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FACULTY OF ENGINEERING AND ARCHITECTURE
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Master Thesis

Second order effects in RC columns: comparative analysis of design approaches

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Abstract

The objective of this project is to analyze different design approaches for RC columns, in particular those cases in which second order effects should be considered when the column is under compression.
Throughout the study, a variety of support cases and different types of section will be presented for columns of different slenderness ratios. These cases will be resolved using the Simplified Methods provided in Eurocode 2 and the evolution of second order effects with the slenderness ratio will be studied.
Within this study, the influence of creep, area of reinforcement and area of concrete in the second order moment will be investigated, to then discern the differences between the Nominal Stiffness and the Nominal Curvature methods.
It will be determined that in the Nominal Stiffness method creep has an increasing effect as the slenderness ratio increments. Creep will have an effect in the Nominal Curvature method until a certain slenderness value is achieved, which depends on the compressive characteristic strength.
When studying the influence of the areas of reinforcement and concrete in taking into account second order effects, it will also be deduced that increasing the area of reinforcement is a good solution if the slenderness only just exceeds the slenderness limit. If the slenderness is much higher than the limit, then it is better to increase the area of concrete since it will increase the limit and reduce the slenderness more effectively.
The differences found between the three types of sections will also be a subject of study. These sections will be rectangular with reinforcement placed in opposite sides, rectangular with uniform distribution of reinforcement and circular with uniform distribution of reinforcement. The first section is the least affected by second order effects whilst the second section is the most affected.
For the calculations, a tool in the form of a spreadsheet for the Simplified Methods will be created with the purpose of being intuitive to use and generate output values automatically.
Finally, columns of different slenderness ratios will be modeled and tested by means of a finite element (FEM) analysis. The results of the evolution of the second order moment with the slenderness ratio will be compared to those obtained through the Simplified Methods and will prove to be quite similar.

Contents

1- Symbols

Latin upper case letters

- A: Total cross-sectional area
- A_c: Cross-sectional area of concrete
- A_s: Cross-sectional area of reinforcement
- E_{cd}: Design value of modulus of elasticity of concrete
- E_{cm}: Secant modulus of elasticity of concrete
- E_s: Design value of modulus of elasticity of reinforcing steel
- I: Second moment of area of the total cross-section
- I_c: Second moment of area of the concrete cross-section
- I_s: Second moment of area of the reinforcement
- M: Bending moment
- M_{01}, M_{02}: First order end moments
- M_{0Eqp}: First order bending moment in quasi-permanent load combination
- M_{0Ed}: First order bending moment in design load combination
- M_{Ed}: Design value of the applied internal bending moment
- N_B: Buckling load
- N_{Ed}: Design value of the applied axial force
- RH: Relative humidity

Latin lower case letters

- b: Cross section width
- c: factor depending on the curvature distribution
- c_0: coefficient which depends on the distribution of first order moment
- d: Effective depth of a cross-section
- e_0: Initial imperfection
- e_2: Deflection
- f_{cd}: Design value of concrete compressive strength
- f_{ck}: Characteristic compressive cylinder strength of concrete at the age of 28 days
- f_{cm}: Mean compressive strength of concrete at the age of 28 days

- f_{yd}: Design yield strength of reinforcement
- f_{yk}: Characteristic yield strength of reinforcement
- h: Cross section depth
- h_0: notional size
- i: Radius of gyration
- k_1: Factor which depends on concrete strength class
- k_2: Factor which depends on axial force and slenderness
- k_c: Factor for effects of cracking, creep etc.
- k_φ: factor for taking account of creep
- k_r: correction factor depending on axial load
- k_s: Factor for contribution of reinforcement
- k_σ: Stress-strength ratio
- l_o: Effective length
- n: Relative normal force
- n_{bal}: Value of n at maximum moment resistance
- r: Radius
- 1/r: Curvature at a particular section
- r_m: Moment ratio
- s_0: Rectangular section with reinforcement laid in opposite sides
- s_1: Rectangular section with uniformly laid reinforcement
- s_2: Circular section with uniformly laid reinforcement
- t_0: Age of concrete at the time of loading, in days
- u: Perimeter of the part exposed to drying

Greek lower case letters

- α_{cc}: coefficient taking account of long term effects on the compressive strength and of unfavorable effects resulting from the way the load is applied
- α_h: Reduction factor for length or height
- α_m: Reduction factor for number of members
- β: Factor which depends on distribution of 1st and 2nd order moments
- $\beta_c(t,t_0)$: Coefficient to describe the development of creep after loading

- $\beta(f_{cm})$: Factor to allow for the effect of concrete strength on the notional creep coefficient (φ_0)
- $\beta(t_o)$: Factor to allow for the effect of concrete age at loading on the notional creep coefficient (φ_0)
- β_H: Coefficient depending on the relative humidity (RH) and the notional member size (h_0)
- θ_i: Inclination
- θ_0: Basic value
- $\varphi(\infty,t_0)$: Final creep coefficient
- φ_{ef}: Effective creep coefficient
- $\varphi_{nl}(\infty,t_0)$: Non-linear notional creep coefficient
- φ_0: Notional creep coefficient
- φ_{RH}: Factor to allow for the effect of relative humidity on the notional creep coefficient
- γ_C: Partial safety factor for concrete
- γ_S: Partial safety factor for reinforcing steel
- μ: Reduced design moment
- λ: Slenderness ratio
- λ_{lim}: Limit for slenderness ratio
- ρ: Longitudinal reinforcement ratio
- σ_c: Compressive stress
- ω: Mechanical reinforcement ratio

2- Preface

Together with new improvements in the quality of reinforced concrete in recent years, new possibilities in the design of structures have appeared. In particular, architects and engineers have embraced these advances in concrete technology, implementing the use of more slender reinforced concrete columns in their designs, resulting in keener structures for the human eye.

Where before other structural solutions had to be found to implement designs, now new possibilities arise in the field of reinforced concrete. Higher and more opened spaces, or simply the use and advantages that reinforced concrete provides in construction, are some of the reasons that have led to the use of more slender columns.

Due to the nature of slender columns, the influence of second order effects is an aspect to take into account in their design. Also, relevant effects like cracking, creep and non-linear material properties influence in a way that calculating the behavior of a column becomes quite a complex matter.

The origin and motivation of this report is to try to research a bit more in the field of second order effects in RC columns, by reviewing the design methods proposed in Eurocode 2 known as the Simplified methods.

3- Introduction

3.1- Objectives

The main objective of this thesis is to analyze second order effects in slender columns, in particular through the Simplified Methods given in Eurocode-2 known as Nominal Stiffness and Nominal Curvature.

There is a special interest in investigating the influence in both methods of a series of parameters that intervene in the calculations, like creep, slenderness, area of reinforcement and of concrete. All this will be done with the objective of studying the evolution of the second order moment as all these parameters vary.

Another objective is to identify general trends and differences in the second order effects on a wide variety of columns in different situations. In order to do this, four different types of columns with different constraints situations will be studied in addition to three different kinds of cross-section.

The creation of a tool that can be easily used to calculate the first and second order moments, taking into account all the above mentioned parameters and situations, is also an aim. This tool must be easy to use and able to do all the calculations automatically.

Finally, the last objective of the thesis is to simulate through a finite element analysis the behavior of a ranging amount of columns with different slenderness ratios, in order to compare the final results with the ones obtained through the Eurocode-2 calculations.

3.2- Scope

This Thesis first gives in chapter 4 some background on the methodology used in Eurocode-2 and, in particular, on the Simplified Methods for the calculation of second order effects. Chapter 5 contains the flow charts of the calculation process that will be used in both of the simplified methods and in Chapter 6, the main parametric study relevant to the second order effects is undertaken. Chapter 7 contains the analysis through finite elements and the comparison with the results obtained through the Simplified Methods. Finally, chapter 8 includes all the concluding remarks.

4- Theoretical Background

4.1- Background on second order effects

One of the hypotheses in structural analysis of lineal elastic structures is that the displacements are finite, but small enough to permit equilibrium in the non-deformed configuration without introducing a very significant error.

When determining the capacity of a structure in design and ultimate states, the effects of the loading acting on the deformed configuration must be studied. These effects increase the internal stresses of the different elements in the structure and the general displacements and they are known as second order effects.

Second order effects are especially influential in structures prone to instabilities, were the displacements are large enough to be magnified by the loads that the structure is suffering. In particular, second order effects are a big issue in columns since they are elements that have a tendency to buckle, a phenomenon where large displacements occur.

Since columns are one of the most common structural elements in construction, Eurocode-2 includes an entire chapter dealing with second order effects in columns and proposes a methodology of calculation through the Simplified Methods. This report will focus on the Nominal Stiffness and Nominal Curvature methods and will study them in depth.

4.2- Background on Eurocode-2 and the Simplified methods

This report makes use and reference of the methods provided in the European standard EN 1992-1-1(2004), Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings. In particular, chapters *5.8* and *5.9* are studied together with the references to other chapters and annexes also contained in the document.

