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Comparison of Finite Difference and Finite Element (Telemac) Models of Dublin Bay

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Comparison of Finite Difference (SW2D) and Finite Element (Telemac) Models of Dublin Bay

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Centre for Water Resources Research

Context of Study

- CWRR, UCD (Centre for Water Resources Research)
- Dublin Bay – Hydrodynamic/Water Quality Models
- Qiang (93) - Dublin Bay Water Quality Management Plan
- Hussey (96) – SW2D - Extended Area
  - Eulerian-Langrangian 2D Finite Difference Model
  - Dublin Bay Water Quality Management Plan Study
  - Howth Outfall Study
- Bedri (07)
  - 3D Hydrodynamic/Water Quality Telemac Model of Inner Bay

SW2D Model Domain

Model Domain

-6° 15’ to -5° 50’ E-W
53° 10’ to 53° 30’ N-S
over 72000 grid boxes

Bathymetry

Admiralty Charts 1447 & 1468
Surveys for Various Studies

- Irish Hydrodata Ltd - Howth
- BKS – Tolka Mudflats
- ESB International – Bull Island

Boundary Conditions

North and South - Elevations for Spring and Neap Tides
East – “Glass Wall”

Objectives of Study

TELEMAC vs SW2D

- Accuracy
- Stability
- Computational Time
- Ease of use - man hour costs.

Schedule of Simulations

Code	Description
SW2D	The original finite difference Extended Dublin Bay model using a grid size of 100.79m by 92.75m.
T0	The Baseline TELEMAC finite element model with a uniform mesh with a resolution of 104m between the nodes.
	The four meshes used in the Telemac Convergence Study to determine the optimum mesh.
T1	12985 nodes
T2	22611 nodes
T3	31653 nodes
T4	49381 nodes

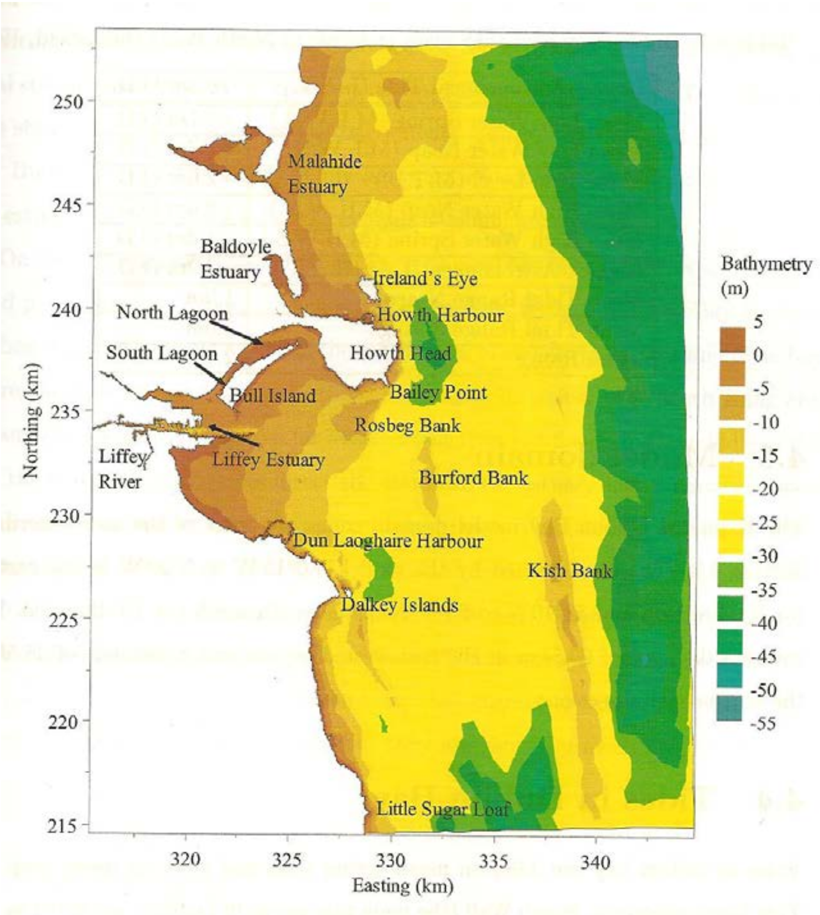
Field Measurements- Tides

Tidal Gauges

October 1998

Tidal Constituents

- North Wall Lighthouse
- North Bank Lighthouse
- Kish Bank Lighthouse
- Howth Harbour
- Dun Laoghaire

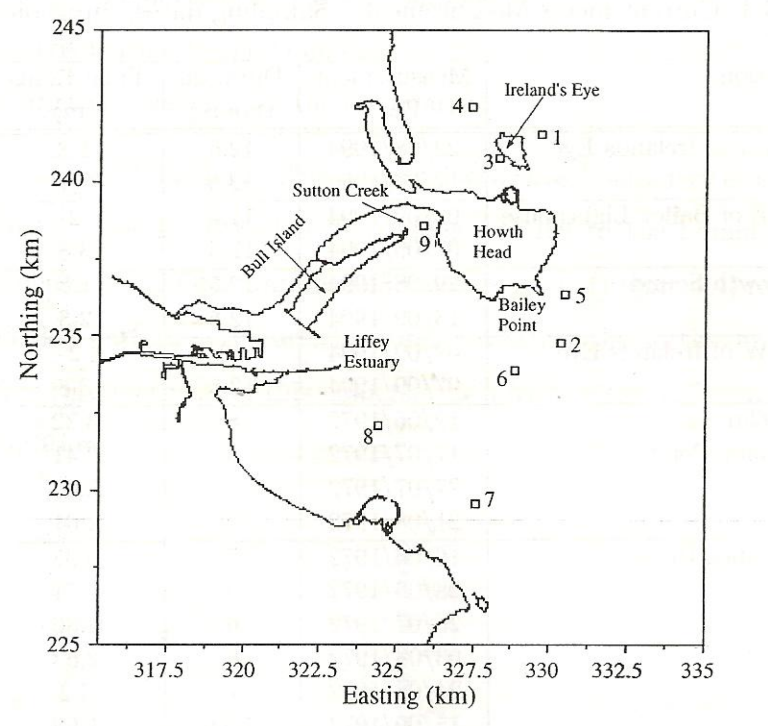


Field Measurements- Currents

- Spring and Neap Tides
- Locations 1-4
  Environmental Study of Howth
  - Irish Hydrodata (98)

- Locations 5 -8
  Environmental Study of Liffey Estuary and Dublin Bay
  -Crisp (76)

Location of Current Meters



Depth Averaged Fluid Equations

Continuity Equation

$$\frac{\partial \xi}{\partial t} + \frac{\partial HU}{\partial x} + \frac{\partial HV}{\partial x} = 0$$

Momentum Equation in *x*-Direction

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} - fV + g\frac{\partial \xi}{\partial x} + \gamma U - \frac{\theta \rho_a W^2 \sin\psi}{\rho H} - k\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial x^2}\right) = 0$$

Momentum Equation in y-Direction

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} + fU + g\frac{\partial \xi}{\partial y} + \gamma V - \frac{\theta \rho_a W^2 \cos\psi}{\rho H} - k\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right) = 0$$

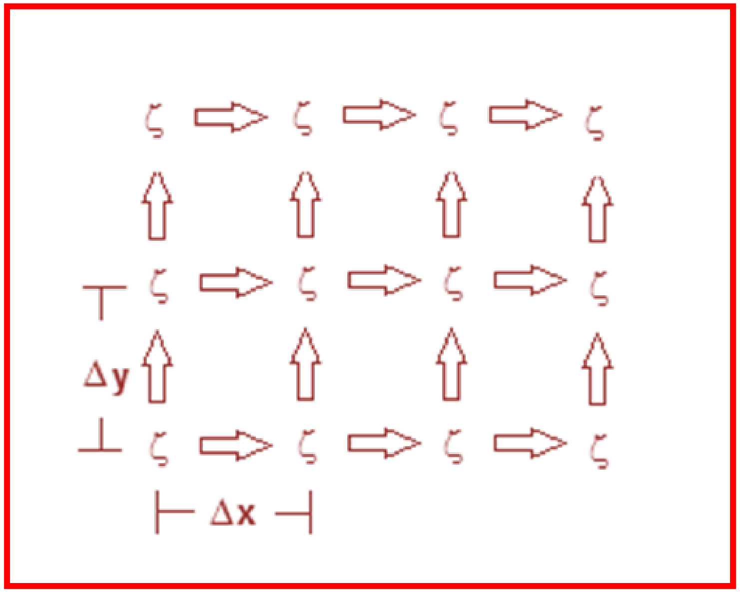
SW2D - Momentum Equation

$$\frac{DU}{Dt} = fV - g\frac{\partial \xi}{\partial x} - \gamma U + \frac{\theta \rho_a W^2 \sin\psi}{\rho H} + k\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial x^2}\right)$$

$$\frac{U^{n+1}_{i+\frac{1}{2},j} - U^{n}_{i+\frac{1}{2}-a,j-b}}{\Delta t} = fV^{n}_{i+\frac{1}{2},j} - \frac{g\alpha_1}{\Delta x}\left(\xi^{n+1}_{i+1,j} - \xi^{n+1}_{i,j}\right) - \frac{g(1-\alpha_1)}{\Delta x}\left(\xi^{n}_{i+1,j} - \xi^{n}_{i,j}\right) +$$

$$k\left(\frac{U^{n}_{i-\frac{1}{2},j} - 2U^{n}_{i+\frac{1}{2},j} + U^{n}_{i+\frac{3}{2},j}}{\Delta x^2} + \frac{U^{n}_{i+\frac{1}{2},j-1} - 2U^{n}_{i+\frac{1}{2},j} + U^{n}_{i+\frac{1}{2},j+1}}{\Delta y^2}\right) -$$

