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Igor Shulman
University of Southern Mississippi

James K. Lewis
Ocean Physics Research and Development

Alan F. Blumberg
HydroQual, Inc.

B. Nicholas Kim
HydroQual, Inc.

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IGOR SHULMAN

Institute of Marine Sciences, University of Southern Mississippi, Stennis Space Center, Mississippi

JAMES K. LEWIS

Ocean Physics Research and Development, Long Beach, Mississippi

ALAN F. BLUMBERG AND B. NICHOLAS KIM

HydroQual, Inc., Mahwah, New Jersey

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ABSTRACT

An optimization approach is derived for assimilating tidal height information along the open boundaries of a numerical model. The approach is then extended so that similar data along transects inside a model domain can also be optimally assimilated. To test the application of such an optimized methodology, M_2 tidal simulations were conducted with a numerical ocean model of the Yellow Sea, an area with a strong tidal influence. The use of the optimized open boundary conditions and internal data assimilation leads to a significant improvement of the predictive skill of the model. Average errors can be reduced by up to 75% when compared to nonoptimized boundary conditions.

1. Introduction

Tides play a significant role in the structure of water characteristics and currents in many regions of the world. As humanity's use of such regions grows, it becomes increasingly important that we understand tidal influences. This can be done through observational programs and numerical simulation studies. The latter approach is becoming increasingly popular, and this study addresses the question of tidal forcing for regional coastal ocean models.

Using a model of the Yellow Sea, M_2 tidal simulations are performed. This region is one with a fairly complex tidal structure, as well as relatively large tidal ranges. Available observations and the tidal simulations performed by Choi (1986) show four major M_2 amphidromic points in the region of the Yellow Sea being modeled (Fig. 1), as well as large tidal amplitudes along the west coast of Korea. As noted in Choi (1986), the results of his simulations are in good agreement with observations. Simulations with a model with greater horizontal resolution showed very similar results with slight differences in the Gulfs of Bohai and Liadong (Choi 1989).

In this study, an adaptation of the optimized open boundary conditions presented by Shulman and Lewis (1994, 1995, 1996) and Shulman (1997) was tested with respect to the improvement of the model prediction skills of the M_2 tidal amplitudes and phases in the Yellow Sea. In addition, this approach was extended to the assimilation of available sea surface elevation data inside the model domain. The results of the simulations are compared to coastal tidal station data.

2. Model

The model used in this study is a version of the Blumberg and Mellor (1987) three-dimensional circulation model. This model is a primitive equation, free surface model. It uses the turbulence closure submodel developed by Mellor and Yamada (1982) and modified by Galperin et al. (1988). The Smagorinsky (1963) formula is used for horizontal mixing. The model uses a curvilinear, orthogonal grid in the horizontal and a bottom-following sigma-coordinate grid in the vertical. A mode-splitting technique is used in the model to separate fast-moving external gravity waves and slow-moving internal gravity waves. In this case, the separation of the vertically integrated governing equations (barotropic, external mode) and the equations governing vertical

Corresponding author address: Dr. Igor Shulman, Institute of Marine Sciences, University of Southern Mississippi, Building 1103, Rm. 249, Stennis Space Center, MS 39529-9904.
E-mail: ims@coam.usm.edu