Chapter *5.8* deals with the analysis of second order effects with axial loads. In *5.8.1*, a series of definitions are initially provided that are important to understand and clarify the terminology used throughout the whole chapter. Some of these definitions are now listed, due to their relevance in understanding the nature of this report:

- *Buckling*: failure due to instability of a member or structure under perfectly axial compression and without transverse load
- *First order effects*: action effects calculated without consideration of the effect of structural deformations, but including geometric imperfections
- *Second order effects*: additional action effects caused by structural deformations

In chapter *5.8.2*, a general criterion is given as a limit to take into account second order effects, and in *5.8.3.1* a more specific criterion is given in a form of a slenderness check for isolated members. The latest will be used in the parameter study contained in chapter 6 of this report.

But what characterizes chapter *5.8* is the inclusion of a general method (*5.8.6*) followed by two simplified methods (*5.8.7* and *5.8.8*).

As explained in *5.8.6(1)*, the general method is based on non-linear analysis including geometric nonlinearities like second order effects. Creep is taken into account and the stress-strain diagrams are based on design values, to obtain a design value of the ultimate load.

The first simplified method is a method based on nominal stiffness, and its use is destined for both isolated members and whole structures. As explained in *5.8.7.1(1)*, nominal values of the flexural stiffness are used taking into account the effects of creep, cracking and material non-linearity, to obtain a resulting design moment.

The second simplified method is based on nominal curvature, and it is destined for isolated members although it can also be used in whole structures if the distribution of curvature is assigned realistically. As explained in *5.8.8.1(1)*, the method is based on the deflection of the member giving a nominal second order moment. Creep is taken into account and the result is a design moment.

4.3- Slenderness Criterion in the simplified methods

For isolated members Eurocode 2 gives a simplified slenderness criterion under *5.8.3.1*. This criterion states that second order effects may be ignored if the slenderness λ is below a certain value λ_{lim}.

$$\lambda < \lambda_{\text{lim}}$$

Where the slenderness ratio λ is defined as:

$$\lambda = l_0/i$$

And the slenderness limit as:

$$\lambda_{\text{lim}} = 20 \cdot \text{A} \cdot \text{B} \cdot \text{C}/\sqrt{n}$$

Where:

$$A = 1/(1 + 0.2 * \varphi_{ef})$$

$$B = \sqrt{1 + 2\omega}$$

$$C = 1.7 - r_m$$

$$n = N_{Ed}/(A_c \cdot f_{cd})$$

$$\omega = As \cdot fyd/(Ac \cdot fcd)$$

$$r_m = M_{01}/M_{02}$$

r_m should be taken positive if the end moments give tension on the same side, negative otherwise. It should also be taken as 1 in braced members in which first order moments arise predominantly due to imperfections or transverse loading and unbraced members, which will be the case of the columns studied in this report.

4.4- Creep in the simplified methods

To take into account the effects of creep, the general conditions of creep must be applied to obtain a final creep coefficient, $\varphi(\infty,t_0)$.

Eurocode 2 specifies in chapter *3.1.4* a threshold, for which if the value of the compressive stress applied at the concrete does not surpass the value of $0.45 \cdot f_{ck}(t_0)$, then the behavior of creep can be considered as lineal. This is specified in Eurocode 2 as:

$$\sigma_c \leq 0.45 \cdot fck\ (t_0)$$

Provided this is true, figure *3.1* included in chapter *3.1.4* can be used as an approximate calculation of the value of creep. Knowing t_0, h_0 and the class of concrete (class R,N or S), $\varphi(\infty,t_0)$ can be obtained from figure 1.

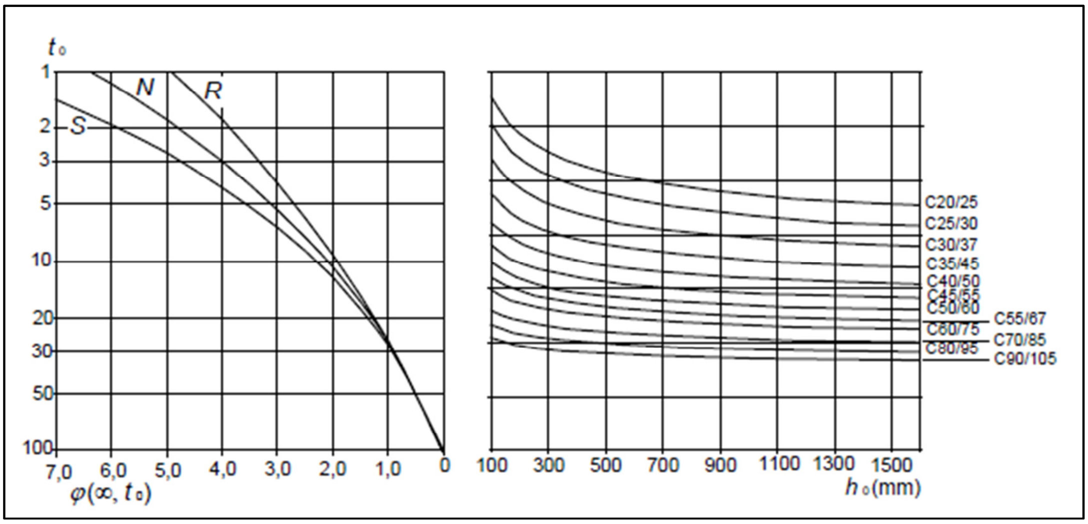


Figure 1: Creep graph included in Eurocode-2, chapter 3.1.4 (3.1)

Where the notional size is described as:

$$h_0 = 2 \cdot A_c/u$$

For a more exact value, the Eurocode offers a more elaborate calculation throughout a series of formulas included in Eurocode-2 *annex B*. Creep coefficient $\varphi(t,t_0)$ is obtained from the product of φ_0 and $\beta_c(t,t_0)$, which are respectively the notional creep coefficient and a coefficient to describe the development of creep with time after loading:

$$\varphi(t, t_0) = \varphi_0 \cdot \beta_c(t, t_0)$$

Where:

$$\varphi_0 = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0)$$

$$\beta_c(t, t_0) = \left[\frac{t - t_0}{\beta_H - t - t_0}\right]^{0.3}$$

φ_{RH} takes into account the effect of relative humidity, $\beta(f_{cm})$ takes into account the effect of concrete strength, $\beta(t_0)$ allows for the effect of concrete age at loading and β_H introduces the effect of relative humidity. There is also an additional adjustment by means of the age of concrete t_o at the time of loading in $\beta(t_0)$, where the type of cement is taken into account (*annex B, (2)* should be reviewed for further details).

If $\sigma_c > 0.45 \cdot fck\ (t_0)$, creep non-linearity should be considered and non-linear method should be used, by means of the formula:

$$\varphi_k(\infty, t_0) = \varphi(\infty, t_0)\exp(1.5(k_\sigma - 0{,}45))$$

Where:

$$k_\sigma = \sigma_c / f_{ck}(t_0)$$

And $\varphi_k(\infty, t_0)$ now substitutes $\varphi(\infty, t_0)$.

Once $\varphi(\infty, t_0)$ is calculated, the duration of the load is taken into account with the introduction of an effective creep coefficient.

$$\varphi_{ef} = \varphi(\infty, t_0) \cdot M_{0Eqp} / M_{0Ed}$$

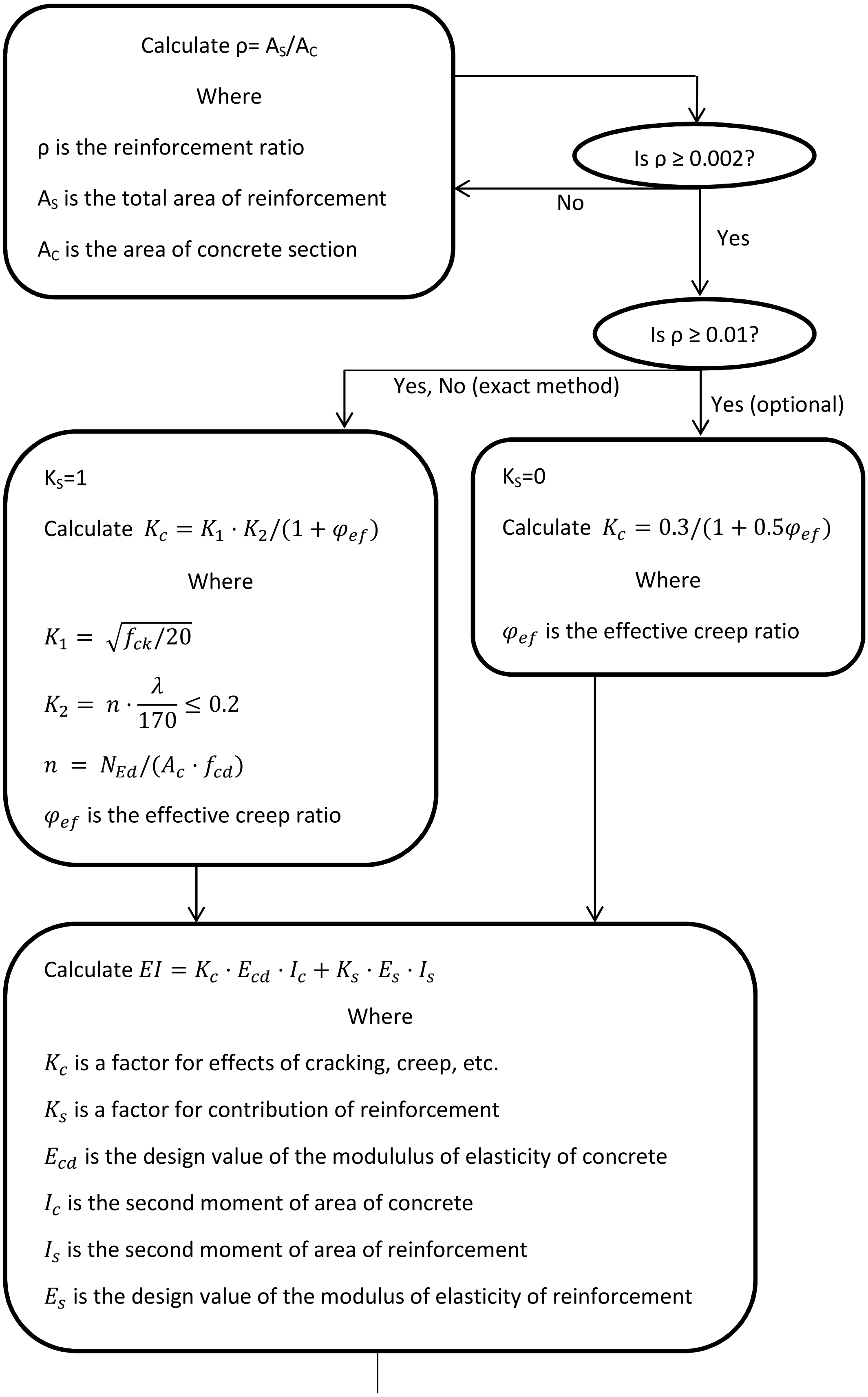
Given the following three conditions are met, the effective creep may be ignored and taken as 0.

