

Spring 5-2016

Applications of the Sierpiński Triangle to Musical Composition

Samuel C. Dent
University of Southern Mississippi

Follow this and additional works at: https://aquila.usm.edu/honors_theses



Part of the [Composition Commons](#), and the [Dynamical Systems Commons](#)

Recommended Citation

Dent, Samuel C., "Applications of the Sierpiński Triangle to Musical Composition" (2016). *Honors Theses*. 415.

https://aquila.usm.edu/honors_theses/415

This Honors College Thesis is brought to you for free and open access by the Honors College at The Aquila Digital Community. It has been accepted for inclusion in Honors Theses by an authorized administrator of The Aquila Digital Community. For more information, please contact Joshua.Cromwell@usm.edu.

The University of Southern Mississippi

Applications of the Sierpiński Triangle to Musical Composition

by

Samuel Dent

A Thesis
Submitted to the Honors College of
The University of Southern Mississippi
in Partial Fulfillment
of the Requirements for the Degree of
Bachelor of Science
in the Department of Mathematics

May 2016

Approved by

Jiu Ding, Ph.D., Thesis Adviser
Professor of Mathematics

Bernd Schroeder, Ph.D., Chair
Department of Mathematics

Ellen Weinauer, Ph.D., Dean
Honors College

Abstract

The present paper builds on the idea of composing music via fractals, specifically the Sierpiński Triangle and the Sierpiński Pedal Triangle. The resulting methods are intended to produce not just a series of random notes, but a series that we think pleases the ear. One method utilizes the iterative process of generating the Sierpiński Triangle and Sierpiński Pedal Triangle via matrix operations by applying this process to a geometric configuration of note names. This technique designs the largest components of the musical work first, then creates subsequent layers where each layer adds more detail.

Key Words: Sierpiński Triangle, Sierpiński Gasket, pedal triangle, music composition, matrix

Acknowledgements

I thank my adviser, Dr. Jiu Ding, for his patience and guidance while helping me prepare this thesis. I would like to thank Dr. Danny Beard for my music theory education and for his helpful critique. I would also like to thank Dr. John Perry for his aid in devising the implementation of the computer code in SAGE. Finally, I thank Dr. Richard Perry for his support and critique of the compositions.

Table of Contents

Figures and Algorithm Pseudocode.....	vii
Definitions and Abbreviations.....	viii
Chapter 1: Introduction.....	1
Description of the problem.....	1
Relevant background.....	2
Chapter 2: Matrix IFS.....	9
Methods.....	9
Results.....	9
Discussion.....	14
Chapter 3: Music Composition.....	16
Methods.....	16
Results.....	16
Discussion.....	18
Bibliography.....	28
Appendices.....	29
Appendix A: SAGE Codes.....	29
Appendix B: Example Graphs.....	36
Appendix C: <i>Sierpiński Pedal Quartet</i>	39

Figures

1.1 Process of feedback.....	2
1.2 Sierpiński Process.....	2
1.3 First iteration of the Sierpiński Triangle IFS.....	5
1.4 Sierpiński Pedal Process.....	6
1.5 An L-System.....	7
1.6 String of notes from Figure 1.5.....	7
1.7 An L-System for a chord progression	8
3.1 Sierpiński L-System on a C maj tonic chord.....	16
3.2 Sierpiński Pedal L-System on E-G-F.....	17

Algorithm Pseudocode

Algorithm 1.....	9
Algorithm 2.....	10
Algorithm 3.....	13
Algorithm 4.....	13

Definitions

Dynamical System.....	3
Feedback.....	2
Fractal.....	1
Fractal Music.....	6
Independent.....	3
Iterated Function System.....	4
L-System.....	6
Limiting Set.....	4
Pedal Triangle.....	6
Sierpiński Pedal Triangle.....	6
Sierpiński Triangle.....	2

Abbreviations

Control Unit - CU

Iterated Function System – IFS

Input Unit - IU

Major Key – maj.

Minor Key – min.

Output Unit - OU

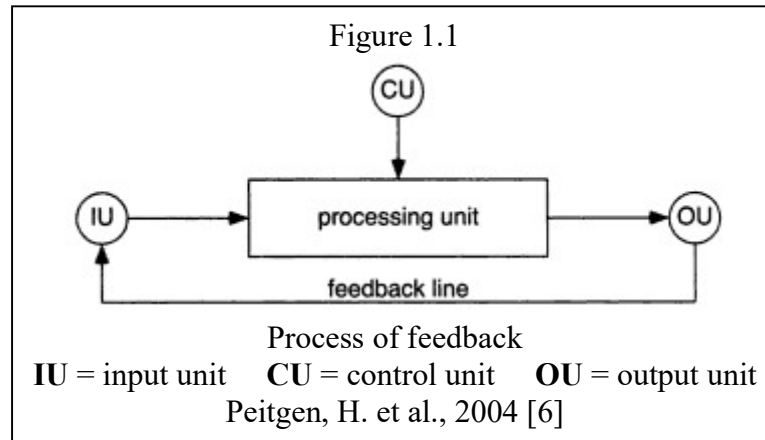
Sierpiński Pedal Triangle – SPT

Chapter 1: Introduction

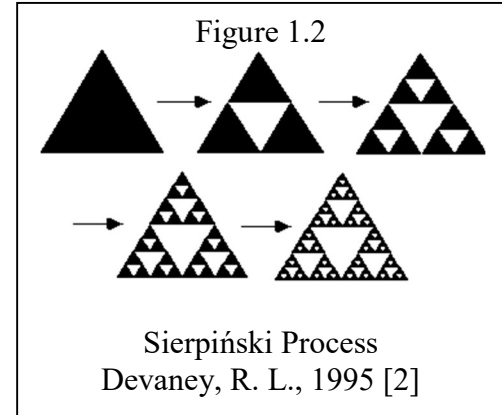
Description of the Problem

This project intends to explore one of the relationships between mathematics and music, specifically through the fractals known as the Sierpiński Triangle and the Sierpiński Pedal Triangle. A **fractal** is a geometric construct that is self-similar throughout its structure. Fractals can appear complex, but they are often generated by the simple process of iterating a pattern, where each iteration reduces the size of the pattern. Music, in essence, is an organized collection of sounds and silences. Composers frequently write music based on a pattern or motif of some kind, resulting in a self-similarity where a musical work can be broken down into sections, then into phrases, and so on. Each level of the music often reflects the nature of the work in its entirety. The self-similarity and patterns are evident in both fractals and music composition, indicating that there is a possible relationship between them. It is this relationship that this project means to explore. The music produced should be more than a series of random notes, but a series that pleases the ear. Additionally, this project seeks to generalize methods of producing the Sierpiński Triangle to the Sierpiński Pedal Triangle.

Relevant Background



Fractals largely depend on the principle of **feedback**: an operation that is repeated many times with the output of one iteration becoming the input for the next iteration. This process is shown in Figure 1.1, where the control unit is a set of parameters to produce a desired result.



The **Sierpiński Triangle** is a particular fractal produced through the feedback process shown in Figure 1.2. For the classic Sierpiński Triangle, the process begins with an equilateral triangle. Form an interior triangle by connecting the midpoints of the sides of the original triangle, and remove this interior triangle to leave three congruent equilateral triangles similar to the original. Take the output as the input and repeat this process indefinitely to produce the Sierpiński Triangle. Beginning with a different type of triangle also generates a Sierpiński Triangle where the n^{th} iteration produces 3^n congruent triangles that are similar to the initial triangle. As a result of the process removing a

triangle equal to a fourth the area of the previous triangle, the area of the construct on the n^{th} iteration is $(0.75)^n$. So, the Sierpiński Triangle itself has an area of 0 because

$$\lim_{n \rightarrow \infty} 0.75^n = 0$$

yet the Sierpiński Triangle can be seen by the human eye [5].

One way to produce fractals such as the Sierpiński Triangle is with an iterated function system, abbreviated IFS. Before discussing the IFS, further background information must be made clear. Lasota and colleagues define a sequence of random variables $\xi_1, \xi_2, \dots, \xi_n$ to be **independent** if, for a sequence of Borel sets B_1, B_2, \dots, B_n , the events $\{\xi_i \in B_i\}$ are independent for all i . That is,

$$\begin{aligned} \text{prob}\{\xi_1 \in B_1, \xi_2 \in B_2, \dots, \xi_n \in B_n\} \\ = \text{prob}\{\xi_1 \in B_1\} \times \text{prob}\{\xi_2 \in B_2\} \times \dots \times \text{prob}\{\xi_n \in B_n\} \end{aligned}$$

A family of transformations $S_t: X \rightarrow X$, for $t \in \mathbb{R}$ on a set X is a **dynamical system** $\{S_t\}_{t \in \mathbb{R}}$ if it satisfies the following properties [5]:

1. $S_0(x) = x, \forall x \in X$.
2. $S_t(S_{t'}(x)) = S_{t+t'}(x), \forall x \in X$ with $t, t' \in \mathbb{R}$.
3. The mapping $(t, x) \rightarrow S_t(x)$ from $\mathbb{R} \times X$ into X is continuous.

Now let X be a closed, non-empty subset of \mathbb{R}^d . Consider N continuous transformations

$$S_i: X \rightarrow X \text{ for } i = 1, \dots, N,$$

the probabilistic vector

$$(p_1, \dots, p_N), \text{ where } p_i \geq 0 \text{ and } \sum_{i=1}^N p_i = 1,$$

and the sequence of independent random variables ξ_1, ξ_2, \dots such that

$$\text{prob}\{\xi_n = i\} = p_i, \text{ for } i = 1, \dots, N.$$

The dynamical system defined by the formula

$$x_{n+1} = S_{\xi_n}(x_n) \text{ for } n = 0, 1, \dots$$

is called an **iterated function system** [5]. For $A \subset X$, we define

$$F(A) = \bigcup_{i=1}^N S_i(A) \quad \text{and} \quad A_n = F^n(A_0)$$

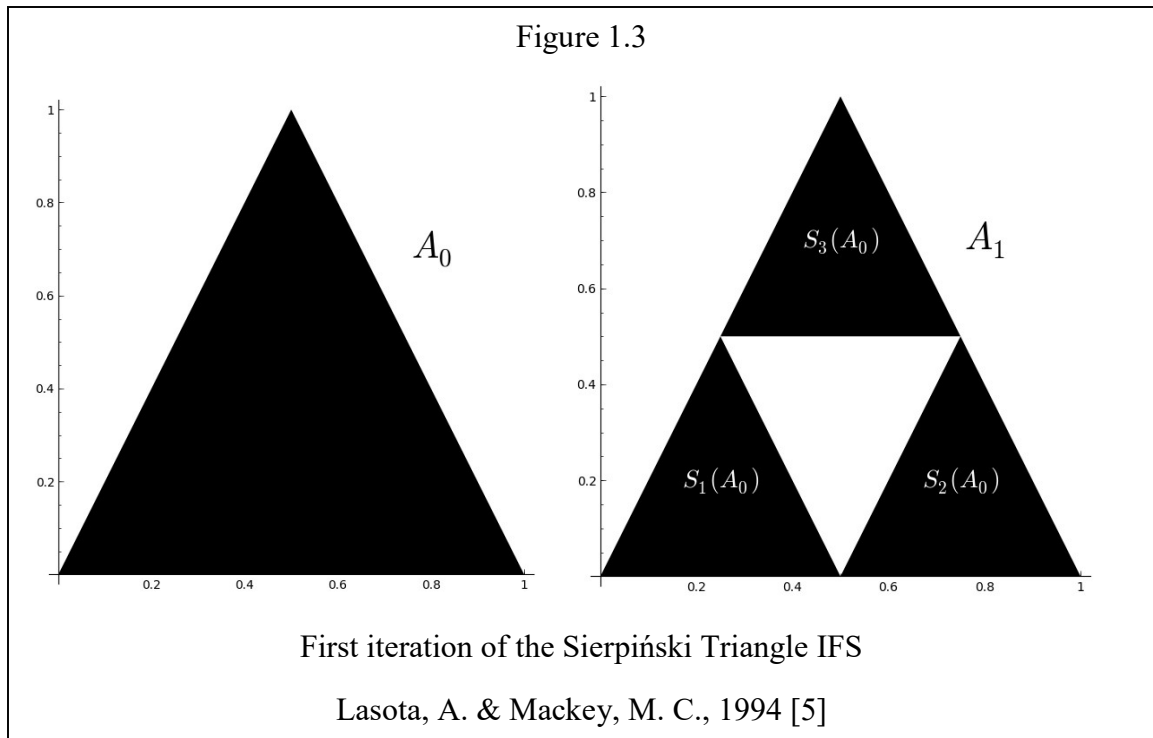
where $A_0 = A$. We also define the **limiting set** $A_* = \lim_{n \rightarrow \infty} F^n(A_0)$ [5].