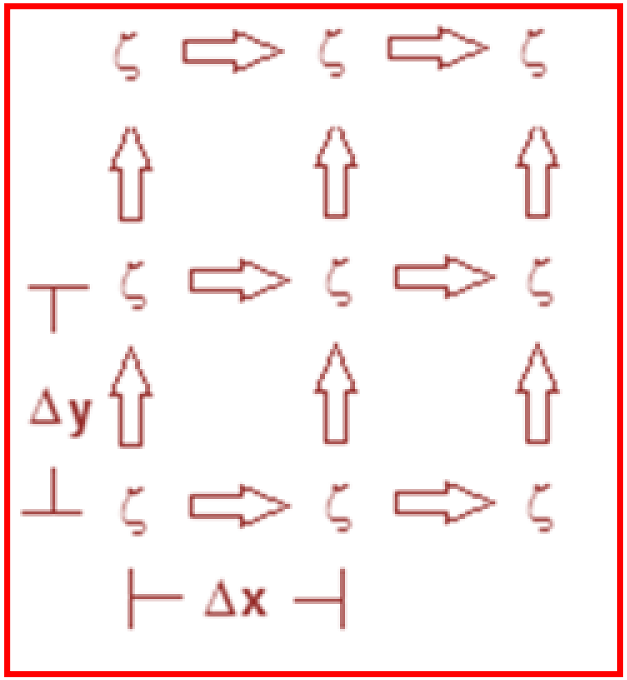
$$\frac{\alpha_2 \gamma^{n}_{i+\frac{1}{2},j}}{H^{n}_{i+\frac{1}{2},j}} U^{n+1}_{i+\frac{1}{2},j} - \frac{(1-\alpha_2)\gamma^{n}_{i+\frac{1}{2},j}}{H^{n}_{i+\frac{1}{2},j}} U^{n}_{i+\frac{1}{2},j} + \left(\frac{\theta \rho_a W^2 \sin\psi}{\rho H}\right)^{n}_{i+\frac{1}{2},j}$$



$$U^{n+1}_{i,j+\frac{1}{2}} = \left(1 + \frac{\alpha_2 \Delta t\, \gamma^{n}_{i+\frac{1}{2},j}}{H^{n}_{i+\frac{1}{2},j}}\right)^{-1}\left[FU^{n}_{i+\frac{1}{2},j} - \frac{g\Delta t \alpha_1}{\Delta x}\left(\xi^{n+1}_{i+1,j} - \xi^{n+1}_{i,j}\right)\right]$$

SW2D - Continuity Equation

$$\left(1 + AAU^{n}_{i+\frac{1}{2},j} + AAU^{n}_{i-\frac{1}{2},j} + AAV^{n}_{i,j+\frac{1}{2}} + AAV^{n}_{i,j-\frac{1}{2}}\right)\xi^{n+1}_{i,j} - AAU^{n}_{i+\frac{1}{2},j}\xi^{n+1}_{i+\frac{1}{2},j} - AAU^{n}_{i-\frac{1}{2},j}\xi^{n+1}_{i-\frac{1}{2},j} - AAV^{n}_{i,j+\frac{1}{2}}\xi^{n+1}_{i,j+\frac{1}{2}} - AV^{n}_{i,j-\frac{1}{2}}\xi^{n+1}_{i,j-\frac{1}{2}} = B^{n}_{i,j}\xi^{n}_{i,j}$$



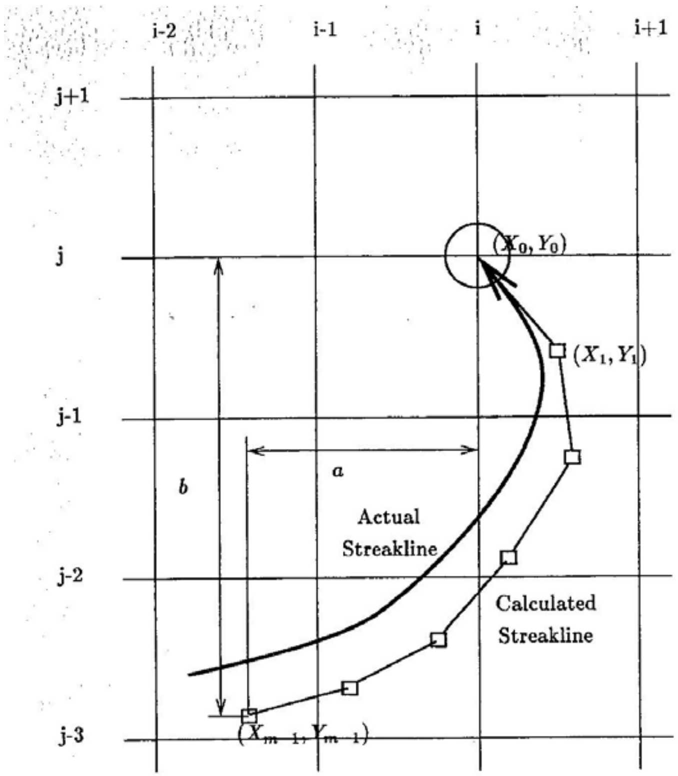
SW2D - Reverse Particle Tracking

- Euler Method

$$U^{n}_{i+\frac{1}{2}-a,j-b}$$

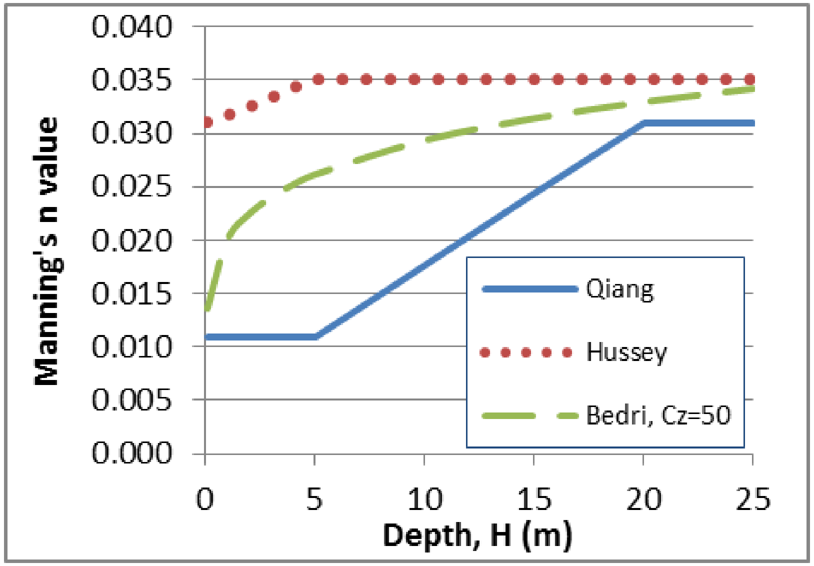
$$x^{n+\frac{1}{10}} = x^{n} + U\Delta t_{s}$$

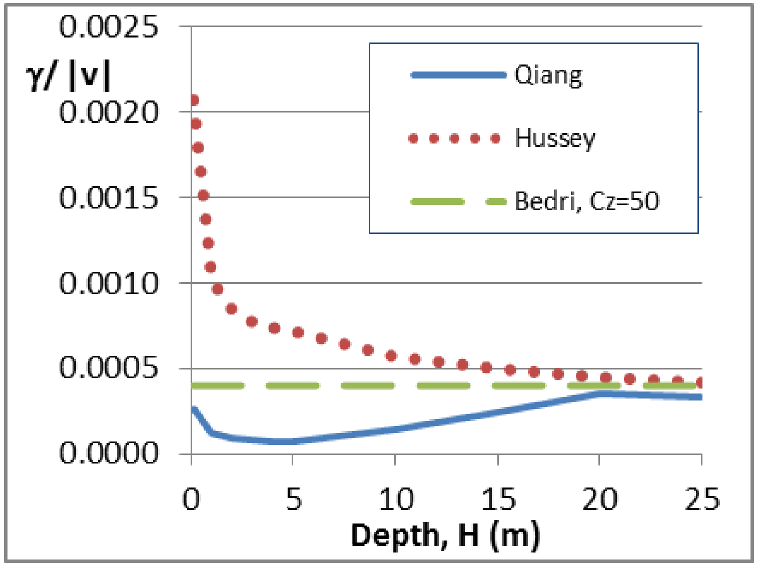
$$y^{n+\frac{1}{10}} = y^{n} + V\Delta t_{s}$$