- $\varphi(\infty, t_0) \leq 2$
- $\lambda \leq 75$
- $M_{0Ed}/N_{Ed} \geq h$

Special attention must be taken if creep is ignored and the slenderness limit λ_{lim} is only underachieved, because this could result in a too unconservative design.

5- Flow charts of the calculation process for the simplified methods

5.1- Nominal Stiffness method



Rectangular section | Circular section

Calculate $M_{Ed} = \mu_d \cdot b \cdot h^2 \cdot f_{cd}$

Where

μ_d is the reduced design moment

b is the depth of the section

h is the width of the section

f_{cd} is the design value of concrete compressive strength

Calculate $M_{Ed} = \mu_d \cdot \pi \cdot r^3 \cdot f_{cd}$

Where

μ_d is the reduced design moment

r is the radius of the section

f_{cd} is the design value of concrete compressive strength

Calculate $M_{0Ed} = \frac{M_{Ed}}{1+\frac{\beta}{N_B/N_{Ed}-1}}$

Where

M_{Ed} is the total design moment

N_B is the buckling load based on nominal stiffness = $\pi^2 \cdot EI/{l_0}^2$

N_{Ed} is the design value of the axial load

$\beta = \pi^2/c_0$ is a factor which depends on distribution of 1st and 2nd order moments

c_0 is a coefficient which depends on the distribution of the first order moment

Calculate $M_2 = M_{Ed} - M_{0Ed}$

Where

M_{Ed} is the total design moment

M_{0Ed} is the first order moment

M_2 is the second order moment

In this case, the first order moment is obtained through an amplification factor and the second order moment is subtracted from the maximum moment the section can withstand. Geometric imperfections may be taken into account by means of a parameter e_0 that contributes to the first order moment through the axial load N_{Ed}.

Calculate $e_0 = \max(\frac{h}{30}; 20; \theta_i \cdot \frac{l_0}{2})$

Where

l_0 is the effective length

$\theta_i = \theta_0 \cdot \alpha_h \cdot \alpha_m$

$\theta_0 = 1/200$

$\alpha_h = 2/\sqrt{l} \qquad 2/3 \leq \alpha_h \leq 1$

$\alpha_m = \sqrt{0.5 \cdot \left(1 + \frac{1}{m}\right)}$

m=1

l is the length or height of the column

5.2- Nominal Curvature method

Calculate $K_r = (n_u - n)/(n_u - n_{bal}) \leq 1$

Where

$n = N_{Ed}/(A_c \cdot f_{cd})$

$n_u = 1 + \omega$

$n = N_{Ed}/(A_c \cdot f_{cd})$

n_{bal} = 0.4

$\omega = A_s \cdot \mathrm{f}_{yd}/(\mathrm{A}_c \cdot f_{cd})$

Calculate $K_\varphi = 1 + \beta \cdot \varphi_{ef} \geq 1$

Where

φ_{ef} is the effective creep ratio

$\beta = 0.35 + \mathrm{f}_{ck}/200 - \lambda/150$

λ is the slenderness ratio

Calculate $1/r = K_r \cdot K_\varphi \cdot 1/r_0$

Where

$1/r$ is the curvature

K_r is a correction factor depending on axial load

K_φ is a factor for taking account of creep

$1/r_0 = \varepsilon_{yd}/(0.45 \cdot d)$

$\varepsilon_{yd} = f_{yd}/E_s$

f_{yd} is the design yield strength of reinforcement

E_s is the design value of modulus of elasticity of reinforcing steel

d is the effective depth of the section

Calculate $M_2 = N_{Ed} \cdot e_2$

Where

N_{Ed} is the design value of axial force

e_2 is the deflection $= 1/r \cdot {l_0}^2/c$

l_0 is the effective length

c is a factor depending on the curvature distribution ($c = \pi^2$for constant cross section)

Rectangular section | Circular section

Calculate $M_{Ed} = \mu_d \cdot b \cdot h^2 \cdot f_{cd}$

Where

μ_d is the total design moment

b is the depth of the section

h is the width of the section

f_{cd} is the design value of concrete compressive strength

Calculate $M_{Ed} = \mu_d \cdot \pi \cdot r^3 \cdot f_{cd}$

Where

μ_d is the total design moment

r is the radius of the section

f_{cd} is the design value of concrete compressive strength

Calculate $M_{0Ed} = M_{Ed} - M2$

Where

M_{Ed} is the total design moment

M_{0Ed} is the first order moment

M_2 is the second order moment

The second order moment is obtained through a calculation of the deflection and the first order moment is subtracted from the maximum moment the section can withstand. Geometric imperfections may be taken into account by means of a parameter e_0 that contributes to the first order moment through the axial load N_{Ed}.

Calculate $e_0 = \max(\frac{h}{30}; 20; \theta_i \cdot \frac{l_0}{2})$

Where

$\theta_i = \theta_0 \cdot \alpha_h \cdot \alpha_m$

$\theta_0 = 1/200$

$\alpha_h = 2/\sqrt{l} \qquad 2/3 \leq \alpha_h \leq 1$

$\alpha_m = \sqrt{0.5 \cdot \left(1 + \frac{1}{m}\right)}$

m=1

l is the length or height of the column

6- Parameter Study

6.1- Introduction

In order to study second order effects in realistic and representative cases, a set of different typical columns with varying boundary conditions are presented. These columns are representative of different structural solutions found in construction and, moreover, can be implemented multiple times in the same construction and work life of a structural engineer.

The four selected type of columns together with their boundary conditions can be seen in the following figure.

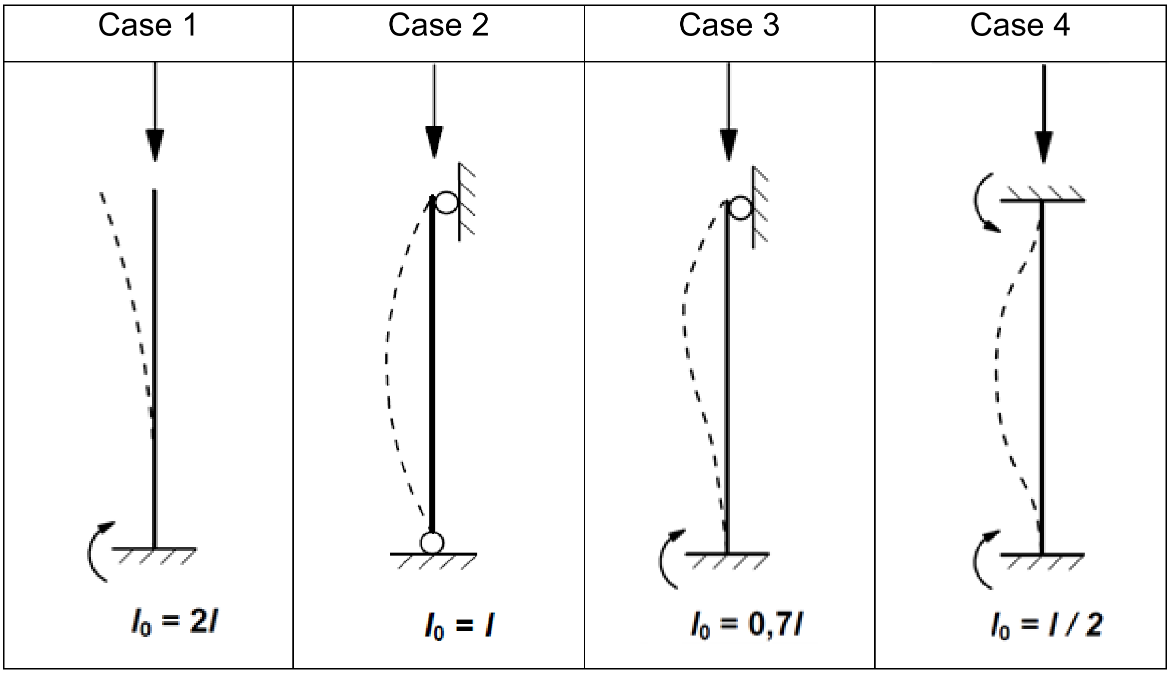


Figure 2: Studied columns with their boundary conditions

For each type of column, an array of three different sections is studied and they will be named as s_0, s_1 and s_2 to facilitate the explanations. The first is a rectangular section with reinforcement laid on opposite sides (s_0), the second is a rectangular section with a uniform distribution of the reinforcement (s_1) and the third consists of a circular section with reinforcement also distributed uniformly (s_2). The main characteristics are shown in the next tables and are represented in the following figures.

