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Reliability-Based Designs Procedure of Earth Retaining Walls in Geotechnical Engineering

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Reliability-based designs procedure of earth retaining walls in geotechnical engineering

Procesos para los diseños por confiabilidad de muros de contención en ingeniería geotécnica

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The design and construction of foundations, retaining structures and slopes are usually based on deterministic formulations that do not allow the distinction between the natural variability and the inherent dispersion in the geotechnical parameters. Due to the inherent variability of the soil properties, there is a growing trend to implement reliability-based designs in geotechnical engineering to reduce design uncertainties by probabilistic methods. The reliability designs require the definition of the probability density functions of the geotechnical properties, as well as knowledge of the spatial variability of soils. This paper identifies the procedures, type of soil investigations, simulations and the most commonly studied areas in geotechnical reliability-based designs. The importance of the correlation length in defining the reduction factors to determine the probabilities of occurrence of events, with Monte Carlo as the most used simulation method in this type of designs, is highlighted. The most commonly studied problems in this regard are related to foundation design and slope stability analyses whereas earth retaining walls and gabion walls are the less studied. Furthermore, no study was found in the determination of the variation of the soil-wall friction nor in the geological influence for this type of structures, which implies a great potential for future research in these areas.

Keywords: earth retaining wall, soil-wall friction, geological influence, probability of failure

El diseño y construcción de cimentaciones, estructuras de contención y taludes se basan generalmente en formulaciones deterministas que no permiten la distinción entre la variabilidad natural y la dispersión inherente de los parámetros geotécnicos. Debido a la variabilidad inherente de las propiedades del suelo, existe una tendencia cada vez mayor en la implementación de diseños basados en la confiabilidad en geotecnia, con el fin de reducir las incertidumbres con métodos probabilísticos. Los diseños por confiabilidad requieren la definición de las funciones de densidad de probabilidad de las propiedades geotécnicas, además de tener el conocimiento de la variabilidad espacial correspondiente a cada tipo de suelo. Este artículo identifica los procedimientos, tipo de investigación del subsuelo, simulaciones y las áreas de estudio más comunes en los diseños geotécnicos basados en la confiabilidad. Se resalta la importancia de la longitud de correlación en la definición de los factores de reducción para determinar las probabilidades de falla y que el método de simulación más utilizado en estos diseños es Monte Carlo. Las áreas de la geotecnia más estudiadas en confiablidad son las de diseño de cimentaciones y análisis de estabilidad de las laderas, mientras que los análisis de muros de contención y muros de gaviones son los menos estudiados. Por otra parte, no se encontraron estudios en la determinación de la variación de la fricción suelo-muro ni en la influencia geológica para este tipo de estructuras, lo que implica un gran potencial para futuras investigaciones.

Palabras clave: muros de contención, fricción suelo muro, influencia geológica, probabilidad de falla

Introduction

Geotechnical engineering is the area of civil engineering that studies the behaviour of soils and rocks for the design and construction of mainly foundations, retaining walls and slopes. The aforementioned designs require the knowledge of the shear strength properties of soils for each of the layers found during ground investigation where the structures are to be located. These properties can be determined with different types of field or laboratory tests, but are commonly used as deterministic values in the different geotechnical models (Phoon and Kulhawy, 1999b). The main problem of the deterministic methods is the inability to assess the probabilities of events for random phenomena that may occur in the soil. Deterministic formulations cannot distinguish between the natural variability and the inherent dispersion of the geotechnical parameters, as these properties are usually obtained by measures of central tendency or are modified subjectively according to field observations and engineering criteria (Phoon and Kulhawy, 1999b). The uncertainties caused by the random behaviour of soil properties and the hypotheses formulated in the interpretation of the parameters may reduce the accuracy of the bearing capacity, earth pressure and slope stability analyses (e.g. Carlsson, 2005). The implementation of statistical models that are able to estimate the uncertainties of the classic methodologies can overcome the lack of accuracy in the deterministic methods.

Previous research

A variety of works deal with the reliability-based designs in geotechnical engineering, being the most common subject the reliability of slope analysis, with over 200 papers, followed by the reliability of foundations with over 90 papers. Figure 1 shows the amount of papers by subarea.

