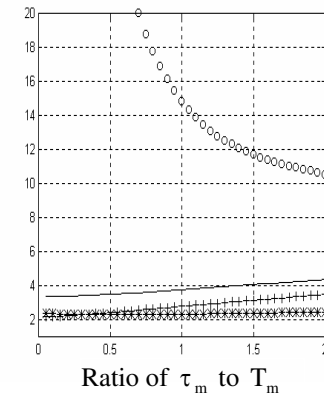


Figure 12: Maximum sensitivity



More such results will be provided in the poster at the conference. No general conclusion can be reached as to the best tuning rule (as expected); it is interesting, though, that the full panorama of simulation results show that many tuning rules may be applied at ratios of time delay to time constant greater than that normally recommended. One example may be seen in Figures 4 to 6, where the gain and phase margin and maximum sensitivity (associated with the use of the PID tuning rule for obtaining minimum IAE in the regulator mode) tends to level out when the ratio of time delay to time constant is greater than 1; normally, the tuning rule is used when the ratio is less than 1 (Murrill [8]). On the other hand, it is clear from the same plots that there is a significant degradation of performance when using the Ziegler and Nichols [1] process reaction PID tuning rule for large ratios of time delay to time constant, which is compatible with application experience.



## 4. Conclusions

The paper has considered the performance and robustness of a PI and PID controlled FOLPD process, with the parameters of the controllers determined by a variety of tuning rules. The original contributions of this work are as follows: (a) an expansion of the analytical approach of Ho *et al.* [6], [7] to determine the approximate gain and phase margin analytically under all operating conditions (b) the analytical determination of the approximate maximum sensitivity of the compensated system and (c) the application of the algorithms using a wide variety of PI and PID tuning rules. The implementation of autotuning algorithms in commercial controllers means that choice of a suitable tuning rule is an important issue; the techniques discussed allow an analytical evaluation to be performed of candidate tuning rules.

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