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Dirt road corrugations

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We consider factors influencing the build-up of corrugations on dirt roads and the reactions of vehicles to them. We suggest that corrugations are (at least in part) a consequence of a natural tangential oscillation of the tread surface of the car tyre that occurs when the vehicle is being driven or braked. Secondly, we suggest that the unpleasant vibration experienced by a vehicle passing over a corrugated road is the result of a beat produced by the difference of the frequency of oscillation of its own tyres and the frequency of the stimulation received by the vehicle due to passage over a corrugated road. We consider how this is affected by actions that the driver can take and conclude with some recommendations and observations of related effects.

The Romans built roads to link the far-flung corners of their empire. Although designed for the rapid deployment of their legions on foot, they were also used by various forms of vehicle — such as carts, wagons and chariots — and remained functional for centuries after the empire collapsed. After the invention of the motor car and the pneumatic tyre, a smoother road surface became a necessity and, in most countries today, cities and towns are connected by means of a network of tarred or concrete roads. At the periphery, however, one eventually reaches the ordinary dirt road with an unconsolidated surface. Such primitive roads are of considerable economic significance and, particularly in developing countries, they tend to carry a substantial proportion of the total traffic. Their characteristics and maintenance remain of great interest and importance.

The dirt road, even when properly drained and well-prepared with a solid foundation of crushed rock so that washways seldom occur, is subject to the development of corrugations (or *washboards* in American terminology). These irregularities, familiar to anyone who has driven on a dirt road, consist typically of a series of peaks and troughs, perpendicular to the road, of amplitude approximately 5 cm and with wavelength Λ between 30 cm and 60 cm. When encountered on a dirt road they present at least an irritation but at their worst a serious risk of accident: they can make it impossible to drive at certain moderate speeds and, while their effect may be reduced by travelling at higher speeds, some degree of stability and controllability is sacrificed.

To the authors' knowledge, supported by an Internet search,[1] no definitive scientific theory accounting for corrugations has been put forward. The article by Mather[2] is informative, describing experiments exhibiting the development of corrugations under controlled conditions in the laboratory, surveying previous attempts to address this problem and including valuable discussion. He was compelled to admit, however, that the level of understanding of their causation left him unable to suggest measures to minimize the effects of corrugations or to reduce the expense of standard road maintenance procedures.

The main object of this article is to describe factors influencing the build-up of corrugations on dirt roads and the reactions of present-day vehicles to them. Specifically, we suggest that corrugations are (at least in part) a consequence of a natural tangential oscillation of the tread surface of the vehicle tyre that occurs when it is being driven or braked and, secondly, that the unpleasant vibration experienced by a vehicle passing over a corrugated road is the result of a beat produced by the difference of the frequency of oscillation of its own tyres and the frequency of the stimulation received by the vehicle due to its speed of motion over corrugations already induced on the road by other vehicles.

First, we analyse the nature and frequency of the tangential oscillation of the tread surface, the evidence for its existence and its dependence on parameters such as inflation pressure and physical properties of the tyre. We then discuss the mechanism by which the oscillation contributes to the corrugations observed in the road and their dependence on the type of traffic carried by the road. We also discuss the interaction between the tyres of a vehicle and a corrugated road and how this is affected by actions that the driver can take.

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The tangential oscillation

We take the following primitive model of an motor car wheel consisting of a central circular disk D and concentric annulus A (Fig. 1):

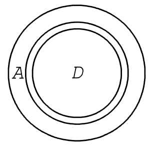

Fig. 1. Schematic representation of a vehicle wheel.

In the first calculation we assume that brakes are applied, so that the body of the wheel (D) is fixed but that the annulus A, representing the outer parts of the tyre (including the tread and adjacent steel reinforcement) is able to rotate elastically but is subject to a restorative torque T proportional to the angular displacement θ, so that we have $T = -k\theta$, where k is a positive constant that depends on the construction of the tyre and also on its inflation pressure. We denote by I_A the moment of inertia of A about its central axis. Then the motion of A is given by the differential equation

$$\theta'' + 2a\theta' + (k/I_A)\theta = 0,$$

where a is a drag coefficient. If $a^2 < k/I_A$, this yields the solution

$$\theta = C_1 e^{-at} \cos \lambda t + C_2 e^{-at} \sin \lambda t,$$

where $\lambda = k/I_A - a^2$.

The parameters k and a are difficult to estimate but we claim that the oscillation can be observed directly and its frequency $f = 2\pi/\lambda$ determined. To do so, jack up the front wheel of a typical vehicle, and engage the footbrake by inserting a piece of wood between the brake pedal and the front seat. If the tyre is now struck tangentially with a light hammer, a dull thud is heard and a low frequency-damped vibration can be detected by the fingers.

The frequency of the oscillation f was observed using a phonograph pick-up applied to the surface connected to an amplifier and oscilloscope. For a Firestone steel-belt radial ATX 215R15 (a light truck tyre) inflated to a pressure of 220 kPa, a frequency of approximately 25 Hz was observed.

Based on empirical evidence, we suggest the following:

Speed-wavelength Law (SWL). *The wavelength* Λ *of the corrugation induced on a road is equal to the product of the mean speed of vehicles and half the period of their mean tangential oscillation frequency (TOF). Specifically, for a mean speed* V *measured in km/h and mean TOF* f, *we have*

$$\Lambda = \frac{5}{36} V / f(\mathrm{m}).$$

Below, we suggest a mechanism for the development of corrugations giving rise to the above law. In the meantime, we note, by way of illustration, that for V = 60 km/h and f = 25 Hz, we obtain Λ = 0.33 m.

The pressure of inflation of a tyre affects *k* and hence also *f*. This effect can be observed directly by recording beat frequencies.