The following is an IFS given by Lasota and colleagues that generates the Sierpiński

Triangle: let $X = \mathbb{R}^2$ and

$$S_i(x) = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} x + \begin{pmatrix} a_i \\ b_i \end{pmatrix}, \text{ for } i = 1, 2, 3 \text{ where}$$

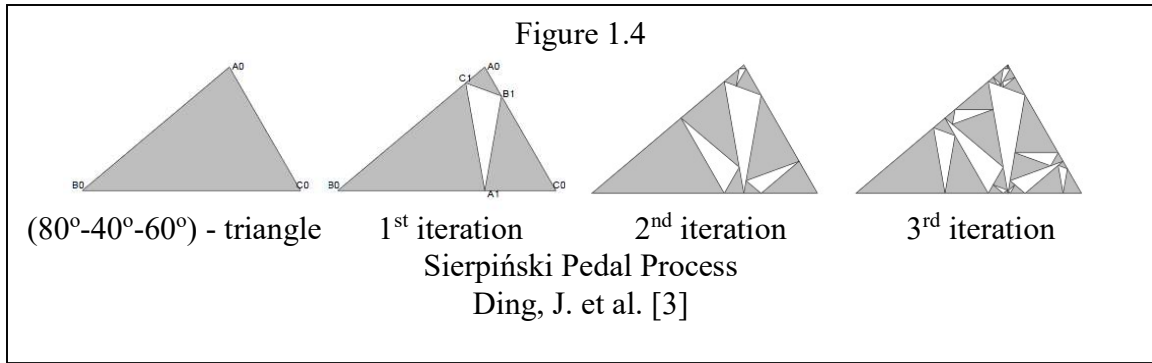
$$a_1 = b_1 = 0; a_2 = \frac{1}{2}, b_2 = 0; a_3 = \frac{1}{4}, b_3 = \frac{1}{2}$$



We choose A_0 to be the isosceles triangle with vertices $(0,0)$, $(1,0)$, $(\frac{1}{2}, 1)$. Because this is a triangle and the transformations produce a similar triangle, each transformation can be calculated on the vertices to find the transformed triangle's vertices, rather than performing the transformations on every point in the triangle. Then $S_1(A_0)$ is a triangle with vertices $(0,0)$, $(\frac{1}{2}, 0)$, $(\frac{1}{4}, \frac{1}{2})$. $S_2(A_0)$ and $S_3(A_0)$ are congruent to $S_1(A_0)$ but translated to the right, and up and to the right, respectively. Then the output is

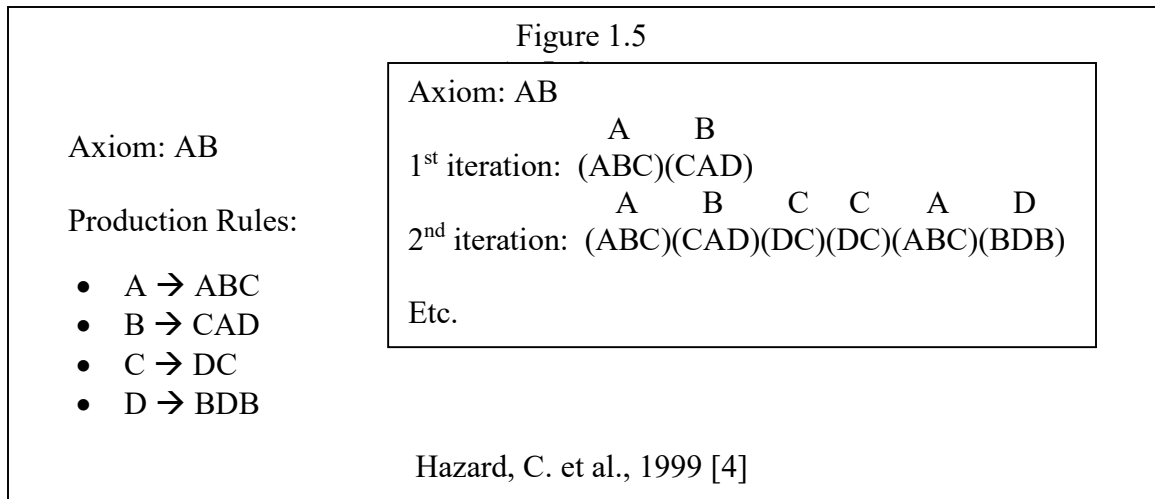
$$A_1 = F(A_0) = S_1(A_0) \cup S_2(A_0) \cup S_3(A_0).$$

This first iteration is shown in Figure 1.3. The limiting set $A_* = \lim_{n \rightarrow \infty} F^n(A_0)$ is the Sierpiński Triangle.



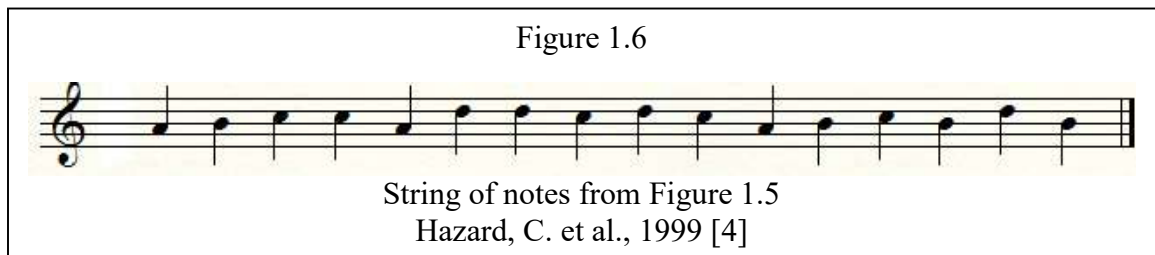
Related to the Sierpiński Triangle is the **Sierpiński Pedal Triangle (SPT)**, which utilizes the pedal triangle rather than the triangle connecting the midpoints. For an initial triangle T_0 , the **pedal triangle** is the triangle formed by connecting the three feet of the altitudes of T_0 . If T_0 is a right triangle, then the pedal triangle is a straight line, and if T_0 is an obtuse triangle, the pedal triangle reaches outside T_0 . For an acute T_0 , the pedal triangle remains inside T_0 and can, therefore, be removed in the Sierpiński process as shown in Figure 1.4. Indefinite iteration results in the SPT. The SPT also produces a total of 3^n triangles similar to the original on the n^{th} iteration, but they are scaled, rotated, and reflected individually [3]. This key difference leads us to believe that the SPT may allow more flexibility concerning applications to music.

Fractal music is the application of methods used to generate fractals in the field of music [4]. An example of this crossover from fractals to music is the L-System defined by Hazard and colleagues. An **L-System** is a repetitive process that transforms a short string or axiom into a longer, more complicated string through a set of production rules. Each symbol in the string has a respective production rule, and with each iteration, the symbols are replaced by their production rule. Figure 1.5 exhibits an example of an L-



System. The resulting string is intended to mimic the self-similarity of fractals. The string can then be interpreted musically as a string of notes, chords, or other objects [4].

A direct approach is to assign each symbol of the string to its corresponding note name, i.e. “A” → A, “B” → B, and so on, and a symbol such as “R” for a rest. The second iteration string from Figure 1.5 would then become the note string in Figure 1.6. The axiom and production rules must be chosen carefully to produce the desired music and effect [4].



With respect to chords, the production rules can replace each symbol (a Roman numeral representing a triad based on that number’s location in a scale) with a short chord progression to create a larger progression. Hazard and colleagues provide the example in Figure 1.7. For maximum effect, the axiom and production rules should be strongly influenced by the guidelines delineated by music theory. Hazard and colleagues do note that this L-System is not particularly effective on its own and should be combined with other methods. For instance, the resulting chordal string can be used as

background chords for a melody or to constrain the melody to sound more like traditional Western music [4].

Figure 1.7
An L-System for a chord progression

Axiom: I

Production Rules:

- $I \rightarrow I IV V I$
- $ii \rightarrow ii V IV$
- $IV \rightarrow IV V I ii$
- $V \rightarrow V I ii V$

Axiom: I

1st iteration: I
 $(I IV V I)$

2nd iteration: I IV V I
 $(I IV V I)(IV V I ii)(V I ii V)(I IV V I)$

Etc.

Hazard, C. et al., 1999 [4]

Chapter 2: Matrix IFS

Method

First, analyze the matrix IFS for the Sierpiński Triangle and generalize it to the SPT. Then, create an algorithm or algorithms to carry out the matrix IFS on an arbitrary acute triangle to generate a given number of iterations in the SPT process. Graph several examples for various choices in initial vertices and iteration levels.

Results

In order to generate the SPT for an arbitrary acute triangle, I broke the task into four algorithms: the first reorganizes the vertices' coordinates, the second performs the first iteration of the Sierpiński Pedal Process, the third performs the Sierpiński Pedal Process a given number of times, and the fourth graphs the appropriate triangles. A description of each is given below.

Algorithm 1

algorithm *redo_points*

inputs

$a_1, a_2, b_1, b_2, c_1, c_2$, the respective x - and y -components of the vertices of an acute triangle

outputs

A, B, C , the vertices in the desired order

do

let A be the vertex with the lowest y -value.

if more than one vertex shares this y -value,

then let A be the vertex that also has the lowest x -value of the remaining points, let B be the vertex with the highest x -value

if both vertices have this x -value,

then let B be the vertex that also has the lowest y -value

let C be the remaining vertex

return A, B, C

Algorithm 1 reorganizes the points so that the other algorithms can work with the vertices of the triangle regardless of their input order. The particular choice of order of output from Algorithm 1 is explained under Algorithm 2. For the purpose of using matrix operations, the implementation of the algorithm in computer code formats each vertex in the output as a 2×1 matrix.

Algorithm 2

algorithm *Pedal_Triangle_Matrix_3*

inputs

$a_1, a_2, b_1, b_2, c_1, c_2$, the respective x - and y -components of the vertices A, B, C of an acute triangle

outputs

$A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, C_3$, the vertices of the triangles produced in one iteration of the Sierpiński Pedal Process

do

use *Algorithm 1* to reorganize the points

for $X \in \{A, B, C\}$,

let $X_0 = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \left[X - \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \right]$ with θ such that side $\overline{A_0B_0}$ will be horizontal

for $X \in \{A_0, B_0, C_0\}$,

for $i \in \{1, 2, 3\}$,

Dilate: let $X' = \begin{pmatrix} D_i & 0 \\ 0 & D_i \end{pmatrix} X$

Reflect: let $X'' = \frac{1}{1+m^2} \begin{pmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{pmatrix} X'$

Rotate: let $X''' = \begin{pmatrix} \cos \theta_i & -\sin \theta_i \\ \sin \theta_i & \cos \theta_i \end{pmatrix} X''$, where θ_i is the angle to rotate

the triangle to the correct position

Translate: let $X'''' = X''' + \begin{pmatrix} x_i \\ y_i \end{pmatrix}$, where x_i, y_i translate the triangle to the

proper vertex on the initial triangle

let $X_i = \begin{pmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{pmatrix} X'''' + \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$

return $A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, C_3$

Algorithm 2 uses Algorithm 1 to reorder the points, then subtracts the coordinates of the new A from all of the vertices and rotates the triangle so that side \overline{AB} is horizontal.

Due to the order of the order of the vertices in the output form Algorithm 1, the triangle should now be in the first quadrant and an acute angle θ is relatively easy to find by

$$\theta = \tan^{-1} \frac{b_2 - a_2}{b_1 - a_1}.$$

With \overline{AB} horizontal, the vertices of the “bottom left” triangle in the Sierpiński Pedal Process do not need to be rotated, and the rotations for the other vertices are easier to visualize and to determine. The remaining steps are repeated three times to make the three dilated triangles from the Sierpiński Pedal Process. The algorithm dilates the triangle by multiplying each vertex by a dilation matrix with a dilation factor D_i . D_i is the ratio of a side of the resulting to the side of an initial triangle which, according to Ding and colleagues can be found by

$$D_i = \cos(\text{angle})$$

where *angle* is the measure of the angle on the initial triangle (A, B, C) to which the dilated triangle will eventually be translated. The algorithm reflects the points about the bisector of angle A where the slope of the bisector is m . To find m , use the fact that $\tan \theta$ is the slope of a line that is θ degrees/radians counterclockwise from the positive x -axis. By the tangent difference identity

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

the tangent of an angle between two lines with slopes m_1 and m_2 , respectively, is equal to

$$\frac{m_1 - m_2}{1 + m_1 m_2}.$$

Thus, the bisector will have a slope m that satisfies

$$\frac{m - k_1}{1 + mk_1} = \frac{k_3 - m}{1 + k_3m}$$

where k_1, k_3 are the slopes of the intersecting lines or, in this case, the sides of the triangle. Because the algorithm makes \overline{AB} horizontal, $k_1 = 0$, and the equation becomes

$$m = \frac{k_3 - m}{1 + k_3m}.$$

According to Andrilli and Hecker, the vertices can then be reflected by the matrix operation shown in Algorithm 2 because the sides of the triangle are part of lines that pass through the origin. Essentially, this operation makes the two sides of the triangle swap locations. The algorithm then rotates the triangle to match the angle to which it will be translated. The “bottom left” triangle does not need to be rotated as stated earlier. The “top” triangle must be rotated $\pi - B$ radians and the “bottom right” triangle must be rotated $C - \pi$ radians. The algorithm then subtracts the coordinates of the vertex (of the dilated triangle) which will be matched to a vertex on the original triangle and adds the coordinates of the proper vertex on the initial triangle. In the algorithm, this is shown as a net matrix addition of $\begin{pmatrix} x_i \\ y_i \end{pmatrix}$. At this point, the angle associated with the vertex of the dilated triangle that was just translated should equal the angle associated with the vertex to which the triangle was moved. The sides of the dilated triangle should also line up with the sides of the initial triangle. At the very end, the list of coordinates is returned as the nine vertices of the dilated triangles in matrix form.

Algorithm 3

algorithm *SPT_Matrix*

inputs

$a_1, a_2, b_1, b_2, c_1, c_2$, the respective x - and y -components of the vertices A, B, C of an acute triangle

n , the number of iterations

outputs

coordinates for the triangles in the next iteration of the Sierpiński Pedal Process

do

use *Algorithm 2* to find $A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, C_3$

let T be the list of these vertices and assign them level n .

if $n - 1 > 0$,

use *Algorithm 3* on A_1, A_2, A_3 with iteration number $n - 1$ and concatenate the list to T

use *Algorithm 3* on B_1, B_2, B_3 with iteration number $n - 1$ and concatenate the list to T

use *Algorithm 3* on C_1, C_2, C_3 with iteration number $n - 1$ and concatenate the list to T

return T

Algorithm 3 enforces n iterations of the Sierpiński Pedal Process. It uses Algorithm 2 to produce one iteration, then calls upon itself to perform the Sierpiński Pedal Process on the generated triangles while decrementing the number of iterations. The decrementation ensures that the process will terminate and not perform an infinite loop. The algorithm returns a list of lists of points and iteration levels.

Algorithm 4

algorithm *SPT_Graph*

inputs

$a_1, a_2, b_1, b_2, c_1, c_2$, the respective x - and y -components of the vertices A, B, C of an acute triangle

n , the number of iterations

outputs

a graph of the Sierpiński Pedal Triangle with n iterations

do

use Algorithm 3 to produce a list T of lists

let SPT be an empty graph

for every list of coordinates $P \in T$,

if the level of iteration (the last entry in P) is 1,

then add the triangle with those vertices to the graph SPT

return SPT

Algorithm 4 sifts through the list of lists produced from Algorithm 3 and produces a graph that only includes the triangles with iteration level 1. These triangles should be the smallest triangles produced from the Sierpiński Pedal Process so no triangles should overlap or cover each other on the graph.

Discussion

Algorithm 2 is named *Pedal_Triangle_Matrix_3* because I created two previous versions. The first version required the computer to compute exact values for the vertices. With multiple instances of trigonometric functions and the introduction of π , the computer attempted to keep track of all of the values exactly and the program was very slow. In combination with Algorithm 3, the slow speed compounded over many uses of Algorithm 2, and the graphs took many hours, especially for more than three iterations. The second version rounded results to several decimal places, losing a small amount of accuracy but exponentially increasing the speed of the program. However, this version retained some redundancies in calculation, and so this third version replaced some of the calculations using simpler equations from Ding and colleagues' work. The entire program is relatively quick and is reasonably accurate for small numbers of iterations. The order of input for the vertices does not affect the process, as desired.