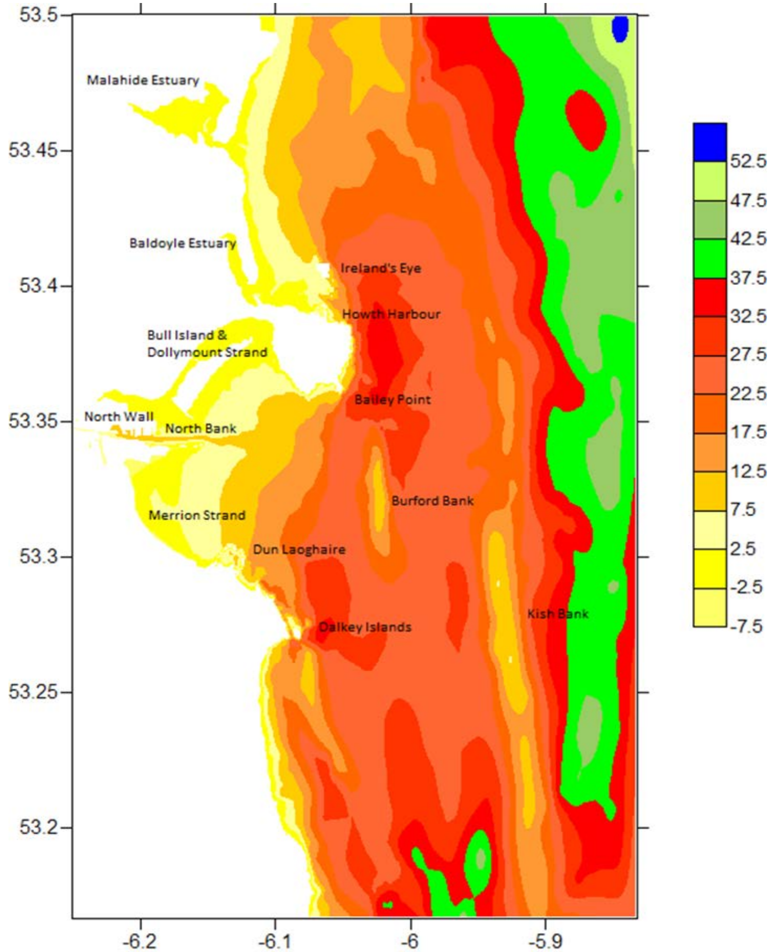
Bottom Friction Parameter

$$\gamma = \frac{\sqrt{U^2+V^2}}{C_z^2} \text{ where } C_z = \frac{H^{\frac{1}{6}}}{n}$$