Parameter	Value	Units
h	500.0	mm
b	500.0	mm
d	450.0	mm
A	250000.0	mm^2
$A_{s,tot}$	7200.0	mm^2
I_s	288000000.0	mm^4
d_1/h	0.1	-

Table 1: Data of s_0

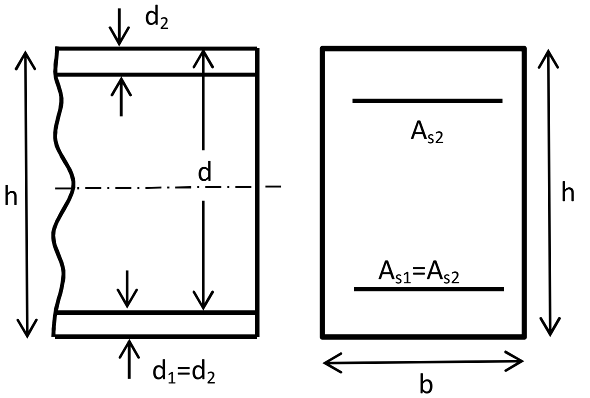


Figure 3: Rectangular section with reinforcement laid in opposite sides (s_0)

Parameter	Value	Units
h	500.0	mm
b	500.0	mm
d	450.0	mm
E	404.5	mm
e	395.5	mm
C	404.5	mm
c	395.5	mm
A	250000.0	mm^2
$A_{s,tot}$	7200.0	mm^2
b_1	50.0	mm
d_1	50.0	mm
I_s	192024300.0	mm^4
d_1/h	0.1	-
b_1/b	0.1	-

Table 2: Data of s_1

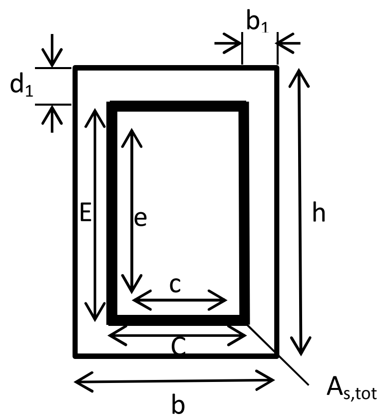


Figure 4: Rectangular section with reinforcement laid uniformly (s_1)

Parameter	Value	Units
d_1	56.4	mm
A	250000.0	mm^2
$A_{s,tot}$	7200.0	mm^2
r	282.1	mm
r_2	228.2	mm
r_1	223.1	mm
I_s	183369699.2	mm^4
d_1/d	0.1	-

Table 3: Data of s_2

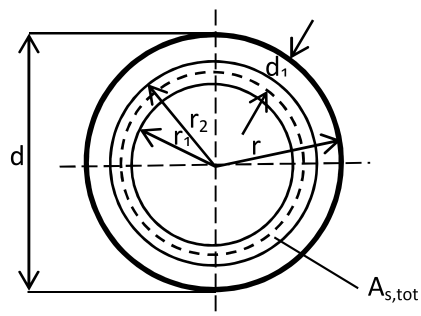


Figure 5: Circular section with reinforcement laid uniformly (s_2)

The total cross sectional area A, the cross sectional area of concrete A_c and the cross sectional area of reinforcement A_s are always equal for the three types of sections throughout the hole study.

Altogether, counting that there are four different cases with varying boundary conditions and that there are three types of section for each case, a total of twelve columns are studied.

For each of these twelve columns, some parameters will be changed in order to study their influence in second order effects. These parameters are the slenderness ratio (λ), creep (φ), the reinforcement area (A_s) and the concrete cross sectional area (A_c). Other parameters like the applied load (N_{Ed}), and type and quality of concrete and of reinforcement are kept constant, in order to enable a correct comparison between the behaviors of the different columns.

Following the methodology proposed in the two simplified methods of Eurocode 2, first and second order moments are obtained for the Nominal Curvature and Nominal Stiffness methods. The evolution of the second order moment when affected by both creep and slenderness and the ability of slender columns, with different boundary conditions and sections, to withstand these second order effects is the core purpose of this parameter study.

In order to perform these analyses in a methodological way, the creation of a tool that can generate the output values of the first and second order moments from a series of controlled input parameters is essential. For this purpose, a spreadsheet seems like the right tool since it allows to easily change one parameter and see how the output changes.

In the spreadsheet, special attention has been put in all the conditions and constraints found in Eurocode 2 e.g, $K_\varphi = 1 + \beta \cdot \varphi ef \geq 1$ then K_φ should be taken as 1 if β is negative and φef is greater than 1. These conditions have all been designed to be taken into account automatically.

The input and output data is shown in a schematic way in the following table.

	Input Data	Output Data
General Data	L	
	N_{Ed}	
	A_s	
	f_{yk}	
	f_{ck}	
	γ_s	
	γ_c	
	α_{cc}	
	E_{cm}	λ_{lim}
	γ_{cE}	
	E_s	λ
	c_0	
	μ	e_0
	l_0	
	M_{01}/M_{02}	e_2
Section 1 Data	h	
	b	M_{tot} (M_{Ed})
	d_1/h	
Section 2 Data	h	M_1 (M_{0Ed})
	b	
	d_1/h	M_2
	b_1/b	
Section 3 Data	A	
	d_1/d	
Creep Data	t	
	t0	
	RH	
	M_{0Eqp}/M_{0Ed}	

Table 4: Spreadsheet Input and Output data

The specific values of the input parameters are shown for each different constraint case and for all the sections:

Input data	Case 1	Case 2	Case 3	Case 4
L	0.5 to 15 meters			
N_{Ed}	1670 kN			
A	250000 mm^2			
A_s	7200 mm^2			
A_c=A-A_s	242800 mm^2			
f_{yk}	400 N/mm^2			
f_{ck}	25 N/mm^2			
γ_s	1.15			
γ_c	1.5			
α_{cc} (n_{bal}=0.4)	1			
Ecm	31Gpa			
γ_{cE}	1.2			
E_s	200 Gpa			
$\beta=\pi^2/c_0$	1	1	$\pi^2/8$=1.234	$\pi^2/8$=1.234
l_0	2L	L	0.7L	0.5L
m	s_0: 0.3434 ; s_1: 0.2700 ; $s_2$0.2272			
C=1.7-r_m	0.7			
b	500 mm			
h	500 mm			
d_1/h	0.10			
b_1/b	0.10			
d_1/d	0.10			
t	∞			
t0	28 days			
RH	60%			
M_{0Eqp}/M_{0Ed}	1/1.35			

Table 5: Values for input data

6.2-Theoretical study of the Influence of creep in the Simplified Methods.

6.2.1- Nominal stiffness

In the Nominal Stiffness method, creep is taken into account by means of a coefficient K_c that is included in the calculation of the rigidity *EI*. The second order moment is calculated from the following expression:

$$M_2 = M_{Ed} - M_{0Ed} = M_{0Ed}\left[1 + \frac{\beta}{\frac{N_B}{N_{Ed}} - 1}\right] - M_{0Ed} = M_{0Ed} \cdot \left[\frac{\beta}{\frac{\pi^2 \cdot EI/{l_0}^2}{N_{Ed}} - 1}\right]$$

Where:

$$EI = K_c \cdot E_{cd} \cdot I_c + K_s \cdot E_s \cdot I_s$$

And:

$$K_c = \frac{K_1 \cdot K_2}{(1 + \varphi_{ef})}$$

K_1 and K_2 are always positive, so if $\varphi_{ef} > 0$ then creep has an influence for any given slenderness λ.