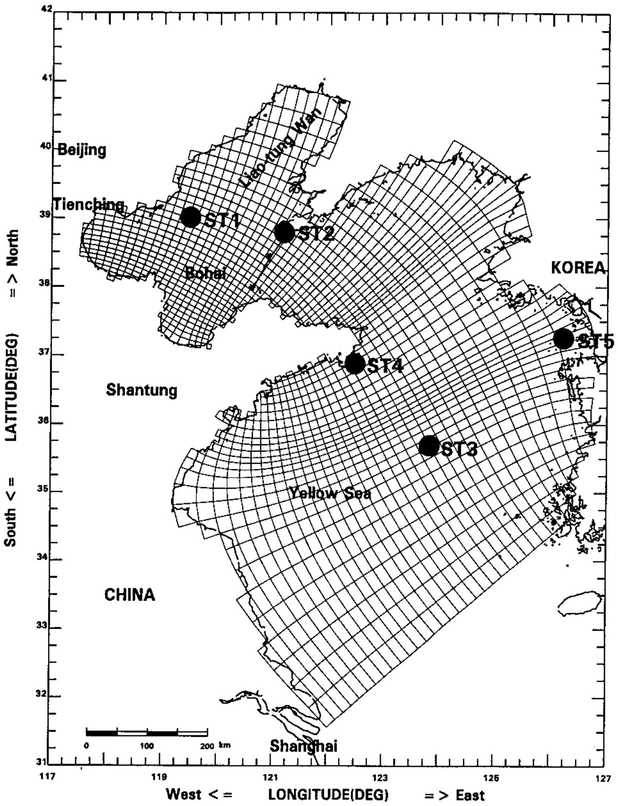

FIG. 1. Curvilinear orthogonal model grid of the Yellow Sea. ST1–ST5 are tidal stations.

structure (baroclinic, internal mode) is introduced. Boundary conditions are formulated for the barotropic and baroclinic modes separately and then adjusted to take into account the different truncation errors for those modes (Blumberg and Mellor 1987). For additional information on the model, the reader is referred to Blumberg and Mellor (1987).

The model domain (Fig. 1) consists of 36 × 72 grid cells in the horizontal (resolution from 3 to 60 km), and 11 equally spaced sigma levels in the vertical. The time step is 30 s for the barotropic mode and 300 s for the baroclinic mode. The bathymetry used by the model is based on the DBDB5 database (National Geophysical Data Center 1985).

3. Open boundary conditions

The model has one southern open boundary. In the existing framework of the model, an open boundary sea surface elevation is specified along this open boundary. The velocities on the boundary are then computed using the elevation and a linearized set of momentum equations. Optimized open boundary conditions developed in Shulman and Lewis (1995, 1996) and Shulman (1997) were incorporated into the model. These conditions combine the available information on the open boundary with the energy flux on the open boundary as determined from the hydrodynamic model. For the barotropic mode, the optimized open boundary conditions are derived from the following optimization problem:

$$\min_{\eta} \left(\frac{g}{2} \int_S (gH)^{1/2} (\eta - \eta^\circ)^2 \, ds \right), \qquad (1)$$

$$-g \int_S H\eta u_n \, ds = P_t, \qquad (2)$$

where P_t is the energy flux on the open boundary S, u_n is the vertically averaged outward normal velocity, η is the sea surface elevation on the open boundary, H is the depth, g is the gravitational constant, and η° are reference values of sea surface elevation on the open boundary. The solution of the optimization problem (1)–(2) will minimize the difference between the model-predicted and observed sea surface elevation on the open boundary and, at the same time, will satisfy the estimate of the flux of energy through the open boundary. To solve the problem (1)–(2) the regularization approach is used. In this case, problem (1)–(2) is reduced to the following minimization problem:

$$\min_{\eta} J = \left[\frac{1}{2} \left(P_t + g \int_S H\eta u_n \, ds \right)^2 + \gamma \frac{g}{2} \int_S (gH)^{1/2} (\eta - \eta^\circ)^2 \, ds \right], \qquad (3)$$

where γ is a parameter of regularization, chosen according to Shulman (1997) by providing the maximum of the entropy integral. The solution of problem (3) can be obtained from the condition of optimality $\delta J/\delta\eta = 0$ and has the following form:

$$\eta - \eta^\circ = \lambda_t (g/H)^{-1/2} u_n, \qquad (4)$$

where

$$\lambda_t = -\frac{P_t + g \int_S H\eta^\circ u_n \, ds}{g^{1/2} \int_S H^{3/2} u_n^2 \, ds + \gamma}. \qquad (5)$$