If the given triangle is acute, no operations involve division by zero or other inconsistencies, and the algorithms correctly execute and produce an appropriate graph. However, there are no safeguards to ensure that the triangle is acute. So, if the entered vertices form a right triangle or an obtuse triangle, the program does not properly execute. A graph may be produced, but the triangles do not form a picture with any significance or meaning. Additionally, it is recommended that the vertices' coordinates

are entered with decimal points, even if the numbers do not require them, to safeguard against the program trying to use exact numbers. Although the program is relatively quick, the IFS is generally more complicated than simply calculating the vertices of the pedal triangle and removing the pedal triangle.

The computer code used in SAGE for these algorithms can be found in the Appendix.

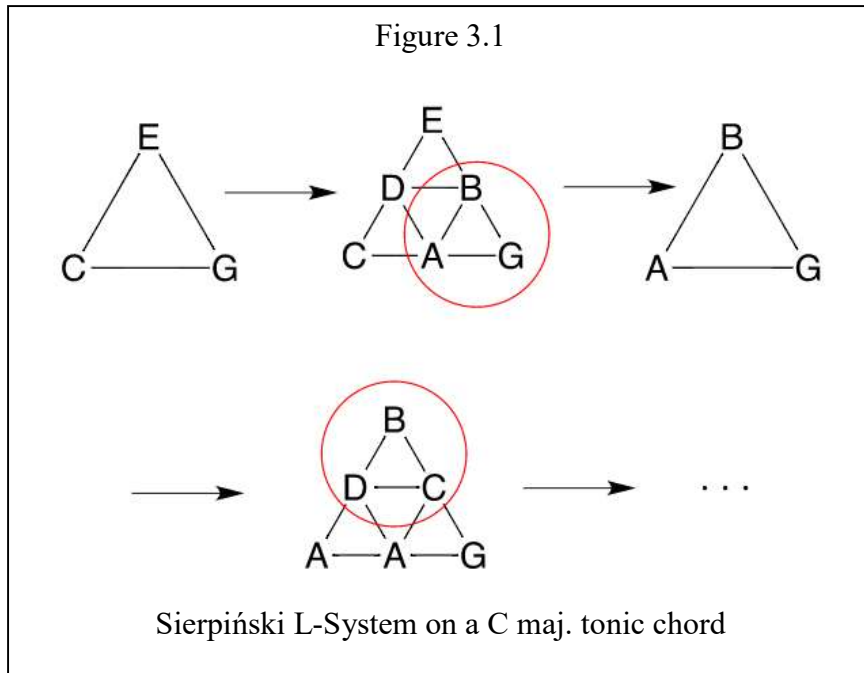
Chapter 3: Music Composition

Methods

Analyze the matrix IFS of the Sierpiński Triangle and SPT for applications to music composition. Also analyze the properties, such as the relevant isometries, of the Sierpiński Triangle and SPT themselves.

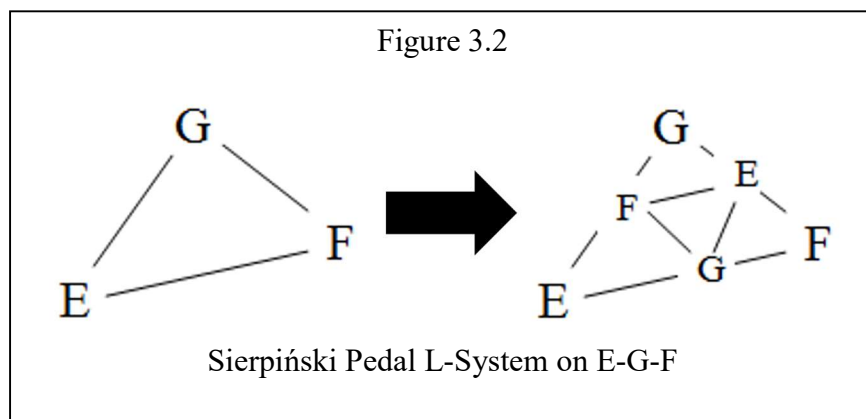
Results

Initially, I created a process of composition that combined the L-System with the Sierpiński Triangle, shown in Figure 3.1. The idea was to arrange three notes into the configuration of a triangle, then create an inner triangle like in the original Sierpiński Process. However, I had difficulty creating rules for choosing notes in this method. The most promising version started with the three notes of a tonic chord in a given key. Then



the inner notes were chosen as notes that could “appropriately” connect the vertices on that side. A random vertex was chosen and the process was performed again on that

triangle. However, this did not prove very effective. So I turned to the Sierpiński Pedal Triangle. Although the matrix IFS is more complicated than other methods, it did emphasize how the initial triangle is related to the dilated triangles through reflections and rotations. To incorporate this idea and mimic the SPT itself, I modified the new Sierpiński L-System by beginning with three chosen notes, then making an interior pedal triangle. This meant the foot of altitude from each vertex would create the same vertex note name, as shown in Figure 3.2. Iterating the process on the three dilated triangles produces more reflections and rotations. I decided to use the three starting notes as a three note pattern, read left-to right as E-G-F. Then, after performing the Sierpiński Pedal Process, any other permutation of the three note pattern present in the L-System



could also be used. In fact, after two iterations, all possible permutations of notes are available. Additionally, sharps or flats can be added to the pattern, depending on the key of the music and the intended effect. For the actual composition of music, the idea is to spread the permutations of the three-note pattern throughout a piece at various levels. “Various levels” refers to having the pattern occur in different parts at different times, across multiple parts at the same time, as part of the chord structure, and other techniques. The intent is to mimic the self-similarity of the SPT. Then between these patterns, compose the rest of the music using musicality and the rules of music theory.

The musicality gives the composer freedom to produce music with desired effects, feelings, and sounds, while the music theory gives some constraints on this freedom. However, for my purposes, I allowed for some instances of musicality to supersede music theory if there was a discrepancy. The impetus behind this decision is that at least one work produced from this project is to be performed by my tuba quartet *Sort of Voce*. As such, the music needed to be interesting to play and hear in addition to interesting to compose and analyze. Furthermore, this situation dictated that at least one work be in four parts, for two tubas and two euphoniums, all of which are in bass clef. The musical work located in the Appendix, titled *Sierpiński Pedal Quartet* is the result of this method under these conditions.

Discussion

The *Sierpiński Pedal Quartet* is divided into three movements and is based on the E-G-F pattern. The first movement, titled *Energy*, is in the key of E^b maj. It utilizes the permutations of the three-note pattern to create quick, moving lines that are full of vigor. The focus on the movement conveys an energetic feel, rather than a focus on chord structure to produce harmonious progressions. As a result, many of the phrases primarily use only three chords, such as I (E^b maj.), IV (A^b maj.), and V (B^b maj.). However, the vast majority of the music is euphonious, rather than many clashing lines of notes.

Energy begins with a Euphonium 2 solo explicitly using the original E-G-F pattern, and in measures 5-16, the other instrument parts are added in layers. Measures 17-28 introduce the main theme of the piece in the Euphonium 1 part. Euphonium 2 and Tuba 1 utilize straight eighth notes to produce the feeling of movement throughout the phrase with a short break from eighth notes in measure 24 so that the players can breathe. Tuba

2 uses a slightly syncopated rhythm that is popular in pop music to generate an even greater feeling of motion. The phrase is essentially repeated in measures 29-40 but with a countermelody in Tuba 1. At measure 41, the movement enters into a bridge-like section which augments the instrument parts from measures 1-16 and adds a more rhythmic aspect in Euphonium 1. Beginning at measure 49, Euphonium 1, Euphonium 2, and Tuba 2 essentially perform the same overall division of eighth notes (dotted quarter note + dotted quarter note + quarter note) every other measure, but in each division, Euphonium 1 or Euphonium 2 (or both) play a grouping of eighth notes, creating a cascade effect. Tuba 1 plays straight eighth notes similar to measures 17-28 while the other parts use the division described previously. Then when the other parts have a whole note, Tuba 1 interjects with the E-G-F pattern or a similar pattern, continuing the motion. Measure 57 reverts back to the last four measures of the theme (e.g. measures 25-28) but with a Euphonium 1 harmony line. All parts are essentially in unison rhythm in measures 61-65, accenting the E-G-F pattern with harmonies. Measures 66-77 are a restatement of the theme from measure 17-28. Measures 78-89 also use the theme. However, the tuba parts wait a measure to come in with an arpeggio-like figure and also introduce different chords than the other theme phrases. Then the tubas, in unison rhythm, accompany the Euphonium 1 part beginning in measure 82. During all of this, Euphonium 2 performs the E-G-F pattern and rhythm from the beginning as straight eighth notes, again keeping the momentum of the piece going. At measure 86, Tuba 1 takes over this line while Euphonium 2 plays a harmony line in unison rhythm. Tuba 2 plays a harmony line to the theme in Euphonium 1. Then measure 90 begins the closing section. The euphoniums trade sixteenth note runs, maintaining the energy of the piece. The tuba parts begin with

four quarter notes, then Tuba 1 switches to a three-eighth-note-pattern in the next measure, and Tuba 2 changes to a two-eighth-note-pattern, intended to make the ending feel like it is speeding up without actually changing tempo. All parts play an eighth note run in octaves in measure 93. In the next measure, Euphonium 1 and Tuba 1 progress to the next note in the run (B^b) while Euphonium 2 and Tuba 2 delay this note with one E-G-F pattern from the introduction. Then all parts end on a tonic E^b major chord on beat four of the last measure.

The second movement is titled *Grief* and exhibits vignettes of some stages of grief a person might experience after a traumatic event. It is broken into three sections, mimicking the overall structure of the whole work. The first section represents the melancholy stage of grief, and so it is in the key of E min. It contains gloomy, flowing lines, reflecting the person's dejected spirit while still maintaining an appearance of control. The middle section, still in E min, features much more dissonant harmonies, polyrhythm, and meter changes to convey a growing anger and intensity. This section represents anger building in the victim as they fight the reality of the traumatic experience until he/she finally loses control and explodes. The final section represents the person finally facing their internal struggle with the traumatic experience, eventually coming to an acceptance of reality and triumphing over grief. The section begins with quiet solo melodic lines that harken back to the melancholy attitude of the first section. Then it transitions into the key of G maj. and builds back intensity, not of anger, but of hope. After reaching the climax of this accumulation of intensity, the piece gradually softens and ends in a sweet whisper.

In measures 1-8 of *Grief*, the instrument parts have staggered entrances that result in dissonance between the three notes E, F[#], and G. The staggered entrances convey the feeling of loneliness of the victim while the dissonance represents the pain of grief. At the end of each short phrase, the dissonance partially resolves, representing the juxtaposition of the victim's collected appearance with his/her interior aching. Beginning in measure 9, the tuba parts form the accompaniment for the forthcoming euphonium melodic material. They maintain the pattern of Tuba 2 downbeat with Tuba 1 eighth notes on beats two and three until measure 30 to keep a moving, flowing feeling that is slower and more lyrical than in *Energy*. Euphonium 2 begins the melodic material in measure 11, and Euphonium 1 takes the mantle in measure 17 while Euphonium 2 provides a harmony. Between each phrase of melody is a measure of rest in the euphonium parts intended to maintain the isolated feeling introduced in the first measures. Permutations of the E-G-F[#] pattern can be readily identified in the melodic material. Tuba 1 picks up the melody in measure 30 while the other parts provide accompaniment. The lugubrious feel is carried all the way to measure 45, where a more intense dissonance emerges, representing the first streak of anger in the victim.

The middle section begins with the euphonium parts continuing the previous dissonance in measures 46-47 while Tuba 1 plays an E-G-F[#] pattern in measure 47. Beginning in measure 48, the euphonium parts clash with tritone dissonance and polyrhythm where Euphonium 1 focuses on the dissonant notes and Euphonium 2 plays a pattern on E-G-F[#]. The tuba parts play a sinister melody in octaves (and briefly in fifths in measure 53) allowing the lower voicing to dominate this section and giving it a darker, angrier tone. In measure 56-57, the tuba parts create a sense of unease by playing the

first note of groupings of three sixteenth notes, rather than on the downbeats of the meter. Meanwhile, the euphonium parts trade sixteenth note runs that are mostly based on thirds and that somewhat matches the rhythm of the tubas. That is, whenever a euphonium starts a sixteenth note run, the tubas also play on the euphonium's starting note. Then in measure 57, the euphoniums play the sixteenth note run simultaneously a second (interval) apart from each other, adding more cacophony to the uneasiness of the measures. Measures 58-59 change the meter to further destabilize the meter, and the euphonium parts continue to clash with one another. Tuba 1 plays a monothematic line in measure 60 which can be broken down into four groupings of three sixteenth notes and a quarter note. The first grouping starts on F[#], uses thirds travel up to the beginning E^b of the next grouping, which uses a second and thirds to reach a G. Once this grouping ends, the final grouping displays a G-F-E pattern. These groupings reflect the three-note pattern with the starting notes, use of thirds, and the direct use of the final G-F-E pattern. The quarter note at the end of the measure with all parts playing functions to inject intense harmonic dissonance back into the measure. Measures 61-62 function the same as measures 58-59. Then, in measure 63, the euphoniums return with the sixteenth note runs from previous measures while Tuba 1 plays a pattern on E-G-F[#]. By measure 64, anger has consumed the victim, represented in the driving E-G-F[#] pattern in Tuba 2. In measures 64-87, Tuba 2 maintains the same driving pattern while the Tuba 1 part becomes more detailed with shorter note values to continue increasing the momentum of the piece. During these phrases, the euphoniums play melodic material that becomes more dissonant as time progresses. The final build to the climax occurs in measures 88-91, where the euphoniums accent every fourth beat while the tubas, staggered two beats

from the euphoniums, also accent every fourth beat. Tuba 1 also plays an E-G-F[#] eighth note pattern while Tuba 2 rests to maintain motion. The climax of anger is reached in measures 92-97, with a final dissonant chord in measure 95 pushing into measure 96. Here Tuba 2 gives a final, explosive pedal E to symbolize the last outburst of anger in the victim, followed by a decay in volume to represent the anger fading away from the victim. A quick moment of silence between measures 97 and 98 returns the feeling of isolation and reflection in the victim. Then Euphonium 2 provides a solo melodic line similar to the melancholy material from measures 11-16, bringing back the lugubrious tone. This is followed by another short pause and a similar line in Tuba 1 that references material from measures 30-45. Another short pause, and Euphonium 1 introduces new material that hints at the key change to G maj. that shortly follows.