6.2.2- Nominal Curvature

In the Nominal Curvature method, creep is taken into account by means of a coefficient K_{φ}. The second order moment is calculated through the following expression:

$$M_2 = N_{Ed} \cdot e_2 = N_{Ed} \cdot \frac{1}{r} \cdot \frac{{l_0}^2}{c} = N_{Ed} \cdot K_r \cdot K_{\varphi} \cdot \frac{1}{r_0} \cdot \frac{{l_0}^2}{c}$$

Where:

$$K_{\varphi} = 1 + \beta \cdot \varphi_{ef} \geq 1$$

- If creep is not taken into account $\varphi_{ef} = 0$ → $K_{\varphi} = 1$ always.
- If creep is taken into account $\varphi_{ef} > 0$ → $K_{\varphi} > 1$ → $\beta > 0$.

$$\beta = 0.35 + \frac{f_{ck}}{200} - \frac{\lambda}{150} > 0$$

From this, it can be derived that creep has an influence on nominal curvature when:

$$\lambda < \left(0.35 + \frac{f_{ck}}{200}\right) \cdot 150$$

and that creep is not taken into account when:

$$\lambda \geq \left(0.35 + \frac{f_{ck}}{200}\right) \cdot 150$$

Given an $f_{ck} = 25$, then creep is taken into account only if $\lambda < 71.25$

6.3- Practical study of the influence of creep and slenderness

In order to evaluate the differences between creep in both methods, a wide set of cases are tested. As mentioned previously, these cases include four different boundary conditions, three different types of section and lengths of columns ranging from 0.5 meters up to 15 meters.

The use of a spreadsheet assures that the calculations are undertaken in a methodological manner and that the variables that change can be controlled.

The following first and second set of graphs, in chapter 6.3.1, show how creep affects the second order moment in both of the simplified methods, and also compare how the three types of section react to the inclusion of creep. This study is undertaken for all four types of columns.

The third graph, in chapter 6.3.2, shows the evolution of the second order moment as the slenderness ratio is increased. The objective is to point out the particular differences in the behavior of the Nominal Stiffness and the Nominal Curvature method when including creep in the model. This test is also undertaken for the different types of cross-section and columns and the influence of the β parameter on the Nominal Stiffness method is also studied when comparing them all.

Chapter 6.3.3 compares the evolution of the second order moment inside every particular case of boundary conditions, analyzing the differences between the simplified method used and the different type of section. Conclusions can be taken on which type of cross section is more prone to second order effects and which method gives a higher second order moment given a certain slenderness.

6.3.1- Influence of the slenderness ratio in creep

The following set of graphs enable to see how much the second order moment is affected by the effect of creep when the slenderness ratio varies. The values in the vertical axis represent the magnitude (kN·m) by which M_2 is increased when creep is considered and, furthermore, allows seeing how the nominal stiffness method includes its effect. In other words:

- Vertical axis: $M_2(\varphi_{ef}>0) – M_2(\varphi_{ef}=0)$ [kN·m]
- Horizontal axis: λ

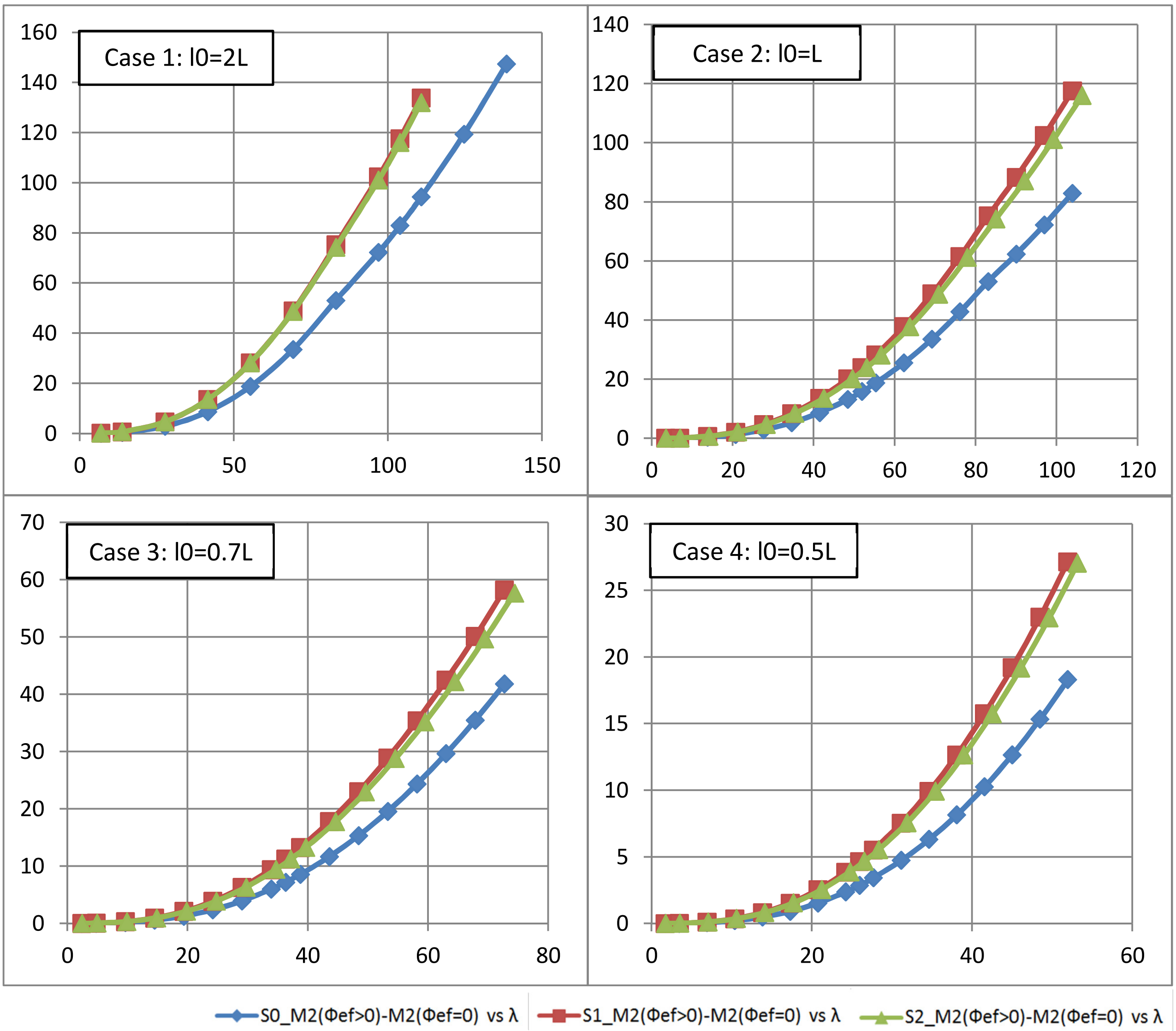


Figure 6: Influence of creep in the second order moment (Nominal Stiffness)

As it can be observed in the graph and as it was explained earlier, if creep is considered it always has an effect on the behavior of the column for any given slenderness λ. But what is of most interest is that creep has an increasing effect on M_2 as the slenderness ratio also increases.

When comparing the three types of cross-section, the uniformly reinforced rectangular section (s_1) is the most affected by second order effects when creep is included in the model followed closely by the circular section (s_2). The rectangular section with opposite reinforcement (s_0) seems to have a considerable delay of the appearance of second order moments compared to the other sections, showing the same M_2 for much higher slenderness ratios and this phenomenon being amplified as λ grows.

The following figure represents the same as the previous one, but it enables to see how creep affects the second order moment in the Nominal Curvature method depending on the slenderness of the column.