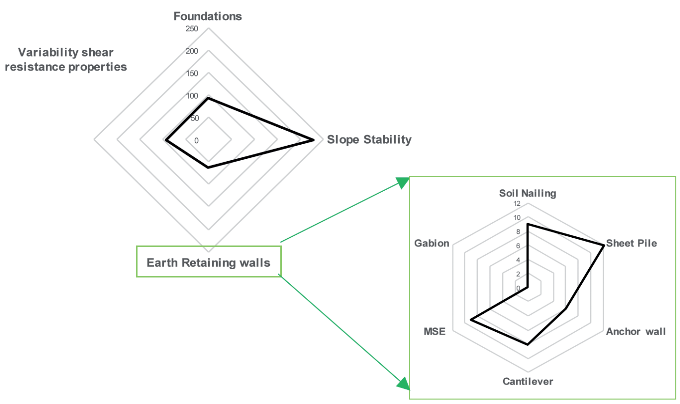

Figure 1: Numbers of papers in reliability based designs in Geotechnics according with the subarea

We made a further refinement with focus on the subject of interest in earth retaining walls. Table 1 presents the state of the art in the reliability design of different type of walls. In the review, pile walls are the most studied structures in reliability-based designs, whilst the anchored walls are the least studied.

Table 1: Summary of reliability based design of earth retaining walls

Type of wall	Reference
Pile wall	Basma, 1991a; Cherubini *et al*., 1992; Goh and Kulhawy, 2005; GuhaRay and Baidya, 2015; Koreta *et al*., 2015; Li *et al*., 2016; Low and Phoon, 2015; Papaioannou and Straub, 2010; Prästings *et al*., 2016; Sessa and D'Urso, 2013; Wang, 2013
Soil nailing	Sivakumar Babu and Singh, 2009, 2010, 2011; Cao *et al*., 2014; Hui and Zhu, 2013; Lu and Jiang, 2012; Zhang *et al*., 2009; Zhao & An, 2011
Cantilever	Fenton *et al*., 2005; Goh *et al*., 2009; Griffiths *et al*., 2008; GuhaRay and Baidya, 2014; Juang *et al*., 2013; Liu *et al*., 2013; Prada *et al*., 2011; Zevgolis and Bourdeau, 2010a; Mandali *et al*., 2011
MSE	Basheer and Najjar, 1996; Chalermyanont and Benson, 2004, 2005a, 2005b; Hegazy *et al*., 2009; Kim and Salgado, 2012; Miyata and Bathurst, 2012; Zevgolis and Bourdeau, 2008, 2010b
Anchor	Basha and Sivakumar Babu, 2008; Basma, 1991b; Chen *et al*., 2011; Cherubini, 1999; Low, 2005

An important feature is the absence of reliability studies in gabion walls. This type of walls is widely used in the world due to their economical and aesthetics characteristics, but they have a variability of dimensions, type of fills and low stiffness. Although this type of walls does not appear in reliability studies, they are commonly found in case history studies and forensic studies. Table 2 shows the quantity of studies carried out on forensic engineering of walls.

Table 2: Number of papers in forensic engineering of wall failures

Wall	Number of papers
Soil nailing	42
Sheetpile	108
Anchor wall	128
Cantilever	93
MSE	57
Gabion	36
Total	464

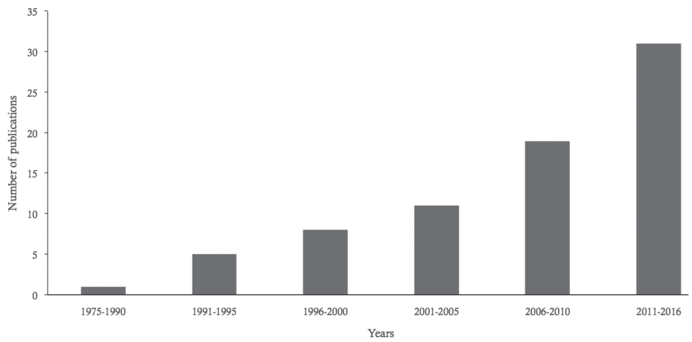

Figure 2: Time line of the evolution in reliability – based designs research works in geotechnical engineering (eje vertical Number of publications)