Measure 111 marks the beginning of the victim's progress of overcoming his/her grief. Euphonium 1 carries the lyrical melody while the tuba parts focus on using chord changes to convey the feeling of change and hope. Euphonium 2 joins in on a harmony line in measure 115, and in measure 118, Euphonium 2 introduces an E-F[#]-G motif that will prevail the remaining portion of the piece. This motif uses the same notes that conveyed pain and anger in the first two sections of *Grief*, but now it intends to inspire positive feelings. This represents the internal change within the victim as he/she transitions from sadness and ire to a more positive outlook. In measures 119-130, the euphonium parts switch roles approximately every four measures, one playing the melody and the other playing the motif, so that the players may catch their breath. Measures 119-122 rebuild the intensity from the middle angry section, but this time the intensity is not found in anger, but in hope and strength. In measures 123-130, the victim

has overcome his/her grief and finds the courage to carry on. The euphoniums play the motif and melody while the tubas provide accompaniment. Tuba 1 joins the melodic material as a harmony line beginning in measure 126. In measures 131-136, the instrument parts retain some dissonance, but now there is more beauty than pain in these sonorities. This represents the victim looking back on the traumatic experience and his/her recent grief with the knowledge that he/she has triumphed over the situation, becoming stronger for it. During this phrase, the Euphonium 1 continues playing the melody from the preceding measures while Euphonium 2 provides a countermelody every other measure. A decay in volume in measure 136 brings all instrument parts to tender whisper for measure 137 to the end, which represents the victim reaching the end of the stages of grief.

The final movement of the work, in F min., is titled *Funk-ish*. It incorporates blues scales and syncopated rhythms to create a funk-like groove throughout. In general, the tubas lay down a bass line with some embellishment while the euphoniums provide the melody and harmony. For some sections, however, no particular part has a “melody,” but instead, the ensemble grooves together as one single unit. That is, every part is of equal “importance” melodically and harmonically. The focus of this movement is to create music where the audience feels the urge to tap their feet or bob their heads to the music.

The first four measures of the movement are introductory material. Measures 1 and 3 have an F in octaves for the first two eighth notes, while measures 2 and 4 have the F-G^b-E^b pattern as the notes distributed across all four parts for the first two eighth notes. Tuba 1 plays for the remaining beats in the first two measures, introducing the rhythm and theme in measure 2 that will occur at the end of every main phrase for the rest of the

movement. Euphonium 1 solos in measure 3 based on the G^b-F-E^b pattern, and in measure 4, Euphonium 2 and Tuba 2 carry the ensemble into measure 5. In measures 5-8, Tuba 2 provides a bass line partially based on the F-G^b-E^b pattern while Tuba 1 focuses on the upbeats. Measure 9 begins the first non-introductory section. For measures 9-12, Tuba 2 continues with a similar bass line as before while Euphonium 2 plays a rhythmic melody. Euphonium 1 and Tuba 1 enter in layers for the last two measures. In measures 13-16, Tuba 2 focuses on playing a C to give the feeling of being on a dominant, rather than tonic, chord. Euphonium 1 takes the melody, and Tuba 1 plays upbeats. Euphonium 2 joins Euphonium 2 as harmony in measure 15, and in measure 16, the Euphoniums play two sixteenth notes on alternating eighth note beats (Euph. 1 on the first eighth note, Euph. 2 on the second eighth note, etc.) so that sixteenth notes are always being played. This builds the intensity going into measure 17 where the upper three instruments play a staggered F-B^b-C^b-E^b theme that reoccurs throughout the piece. In measure 18, the upper three voices play a harmonized version of the line from the Tuba 1 part in measure 2. During these two measures, Tuba 2 plays the F-F-G^b-E^b pattern in quarter notes.

The next phrase, beginning in measure 19, is one of the sections that focuses less on melodic material and more on the ensemble grooving together. In measures 19-22, the groove is generally created by the tubas playing the first two eighth notes with the euphoniums playing a quarter note on beat 2. Then the tubas play a bass rhythm in tandem during the remaining two beats. The tubas enter a sixteenth note apart from each other in fifths to mimic an electric bass playing on two different strings. The exception is measure 20, where the tubas play a different funk line in octaves. The F-G^b-E^b pattern is

found in these measures with the F in the tuba parts on beat one and the E^b and G^b in the euphonium parts on beat two. Measures 23-26 are similar but focus on the dominant chord again. In these measures, Tuba 1 plays four sixteenth notes on beat one, Euphonium 2 plays four sixteenth notes on beat two, and Euphonium 1 plays a sixteenth note run in the remaining beats. The sixteenth notes in Tuba 1 focus on a G, and Tuba 2 plays E^b and F in beats three and four, forming a G-E^b-F pattern. Measures 27-30 resemble measures 19-22 but with Tuba 1 playing the F-B^b-C^b-E^b theme on beat one, and the euphoniums playing sixteenth notes on beats three and four. Measures 31-32 mimic measures 17-18 but with a modified Tuba 2 resembling the bass material from measures 19-30.

The next phrase in measures 33-46 follows the same general form as the previous phrase with a section in F, a section in B^b instead of C, another section in F, and the closing two measures. It maintains constant sixteenth notes scattered throughout the Euphonium 1, Euphonium 2, and Tuba 2 parts. In measures 33-36, Tuba 1 provides the basic bass line while the other parts provide the constant sixteenth notes. Tuba 2 picks up the bass line in measure 37 while the euphonium parts maintain the sixteenth notes; Euphonium 2 plays two sixteenth notes, then Euphonium 1 plays two sixteenth notes, and they continue alternating. Tuba 1 contrasts the quick sixteenth notes with a melodic line made of longer note values, namely half notes and quarter notes. This melody also contains instances of the F-E^b-G^b pattern. In measures 41-44, Tuba 1 continues the melody and Tuba 2 continues the bass line. The euphonium parts play the F-B^b-C^b-E^b pattern as sixteenth notes in a general arch shape. That is, the parts follow the pattern going up in pitch the first two beats of each measure, and follow the pattern down the last

two beats. However, in measure 44, the euphoniums mimic the material from measures 33-36 to build intensity going into the concluding two measures of the phrase. Again the last two measures resemble measures 17-18 with a modified bass line.

Measure 47 marks the final phrase of the movement. The euphonium parts play a rhythmic pattern based on a repetition of an eighth note followed by a sixteenth note with the Euphonium 1 part containing an F-E^b-G^b pattern. The tubas play F in octaves as two strong eighth notes on beat one of each measure, then fill in the rest of the measure while the euphoniums rest. Tuba 1 plays the F-B^b-C^b-E^b pattern on beat four. At measure 51, the euphoniums switch parts, and the tubas echo in fifths the pattern in the euphoniums. Tuba 2 echoes the pattern three times: the first beginning on E^b, the second on G^b, and the third on F. This forms an E^b-G^b-F pattern across the three measures. The final measure follows the form of the other phrases and resembles measure 18.

The overall structure of the movements also reflects the three-note pattern. *Energy* is in the key of E^b maj., *Grief* is in E min./G maj., and *Funk-ish* is in F min. This resembles Figure 3.2 in that the triangle on the E vertex is the largest, so most of the musical work is in E^b maj. or E min. The next largest triangle is on the F vertex, so F min. covers the next largest section of the work. Finally, the triangle on the G vertex is smallest, so the smallest portion of the work is in G maj. All of the movements use similar permutations of E-G-F from the Sierpiński Pedal L-System at various levels of the music, but the effects created by these permutations vastly differ from each other between the individual movements.

Bibliography

1. Andrilli, S. and Hecker, D (2016). Computer Graphics. *Elementary Linear Algebra*. Elsevier Inc.
2. Devaney, R. L. (1995, April). *The Sierpinski Triangle*. Retrieved from Boston University, <http://math.bu.edu/DYSYS/chaos-game/node2.html>.
3. Ding, J., Hitt, L. R., Wang, B., & Zhang, X. *Sierpinski Pedal Triangle*.
4. Hazard, C., Johnson, D., & Kimport, C. (1999). *Fractal Music*. Retrieved from <http://www.tursiops.cc/fm/>.
5. Lasota, A. & Mackey, M. C. (1994). Iterated Function Systems and Fractals. *Chaos, fractals, and noise* (2nd ed.). New York: Springer-Verlag New York, Inc.
6. Peitgen, H., Jürgens, H., & Saupe, D. (2004). The Backbone of Fractals. *Chaos and fractals: New frontiers of science* (2nd ed.). New York: Springer-Verlag, New York, Inc.

Appendices

Appendix A: SAGE Codes

Algorithm 1: *redo_points*

```
def redo_points(a1, a2, b1, b2, c1, c2):
    if a2 < b2 and a2 < c2:
        if c1 < b1:
            A = matrix([[a1], [a2]])
            B = matrix([[b1], [b2]])
            C = matrix([[c1], [c2]])
        elif b1 < c1:
            A = matrix([[a1], [a2]])
            B = matrix([[c1], [c2]])
            C = matrix([[b1], [b2]])
        elif b1 == c1:
            if b1 < a1:
                if b2 < c2:
                    A = matrix([[a1], [a2]])
                    B = matrix([[c1], [c2]])
                    C = matrix([[b1], [b2]])
                else:
                    A = matrix([[a1], [a2]])
                    B = matrix([[b1], [b2]])
                    C = matrix([[c1], [c2]])
            else:
                if c2 < b2:
                    A = matrix([[a1], [a2]])
                    B = matrix([[c1], [c2]])
                    C = matrix([[b1], [b2]])
                else:
                    A = matrix([[a1], [a2]])
                    B = matrix([[b1], [b2]])
                    C = matrix([[c1], [c2]])
        elif a2 == b2 and a2 < c2:
            if a1 < b1:
                A = matrix([[a1], [a2]])
                B = matrix([[b1], [b2]])
                C = matrix([[c1], [c2]])
            else:
                A = matrix([[b1], [b2]])
                B = matrix([[a1], [a2]])
                C = matrix([[c1], [c2]])
        elif a2 == c2 and a2 < b2:
            if a1 < c1:
```

```

        A = matrix([[a1], [a2]])
        B = matrix([[c1], [c2]])
        C = matrix([[b1], [b2]])
    else:
        A = matrix([[c1], [c2]])
        B = matrix([[a1], [a2]])
        C = matrix([[b1], [b2]])
elif b2 < a2 and b2 < c2:
    if c1 < a1:
        A = matrix([[b1], [b2]])
        B = matrix([[a1], [a2]])
        C = matrix([[c1], [c2]])
    elif a1 < c1:
        A = matrix([[b1], [b2]])
        B = matrix([[c1], [c2]])
        C = matrix([[a1], [a2]])
    elif a1 == c1:
        if a1 < b1:
            if a2 < c2:
                A = matrix([[b1], [b2]])
                B = matrix([[c1], [c2]])
                C = matrix([[a1], [a2]])
            else:
                A = matrix([[b1], [b2]])
                B = matrix([[a1], [a2]])
                C = matrix([[c1], [c2]])
        else:
            if a2 < c2:
                A = matrix([[b1], [b2]])
                B = matrix([[a1], [a2]])
                C = matrix([[c1], [c2]])
            else:
                A = matrix([[b1], [b2]])
                B = matrix([[c1], [c2]])
                C = matrix([[a1], [a2]])
elif b2 == c2 and b2 < a2:
    if b1 < c1:
        A = matrix([[b1], [b2]])
        B = matrix([[c1], [c2]])
        C = matrix([[a1], [a2]])
    else:
        A = matrix([[c1], [c2]])
        B = matrix([[b1], [b2]])
        C = matrix([[a1], [a2]])
elif c2 < a2 and c2 < b2:
    if a1 < b1:
        A = matrix([[c1], [c2]])
        B = matrix([[b1], [b2]])
        C = matrix([[a1], [a2]])

```



```

elif b1 < a1:
    A = matrix([[c1], [c2]])
    B = matrix([[a1], [a2]])
    C = matrix([[b1], [b2]])
elif a1 == b1:
    if a1 < c1:
        if a2 < b2:
            A = matrix([[c1], [c2]])
            B = matrix([[b1], [b2]])
            C = matrix([[a1], [a2]])
        else:
            A = matrix([[c1], [c2]])
            B = matrix([[a1], [a2]])
            C = matrix([[b1], [b2]])
    if c1 < a1:
        if a2 < b2:
            A = matrix([[c1], [c2]])
            B = matrix([[a1], [a2]])
            C = matrix([[b1], [b2]])
        elif b2 < a2:
            A = matrix([[c1], [c2]])
            B = matrix([[b1], [b2]])
            C = matrix([[a1], [a2]])
return A, B, C

```

Algorithm 2: Pedal_Triangle_Matrix_3

```

def Pedal_Triangle_Matrix_3(a1, a2, b1, b2, c1, c2):

    A0, B0, C0 = redo_points(a1, a2, b1, b2, c1, c2)
    j = A0[0, 0]
    k = A0[1, 0]

    B0 = matrix([[B0[0, 0]-A0[0, 0]], [B0[1, 0]-A0[1, 0]]])
    C0 = matrix([[C0[0, 0]-A0[0, 0]], [C0[1, 0]-A0[1, 0]]])
    A0 = matrix([[A0[0, 0]-A0[0, 0]], [A0[1, 0]-A0[1, 0]]])
    rotation = arctan((B0[1, 0]-A0[1, 0])/(B0[0, 0]-A0[0, 0]))
    #R(theta) = matrix([[cos(theta), -sin(theta)], [sin(theta), cos(theta)]])
    A0 = matrix([[cos(-rotation), -sin(-rotation)], [sin(-rotation), cos(-rotation)]])*A0
    B0 = matrix([[cos(-rotation), -sin(-rotation)], [sin(-rotation), cos(-rotation)]])*B0
    C0 = matrix([[cos(-rotation), -sin(-rotation)], [sin(-rotation), cos(-rotation)]])*C0
    #return show(polygon([(A0[0, 0], A0[1, 0]), (B0[0, 0], B0[1, 0]), (C0[0, 0], C0[1, 0])]),
    #aspect_ratio = 1)