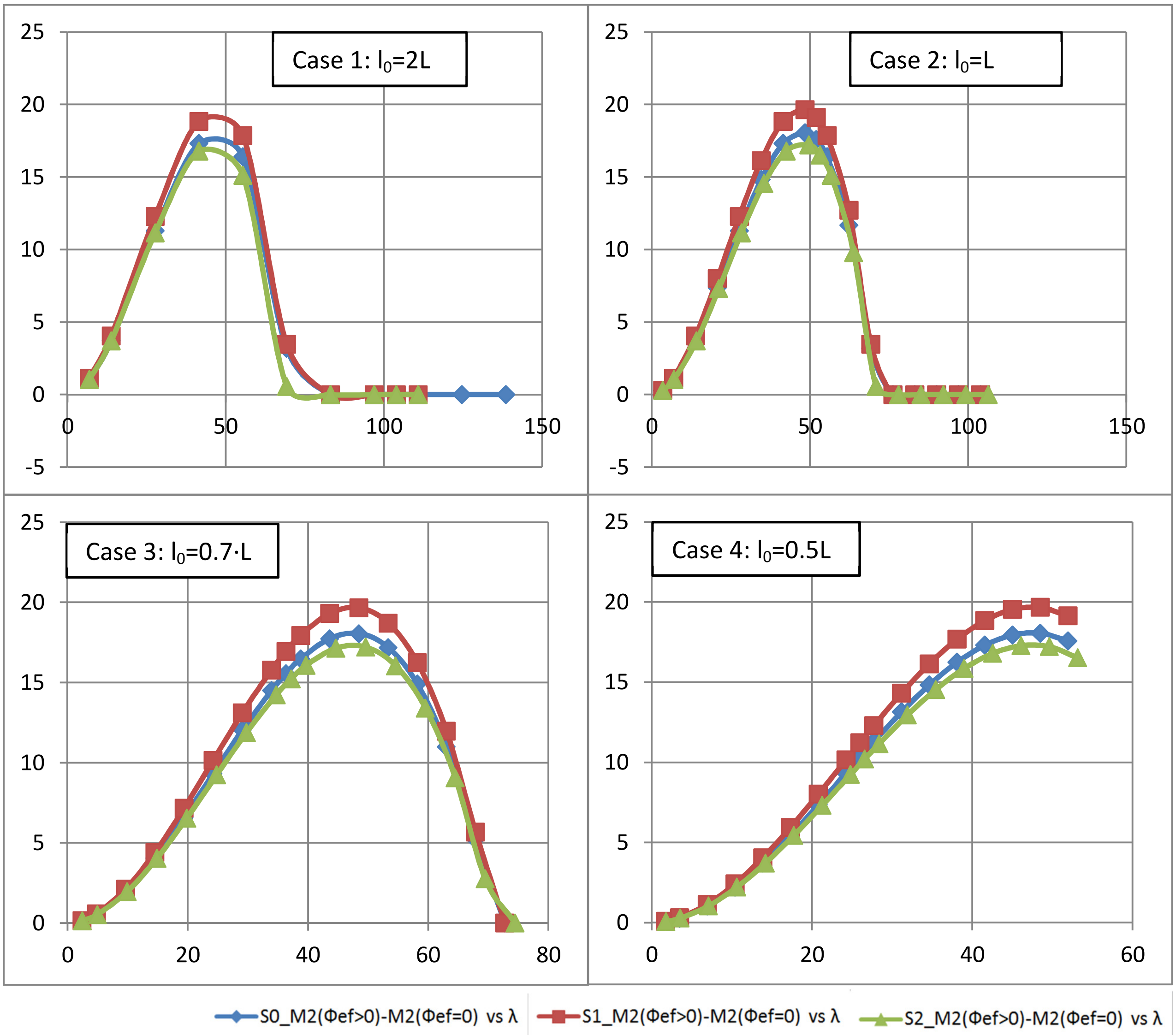


Figure 7: Influence of creep in the second order moment (Nominal Curvature)

As it can be observed, creep has an effect on columns of little to moderate slenderness and seems not to have an effect on columns with very high slenderness ratios. As explained earlier, the slenderness value that indicates when creep is taken into account is:

$$\lambda < \left(0.35 + \frac{f_{ck}}{200}\right) \cdot 150 = 71.5$$

The slenderness limit λ_{lim} states that second order effects should be taken into account for slenderness ratios higher than the limit, and in this study and for all the four cases, the slenderness limit averages λl_{im}=33 when including creep and λl_{im}=25 when not, so it can be said that creep starts with a slightly increasing effect and then continues with a decreasing effect on the nominal curvature method.

If the behavior of the three types of section is compared, the uniformly reinforced rectangular section (s_1) is the most affected by the influence of creep, followed then by the rectangular section with opposite reinforcement (s_0) and finally by the circular section (s_2), with a notable decrease at maximum point averaging 3kN·m between the first and this last section. This behavior is clearly different than the one shown in the Nominal Curvature method, with the influence of creep being greater in s_1 but lower in s_0.

Cases 3 and 4 correspond to columns with lower effective lengths and only the results for columns with heights up to 15 m are compared. Due to this, slenderness values lower than those of cases 1 and 2 are represented but the columns dimensions are identical. This is why for cases 3 and 4 the graphs may look incomplete, although if higher slenderness columns were introduced the shapes would be the same as in the first two figures.

In chapters 6.3.2 and 6.3.3 an extra calculation relative to a 21 meter column length is done for case 4, since the slenderness for the 15 meter length column is lower than λ=71.25 and the effect of creep in the Nominal Curvature method cannot be fully appreciated.

6.3.2- Evolution of the second order moment with creep

The following graph shows the evolution of the second order moment when the slenderness ratio is increased.

When comparing the two simplified methods, some similar behavior trends can be found for the three types of sections and for the four studied cases. First, case 1 with the rectangular section and opposite reinforcement is studied to observe in greater detail these similarities.

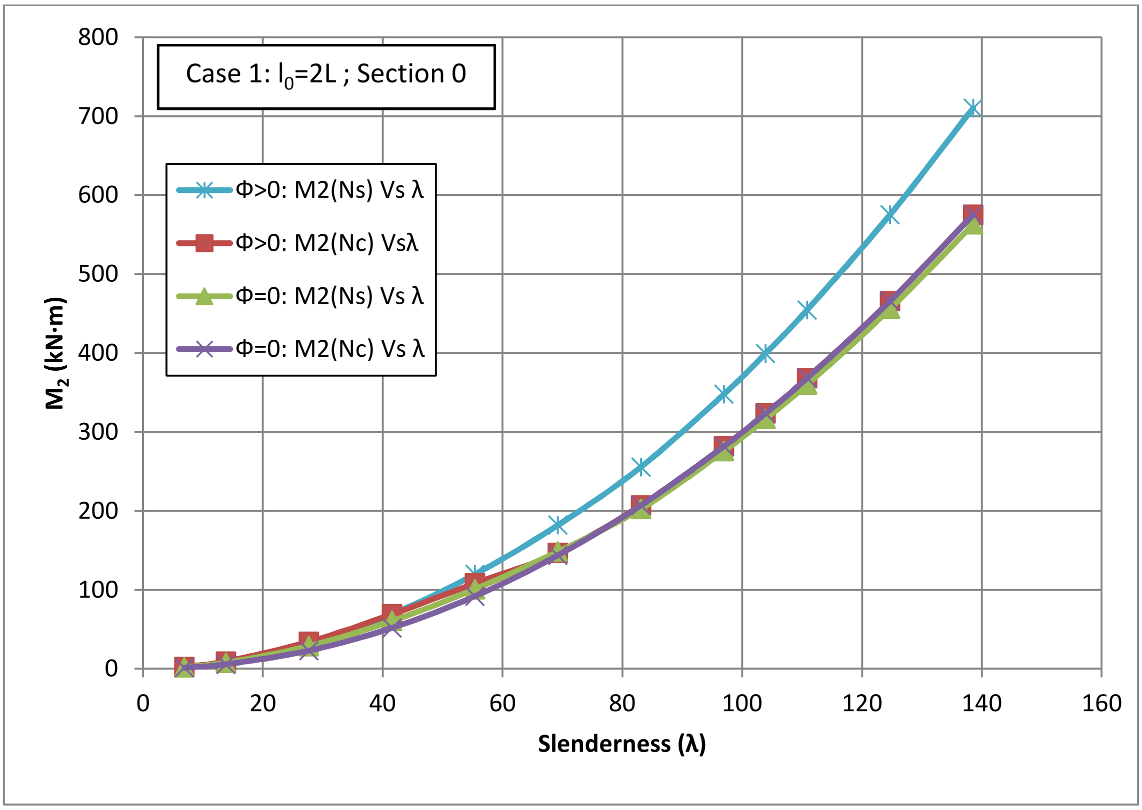


Figure 8: Evolution of M_2 vs λ for Case 1 and s_0

For the nominal stiffness method, it is clear that the effect of creep results in a higher second order moment value for each slenderness ratio. On the other hand, for the nominal curvature method, it can be observed that for slenderness below a certain value this is also certain, but from this value onwards creep is not taken into account. Note that when the Nominal Curvature method is affected by creep and for columns bellow the slenderness limit, the value of M_2 is higher than that of the Nominal Stiffness Method, but after a certain λ value this is not true and the creep branch joins the non-creep branch. This again, is due to the deflection adjustment through the coefficient K_φ (see chapter 6.2.2) and it is the direct consequence of what has been shown in the previous set of graphs.

When comparing the 12 graphs (3 sections and 4 constraints situations) the effect of the parameter β on the nominal stiffness can be observed. For cases 1 (l_0=2L) and 2 (l_0=L) Beta is taken as β=1, whereas in cases 3 (l_0=0.7) and 4 (l_0=0.5) Beta is taken as $\beta = \pi^2/c_0 = \pi^2/8 = 1.234$. This is because as it is stated in the Bo Westerberg report (see chapter 9, Bibliography) and as it is indicated in the Eurocode 2 under chapter *5.8.7.3*, members bent in double curvature may present in some cases unfavorable values for second order moments. When β is increased, higher values for the second order moment are obtained and for this reason it is recommended to take c_0 as 8 for cases 3 and 4.

All this finally results in an increase of the second order moments in the Nominal stiffness method for cases 3 and 4, compared to cases 1 and 2.

In the following graphs, the vertical axis also represent the second order moment M_2 (kN·m) and the horizontal axis the slenderness ratio λ.