Procedure for reliability-based design

The reliability-based designs, as the regular deterministic geotechnical ones, require a series of steps to define the different geotechnical properties and field characteristics that affect the probability of failure of geotechnical structures. From 75 references published in the course of four decades, between 1977 and 2016 (Figure 2), the increase of research in this area with time become evident, especially after the year 2000. This increase may be explained due to the advances in computational modelling where more complex statistical analyses were made possible. According with these results, we drafted the following general procedure for reliability-based designs in geotechnical engineering as a common baseline for different subareas of work (Figure 3).

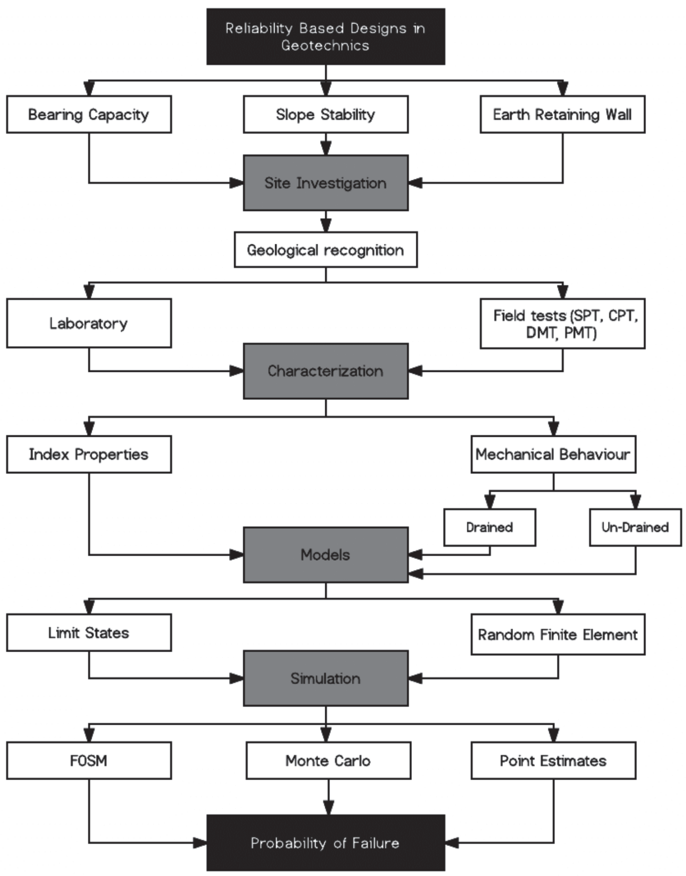

Figure 3: General procedure for reliability-based designs in geotechnical engineering

Site investigation

This task involves undertaking different field test, such as SPT, CPT, DMT or PMT complemented by different laboratory tests, like triaxial and direct shear stress, in order to identify the different soil layers and their mechanical properties. The site investigation not only limits to the definition of soil properties, but may also include the definition of the soil spatial variability. The spatial variability is commonly estimated by the correlation length or scale of fluctuation, originally described by Vanmarcke (1977). This length corresponds to the distance at which the values are significantly correlated, where values separated over this distance will have no correlation (*e.g.* Fenton and Griffiths, 2008). Some authors work with this type of correlation to determine the spatial variability of the soil, like Zou *et al.* (2015), who used it with cone penetration test results and Cao *et al.* (2013), who analyzed the effect of the spatial variability in the designs of drilled shafts. The scale of fluctuation will determine the influence of the test location in the field and in the determination of the mechanical properties, according to their spatial location, creating a more realistic model of the soil properties (*e.g.* Fenton and Griffiths, 2010). The spatial variability in geotechnical engineering have been described by a number of authors (Baecher and Christian, 2003; Basarir *et al*., 2010; Cao *et al*., 2013; Fenton, 1999; Firouzianbandpey *et al*., 2014; Gambino and Gilbert, 1999; Jha, 2014; Papaioannou and Straub, 2012).