```

```

m1 = ((B0[1,0]-C0[1,0])/(B0[0,0]-C0[0,0])) #for finding A1 (of pedal triangle)
m2 = ((C0[1,0]-A0[1,0])/(C0[0,0]-A0[0,0])) #for finding B1
#m3 should be zero since we rotated the triangle

a0 = sqrt((B0[0,0]*1.-C0[0,0])**2+(B0[1,0]*1.-C0[1,0])**2) #distance from B0 to C0
b0 = sqrt((A0[0,0]*1.-C0[0,0])**2+(A0[1,0]*1.-C0[1,0])**2) #distance from A0 to C0
c0 = sqrt((A0[0,0]*1.-B0[0,0])**2+(A0[1,0]*1.-B0[1,0])**2)#distance from A0 to B0

theta_A = arccos((a0**2 - b0**2 - c0**2)/(-2*b0*c0)) #the measure of angle A
theta_B = arccos((b0**2 - a0**2 - c0**2)/(-2*a0*c0)) #the measure of angle B
theta_C = arccos((c0**2 - a0**2 - b0**2)/(-2*a0*b0)) #the measure of angle C

a_1 = a0*cos(theta_A)
b_1 = b0*cos(theta_B)
c_1 = c0*cos(theta_C)

#producing the bottom left triangle

#We shrink the original triangle
Shrink1 = matrix([[a_1/a0, 0], [0, a_1/a0]])
A_1 = Shrink1*A0 # this should result in [[0], [0]]
A_2 = Shrink1*B0
A_3 = Shrink1*C0

#We reflect the triangle about a line First we need to find the slope of the line
#about which we will reflect the triangle
k1 = 0. #the slope between A0 and B0
k3 = m2 #the slope between A0 and C0
k2 = var('k2')
sols3 = solve([(k2-k1)/(1.+k2*k1) == (k3-k2)/(1.+k3*k2)], k2)
k2 = round(sols3[1].right_hand_side(), 10)
M = 1/(1.+k2**2)*matrix([[1-k2**2, 2*k2], [2*k2, k2**2 -1]])
A_1 = M*A_1
A_2 = M*A_2
A_3 = M*A_3
#return show(polygon([(A0[0,0], A0[1,0]), (B0[0,0], B0[1,0]), (C0[0,0], C0[1,0])])+
#polygon([(A_1[0,0], A_1[1,0]), (A_2[0,0], A_2[1,0]), (A_3[0,0], A_3[1,0])], color =
#'red', zorder = 5), aspect_ratio = 1) due to the way we repositioned the original
#triangle, this smaller triangle is done

#producing the bottom right triangle

#Shrink
Shrink2 = matrix([[b_1/b0, 0], [0, b_1/b0]])
B_1 = Shrink2*A0 # this should result in [[0], [0]]
B_2 = Shrink2*B0
B_3 = Shrink2*C0

```

```

#Reflect
B_1 = M*B_1
B_2 = M*B_2
B_3 = M*B_3
#return show(polygon([(A0[0,0],A0[1,0]), (B0[0,0],B0[1,0]), (C0[0,0],C0[1,0])])+
#polygon([(B_1[0,0],B_1[1,0]), (B_2[0,0],B_2[1,0]), (B_3[0,0],B_3[1,0])], color =
#'red',zorder = 5), aspect_ratio = 1)

#Now we need to rotate the triangle to the correct position
B_1 = matrix([[round(cos(theta_C - pi),10),round(-sin(theta_C -
pi),10)], [round(sin(theta_C - pi),10),round(cos(theta_C - pi),10)]])*B_1
B_2 =matrix([[round(cos(theta_C - pi),10),round(-sin(theta_C -
pi),10)], [round(sin(theta_C - pi),10),round(cos(theta_C - pi),10)]])*B_2
B_3 = matrix([[round(cos(theta_C - pi),10),round(-sin(theta_C -
pi),10)], [round(sin(theta_C - pi),10),round(cos(theta_C - pi),10)]])*B_3

#Now we translate the triangle to the correct position
B_1 = B_1 - B_2 + B0
B_3 = B_3 - B_2 + B0
B_2 = B_2 - B_2 + B0
#return show(polygon([(A0[0,0],A0[1,0]), (B0[0,0],B0[1,0]), (C0[0,0],C0[1,0])])+
#polygon([(B_1[0,0],B_1[1,0]), (B_2[0,0],B_2[1,0]), (B_3[0,0],B_3[1,0])], color =,
#'red',zorder = 5), aspect_ratio = 1)

#producing the top triangle

#Shrink
Shrink3 = matrix([[c_1/c0, 0],[0,c_1/c0]])
C_1 = Shrink3*A0 # this should result in [[0],[0]]
C_2 = Shrink3*B0
C_3 = Shrink3*C0

#Reflect
C_1 = M*C_1
C_2 = M*C_2
C_3 = M*C_3
#return show(polygon([(A0[0,0],A0[1,0]), (B0[0,0],B0[1,0]), (C0[0,0],C0[1,0])])+
#polygon([(B_1[0,0],B_1[1,0]), (B_2[0,0],B_2[1,0]), (B_3[0,0],B_3[1,0])], color =
#'red',zorder = 5), aspect_ratio = 1)

#Now we need to rotate the triangle to the correct position

C_1 = matrix([[round(cos(pi - theta_B),10),-round(sin(pi - theta_B),10)],
[round(sin(pi - theta_B),10),round(cos(pi - theta_B),10)]])*C_1
C_2 = matrix([[round(cos(pi - theta_B),10),-round(sin(pi - theta_B),10)],
[round(sin(pi - theta_B),10),round(cos(pi - theta_B),10)]])*C_2
C_3 = matrix([[round(cos(pi - theta_B),10),-round(sin(pi - theta_B),10)],

```

```

[round(sin(pi - theta_B),10),round(cos(pi - theta_B),10)]]*C_3

#Now we translate the triangle to the correct position
C_1 = C_1 - C_3 + C0
C_2 = C_2 - C_3 + C0
C_3 = C_3 - C_3 + C0
#return show(polygon([(A0[0,0],A0[1,0]), (B0[0,0],B0[1,0]), (C0[0,0],C0[1,0])])+
#polygon([(C_1[0,0],C_1[1,0]), (C_2[0,0],C_2[1,0]), (C_3[0,0],C_3[1,0])], color =
#'red',zorder = 5), aspect_ratio = 1)

#rotate all points to the original triangle's position
A_1 = matrix([[cos(rotation),-sin(rotation)],[sin(rotation),cos(rotation)]])*A_1
A_2 = matrix([[cos(rotation),-sin(rotation)],[sin(rotation),cos(rotation)]])*A_2
A_3 = matrix([[cos(rotation),-sin(rotation)],[sin(rotation),cos(rotation)]])*A_3
B_1 = matrix([[cos(rotation),-sin(rotation)],[sin(rotation),cos(rotation)]])*B_1
B_2 = matrix([[cos(rotation),-sin(rotation)],[sin(rotation),cos(rotation)]])*B_2
B_3 = matrix([[cos(rotation),-sin(rotation)],[sin(rotation),cos(rotation)]])*B_3
C_1 = matrix([[cos(rotation),-sin(rotation)],[sin(rotation),cos(rotation)]])*C_1
C_2 = matrix([[cos(rotation),-sin(rotation)],[sin(rotation),cos(rotation)]])*C_2
C_3 = matrix([[cos(rotation),-sin(rotation)],[sin(rotation),cos(rotation)]])*C_3

#translate triangles to position of the original
A = matrix([[j],[k]])
A_1 = A_1 + A
A_2 = A_2 + A
A_3 = A_3 + A
B_1 = B_1 + A
B_2 = B_2 + A
B_3 = B_3 + A
C_1 = C_1 + A
C_2 = C_2 + A
C_3 = C_3 + A

#return show(polygon([(A_1[0,0],A_1[1,0]), (A_2[0,0],A_2[1,0]), (A_3[0,0],
# A_3[1,0])], color = 'black') +polygon([(B_1[0,0],B_1[1,0]), (B_2[0,0],B_2[1,0]),
#(B_3[0,0],B_3[1,0])], color = 'black')\
#+ polygon([(C_1[0,0],C_1[1,0]), (C_2[0,0],C_2[1,0]), (C_3[0,0],C_3[1,0])], color =
#'black'), aspect_ratio = 1)
return A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2, C_3

```

Algorithm 3: *SPT_Matrix*

```

def SPT_Matrix(a1, a2, b1, b2, c1, c2, level):
    P = Pedal_Triangle_Matrix_3(a1,a2,b1,b2,c1,c2)
    p11 = P[0][0][0]
    p12 = P[0][1][0]
    p21 = P[1][0][0]
    p22 = P[1][1][0]

```

```

p31 = P[2][0][0]
p32 = P[2][1][0]
p41 = P[3][0][0]
p42 = P[3][1][0]
p51 = P[4][0][0]
p52 = P[4][1][0]
p61 = P[5][0][0]
p62 = P[5][1][0]
p71 = P[6][0][0]
p72 = P[6][1][0]
p81 = P[7][0][0]
p82 = P[7][1][0]
p91 = P[8][0][0]
p92 = P[8][1][0]
#print (p11, p12), (p21, p22), (p31, p32), (p41, p42), (p51, p52), (p61, p62),
#(p71, p72), (p81, p82), (p91, p92)
T = [(p11, p12, p21, p22, p31, p32, level), (p41, p42, p51, p52, p61, p62, level),
      (p71, p72, p81, p82, p91, p92, level)]

if level - 1 > 0:
    print "Processing.  ", level
    T += SPT_Matrix(p11, p12, p21, p22, p31, p32, level-1)
    print "Processing. .  ", level
    T += SPT_Matrix(p41, p42, p51, p52, p61, p62, level-1)
    print "Processing. . .  ", level
    T += SPT_Matrix(p71, p72, p81, p82, p91, p92, level-1)
    print "Moving to next one"

return T

```

Algorithm 4: *SPT_Graph*

```

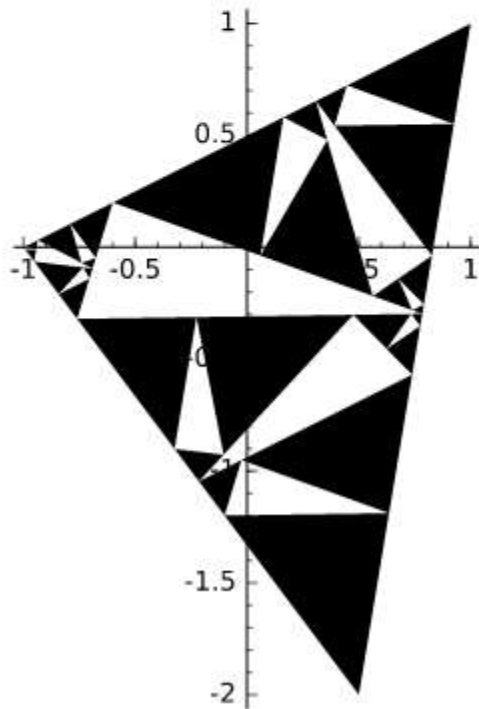
def SPT_Graph(a1, a2, b1, b2, c1, c2, level):
    T = SPT_Matrix(a1, a2, b1, b2, c1, c2, level)
    #print "I have a matrix"
    p = Graphics()
    for P in T:
        if P[6] == 1:
            p += polygon([(P[0], P[1]), (P[2], P[3]), (P[4], P[5])], color = 'black')
    #print "I have a polygon"
    return show(p, aspect_ratio = 1)

```

Appendix B: Example Graphs

SPT_Graph(-1., 0., 1., 1., 1./2., -2., 3)

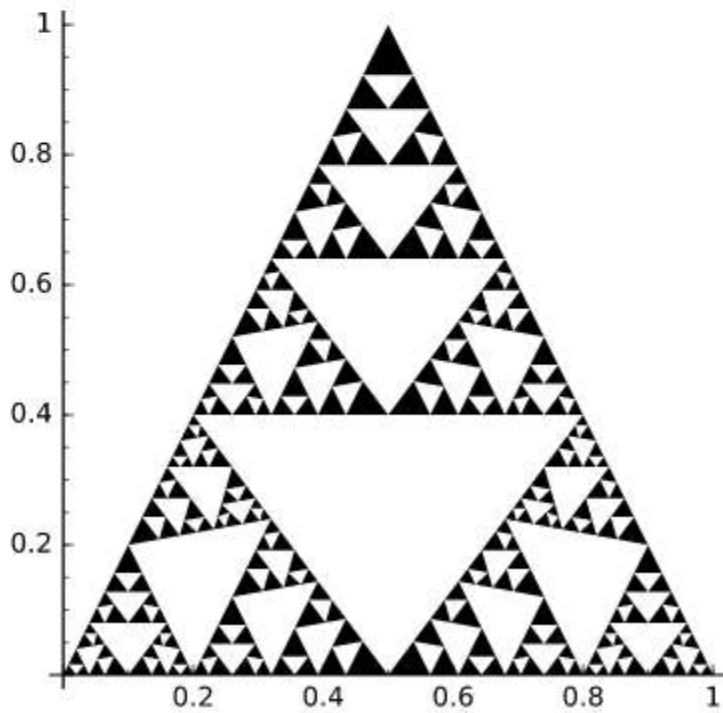
```
Processing. 3
Processing. 2
Processing.. 2
Processing... 2
Moving to next one
Processing.. 3
Processing. 2
Processing.. 2
Processing... 2
Moving to next one
Processing... 3
Processing. 2
Processing.. 2
Processing... 2
Moving to next one
Moving to next one
```



SPT_Graph(0., 0., 1., 0., 1./2., 1., 5)

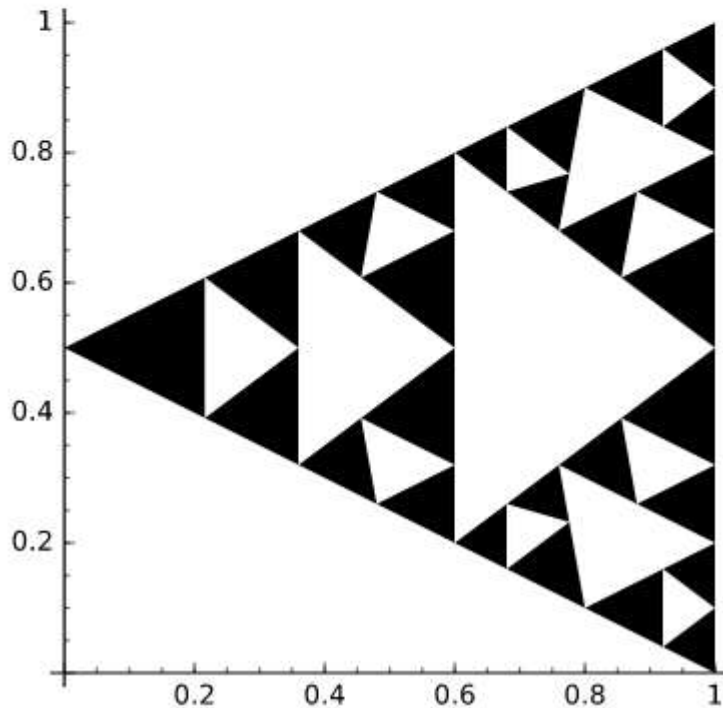
Processing.	5	Processing...	2	Processing..	2
Processing.	4	Moving to next	one	Processing...	2
Processing.	3	Processing...	3	Moving to next	one
Processing.	2	Processing.	2	Processing..	3
Processing..	2	Processing..	2	Processing.	2
Processing...	2	Processing...	2	Processing..	2
Moving to next	one	Moving to next	one	Processing...	2
Processing..	3	Moving to next	one	Moving to next	one
Processing.	2	Moving to next	one	Processing...	3
Processing..	2	Processing..	5	Processing.	2
Processing...	2	Processing.	4	Processing..	2
Moving to next	one	Processing.	3	Processing...	2
Processing...	3	Processing.	2	Moving to next	one
Processing.	2	Processing..	2	Moving to next	one
Processing..	2	Processing...	2	Moving to next	one
Processing...	2	Moving to next	one	Processing...	5
Moving to next	one	Processing..	3	Processing.	4
Moving to next	one	Processing.	2	Processing.	3
Processing..	4	Processing..	2	Processing.	2
Processing.	3	Processing...	2	Processing..	2
Processing.	2	Moving to next	one	Processing...	2
Processing..	2	Processing...	3	Moving to next	one
Processing...	2	Processing.	2	Processing..	3
Moving to next	one	Processing..	2	Processing.	2
Processing..	3	Processing...	2	Processing..	2
Processing.	2	Moving to next	one	Processing...	2
Processing..	2	Moving to next	one	Moving to next	one
Processing...	2	Processing..	4	Moving to next	one
Moving to next	one	Processing.	3	Processing...	3
Processing...	3	Processing.	2	Processing.	2
Processing.	2	Processing..	2	Processing..	2
Processing..	2	Processing...	2	Processing...	2
Processing...	2	Moving to next	one	Moving to next	one
Moving to next	one	Processing..	3	Moving to next	one
Moving to next	one	Processing.	2	Processing..	4
Processing...	4	Processing..	2	Processing.	3
Processing.	3	Processing...	2	Processing.	2
Processing.	2	Moving to next	one	Processing..	2
Processing..	2	Processing...	3	Processing...	2
Processing...	2	Processing.	2	Moving to next	one
Moving to next	one	Processing..	2	Processing..	3
Processing..	3	Processing...	2	Processing.	2
Processing.	2	Moving to next	one	Processing..	2
		Moving to next	one	Processing...	2
		Processing...	4	Moving to next	one
		Processing.	3	Processing...	3
		Processing.	2	Processing.	2

```
Processing... 2
Moving to next one
Moving to next one
Processing... 4
Processing. 3
Processing. 2
Processing.. 2
Processing... 2
Moving to next one
Processing.. 3
Processing. 2
Processing.. 2
Processing... 2
Moving to next one
Processing... 3
Processing. 2
Processing.. 2
Processing... 2
Moving to next one
Moving to next one
Moving to next one
Moving to next one
```