Manufacturers of tyres and of motor cars recommend strongly that the tyres used on the pair of drive wheels be of similar make and type. When this advice is ignored and the car driven on a smooth tarred road, a beat is usually audible, emanating from the differential. We suggest that the beat produced is the difference of the tangential oscillation frequencies for the respective tyres, resulting presumably from minor differences in *k*.

In a similar way one may record the beat frequency produced by tyres of identical type but with differing inflation pressures.

Build-up of corrugations

A vehicle travelling on a perfectly smooth dirt road (for example, one that had recently been well scraped) might not begin to initiate irregularities in any detectable way. In practice, dirt roads do have uneven surfaces. In consequence the contact between tyres and road will not be perfect. At times the drive wheels will bounce. Before contact of the tyres with the road is lost, they will be under tension (assuming that the vehicle is under power). At the instant of contact loss, the tension being released, the tread surface begins to oscillate at the TOF. Loose particles of dirt are brushed one way or the other. There are two possible states: firm contact of tread with the ground, causing no dispersion; and lack of contact with oscillating tread surface. In time high spots are created, where the contact is more likely to be firm, and troughs, where the contact is less likely to be firm and so the troughs are deepened. At a node of the tread oscillation, however, no dust is disturbed so a higher spot can develop here. If many vehicles with the same TOF pass at the same speed, a stable state will be approached at which the tyres make contact at each high spot and no oscillation nodes are present in the troughs. When this is achieved the corrugation is complete. The weight of the vehicles consolidate the loose dirt at the high spots, so that these become (semi-) permanent features of the road.

Note that the stable state envisaged satisfies the predictions of the SWL.

If the above mechanism for the formation of corrugations is accepted, it becomes apparent that their characteristics will depend on the nature of the traffic, in particular on the predominant types of vehicle and on the mean speed at which they travel.

On braking, one might also expect corrugations to be generated. Indeed, corrugations often seem to be more severe on approaches to a corner of the road and on the corner itself. It is noteworthy that the observed wavelength at such places tends to be shorter, in line with what one would expect from the SWL. On the other hand, it is not clear to what extent free wheels contribute to corrugation formation, although, according to Mather,[2] they certainly do.

Vehicle response to corrugation

Corrugations tend not to be uniformly distributed on a dirt road. An experienced driver can often avoid them for a while by selecting a particular route for his wheels. However, inevitably they are encountered and a heavy vibration, usually most noticeable on the front wheels, occurs whether or not these are the driving wheels. The severity of the reaction will depend, of course, on the effectiveness of the vehicle's shock absorbers but these may not protect completely. The usual impression of the driver is that if he can travel fast enough, the car proceeds smoothly but the presence of potholes or other obstructions may make this impossible.

What determines the speed at which the driver can proceed comfortably? We propose the following:

Smooth Speed Law (SSL). *A motor car can travel smoothly on a corrugated dirt road provided the driver maintains a speed* V *approximately equal to the product of the wavelength of the corrugations and twice the TOF of his tyres. However, depending on the state of the vehicle's shock absorbers, serious vibrations may occur at speeds between 0.6*V *and 0.9*V.

At the speed V given by the SSL, the tyres make firm contact with the road only on the near side of the crests of the corrugation and these arrive in tune with the tangential oscillation of the tyres.

At other speeds it is not immediately obvious why or how the corrugations should convert energy of forward motion into a transient vertical component. To explain this phenomenon we need to consider an alternative model for the pneumatic tyre as a shock-absorbing system specifically designed to filter out forced oscillations arriving at a fixed frequency.

Consider the case of a free wheel of a vehicle travelling at a speed of $0.8V$. Suppose that at time $t = 0$ the wheel is making firm contact with a corrugation. Besides receiving a vertical thrust (to support the weight of the vehicle), the portion of tread making contact receives also a small tangential thrust (to maintain the rolling state). This begins a cycle of the tangential oscillation that, because of our assumption concerning speed, is half completed *before* the wheel makes contact with the next corrugation crest. The tread then receives a second tangential thrust. In this way the tangential oscillation can build up even on a free wheel.

Note that on contact with a crest (essentially an inclined plane), the tangential oscillation of the tread converts to a vertical impetus.

The axle of the wheel then receives a series of intermittent vertical thrusts but the component due to the tangential oscillation varies, because of the beat effect, at a frequency equal to the difference $(5/4)f - f = f/4$. Because of the characteristics of the normal suspension system of the vehicle, it is this smaller frequency that can cause the problems sometimes encountered.

For a vehicle travelling at a greater speed than the Smooth Speed (SS), the tangential oscillation effect seems to be less noticeable. This may be due to a bouncing effect that destroys the phase shifts. In any case, at speeds greater than the SS the driver often has the feeling that he has less control over the vehicle.

For travel on dirt roads drivers are often advised to reduce slightly the inflation pressure of their tyres. Since the constant *k* depends on this pressure, doing so has the effect of reducing the TOF. Our observations go some way to explain why reducing tyre pressure is effective.

Discussion and recommendations

The tyres of a vehicle are not always recognized as part of its suspension system. However, it is clear that they play a significant role in filtering out high-frequency perturbations derived from the road. No full discussion of the characteristics of the system can afford to neglect either the tyres or the spring/shock absorber action. Bekker *et al.*[3] considered only the latter. It is interesting to compare their results with those of the present article in which we have concentrated on the tyres; there is certainly a case for a more inclusive approach that considers the linkage.

Vehicle tyres are mostly used on the rigid surfaces of tarred and concrete roads. It is possible that their design is not optimal for performance on dirt roads and that if the tangential oscillation of the tread was further damped their tendency to generate corrugations might be reduced. If so, there would be some advantage to the road authorities to encourage the use of special tyres in country areas.

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