Soil characterization

According to Fenton and Griffiths (2010), the ground is a complex engineering material that is difficult to characterize realistically. Its parameters can vary greatly from site to site, evidenced in the coefficient of variation (standard deviation divided by the mean) as illustrated by Phoon and Kulhawy (1999a) and Uzielli *et al.* (2006). This coefficient of variation C_v is useful when trying to compare the uncertainties of more than one group of data, which differ considerably in the extent of the observation (Montgomery and Runger, 2003). Table 3 and 4 present the values of the coefficient of variation for several properties of different types of soil.

Table 3: Inherent variability of soil properties (Phoon *et al*., 1995)

Property	Type of soil	Number of data	Value		C_v, %	
			range	mean	range	mean
s_u, kPa	Clay	42	8-638	112	6-80	32
ϕ', °	Sand	7	35-41	37.6	5-11	9
ϕ', °	Clay, silt	12	0-33	15.3	10-50	21
ϕ', °	Clay, silt	9	17-41	33.3	4-12	9

Due to this uncertainty, the use of response prediction models, which can take into account the spatial variability, are defined as a Probability Density Function PDF of the geotechnical parameters. The probability functions used in the reliability based design identifies the probabilities of occurrence of the soil properties that are susceptible to great change and whose behaviour can only be described by inductive statistical analysis (Baecher and Christian, 2003).

Table 4: Inherent variability of soil properties (Uzielli *et al*., 2007)

Test	Property	Type of soil	Mean range	C_v, %
Triaxial UC	s_u, kPa	Clay	1-40	20-55
UU	s_u, kPa	Clay	1-35	10-30
CIUC	s_u, kPa	Clay	15-70	20-40
CID	ϕ', º	Clay and sand	20-40	5-15
CPT	q_t, MPa	Clay	0.5-2.5	<20
	q_c, MPa	Clay	0.5-2.0	20-40
		Sand	0.5-3.0	20-60
SPT	N, bpf	Clay and sand	10-70	25-50
Index properties	w_n, %	Clay and silt	13-100	8-30
	w_L, %	Clay and silt	30-90	6-30
	w_P, %	Clay and silt	15-25	6-30
	I_P, %	Clay and silt	10-40	-
	I_L, %	Clay and silt	10	-
	γ, kN/m^3	Clay and silt	13-20	<10
	D_R, %	Sand	30-70	10-40
				50-70

How to obtain the PDF

The determination of the PDFs for the drained and undrained shear strength parameters require enough number of laboratory and field tests, which are rarely available to prescribe a full joint distribution (Fenton and Griffiths, 2008). A procedure to define the PDF for the shear strength properties, adapted from Griffiths and Fenton (2008), is described below:

1. If sufficient data are available, select the functions that best fit the histogram of the data.
2. Conduct a goodness of fit test for all the selected PDF's to evaluate how well the fitted distribution represents the true underlying distribution of data. Commonly used tests are Chi–Square, Kolmogorov–Smirnov and Anderson–Darling tests.
3. After defining the PDF's, ensure that the distribution represents the soil property realistically. For example, some variables cannot

have negative values (*e.g.* modulus of elasticity or friction angle); nonetheless, some functions, such as the normal distribution, may present negative tails that can affect the results. Distributions with negative tails are allowed, as long as the probability of occurrence of these values is very low.

4. Compare the results with distributions available in the literature, especially when making assumptions or using a database.
5. If there is more than one possible PDF, choose the simplest one.
6. Estimate the final PDF parameters by the Method of Moments or Maximum likelihood.

The normal or lognormal probability functions can usually represent random processes that occur in the soil. However, under specific conditions, different functions are applicable in order to adjust the sampling distributions more accurately as is shown in Table 5. Normally, the grain size distribution is used to explain the parameters, distribution and behaviour of soil properties. However, the influence of the geological origin in the soil properties has not yet been considered. The geological origin not only generates a particular grain size distribution, but it can correlate to the state of weathering, type of formation (residual or transported) and to the previous state of stresses and how these factors can affect the variability of the geotechnical parameters.