SPT_Graph(0., 1./2, 1., 0, 1., 1., 3)

```
Processing. 3
Processing. 2
Processing.. 2
Processing... 2
Moving to next one
Processing.. 3
Processing. 2
Processing.. 2
Processing... 2
Moving to next one
Processing... 3
Processing. 2
Processing.. 2
Processing... 2
Moving to next one
Moving to next one
```



Appendix C: *Sierpiński Pedal Quartet*

The following pages contain the *Sierpiński Pedal Quartet*. At the beginning of each movement, the lines of the staff system are labeled with the beginning letter of the instrument part and the part number (e.g. E 1 = Euphonium 1, T 1 = Tuba 1).

Energy

Sam Dent

Ebullient! (♩ = 155)

The musical score is written for four staves, labeled E 1, E 2, T 1, and T 2. The key signature is two flats (Bb, Eb) and the time signature is 4/4. The tempo is marked as **Ebullient!** with a quarter note equal to 155 (♩ = 155). The first system shows E 1, T 1, and T 2 with whole rests. E 2 has a melodic line starting with a forte (*f*) dynamic. The second system continues the E 2 and T 2 parts, with E 1 and T 1 remaining silent.

Energy

8

f

This musical system covers measures 8 through 11. It features four staves in bass clef with a key signature of two flats. The first staff begins with a whole rest in measure 8, followed by a melodic line in measures 9-11. The second staff has a dynamic marking of *f* in measure 9 and contains a steady eighth-note accompaniment. The third staff continues the eighth-note accompaniment. The fourth staff has whole rests throughout the system.

12

13

f

This musical system covers measures 12 through 15. It features four staves in bass clef with a key signature of two flats. The first staff has a melodic line starting in measure 12 and becoming more complex in measure 13. The second staff has an eighth-note accompaniment with accents (>) in measures 13-15. The third staff continues the eighth-note accompaniment with accents (>) in measures 13-15. The fourth staff has a rhythmic accompaniment of eighth notes with accents (>) in measures 13-15, starting with a dynamic marking of *f*.

Energy

15

fp *f*
fp *mp*
fp *mp*
fp *mf*

18

fp *mf*

Energy

21

Musical score for measures 21-23. The score is written in bass clef with a key signature of two flats (B-flat and E-flat). Measure 21 features a melodic line in the upper voice with eighth notes and a half note, and a rhythmic accompaniment in the lower voices consisting of eighth notes and quarter notes. Measure 22 continues the melodic line with a half note and eighth notes. Measure 23 concludes the melodic phrase with a half note and eighth notes.

24

Musical score for measures 24-26. The score is written in bass clef with a key signature of two flats. Measure 24 begins with a melodic line in the upper voice and a rhythmic accompaniment in the lower voices. Measure 25 features a melodic line with accents (>) and a rhythmic accompaniment. Measure 26 concludes the melodic phrase with a half note and eighth notes.

Energy

27

29

Musical score for measures 27-29. The score is written in bass clef with a key signature of two flats (B-flat and E-flat). Measure 27 begins with a whole note chord consisting of G2, B-flat2, and E-flat3. The bass line in measure 27 consists of quarter notes G2, B-flat2, and E-flat3. Measure 28 features a sixteenth-note triplet in the upper voice, a whole note chord G2-B-flat2-E-flat3 in the second voice, and a whole note chord G2-B-flat2-E-flat3 in the third voice. The bass line continues with quarter notes G2, B-flat2, and E-flat3. Measure 29 starts with a dynamic marking of *f* (forte). The upper voice has a sixteenth-note triplet, the second voice has a sixteenth-note triplet, and the third voice has a sixteenth-note triplet. The bass line continues with quarter notes G2, B-flat2, and E-flat3.

30

Musical score for measures 30-32. The score is written in bass clef with a key signature of two flats (B-flat and E-flat). Measure 30 begins with a sixteenth-note triplet in the upper voice, a whole note chord G2-B-flat2-E-flat3 in the second voice, and a whole note chord G2-B-flat2-E-flat3 in the third voice. The bass line consists of quarter notes G2, B-flat2, and E-flat3. Measure 31 features a sixteenth-note triplet in the upper voice, a sixteenth-note triplet in the second voice, and a sixteenth-note triplet in the third voice. The bass line continues with quarter notes G2, B-flat2, and E-flat3. Measure 32 starts with a sixteenth-note triplet in the upper voice, a sixteenth-note triplet in the second voice, and a sixteenth-note triplet in the third voice. The bass line continues with quarter notes G2, B-flat2, and E-flat3.

Energy

33

Musical score for measures 33-35. The score is written in bass clef with a key signature of two flats (B-flat and E-flat). It consists of four staves. The top staff features a melodic line with eighth and sixteenth notes, including a fermata over the final note of the first measure. The second staff contains a rhythmic accompaniment of eighth notes. The third staff has a melodic line with some rests and eighth notes. The bottom staff provides a bass line with dotted and eighth notes.

36

Musical score for measures 36-38. The score is written in bass clef with a key signature of two flats (B-flat and E-flat). It consists of four staves. The top staff features a melodic line with eighth and sixteenth notes, including a fermata over the first note of the first measure. The second staff contains a rhythmic accompaniment of eighth notes. The third staff has a melodic line with eighth notes and rests. The bottom staff provides a bass line with dotted and eighth notes.

Energy

39

41

mf

p cresc.

mf

p cresc.

p

cresc.

p

Detailed description: This block contains the musical notation for measures 39 through 41. It features five staves. The top staff (treble clef) has a whole rest in measure 39 and a melodic line starting in measure 40. The second and third staves (bass clef) have whole notes in measure 39 and a melodic line starting in measure 40. The fourth staff (bass clef) has a whole note in measure 39 and a melodic line starting in measure 40. The bottom staff (bass clef) has a whole note in measure 39 and a melodic line starting in measure 40. Dynamics include *mf* and *p cresc.* with hairpins. A boxed measure number '41' is placed above the first staff.

43

f

Detailed description: This block contains the musical notation for measures 43 through 46. It features five staves. The top staff (treble clef) has a melodic line starting in measure 43. The second and third staves (bass clef) have whole notes in measure 43 and a melodic line starting in measure 44. The bottom staff (bass clef) has a whole note in measure 43 and a melodic line starting in measure 44. Dynamics include *f*. A measure number '43' is placed above the first staff.

Energy

46

Musical score for measures 46-48. The score is written in bass clef with a key signature of two flats (B-flat and E-flat). It consists of four staves. The top staff features a continuous eighth-note melody with a slur. The second staff has a few notes with a slur. The third staff contains a few notes with a slur. The bottom staff has a few notes with a slur.

49

Musical score for measures 49-51. The score is written in bass clef with a key signature of two flats (B-flat and E-flat). It consists of four staves. The top staff has notes with accents (>) and slurs. The second staff has notes with accents (>) and slurs, with a dynamic marking of *f*. The third staff has notes with accents (>) and slurs, with a dynamic marking of *f*. The bottom staff has notes with accents (>) and slurs, with a dynamic marking of *mf*.

Energy

52

Musical score for measures 52-54. The score is written in bass clef with a key signature of two flats (B-flat and E-flat). It consists of four staves. The top staff contains a melodic line with accents and slurs. The second staff contains a similar melodic line. The third staff features a rhythmic pattern of eighth notes with accents. The bottom staff contains a bass line with quarter notes and rests.

55

Musical score for measures 55-57. The score is written in bass clef with a key signature of two flats (B-flat and E-flat). It consists of four staves. The top staff contains a melodic line with accents and slurs. The second staff contains a similar melodic line. The third staff features a rhythmic pattern of eighth notes with accents. The bottom staff contains a bass line with quarter notes and rests.

Energy

58

Musical score for measures 58-60. The score is written in bass clef with a key signature of two flats (B-flat and E-flat). It consists of four staves. The first staff has a whole note G2 in measure 58, followed by a whole rest in measure 59 and a whole note G2 in measure 60. The second staff has a dotted quarter note G2, an eighth note A2, a quarter note B-flat2, and a quarter note C3 in measure 58; a dotted quarter note D3, an eighth note E3, a quarter note F3, and a quarter note G3 in measure 59; and a dotted quarter note A3, an eighth note B-flat3, a quarter note C4, and a quarter note D4 in measure 60. The third staff has a dotted quarter note E3, an eighth note F3, a quarter note G3, and a quarter note A3 in measure 58; a dotted quarter note B-flat3, an eighth note C4, a quarter note D4, and a quarter note E4 in measure 59; and a dotted quarter note F4, an eighth note G4, a quarter note A4, and a quarter note B-flat4 in measure 60. The fourth staff has a dotted quarter note G2, an eighth note A2, a quarter note B-flat2, and a quarter note C3 in measure 58; a dotted quarter note D3, an eighth note E3, a quarter note F3, and a quarter note G3 in measure 59; and a dotted quarter note A3, an eighth note B-flat3, a quarter note C4, and a quarter note D4 in measure 60.

61

Musical score for measures 61-63. The score is written in bass clef with a key signature of two flats (B-flat and E-flat). It consists of four staves. The first staff has a dotted quarter note G2 with an accent (>), an eighth note A2 with an accent (>), a quarter note B-flat2 with an accent (>), and a quarter note C3 with an accent (>) in measure 61; a dotted quarter note D3 with an accent (>), an eighth note E3 with an accent (>), a quarter note F3 with an accent (>), and a quarter note G3 with an accent (>) in measure 62; and a dotted quarter note A3 with an accent (>), an eighth note B-flat3 with an accent (>), a quarter note C4 with an accent (>), and a quarter note D4 with an accent (>) in measure 63. The second staff has a dotted quarter note E3 with an accent (>), an eighth note F3 with an accent (>), a quarter note G3 with an accent (>), and a quarter note A3 with an accent (>) in measure 61; a dotted quarter note B-flat3 with an accent (>), an eighth note C4 with an accent (>), a quarter note D4 with an accent (>), and a quarter note E4 with an accent (>) in measure 62; and a dotted quarter note F4 with an accent (>), an eighth note G4 with an accent (>), a quarter note A4 with an accent (>), and a quarter note B-flat4 with an accent (>) in measure 63. The third staff has a dotted quarter note G2 with an accent (>), an eighth note A2 with an accent (>), a quarter note B-flat2 with an accent (>), and a quarter note C3 with an accent (>) in measure 61; a dotted quarter note D3 with an accent (>), an eighth note E3 with an accent (>), a quarter note F3 with an accent (>), and a quarter note G3 with an accent (>) in measure 62; and a dotted quarter note A3 with an accent (>), an eighth note B-flat3 with an accent (>), a quarter note C4 with an accent (>), and a quarter note D4 with an accent (>) in measure 63. The fourth staff has a dotted quarter note G2 with an accent (>), an eighth note A2 with an accent (>), a quarter note B-flat2 with an accent (>), and a quarter note C3 with an accent (>) in measure 61; a dotted quarter note D3 with an accent (>), an eighth note E3 with an accent (>), a quarter note F3 with an accent (>), and a quarter note G3 with an accent (>) in measure 62; and a dotted quarter note A3 with an accent (>), an eighth note B-flat3 with an accent (>), a quarter note C4 with an accent (>), and a quarter note D4 with an accent (>) in measure 63.

Energy

65

Musical score for measures 65-68. The score is written in bass clef with a key signature of two flats. It consists of five staves. The top staff contains a melodic line with a fermata over the first measure and a triplet of eighth notes in the second measure. The second and third staves contain a rhythmic accompaniment of eighth notes. The fourth and fifth staves contain a bass line with a fermata over the first measure and a sequence of eighth notes. Dynamics include *fp*, *f*, *mp*, and *mf*.

69

Musical score for measures 69-72. The score is written in bass clef with a key signature of two flats. It consists of five staves. The top staff contains a melodic line with a fermata over the first measure and a sequence of eighth notes. The second and third staves contain a rhythmic accompaniment of eighth notes. The fourth and fifth staves contain a bass line with a sequence of eighth notes. Dynamics include *fp* and *mf*.

Energy

72

Musical score for measures 72-74. The score consists of four staves. The top staff is in bass clef with a key signature of two flats and a common time signature. It features a melodic line with a slur over measures 72 and 73, and a fermata over the final note in measure 74. The second and third staves are in bass clef and contain rhythmic accompaniment with eighth notes and accents (>) in measure 73. The bottom staff is in bass clef and contains a bass line with quarter and eighth notes.

75

Musical score for measures 75-77. The score consists of four staves. The top staff is in bass clef with a key signature of two flats and a common time signature. It features a melodic line with a slur over measures 75 and 76, and a fermata over the final note in measure 77. The second and third staves are in bass clef and contain rhythmic accompaniment with eighth notes. The bottom staff is in bass clef and contains a bass line with quarter and eighth notes. Dynamic markings *fp* are present in measures 76 and 77 on the second, third, and bottom staves, with corresponding hairpins.