The implementation of boundary condition (4)–(5) is straightforward. The hydrodynamic model uses the staggered Arakawa C-grid; sea surface elevation is calculated at the center of the grid cell, while the velocities are calculated on sides of the grid box. The numerical model uses two previous time steps (t and $t - 1$) for calculating variables for the $t + 1$ time step. In the optimization approach, the η for the $t + 1$ time step at the open boundary is calculated from (4)–(5) using the specified η° (perhaps from observations) and u_n from the next interior model grid cell at the time step t. The estimate of P_t is obtained by using η and u_n at time t. Then the velocity on the open boundary is calculated using the linearized momentum equation. Note that condition (4)–(5) was used slightly differently in Shulman and Lewis (1994, 1995, 1996) and Shulman (1997). In those studies, the version of the model was used with the open boundary crossing the location of velocity. In such a setup, the velocity on the open boundary is specified from Eq. (4) by using the sea surface elevation located half of a grid inside of the open boundary.

4. Assimilation of the sea surface data along the transects

The proposed optimization scheme can be considered as a local data assimilation process (local in time and space) on the open boundary. The available information about sea surface elevation is assimilated into the model to specify the open boundary conditions. In the same fashion, we can assimilate information about sea surface elevation inside the model domain along, for example, the transects of satellite altimeter data. To do this, the H, η, and u_n information could come from either side of the location of the transect or be determined as an average of these variables from the two sides of the transect. The sea elevation along the transect that is calculated from the continuity equation is replaced by that calculated using (4)–(5). In this study, the results of the assimilation of available tidal information along a transect inside the domain of the Yellow Sea model are shown.

5. Model simulations

In this section, the performance of the optimized barotropic open boundary condition (4)–(5) is demonstrated for different configurations of the numerical model. Simulations using the vertically averaged mode of the model (called barotropic in the paper) with different model parameters, as well as simulations using the full three-dimensional capability of the model, have been conducted.

The results of model simulations were compared with the observations of M_2 tidal amplitudes and phases for five tidal stations (ST1–ST5) located in the Yellow Sea model domain (Fig. 1). To quantify the differences between the observed and model-predicted tidal amplitudes, we calculated the total average percent error of the predicted amplitudes for all five tidal stations.

First, results are presented for three simulations that were performed on the M_2 tide in the vertically averaged mode of the model (runs A1–A3, Table 1).

The reference values of sea surface elevation on the open boundary (function η°) were determined from the results of Choi (1986). The amplitudes and phases for each grid cell on the open boundary were derived by using the cotidal and corange plots for the Yellow Sea area in Choi (1986). The bottom friction is parameterized by the standard quadratic drag law, with the drag coefficient C_d equaling 0.0025. For run A1, λ_r was set to zero, meaning that no optimization was being performed and the open boundary was ''clamped'' to η°. The results of run A1 (Fig. 2, upper panel) indicated that the tidal amplitudes were underestimated for all stations, some of them significantly. The phases (Fig. 2, upper panel) were predicted reasonably well for three of the five tidal stations; however, there was a significant phase shift for the station in Bohai Bay (station ST1). The average error for the prediction of amplitudes was 42.4%.

TABLE 1. Description of the runs with the Yellow Sea model. Here C_d is the drag coefficient in the bottom friction formulation. Open boundary conditions (OBCs): the clamped one is when sea surface is specified on open boundary, and the optimized one corresponds to Eqs. (4)–(5). Assimilation means the run with assimilation of the transect at the entrance to the Bohai Bay (Yes) or without assimilation (No).

Run	Mode	Drag (C_d)	OBCs	Assimilation
A1	Barotropic	0.0025	Clamped	No
A2	Barotropic	0.0025	Optimized	No
A3	Barotropic	0.0025	Optimized	Yes
B1	Barotropic	0.0015	Clamped	No
B2	Barotropic	0.0015	Optimized	No
B3	Barotropic	0.0015	Optimized	Yes
C1	Diagnostic	—	Clamped	No
C2	Diagnostic	—	Optimized	No
C3	Diagnostic	—	Optimized	Yes

In the next numerical experiment (run A2, Table 1), the model was run using the optimization approach on the open boundary. The results showed better prediction of amplitudes and phases for all five stations (Fig. 2, upper panel), but the prediction was still poor in the Bohai Bay. The average error was reduced to 37.8% for the prediction of amplitudes.