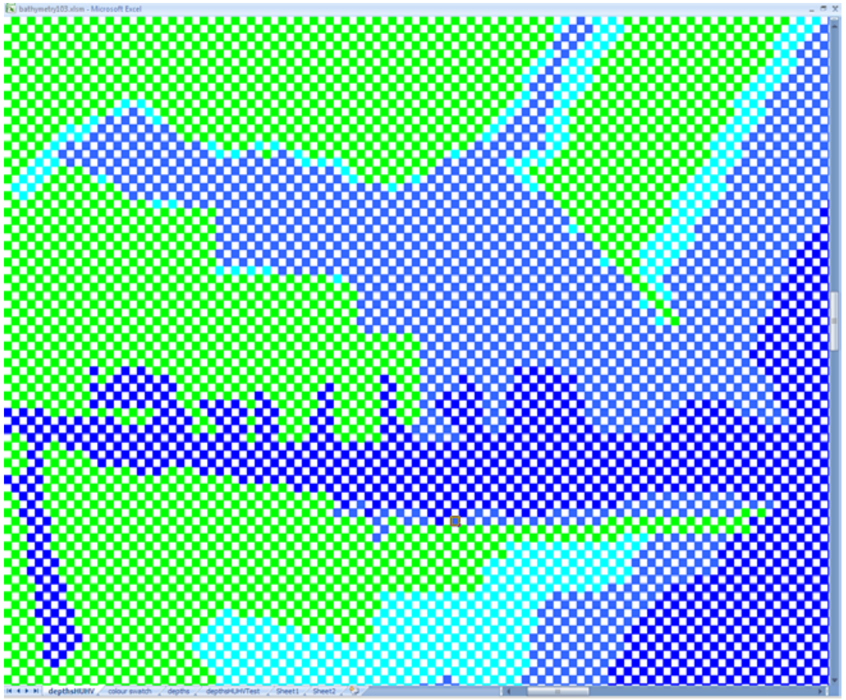




Surfer Grid & SW2D Pre-Processor



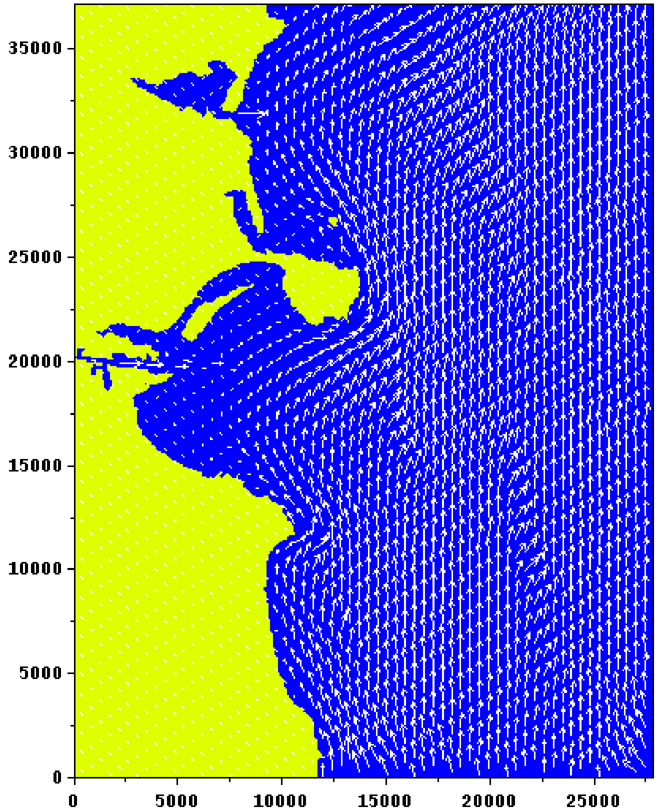
Bathymetry - Surfer Grid



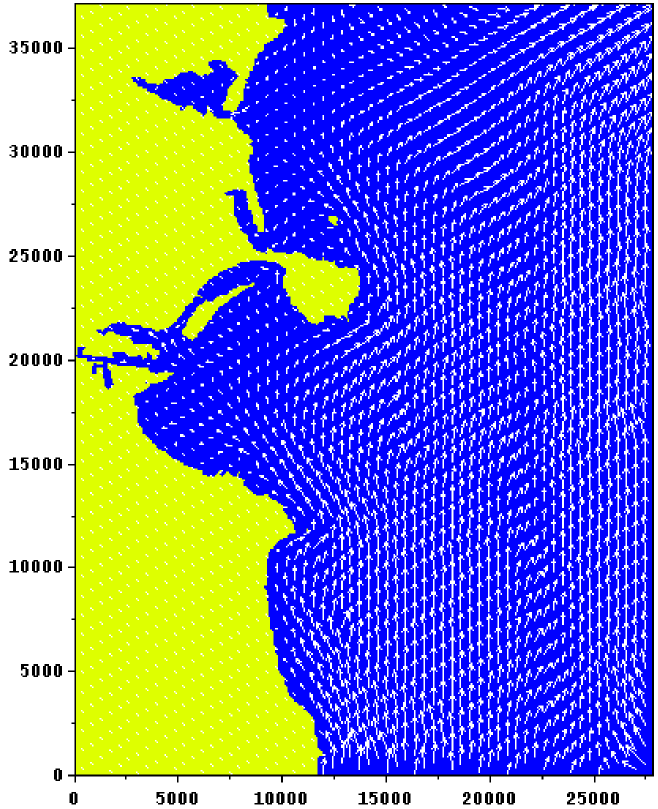
Excel VBA – Preprocessor

SW2D – Vector Plots

Low Water

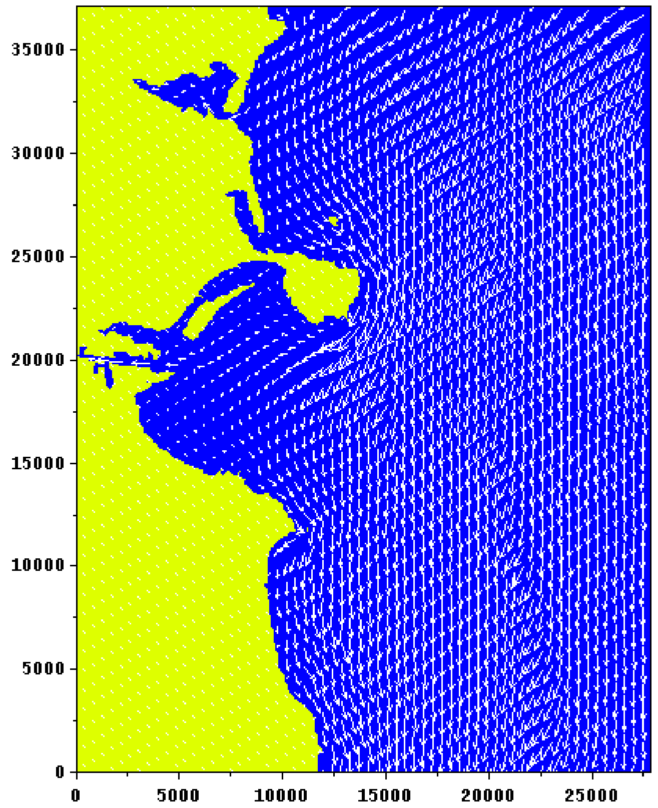


Mid-Flood

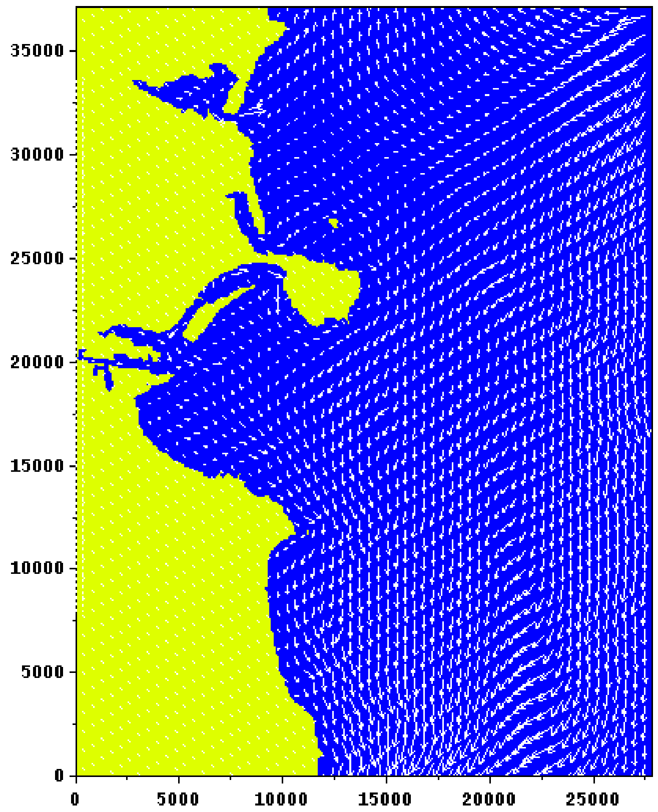


SW2D – Vector Plots

High Water



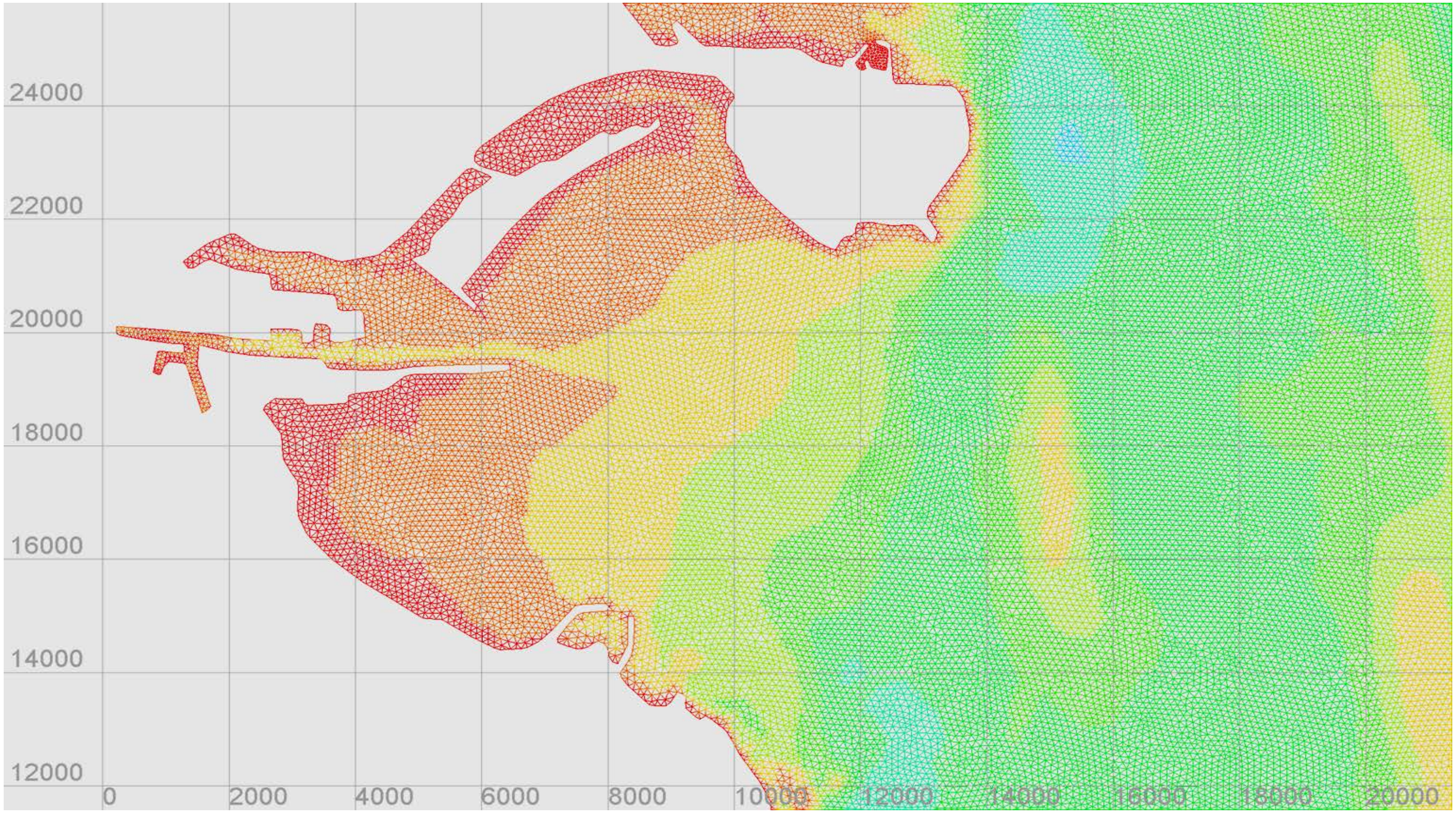
Mid-Ebb



SW2D- Tabulated Results

		North Wall	North Bank	Dun Laoghaire	Kish Lighthouse	Howth Harbour
Low Tide (m)	model	-1.607	-1.575	-1.597	-1.542	-1.750
	measured	-1.720	-1.560	-1.665	-1.530	-1.730
	% diff	6.6%	-1.0%	4.1%	-0.8%	-1.2%
High Tide (m)	model	1.890	1.880	1.870	1.800	2.025
	measured	2.060	1.900	1.93	1.900	2.080
	% diff	8.3%	1.1%	3.1%	5.3%	2.6%
Tidal Range (m)	model	3.497	3.455	3.467	3.342	3.775
	measured	3.780	3.460	3.595	3.430	3.810
	% diff	7.5%	0.1%	3.6%	2.6%	0.9%
Time of Low Tide	model	09:11	09:20	09:11	09:06	09:05
	measured	09:07	09:06	09:01	09:11	09:45
	diff (mins)	+4	+14	+10	+5	**-40**
Time of High Tide	model	15:16	15:18	15:18	15:14	15:13
	measured	15:20	15:16	15:23	15:11	15:56
	diff (mins)	-4	-2	-5	+3	**-43**