Energy

78

Musical score for measures 78-80. The score is written in bass clef with a key signature of two flats (B-flat and E-flat). It consists of four staves. The top staff has a treble clef and contains a melodic line with a fermata at the end of measure 78 and a fermata at the end of measure 80. The second staff contains a rhythmic accompaniment of eighth notes. The third staff contains a melodic line starting in measure 79 with a forte (*f*) dynamic marking. The fourth staff contains a bass line with a forte (*f*) dynamic marking and a fermata at the end of measure 80.

81

Musical score for measures 81-84. The score is written in bass clef with a key signature of two flats (B-flat and E-flat). It consists of four staves. The top staff has a treble clef and contains a melodic line with a fermata at the end of measure 81 and a fermata at the end of measure 84. The second staff contains a rhythmic accompaniment of eighth notes. The third staff contains a melodic line starting in measure 81 with a fermata at the end of measure 81 and a fermata at the end of measure 84. The fourth staff contains a bass line with a fermata at the end of measure 81 and a fermata at the end of measure 84.

Energy

84

Musical score for measures 84-86. The score is written in bass clef with a key signature of two flats (B-flat and E-flat). It consists of four staves. The top staff features a melodic line with a slur over the first two measures and a fermata over the second measure. The second and third staves contain rhythmic patterns of eighth and sixteenth notes. The bottom staff has a bass line with slurs and accents. A dynamic marking of *mf* is present in the third measure of the third staff.

87

Musical score for measures 87-89. The score is written in bass clef with a key signature of two flats. It consists of four staves. The top staff has a melodic line with a fermata over the first measure. The second and third staves contain rhythmic patterns of eighth and sixteenth notes. The bottom staff has a bass line with slurs and accents.

Energy

90

Musical score for measures 90-91. The score consists of four staves in bass clef with a key signature of two flats. The first staff features a rhythmic pattern of eighth notes with accents, starting with a rest. The second staff has a similar pattern but with a rest in the first measure. The third staff plays a steady eighth-note line with accents. The fourth staff plays a steady eighth-note line with accents. The dynamic marking *ff* is present in the first two staves.

92

Musical score for measures 92-94. The score consists of four staves in bass clef with a key signature of two flats. The first staff continues the eighth-note pattern with accents. The second staff has a more complex eighth-note pattern with accents. The third staff continues the eighth-note line with accents. The fourth staff continues the eighth-note line with accents. The dynamic marking *ff* is present in the first staff.

Grief

Sam Dent

The first system of the score consists of four staves, labeled E 1, E 2, T 1, and T 2. The key signature is one sharp (F#) and the time signature is 3/4. The music is written in bass clef. The first two staves (E 1 and E 2) feature a melodic line with a dynamic marking of *p* (piano). The third staff (T 1) has a dynamic marking of *p* starting in the fourth measure. The fourth staff (T 2) has a dynamic marking of *p* at the beginning. The system concludes with a double bar line.

The second system of the score begins with a measure number '8' above the first staff. A boxed measure number '11' is positioned above the second staff. The system contains four staves. The first staff has a dynamic marking of *mf* (mezzo-forte) in the third measure. The second staff has a dynamic marking of *mp* (mezzo-piano) in the second measure. The third staff has a dynamic marking of *mp* at the beginning. The fourth staff has a dynamic marking of *mp* at the beginning. The system concludes with a double bar line.

Grief

13

Musical score for measures 13-17. The score is written in bass clef with a key signature of one sharp (F#). It consists of four staves. The top staff contains rests for measures 13-14 and notes for measures 15-17. The second staff contains melodic lines with slurs and a dynamic marking of *mf* in measure 15. The third staff contains a rhythmic accompaniment of eighth notes with slurs. The bottom staff contains a bass line with dotted half notes.

18

Musical score for measures 18-22. The score is written in bass clef with a key signature of one sharp (F#). It consists of four staves. The top staff contains a melodic line with slurs and a fermata in measure 22. The second staff contains a melodic line with slurs. The third staff contains a rhythmic accompaniment of eighth notes with slurs. The bottom staff contains a bass line with dotted half notes.

Grief

23

Musical score for measures 23-27. The score is written for four staves in bass clef with a key signature of one sharp (F#). The first staff contains a melodic line with slurs and ties. The second staff contains a bass line with slurs and ties. The third staff contains a rhythmic accompaniment of eighth notes with slurs. The fourth staff contains a bass line with half notes.

28

30

Musical score for measures 28-32. The score is written for four staves in bass clef with a key signature of one sharp (F#). The first staff contains a melodic line with slurs and ties. The second staff contains a bass line with slurs and ties. The third staff contains a rhythmic accompaniment of eighth notes with slurs. The fourth staff contains a bass line with half notes. A dynamic marking of *f* is present in measure 30, and a dynamic marking of *mf* is present in measure 31. A triplet of eighth notes is marked with a '3' in measure 32.

Grief

33

Musical score for measures 33-37. The score is written for four staves in bass clef with a key signature of one sharp (F#). Measure 33 features a melodic line in the top staff with eighth notes and a half note, while the other staves have rests. Measure 34 continues the melodic line in the top staff. Measure 35 has a whole rest in the top staff and a half note in the second staff. Measure 36 has a whole rest in the top staff and a half note in the second staff. Measure 37 has a whole rest in the top staff and a half note in the second staff. The bottom two staves provide a harmonic accompaniment with eighth and quarter notes.

38

Musical score for measures 38-42. The score is written for four staves in bass clef with a key signature of one sharp (F#). Measure 38 has a whole rest in the top staff and a half note in the second staff. Measure 39 has a whole rest in the top staff and a half note in the second staff. Measure 40 has a whole rest in the top staff and a half note in the second staff. Measure 41 has a whole rest in the top staff and a half note in the second staff. Measure 42 has a whole rest in the top staff and a half note in the second staff. The bottom two staves provide a harmonic accompaniment with eighth and quarter notes.

Grief

43

Musical score for measures 43-46. The score is written for four staves in bass clef with a key signature of one sharp (F#). The time signature is 4/4. Measure 43 shows a whole rest in the top two staves and a quarter note in the bottom two. Measure 44 has a whole rest in the top two staves and a half note in the bottom two. Measure 45 features a half note in the top two staves and a half note in the bottom two. Measure 46 contains a sixteenth-note triplet in the top two staves and a whole rest in the bottom two.

47

48

Musical score for measures 47-50. The score is written for four staves in bass clef with a key signature of one sharp (F#). The time signature is 4/4. Measure 47 contains a sixteenth-note triplet in the top two staves and a whole note in the bottom two. Measure 48 features a sixteenth-note triplet in the top two staves and a sixteenth-note triplet in the bottom two. Measure 49 has a sixteenth-note triplet in the top two staves and a half note in the bottom two. Measure 50 contains a sixteenth-note triplet in the top two staves and a half note in the bottom two. Dynamics include *f* and *ff*.

Grief

49

Musical score for measures 49-50. The score is written for four staves in bass clef with a key signature of one sharp (F#). The first staff contains a melodic line with triplet markings (3) over groups of three notes. The second staff contains a rhythmic accompaniment of eighth notes. The third and fourth staves contain a bass line with triplet markings (3) over groups of three notes. The music is divided into two measures by a vertical bar line.

51

Musical score for measures 51-52. The score is written for four staves in bass clef with a key signature of one sharp (F#). The first staff contains a melodic line with triplet markings (3) over groups of three notes. The second staff contains a rhythmic accompaniment of eighth notes. The third and fourth staves contain a bass line with triplet markings (3) over groups of three notes. The music is divided into two measures by a vertical bar line.

Grief

53

Musical score for measures 53-54. The score is written for four staves in bass clef with a key signature of one sharp (F#). The first staff contains a melodic line with triplet markings (3) over groups of three notes. The second staff contains a rhythmic accompaniment of eighth notes. The third and fourth staves contain more complex melodic lines with triplet markings and dynamic markings (accents) under the notes.

55

Musical score for measures 55-56. The score is written for four staves in bass clef with a key signature of one sharp (F#). The first staff continues the melodic line with triplet markings (3). The second staff continues the rhythmic accompaniment. The third and fourth staves feature a change in dynamics, marked with *ff* (fortissimo), and contain more complex melodic lines with accents and slurs.

Grief

57

Musical score for measures 57-58. The score consists of four staves. The first two staves are bass clefs, and the last two are also bass clefs. The key signature is one sharp (F#). Measure 57 shows complex rhythmic patterns with many beamed notes. Measure 58 features a 7/16 time signature and continues the rhythmic complexity.

59

Musical score for measures 59-61. The score consists of four staves. The first two staves are bass clefs, and the last two are also bass clefs. The key signature is one sharp (F#). Measure 59 shows complex rhythmic patterns with many beamed notes. Measure 60 is marked with a 4/4 time signature and includes dynamic markings *f* and *ff*. Measure 61 features a 7/16 time signature and continues the rhythmic complexity. Dynamic markings *f* and *ff* are present in measures 60 and 61.

Grief

62

Musical score for measures 62-64. It consists of four staves in bass clef with a key signature of one sharp (F#). The time signature changes from 4/4 to 3/4. The music features various rhythmic patterns, including eighth and sixteenth notes, and rests. The first staff has a melodic line with some accidentals. The second staff has a similar melodic line. The third staff has a steady eighth-note accompaniment. The fourth staff has a sparse accompaniment with rests.

64

(♩=♩) ♩ = 160

Musical score for measures 64-68. It consists of four staves in bass clef with a key signature of one sharp (F#) and a 3/4 time signature. The music features a variety of dynamics including forte (*f*), mezzo-forte (*mf*), and accents (>). The first two staves have a melodic line with accents and dynamics. The third staff has a steady eighth-note accompaniment with dynamics. The fourth staff has a triplet accompaniment with dynamics.

Grief

69

Musical score for measures 69-73. The score is written in bass clef with a key signature of one sharp (F#). It consists of four staves. The top staff contains a melodic line with eighth and quarter notes, some with accents (>). The second staff contains a similar melodic line. The third staff contains a rhythmic accompaniment of eighth notes with accents. The bottom staff contains a triplet accompaniment of eighth notes, with a '3' above each group. The music concludes with a whole rest in the top two staves.

74

Musical score for measures 74-78. The score is written in bass clef with a key signature of one sharp (F#). It consists of four staves. The top staff contains a melodic line with eighth and quarter notes, some with accents (>), and a slur over a half note. The second staff contains a similar melodic line. The third staff contains a rhythmic accompaniment of eighth notes with accents. The bottom staff contains a triplet accompaniment of eighth notes, with a '3' above each group. The music concludes with a whole rest in the top two staves.

Grief

79

79

cresc.

cresc.

cresc.

cresc.

3

3

3

3

3

cresc.

Detailed description: This system contains measures 79 through 83. It features four staves in bass clef with a key signature of one sharp (F#). The top two staves have sparse notes with accents (>) and a *cresc.* marking in the fourth measure. The third staff has a melodic line with accents and a *cresc.* marking in the fourth measure. The bottom staff has a triplet accompaniment of eighth notes with accents and a *cresc.* marking in the fourth measure.

84

84

ff

ff

ff

3

3

3

3

ff

Detailed description: This system contains measures 84 through 88. It features four staves in bass clef with a key signature of one sharp (F#). The top two staves have notes with accents (>) and a *ff* marking in the fifth measure. The third staff has a melodic line with accents and a *ff* marking in the fifth measure. The bottom staff has a triplet accompaniment of eighth notes with accents and a *ff* marking in the fifth measure. The system concludes with a final *ff* marking on a single note in the bottom staff.

Grief

89

Musical score for measures 89-94. The score is written for four staves in bass clef with a key signature of one sharp (F#). The music features a melodic line in the upper staves and a rhythmic accompaniment in the lower staves. Measure 89 starts with a half note G2. Measures 90-91 show a melodic line with eighth notes and a bass line with eighth notes. Measure 92 has a melodic line with a dotted half note and a bass line with eighth notes. Measure 93 has a melodic line with a dotted half note and a bass line with eighth notes. Measure 94 has a melodic line with a dotted half note and a bass line with eighth notes. The score includes various musical notations such as slurs, accents, and triplets.

95

$\text{♩} = 80$

98

mf

Musical score for measures 95-98. The score is written for four staves in bass clef with a key signature of one sharp (F#). The music features a melodic line in the upper staves and a rhythmic accompaniment in the lower staves. Measure 95 starts with a half note G2. Measures 96-97 show a melodic line with eighth notes and a bass line with eighth notes. Measure 98 has a melodic line with a dotted half note and a bass line with eighth notes. The score includes various musical notations such as slurs, accents, and triplets.

Grief

101

Musical score for measures 101-106. The score is written in bass clef with a key signature of one sharp (F#). It consists of four staves. The first staff has rests for measures 101-105 and a quarter note G4 with a fermata in measure 106, marked *mf*. The second staff has a quarter note G3 in measure 101, a quarter note A3 in measure 102, a quarter note B3 in measure 103, a half note C4 with a fermata in measure 104, and rests in measures 105 and 106. The third staff has rests in measures 101-102, then a series of eighth notes (D4, E4, F#4, G4, A4, B4, C5) with a slur and a fermata over the last two notes in measure 106, marked *mf*. The fourth staff has rests for all measures.

107

111

rit.

$\bullet = 80$

cresc.

pp cresc.

Musical score for measures 107-111. The score is written in bass clef with a key signature of one sharp (F#). It consists of four staves. The first staff has a quarter note G3 in measure 107, followed by eighth notes (A3, B3, C4, D4, E4, F#4, G4) with a slur and a fermata over the last two notes in measure 111, marked *rit.*. The second staff has rests for all measures. The third staff has rests for all measures. The fourth staff has rests for all measures. At measure 111, there is a tempo marking $\bullet = 80$ and a dynamic marking *pp cresc.*. The score ends with a double bar line at measure 111.

Grief

112

Musical score for measures 112-116. The score is written for four staves in bass clef with a key signature of one sharp (F#). Measure 112 features a melodic line in the top staff with eighth notes and a slur. The second staff has a fermata. The third staff has a half note. The fourth staff has a half note. Measure 113 has a fermata in the second staff and a half note in the third. Measure 114 has a fermata in the second staff and a half note in the third. Measure 115 has a fermata in the second staff, a half note in the third, and a *cresc.* marking. Measure 116 has a fermata in the second staff, a half note in the third, and a half note in the fourth.