To improve the prediction in Bohai Bay, a run was conducted with the assimilation of sea surface elevation data along a transect just outside of the Bohai Bay (Fig. 1). Tidal data for the three stations at the entrance to Bohai were used to calculate the reference sea surface elevations along the bay's entrance. These reference data were assimilated into the model according to Eqs. (4) and (5), using the same framework as for the open boundary. The prediction of the M_2 tides improved significantly due to data assimilation, especially in the Bohai Bay (run A3, Fig. 2, upper panel). The average error for the amplitudes was reduced to 27% (15% reduction in comparison to the run without the optimized open boundary conditions and assimilation). It is clear that model simulations with local data assimilation on the open boundary and along the interior transect significantly improve the predictive skill of the model.

The errors in the prediction of the tidal characteristics of the Yellow Sea are a result of the use of the DBDB5 bathymetry, the coarse-resolution grid, and the parameterization of bottom friction stress. The next set of barotropic experiments (B1–B3 experiments, Table 1) used a value of 0.0015 for the bottom frictional drag

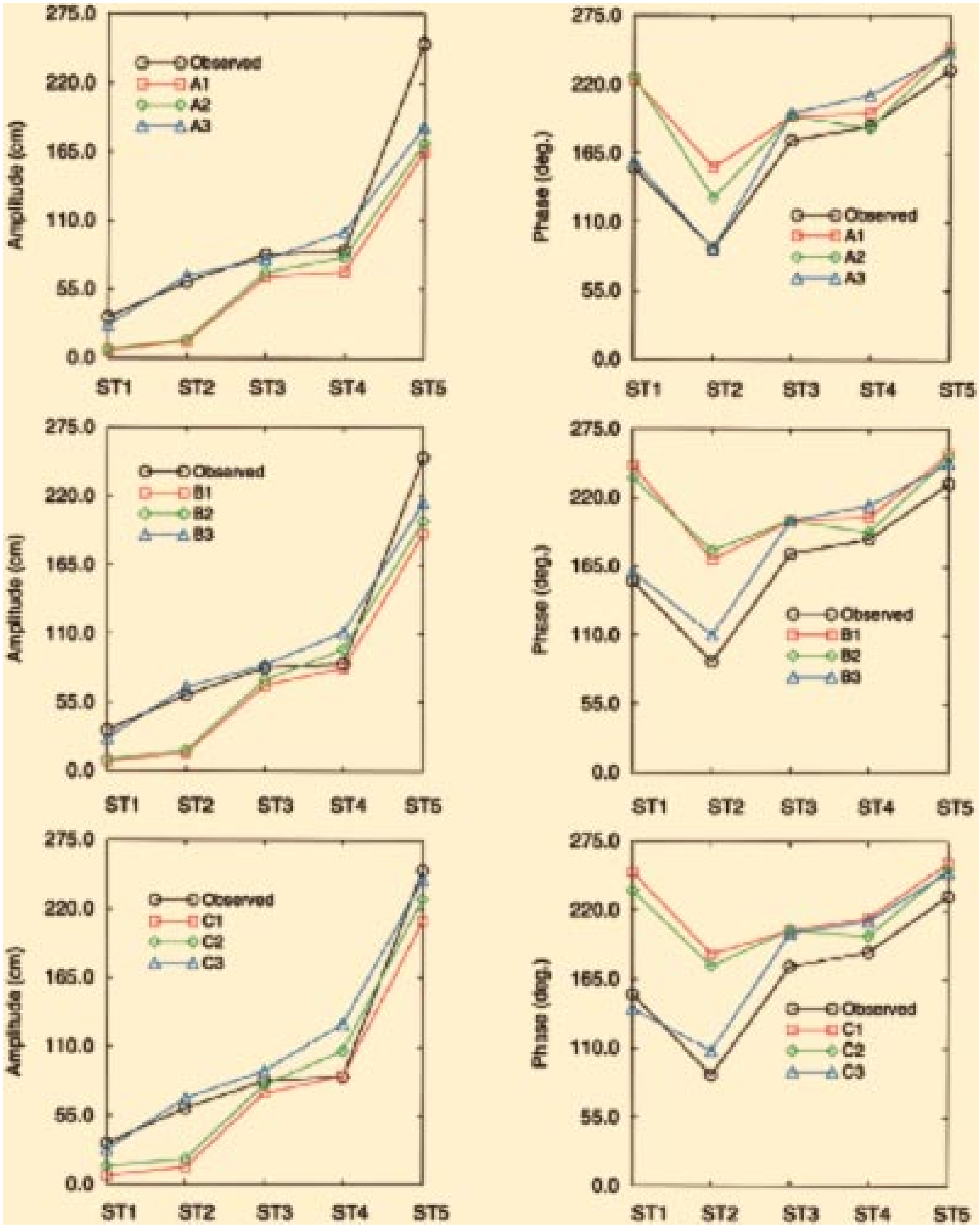

FIG. 2. The results of simulations with the Yellow Sea model for selected tidal stations (ST1–ST5). Amplitudes (left) and phases (right) for runs A1–A3 (Table 1) are presented on the upper panel; for runs B1–B3, the middle panel; and for C1–C3, the bottom panel. The curves marked with a circle are the observed amplitudes and phases (relative to UTC); the curves marked with a square are the results of runs A1, B1, and C1; the curves marked with a rhombus are the results of runs A2, B2, and C2; and the curves marked with a triangle are the results of runs A3, B3, and C3.

coefficient. The results of the simulation without optimized open boundary conditions and assimilation (B1) are given in Fig. 2, middle panel. The prediction of amplitudes is better in comparison to the results of the simulation with the drag coefficient equaling 0.0025 (run A1, Table 1), but the prediction of phases is worse. The corresponding average error is 34.8% for amplitudes. The use of the optimized open boundary conditions (B2) showed a slight improvement in the prediction of amplitudes and phases in comparison to run B1 (Fig. 2, middle panel). The average