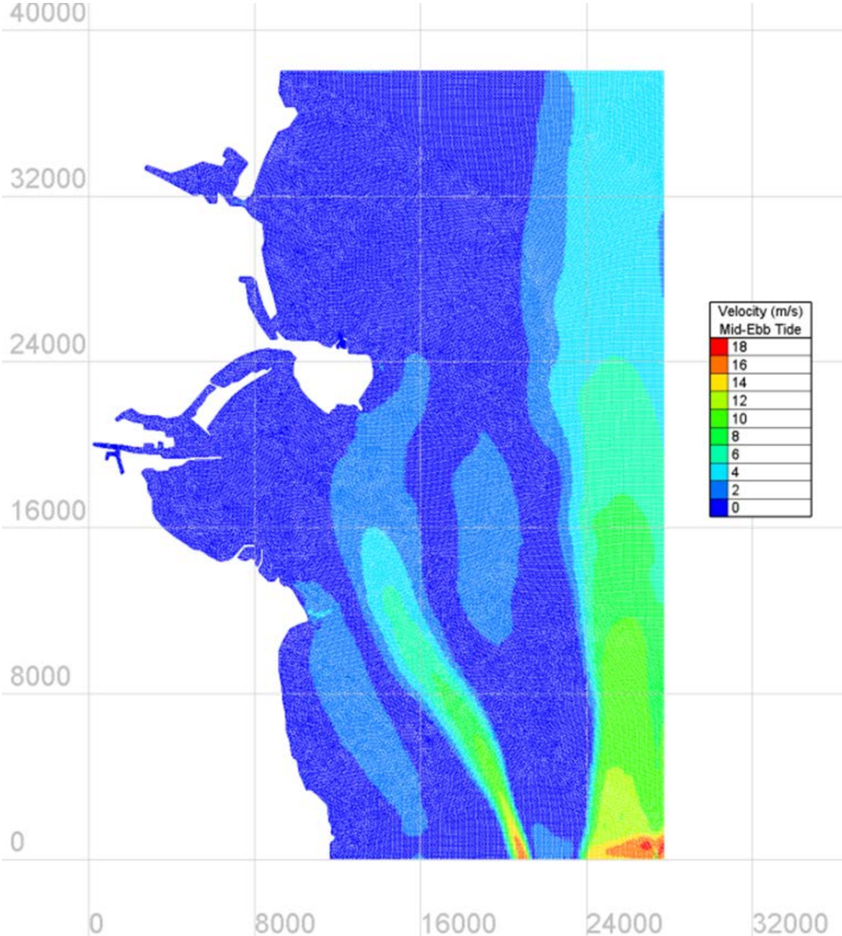
Telemac – Structured Mesh



Prepared with Blue Kenue

Canadian Hydraulics Centre of the National Research Council Canada

Telemac Structured Mesh - Results



- A finite element solution is generally considered to be unique if the "entropy" condition is satisfied (Hervouet, 2007). In the Saint-Venant equations, the entropy is equal to the total energy of a column of water written as:

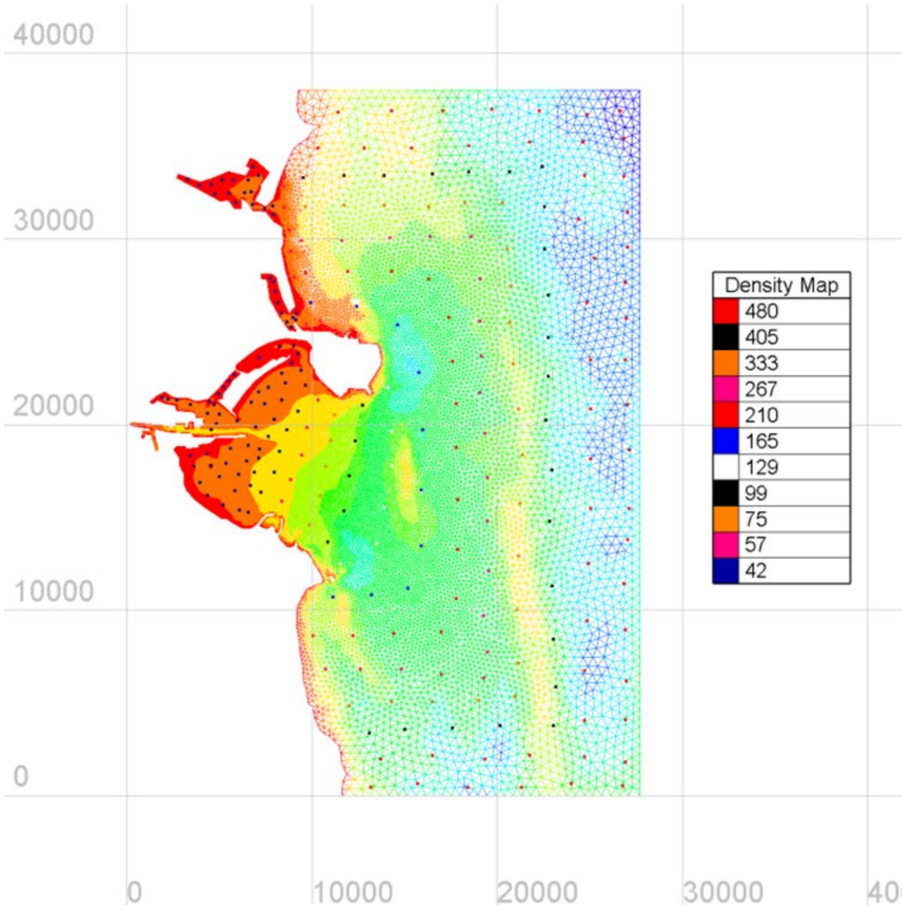
$$E = \frac{hu^2}{2} + g\frac{h^2}{2} + ghZ_f$$

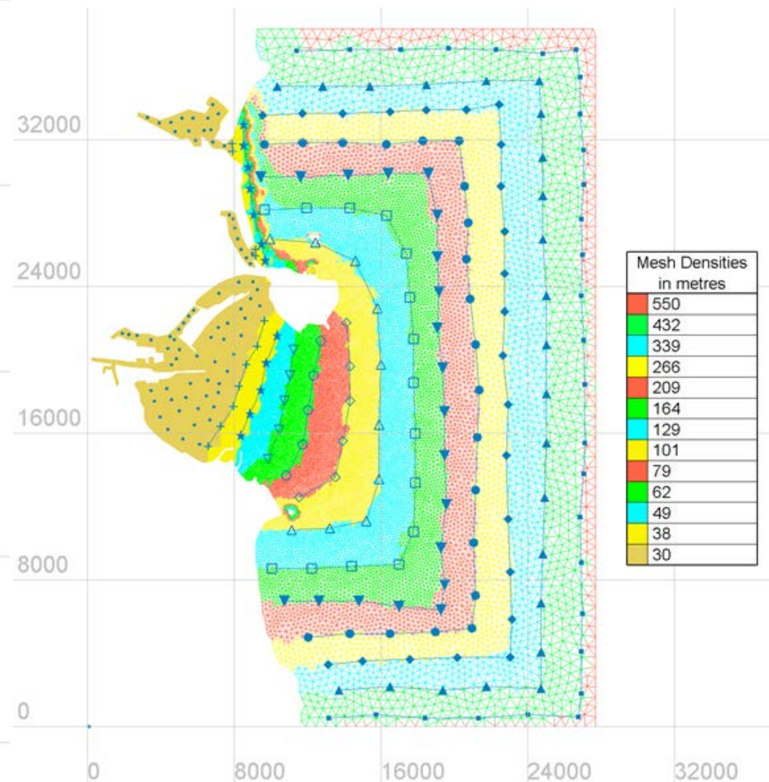
- The entropy condition is given by:

$$\frac{\partial E}{\partial t} + div\left[u\left(E + g\frac{h^2}{2}\right)\right] \leq 0$$

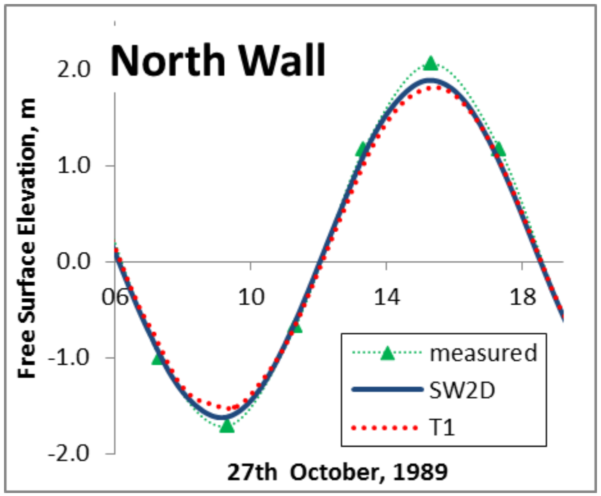
- The energy flux into the model domain increases as the mesh is refined at an open boundary. The entropy condition is not satisfied if the energy flux becomes too large, resulting in the possibility of an infinite number of solutions of the Saint-Venant equations and instability.