117

Musical score for measures 117-119. The score is written for four staves in bass clef with a key signature of one sharp (F#). Measure 117 features a melodic line in the top staff with eighth notes and a slur. The second staff has a half note. The third staff has a half note. The fourth staff has a half note. Measure 118 has a melodic line in the top staff with eighth notes and a slur. The second staff has a half note. The third staff has a half note. The fourth staff has a half note. Measure 119 has a melodic line in the top staff with eighth notes and a slur. The second staff has a half note. The third staff has a half note. The fourth staff has a half note.

Grief

119

Musical score for measures 119-120. The score is written for four staves in bass clef with a key signature of one sharp (F#). The first staff contains a melodic line with a half note, a quarter note, and a dotted quarter note, followed by a half note. The second staff features a piano (*f*) accompaniment with a triplet of eighth notes and a continuous sixteenth-note pattern. The third staff has a melodic line with a half note, a quarter note, and a dotted quarter note, followed by a half note. The fourth staff features a piano (*f*) accompaniment with a triplet of eighth notes and a continuous sixteenth-note pattern. The dynamic *f* is marked at the beginning of the first and third staves.

121

Musical score for measures 121-122. The score is written for four staves in bass clef with a key signature of one sharp (F#). The first staff contains a melodic line with a half note, a quarter note, and a dotted quarter note, followed by a half note. The second staff features a piano (*f*) accompaniment with a triplet of eighth notes and a continuous sixteenth-note pattern. The third staff has a melodic line with a half note, a quarter note, and a dotted quarter note, followed by a half note. The fourth staff features a piano (*f*) accompaniment with a triplet of eighth notes and a continuous sixteenth-note pattern. The dynamic *f* is marked at the beginning of the first and third staves.

Grief

123

Musical score for measures 123-124. The score consists of five staves. The top staff is a treble clef with a key signature of one sharp (F#) and contains a continuous sixteenth-note scale. The second staff is a bass clef with a key signature of one sharp (F#) and contains a melody starting with a quarter rest, followed by a quarter note, a half note with a slur, and a dotted half note with a slur. The third staff is a bass clef with a key signature of one sharp (F#) and contains a single half note with an accent (>) and a slur. The fourth staff is a bass clef with a key signature of one sharp (F#) and contains a single half note with a slur. The fifth staff is a bass clef with a key signature of one sharp (F#) and contains a single half note with a slur. The dynamic marking *ff* is present in the second, third, and fourth staves.

125

Musical score for measures 125-126. The score consists of five staves. The top staff is a treble clef with a key signature of one sharp (F#) and contains a continuous sixteenth-note scale. The second staff is a bass clef with a key signature of one sharp (F#) and contains a melody starting with a quarter rest, followed by a quarter note, a half note with a slur, and a dotted half note with a slur. The third staff is a bass clef with a key signature of one sharp (F#) and contains a melody starting with a quarter note, followed by a quarter note, a half note with a slur, and a dotted half note with a slur. The fourth staff is a bass clef with a key signature of one sharp (F#) and contains a single half note with a slur. The fifth staff is a bass clef with a key signature of one sharp (F#) and contains a single half note with a slur, followed by a quarter note and an eighth note in the next measure. The dynamic marking *ff* is present in the second, third, and fourth staves.

Grief

127

Musical score for measures 127-128. The score is written in bass clef with a key signature of one sharp (F#). It consists of four staves. The top staff contains a melodic line with a quarter rest, followed by quarter notes G2, A2, and B2, and a half note C3. The second staff contains a dense sixteenth-note accompaniment. The third staff contains a melodic line with a quarter note G2, a quarter note A2, a half note B2, and a quarter note C3. The bottom staff contains a bass line with a whole note G2 in the first measure and a whole note C3 in the second measure.

129

Musical score for measures 129-130. The score is written in bass clef with a key signature of one sharp (F#). It consists of four staves. The top staff contains a melodic line with a quarter rest, followed by quarter notes G2, A2, and B2, and a half note C3. The second staff contains a dense sixteenth-note accompaniment. The third staff contains a melodic line with a quarter note G2, a quarter note A2, a half note B2, and a quarter note C3. The bottom staff contains a bass line with a whole note G2 in the first measure, and whole notes C3 in the second and third measures.

Grief

131

Musical score for measures 131-133. The score is written for four staves in bass clef with a key signature of one sharp (F#). Measure 131: The top staff has a half note G4, a quarter note A4, and a quarter note B4, all beamed together. The second staff has a quarter note G4, a quarter note A4, and a quarter note B4. The third staff has a half note G4. The fourth staff has a half note G4. Measure 132: The top staff has a half note G4, a quarter note A4, and a quarter note B4, all beamed together. The second staff has a quarter note G4, a quarter note A4, and a quarter note B4, all beamed together. The third staff has a half note G4. The fourth staff has a half note G4. Measure 133: The top staff has a half note G4, a quarter note A4, and a quarter note B4, all beamed together. The second staff has a quarter note G4, a quarter note A4, and a quarter note B4. The third staff has a half note G4. The fourth staff has a half note G4.

134

Musical score for measures 134-136. The score is written for four staves in bass clef with a key signature of one sharp (F#). Measure 134: The top staff has a half note G4. The second staff has a quarter note G4, a quarter note A4, and a quarter note B4, all beamed together. The third staff has a half note G4. The fourth staff has a half note G4. Measure 135: The top staff has a half note G4, a quarter note A4, and a quarter note B4, all beamed together. The second staff has a quarter note G4, a quarter note A4, and a quarter note B4, all beamed together. The third staff has a half note G4. The fourth staff has a half note G4. Measure 136: The top staff has a half note G4, a quarter note A4, and a quarter note B4, all beamed together. The second staff has a quarter note G4, a quarter note A4, and a quarter note B4, all beamed together. The third staff has a half note G4. The fourth staff has a half note G4.

Grief

137 *rit.*

mp

p

p

p

Funk-ish

Sam Dent

Feel the groove! ♩ = 92

The musical score is written for four staves, labeled E 1, E 2, T 1, and T 2, all in bass clef. The time signature is 4/4, and the key signature has three flats (B-flat, E-flat, A-flat). The tempo is marked as ♩ = 92. The first system consists of two measures. In the first measure, E 1 and E 2 play a pair of eighth notes with accents, while T 1 plays a rhythmic eighth-note pattern. In the second measure, E 1 and E 2 play another pair of eighth notes, and T 1 continues its pattern. The second system also consists of two measures. The first measure features a triplet of eighth notes in E 1, eighth notes in E 2, and a pair of eighth notes in T 1. The second measure continues the triplet in E 1, eighth notes in E 2, and eighth notes in T 1. Dynamic markings of *f* (forte) are present in the first measure of both systems. Accents (>) are used throughout to emphasize specific notes.

Funk-ish

5

Musical score for measures 5-7. The score is written in bass clef with a key signature of three flats (B-flat, E-flat, A-flat). It consists of four staves. The top staff is mostly empty with a few notes. The second staff has a few notes. The third staff features a complex rhythmic pattern with many notes and accents (>). The bottom staff has a steady bass line with notes and accents (>).

9

8

Musical score for measures 8-9. The score is written in bass clef with a key signature of three flats (B-flat, E-flat, A-flat). It consists of four staves. The top staff is mostly empty. The second staff has a few notes. The third staff features a complex rhythmic pattern with many notes and accents (>). The bottom staff has a steady bass line with notes and accents (>). A dynamic marking *f* is present in measure 9, and *mf* is present in measure 8.

Funk-ish

10

Musical score for measures 10-11. The score is in bass clef with a key signature of two flats (B-flat and E-flat). Measure 10 features a bass line with eighth notes and a piano accompaniment with a melodic line. Measure 11 features a piano accompaniment with a melodic line and a bass line with eighth notes. The dynamic marking *mf* is present in measure 11.

12

Musical score for measures 12-13. The score is in bass clef with a key signature of two flats (B-flat and E-flat). Measure 12 features a piano accompaniment with a melodic line and a bass line with eighth notes. Measure 13 features a piano accompaniment with a melodic line and a bass line with eighth notes. The dynamic marking *f* is present in measure 13, and *mf* is present in measure 12.

Funk-ish

14

Musical score for measures 14 and 15. The score is written in bass clef with a key signature of two flats (B-flat and E-flat). It consists of four staves. The top staff contains a melodic line with eighth and sixteenth notes. The second staff contains a melodic line with eighth notes and rests. The third staff contains a rhythmic pattern of eighth notes with accents. The bottom staff contains a bass line with eighth notes and rests.

16

Musical score for measures 16 and 17. The score is written in bass clef with a key signature of two flats (B-flat and E-flat). It consists of four staves. The top staff contains a melodic line with eighth and sixteenth notes. The second staff contains a melodic line with eighth notes and rests. The third staff contains a rhythmic pattern of eighth notes with accents. The bottom staff contains a bass line with eighth notes and rests.

Funk-ish

18 19

mf

mf

mf

mf

20

mf

Funk-ish

22

Musical score for measures 22-23. The score is in bass clef with a key signature of three flats (B-flat, E-flat, A-flat). It consists of four staves. The first two staves are for a piano, with dynamics *fp* and *f*. The third and fourth staves are for a bass line, with dynamics *f*. The music features a mix of eighth and sixteenth notes, with some rests and accents.

24

Musical score for measures 24-25. The score is in bass clef with a key signature of three flats (B-flat, E-flat, A-flat). It consists of four staves. The first two staves are for a piano, and the third and fourth staves are for a bass line. The music continues with eighth and sixteenth notes, including rests and accents.

Funk-ish

26

Musical score for measures 26-27. The score is written in bass clef with a key signature of two flats (B-flat and E-flat). It consists of four staves. The first staff has a rest in measure 26 and a melodic line in measure 27. The second staff has a melodic line in measure 26 and a rest in measure 27. The third staff has a melodic line in measure 26 and a rest in measure 27. The fourth staff has a melodic line in measure 26 and a rest in measure 27. Dynamics include *mf* and *cresc.* in the first two staves, and *mp* and *cresc.* in the last two staves.

28

Musical score for measures 28-29. The score is written in bass clef with a key signature of two flats (B-flat and E-flat). It consists of four staves. The first staff has a melodic line in measure 28 and a rest in measure 29. The second staff has a melodic line in measure 28 and a rest in measure 29. The third staff has a melodic line in measure 28 and a rest in measure 29. The fourth staff has a melodic line in measure 28 and a rest in measure 29.

Funk-ish

30

Musical score for measures 30-32. The score is written in bass clef with a key signature of three flats (B-flat, E-flat, A-flat). It consists of four staves. Measure 30 features a series of eighth notes with accents in the top staff, followed by a rest. Measure 31 continues with similar eighth notes and accents. Measure 32 shows a more complex rhythmic pattern with eighth and sixteenth notes. Dynamics include *f* (forte) in measures 30, 31, and 32.

32

33

Musical score for measures 32-33. The score is written in bass clef with a key signature of three flats. It consists of four staves. Measure 32 features a series of eighth notes with accents in the top staff, followed by a rest. Measure 33 continues with similar eighth notes and accents. Dynamics include *mf* (mezzo-forte) in measures 32 and 33.

Funk-ish

34

mf

This musical system covers measures 34 and 35. It consists of four staves. The top staff features a melodic line with eighth-note runs and rests. The second staff has a rhythmic accompaniment with eighth notes and rests. The third staff contains a steady eighth-note bass line. The bottom staff provides a bass line with eighth notes and rests. A dynamic marking of *mf* is placed below the first staff.

36

f

This musical system covers measures 36 and 37. It consists of four staves. The top staff continues the melodic line with eighth-note patterns. The second staff has a rhythmic accompaniment with eighth notes and rests. The third staff contains a steady eighth-note bass line. The bottom staff provides a bass line with eighth notes and rests. A dynamic marking of *f* is placed below the third staff.

Funk-ish

38

Musical score for measures 38-39. The score is written in bass clef with a key signature of three flats (B-flat, E-flat, A-flat). It consists of four staves. The top staff features a complex rhythmic pattern of eighth and sixteenth notes with slurs. The second staff continues this pattern with a different rhythmic grouping. The third staff contains a simple bass line with quarter notes. The bottom staff features a steady eighth-note accompaniment with accents (>) under the notes.

40

Musical score for measures 40-41. The score is written in bass clef with a key signature of three flats. It consists of four staves. The top staff has a complex rhythmic pattern in measure 40, followed by a measure 41 with a dynamic marking of *f* and a more complex rhythmic pattern. The second staff continues the pattern from measure 40, with a dynamic marking of *f* in measure 41. The third staff contains a simple bass line with quarter notes. The bottom staff features a steady eighth-note accompaniment with accents (>) under the notes and a dynamic marking of *f* in measure 41.

Funk-ish

42

Musical score for measures 42-43. The score is written in bass clef with a key signature of three flats (B-flat, E-flat, A-flat). It consists of four staves. The top staff features a melodic line with eighth and sixteenth notes, including a triplet of eighth notes. The second staff continues the melodic line with similar rhythmic patterns. The third staff provides a harmonic accompaniment with a steady eighth-note bass line. The fourth staff features a more complex rhythmic pattern with eighth and sixteenth notes, including accents and slurs.

44

Musical score for measures 44-45. The score is written in bass clef with a key signature of three flats (B-flat, E-flat, A-flat). It consists of four staves. The top staff features a melodic line with eighth and sixteenth notes, including a triplet of eighth notes. The second staff continues the melodic line with similar rhythmic patterns. The third staff provides a harmonic accompaniment with a steady eighth-note bass line. The fourth staff features a more complex rhythmic pattern with eighth and sixteenth notes, including accents and slurs.

Funk-ish

46 47

mp cresc.
mp cresc.
p cresc.
p cresc.

48

Funk-ish

50

Musical score for measures 50-51. The score consists of four staves in bass clef, all in the key of B-flat major. The first two staves feature a melodic line with eighth notes and a slur over the first two measures. The third staff has a rhythmic pattern with accents (>) on the first two notes. The fourth staff has a rhythmic pattern with accents (>) on the first two notes. The dynamic marking *ff* (fortissimo) is present in the second measure of each staff.

52

Musical score for measures 52-53. The score consists of four staves in bass clef, all in the key of B-flat major. The first two staves feature a melodic line with eighth notes and a slur over the first two measures. The third staff has a rhythmic pattern with accents (>) on the first three notes. The fourth staff has a rhythmic pattern with accents (>) on the first three notes.

Funk-ish

53

Musical score for measures 53-54, featuring four staves in bass clef with a key signature of three flats. The notation includes eighth and sixteenth notes, rests, and dynamic markings such as accents (>) and hairpins ($\bar{}$ and $\underline{}$).

54

Musical score for measures 55-56, continuing from the previous system. It features four staves in bass clef with a key signature of three flats. The notation includes eighth and sixteenth notes, rests, and dynamic markings such as accents (>) and hairpins ($\bar{}$ and $\underline{}$).