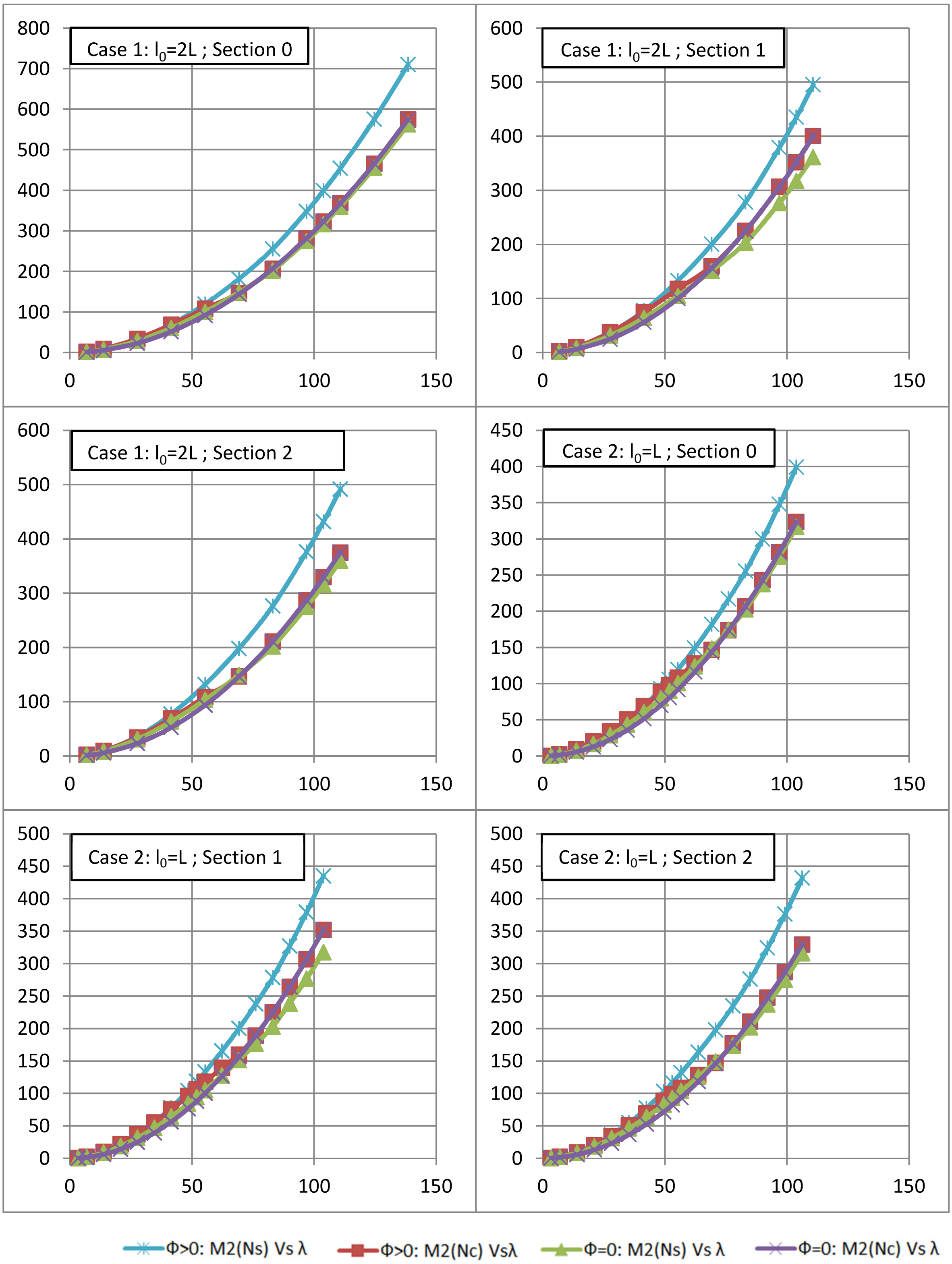


Figure 9: Evolution of the second order moment M_2 vs λ for cases 1 and 2, all sections

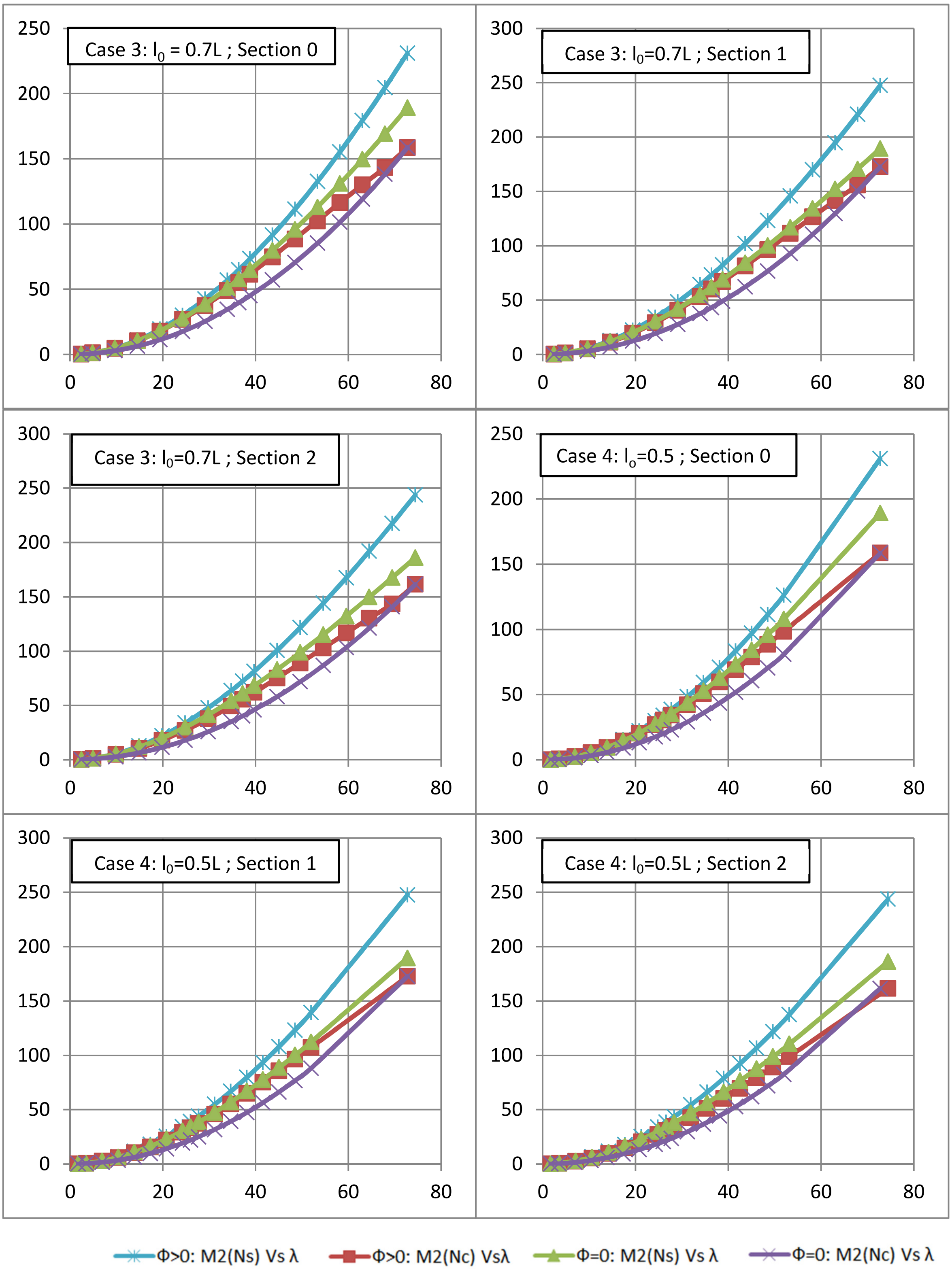


Figure 10: Evolution of the second order moment M_2 vs λ for cases 3 and 4, all sections

6.3.3- Evolution of the second order moment depending on the type of section

These graphs represent the evolution of the second order moment M_2 versus the slenderness λ for both simplified methods and for all the sections when including creep.