error was 32.4% for amplitudes. The last simulation was performed with the optimization and assimilation of transect data at the entrance to the Bohai Bay (run B3). As before, the prediction skill was significantly improved for Bohai Bay (station ST1). The average error in the prediction of amplitudes was around 10%. This is less in comparison to the run without the optimization on the open boundary and without the assimilation of transect data.

The results of the simulations indicate the sensitivity of the Yellow Sea model to the value of the drag coefficient in the bottom friction formulation if the model is used in the barotropic mode. To better resolve boundary layer physics, M_2 tidal simulations were performed using the model in a three-dimensional mode (C1–C3, Table 1). In the three-dimensional simulations, the temperature and salinity were fixed (see Blumberg and Mellor 1987). In this case, the bottom boundary layer velocity is used in the bottom friction formulation instead of the vertically averaged velocity. The bottom friction coefficient is calculated through the use of a logarithmic boundary layer formulation with the bottom roughness of 1 mm and a minimum frictional coefficient of 0.0025. A single vertical profile (no variation in the horizontal directions) of temperature and salinity was used in these simulations. The temperature was chosen to be the same from top to bottom, 18°, while salinity increased from 31.5 psu at the top to 36.5 psu at the bottom.

For run C1 (without optimized open boundary conditions or assimilation of the transect of data), the amplitudes of the M_2 tide (see Fig. 2, bottom panel) were better predicted in comparison to the corresponding barotropic simulations (runs A1 and B1), but phases were slightly worse. The average error for the prediction of amplitudes was 30.6%. An improvement in the prediction of amplitudes and phases for many stations was achieved with the use of the optimization on the open boundary (run C2, Fig. 2, bottom panel). The average error for the prediction of amplitudes was 24.8% (a 6% reduction). It is clear that the model performs better with the optimization on the open boundary in the two- as well as in the three-dimensional cases. The last run (C3) was performed with the optimization on the open boundary and assimilation of the transect at the entrance to the Bohai Bay. In this case, the error was reduced to 21% for amplitudes.