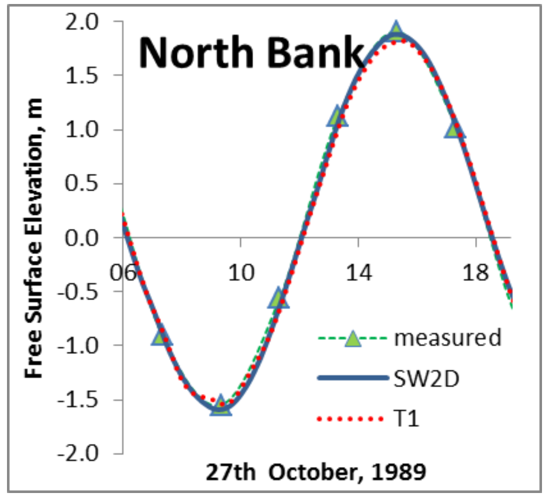
Telemac – Unstructured Mesh

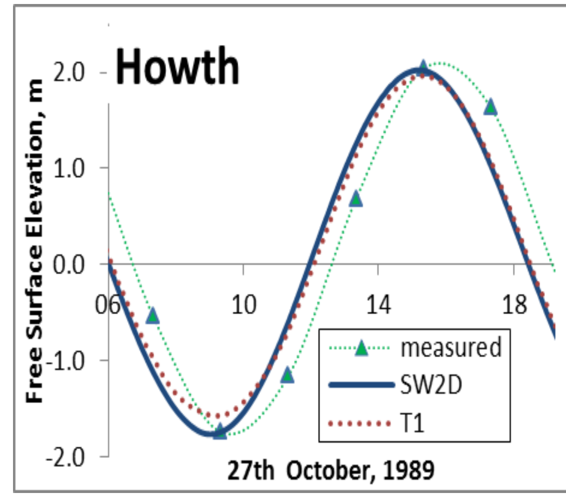


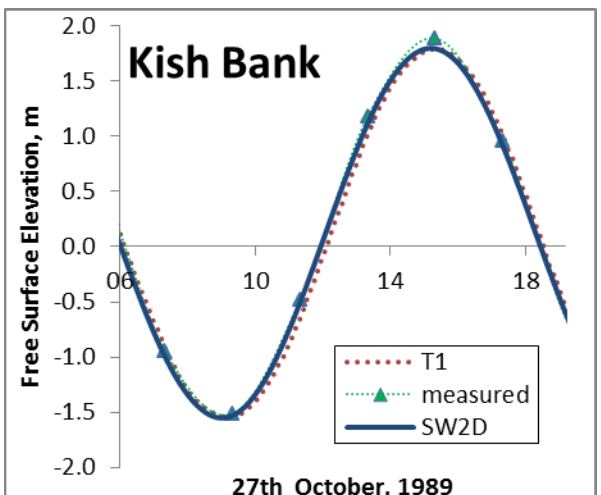


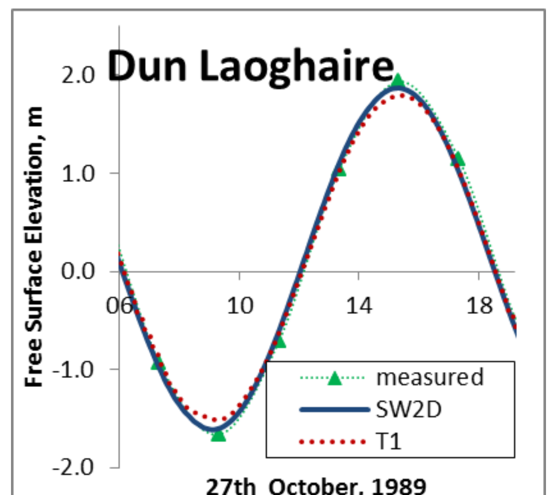
Tidal Elevations
Measured, SW2D & Telemac