Influence of the geological origin

The geological environment not only determines the type of site investigation to conduct on any site, but it can also explain the discrepancies usually observed in the mechanical behaviour of different types of soils. Mud or debris flows, for instance, behave in very different manners when compared to residual soils. The former, for example, is comprised by rock fragments with different states of weathering embedded in a homogenous matrix, and this type of formation affects the results from the different field test that can be performed during ground exploration. These tests might evaluate the resistance of a particular rock fragment that do not represent the behaviour of the soil mass as shown in Figure 4.

On the other hand, geotechnical engineering is less developed when it comes to analyze residual soils, since it has advanced mostly in sedimentary formations (Mitchell and Soga, 2005; Wesley, 2009, 2011). However, residual soils tend to have a similar grain size distribution with the presence of

Table 5: Probability functions used in reliability designs in geotechnics (Mattos and Viviescas, 2015)

Friction angle ϕ’	Cohesion c	Unit weight γ	Surcharge R	Reference
Normal, log normal, Beta, Gamma and Uniform	x	Normal, lognormal, Beta and Gamma	x	Baecher and Christian (2003)
Normal	Normal	Normal	x	Sivakumar Babu and Srivastava (2007)
Lognormal	x	x	x	Cherubini *et al.* (2007)
Normal	x	Normal	x	Uzielli *et al.* (2007)
Normal y Lognormal	Normal and lognormal	Normal and lognormal	Gamma and Weibull	Phoon (2008)
X	Beta	x	x	Qingnian and Yuzhou (2011)
Normal	Lognormal	Normal	x	Prada *et al.* (2011)
Beta	x	Normal	x	Papaioannou and Straub (2012)
X	Lognormal	Lognormal	Gamma	Fan and Liang (2013)
Normal	Normal	Normal	x	Wu (2013)

some randomly distributed, partly weathered, rock fragment. Nonetheless, depending on the type of formation and the influence of the weathering with depth, the soil properties may have different scale of fluctuations that can make the definition of the soil parameters difficult. Therefore, the integration of geology with reliability-based designs, will reduce to some extent the gap between the uncertainty of the designs and the field behaviour of the geo-structures.