Simulations A–C were compared to the results described in Choi (1986). In that study, a two-dimensional model was used with a resolution of 1/5° lat × 1/4° long, which corresponds to about 15-km horizontal resolution. A bottom friction drag coefficient of 0.0025 was used in the standard quadratic drag law (the same as in our A1–A3 runs). The results of the comparisons are presented in Table 2 for stations ST1, ST3, and ST5. The predictions for station ST1 were compared to the predictions for station Chinwangto, 39°54′N, 119°10′E (Choi 1986), which is also located in the Bohai Bay, northwest from ST1. Stations ST3 and ST5 are the same as those used in Choi's paper. The accuracy of prediction of amplitudes and phases of the M_2 tide in Bohai Bay (station ST1) is similar for Choi's and our models (Table 2) when no optimization on the open boundary and assimilation of transect data are performed (runs A1, B1, and C1). However, the use of the optimized open boundary condition and the assimilation of the transect data significantly increased the accuracy of the predictions for all runs, A, B, and C, in comparison to Choi's results. The error of prediction of the amplitude is reduced from 78.5% for run C1 to 14.2% for run C3, and the offset in the prediction of the phase is reduced from 97.3° to 11.3°. Similar results were obtained for the prediction of the tidal amplitude for the station located in the central part of the Yellow Sea (ST3) (Table 2). The optimization and assimilation of the transect reduced the error to 2.4% for the B3 run in comparison to 20.5% from Choi (1986). However, the predicted phases are worse in comparison with Choi's predictions. This is probably a result of the coarser resolution of our model in this area (18.5 km) and the differences in bathymetry.

TABLE 2. Comparison between *M*2 tidal simulations with the Yellow Sea model and Choi's (1986) results at three tidal stations (ST1, ST3, and ST5). Amplitudes are presented as percent errors, and phases are presented as the difference (degrees) between observed and model values.

	ST1		ST3		ST5	
Run	Ampl. (%)	Phase	Ampl. (%)	Phase	Ampl. (%)	Phase
Choi's	86.0	−73.0	20.5	3.0	3.1	−12.0
A1	80.3	−70.7	21.8	−19.3	34.3	−18.8
A2	78.5	−73.1	21.2	−19.3	31.4	−16.4
A3	18.8	−5.3	5.2	−21.7	26.3	−13.9
B1	75.0	−92.4	17.3	−26.6	24.1	−23.6
B2	70.6	−82.9	12.2	−26.6	20.2	−21.4
B3	18.8	−7.9	2.4	−26.6	14.3	−16.4
C1	78.5	−97.3	12.1	−28.7	16.3	−26.
C2	54.6	−82.9	4.1	−29.0	9.2	−21.2
C3	14.2	11.3	9.2	−26.6	2.7	−18.8

For station ST5, Choi's results are better than the results of two-dimensional barotropic runs A and B. This is certainly a result of the much coarser resolution of our model in this area (37 km in comparison to around 15 km for Choi's model). At the same time, for the three-dimensional run C3 (see Table 2), the use of the optimized open boundary condition and data assimilation reduced the error in the prediction of the amplitude to 2.7% in comparison to 3.1% for Choi's model, and the offset for the phase is only 18.8° in comparison to 12° for Choi's model.

6. Conclusions

An optimization approach proposed for the specification of open boundary conditions was tested for the M_2 tides in a model of the Yellow Sea. The comparison of simulation results with and without optimization on the open boundary shows that local data assimilation in the vicinity of the open boundary improves the predictive skill of the model. The optimization approach was extended to the assimilation of available sea surface data along transects inside the model domain. In this case, the model prediction skill was improved significantly in the areas far from the boundary. The results support the simple approach of assimilating sea surface data along transects using the optimization technique developed for the specification of open boundary conditions. The proposed data assimilation approach was tested in simulations of periodic tidal phenomenon. For this reason, future research is needed in order to make an assessment of the stability properties of the approach for long time integrations and for the assimilation of satellite-derived data. This method can be used for the assimilation of satellite sea surface data into hydrodynamic models as well.

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