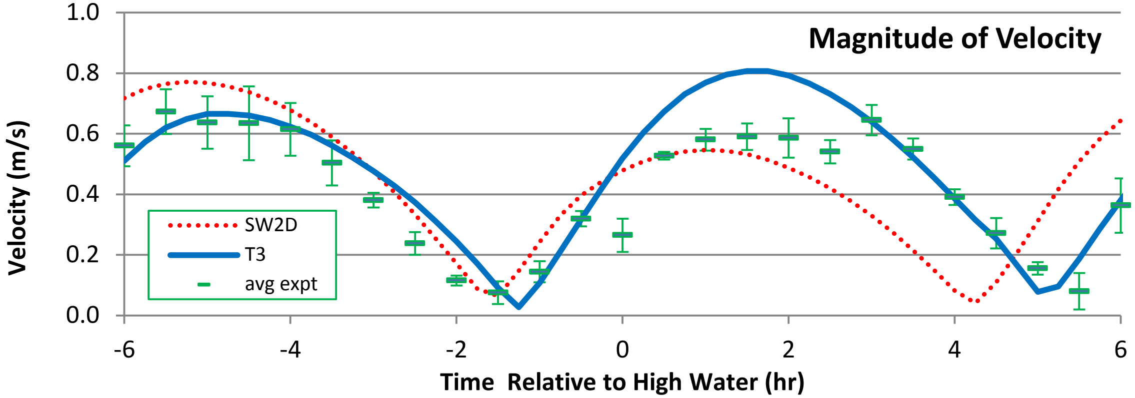


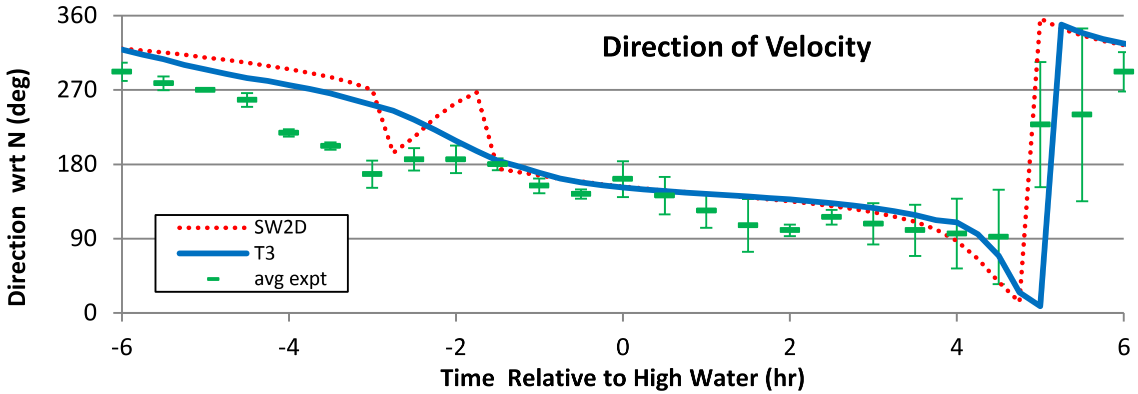




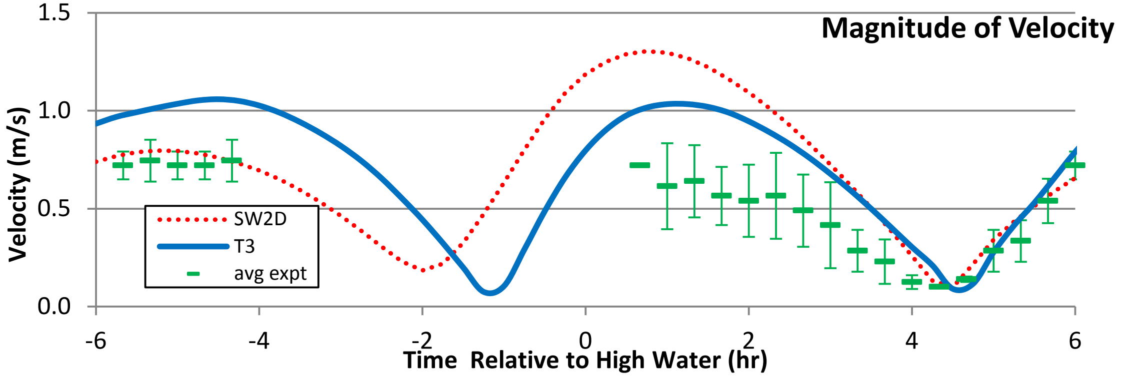


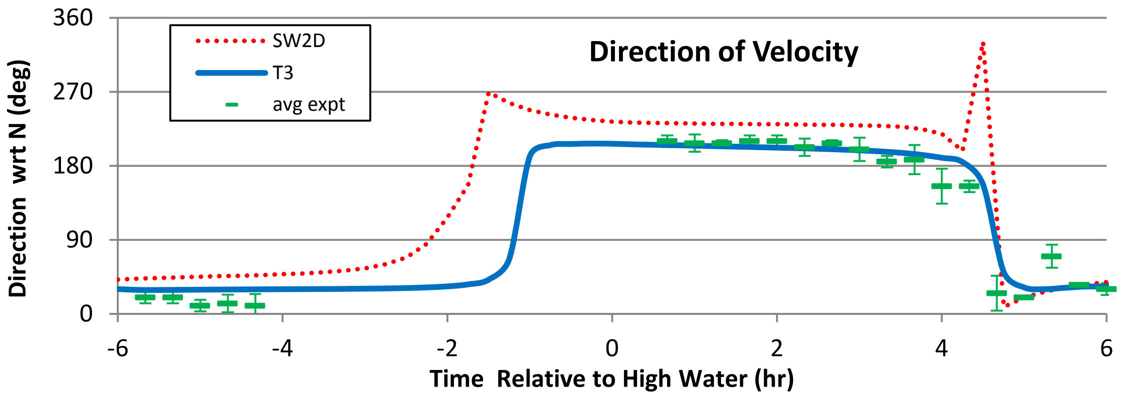
Location 1 – Spring Tide



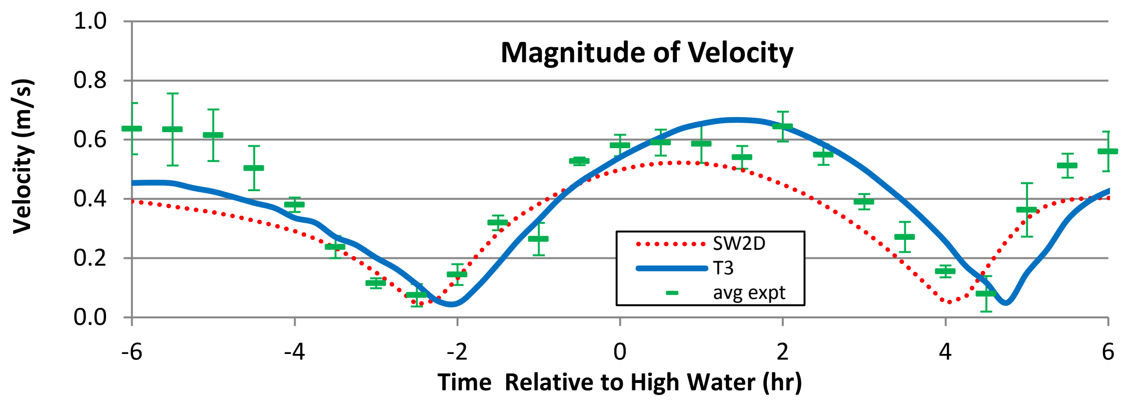


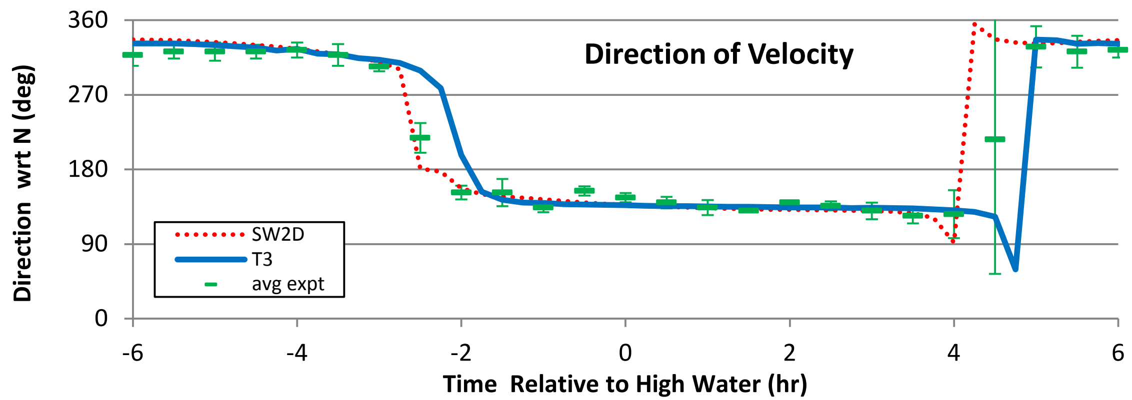
Location 2 – Spring Tide



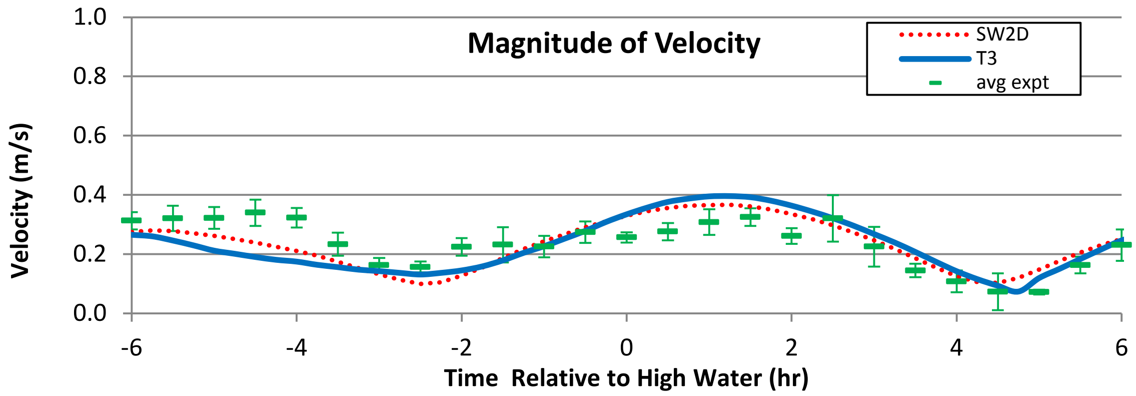


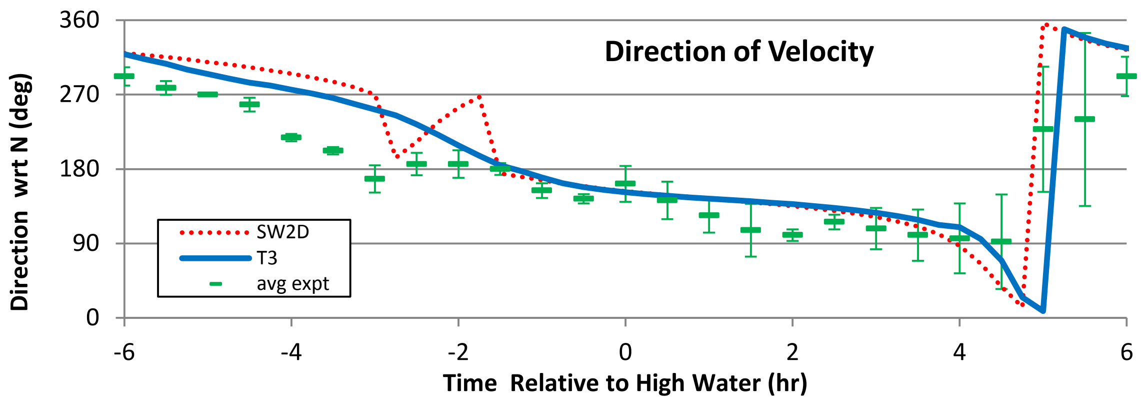
Location 3 – Spring Tide



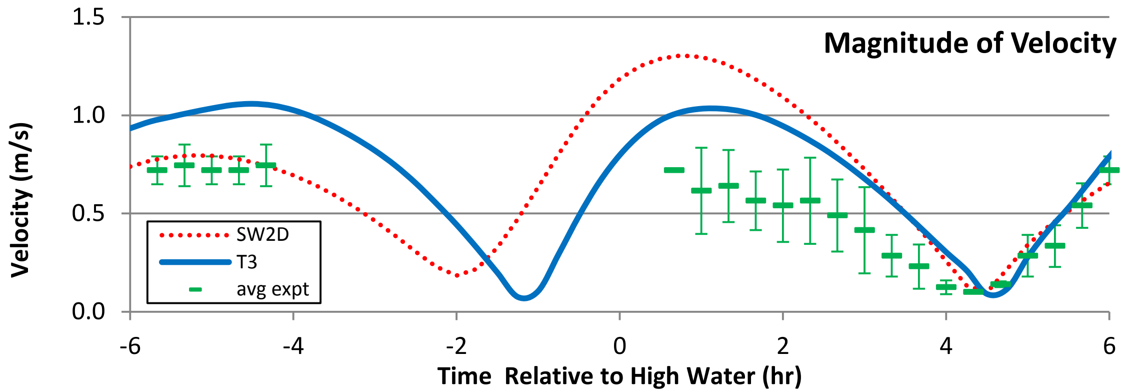


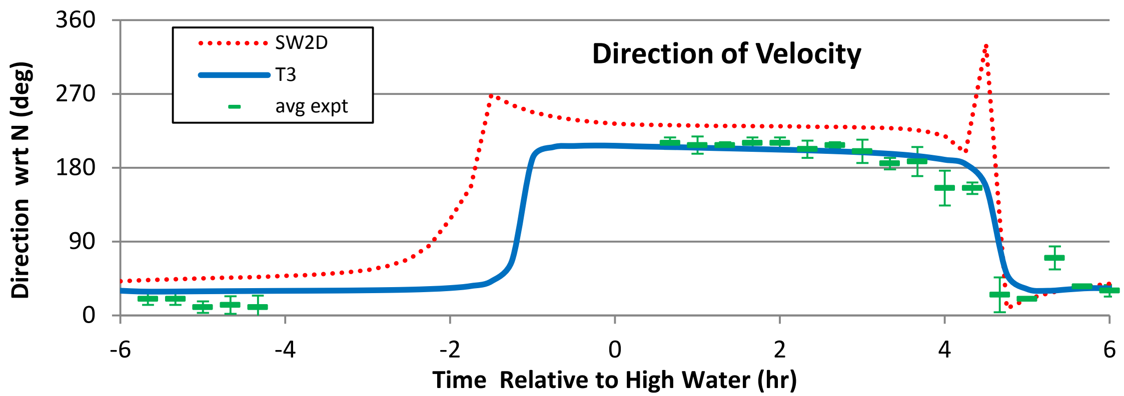
Location 4 – Spring Tide