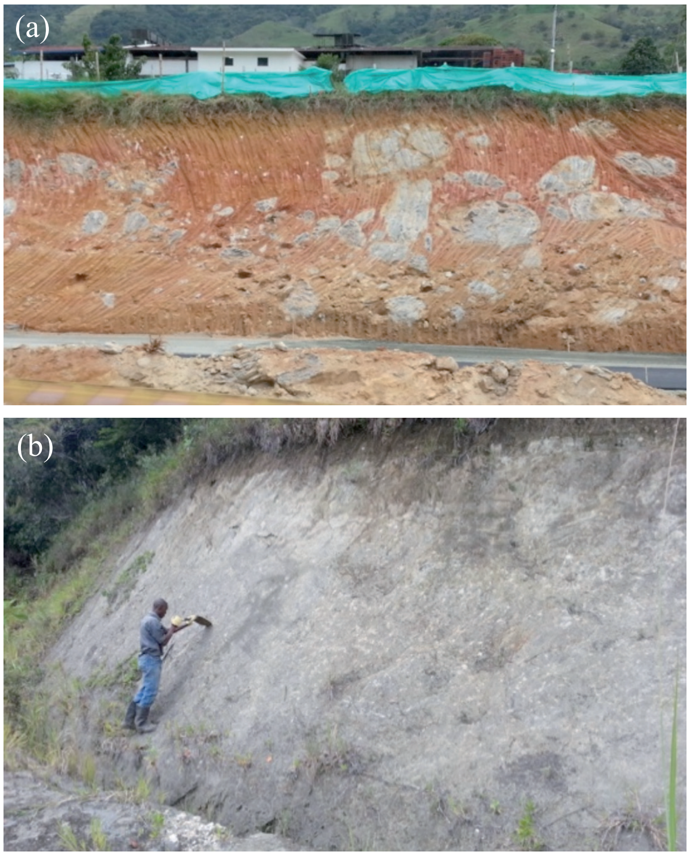

Figure 4: a) Mud/debris flow and b) residual soils characteristics

Geotechnical models

The most commonly used models in geotechnics are the limit state (failure or serviceability) and finite elements models. The former is based on the classical theory of failure and elastic deformations in order to define the bearing capacity of foundations, slopes stability and earth pressures. The latter is based on stress-strain equations discretized in small nodes (Smith *et al*., 2013). The finite elements model is widely used in geotechnical engineering, because it determines the failure by the weak areas and not by a preset area (*e.g.* Griffiths and Lane, 1999). Because of the uncertainty of the parameters represented by the PDF, Random Finite Element Models RFEM arise, allowing the determination of fault conditions and the probability of success in the finite element method through a simulation method (Griffiths and Fenton, 1993).

Simulation methods

Several statistical methods involve various numerical analysis techniques for assessing the reliability of the geotechnical designs under a probabilistic approach. The most used simulation methods are the Method of First Order Second Moment FOSM, method of point estimates (Rosenblueth, 1975) and Monte Carlo method (e.g. Hidalgo and Pacheco, 2011). According to Lacasse and Nadim (1998), the applicability of the Monte Carlo method in geotechnical engineering is widely known for its conceptual simplicity compared to other methods. As shown in Figure 5, the use of Monte Carlo more than doubles the use of other methods in the specialized literature.

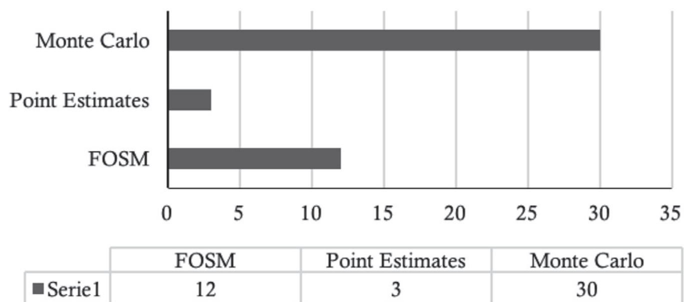

Figure 5: Simulation methods used on different papers in reliability-based designs in Geotechnics

Considering the above, we summarize a procedure for the reliability-based designs in the flowchart of Figure 6. The flowchart covers the type of investigation, the definition of a PDF that adequately represents soil properties and the analysis of the model results, in order to obtain the best estimate of the probability of failure.