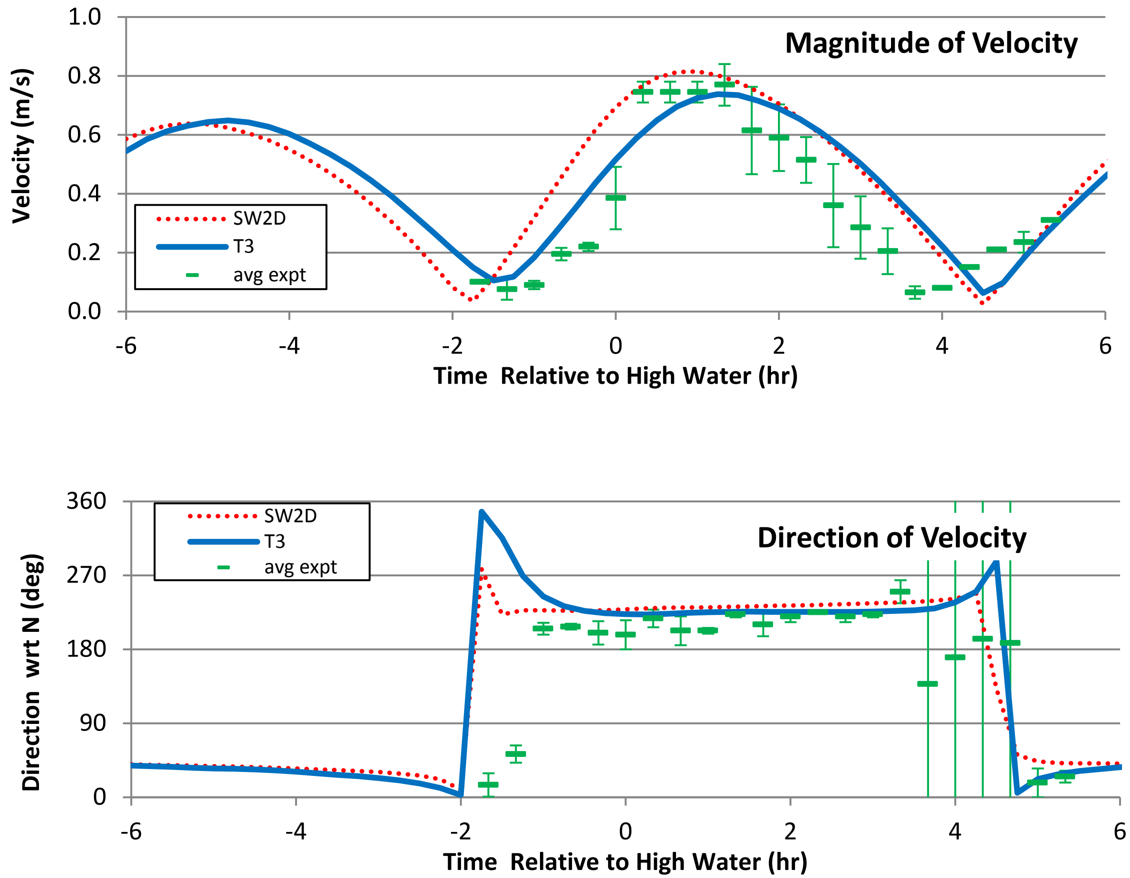


Location 5 – Spring Tide

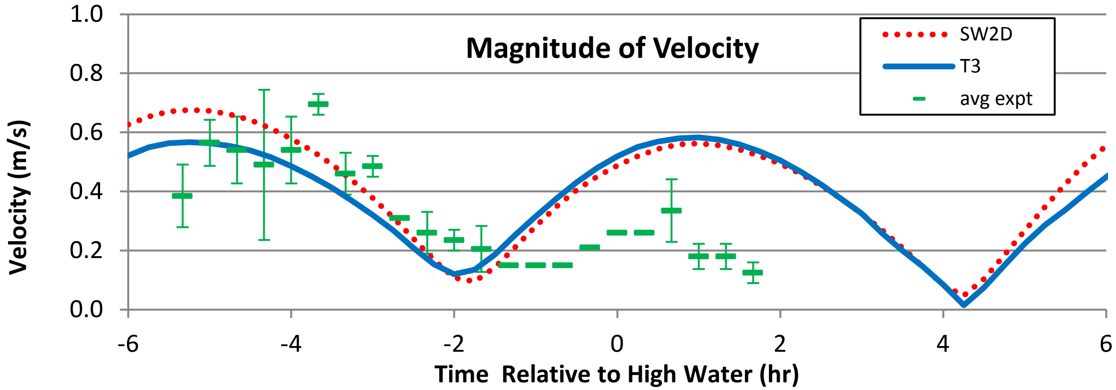


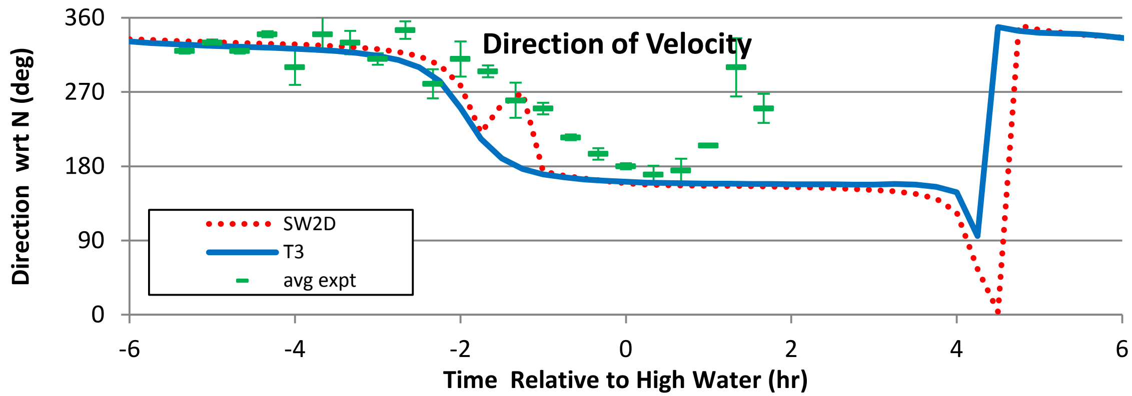


Location 6 – Spring Tide

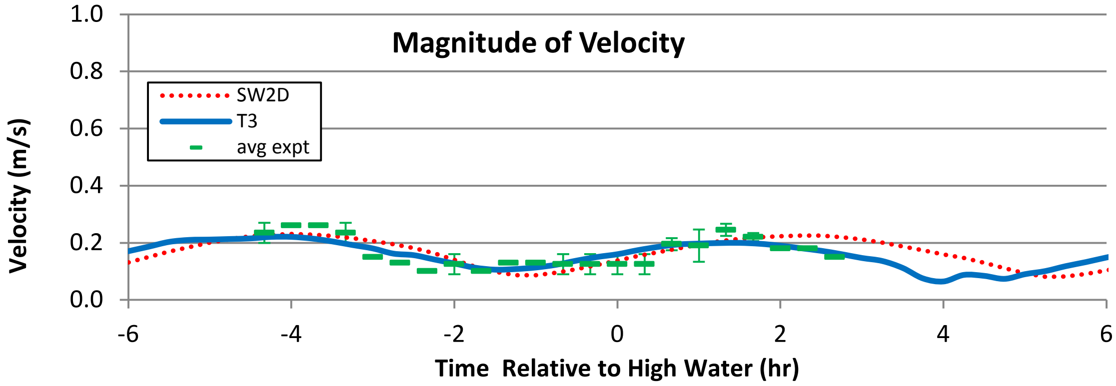


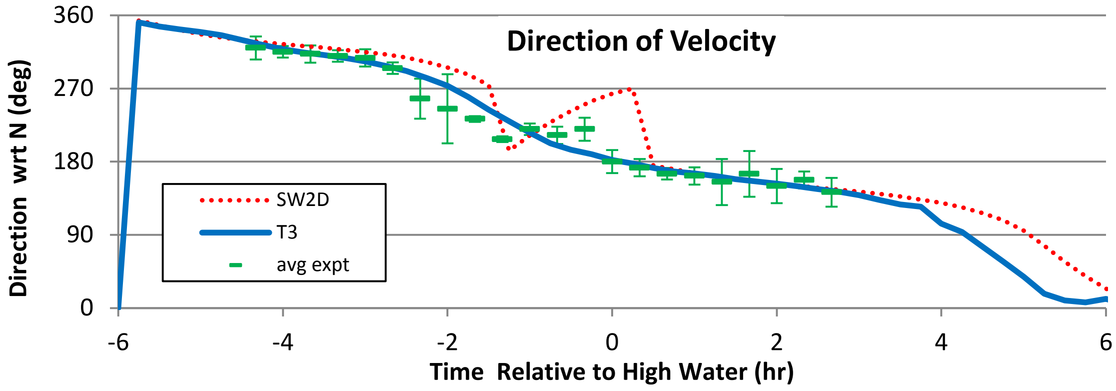
Location 7 – Spring Tide





Location 8 – Spring Tide





Simulation Times

- Dell OptiPlex 780 – Intel Pentium CPU G840 @ 2.80GHz chip.
- Equivalent Simulation Time – 5 cycles.

Model	Nodes (1000)	Timestep (s)	CPU time (s)
SW2D	72	30	121,000
T5	75	5	5428
T5	75	15	2007
T3	25	5	1645
T3	25	15	601
T3	25	30	341