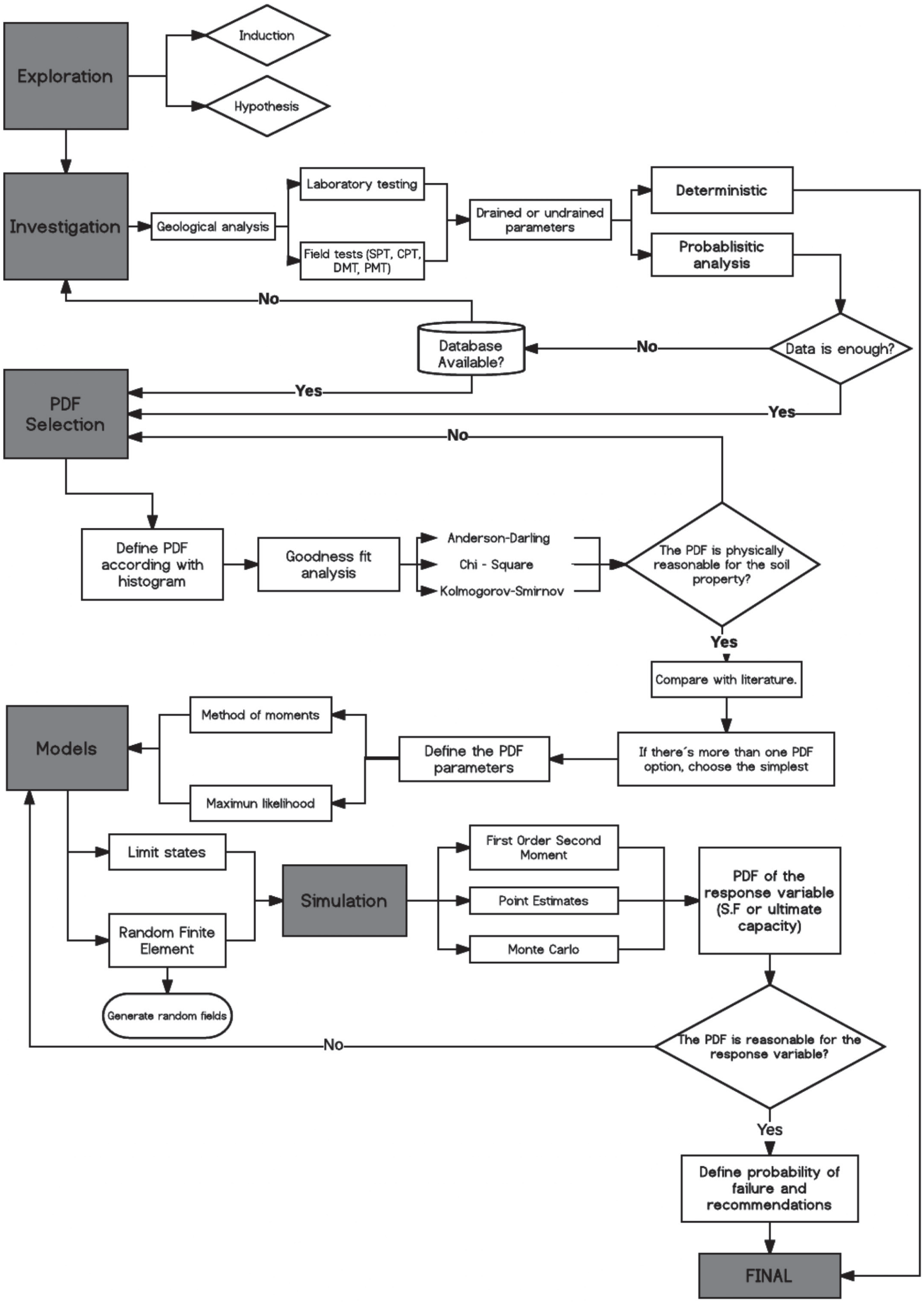

Figure 6: Flowchart to define the probability of failure in geotechnical engineering

Conclusions

It was found that pile walls are the most commonly studied retaining structures in reliability-based designs in geotechnical engineering. This can be explained by the fact that this type of structures are more frequently implemented compared to other type of walls. However, economy and new needs to find faster solutions to the earth retaining structures, it is expected that the anchor walls will take more ground in the reliability based designs. On the other hand, there is an absence of reliability studies in Gabion walls, which may be explained by the variability of the dimensions, type of fills and low stiffness of this type of walls, which make them highly complex to study and a less desirable choice for reliability-designs, despite its widespread use in several countries. There is also a lack of studies regarding the determination of the probability density functions of the soil-wall friction. This is an important variable in defining the stability of retaining structures. Soil-wall friction is commonly defined deterministically as a fraction of the friction angle, regardless of the contact material and the geological origin of the soil.

Studies that relate the geotechnical variables and models with the geological origin are not yet available. The consideration of the relationship between geological origin and mechanical properties in reliability-based designs will reduce uncertainties among the designs and better represent the real behaviour of geo-structures.

The definition of the PDF is among the most difficult tasks in the reliability-based designs, as it requires a large amount of data or access to reliable databases on similar soils, something commonly non-existent. The definition of these functions must also consider the shape of the function and the value of the parameters, because if these functions present a high content of negative values, a large amount of non-real values will arise in the models. Monte Carlo is the most implemented simulation method due to its conceptual simplicity compared to other methods. This has led to its adoption in different commercial and non-commercial software for geotechnical designs. Finally, a reliability-design procedure is proposed, which aims at generating the best representation of each variable and model from the start of the project and propending to define the probability of failure more accurately. Erroneous estimates of the probability of failure may lead to accept high-risk structures or to reject safe structures, affecting the economy and safety of different types of geotechnical structures.

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