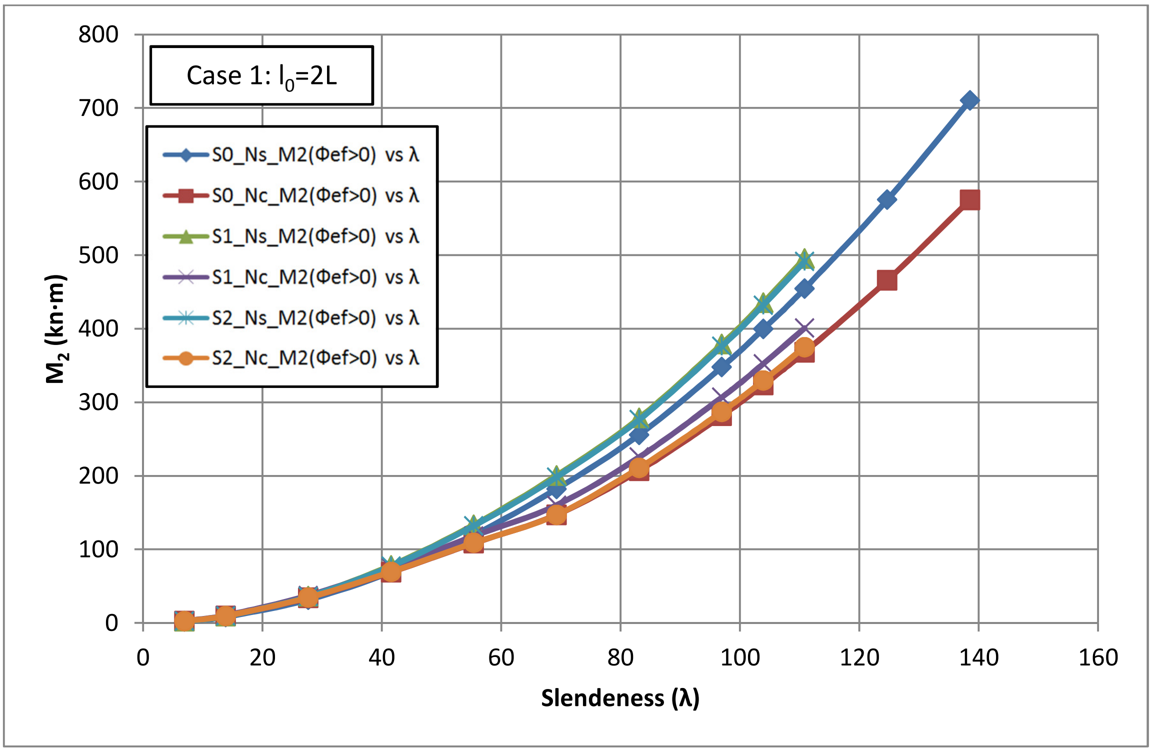


Figure 11: M_2(φef>0) vs λ for Case 1

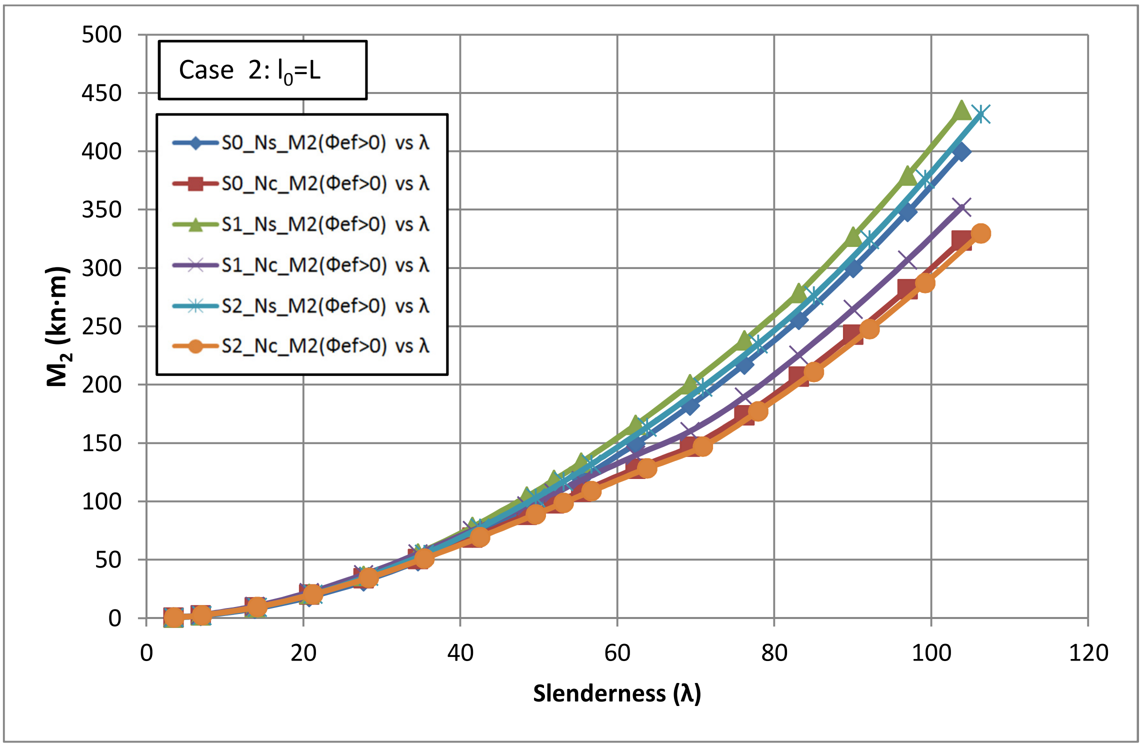


Figure 12: $M_2(\varphi ef>0)$ vs λ for case 2

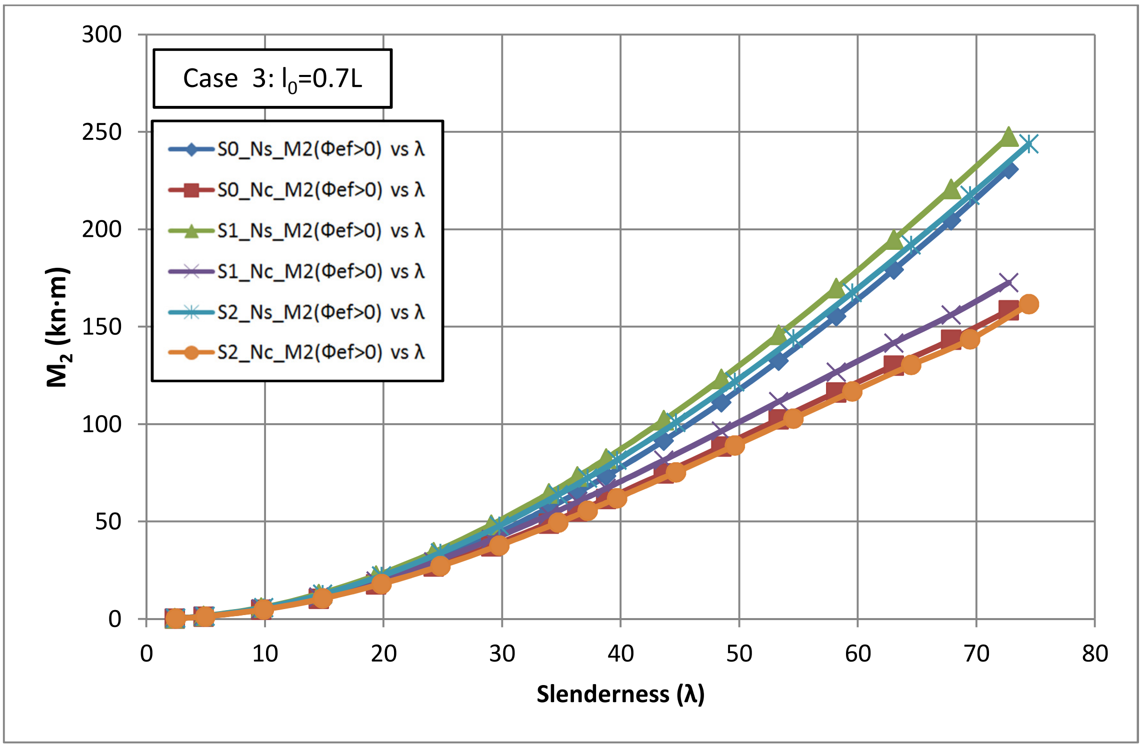


Figure 13: $M_2(\varphi ef>0)$ vs λ for case 3

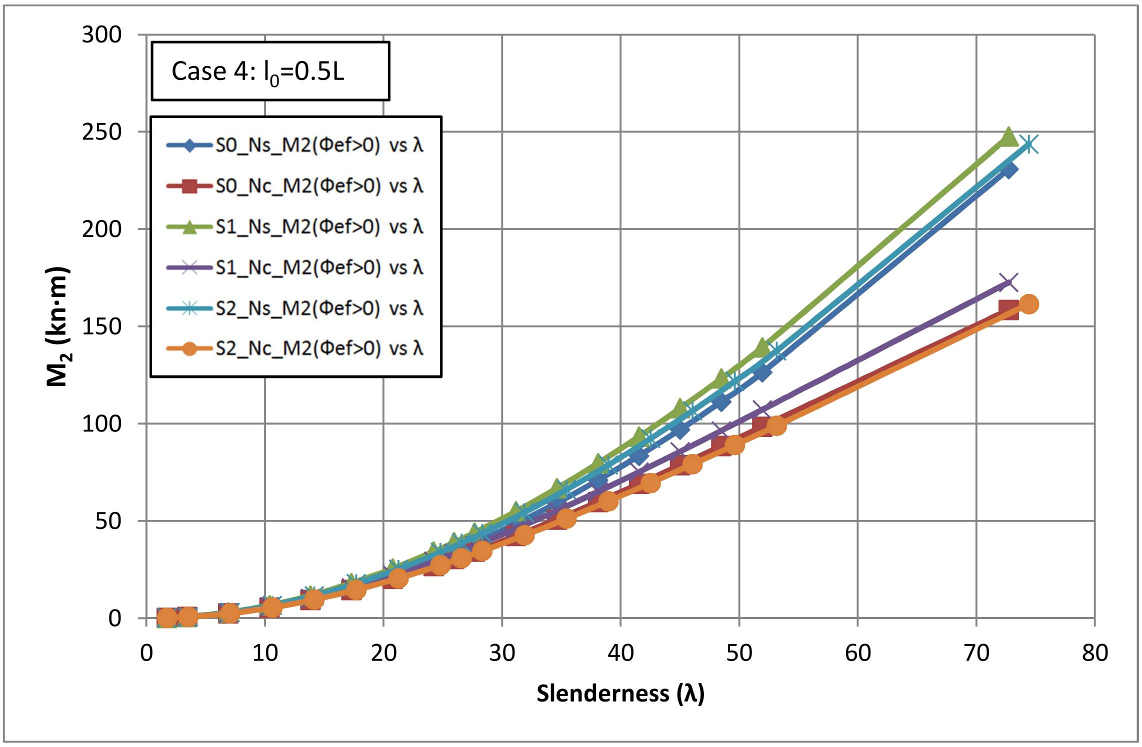


Figure 14: M_2(φef>0) vs λ for case 4

For high slenderness ratios, the Nominal Stiffness method is more affected by second order effects than the Nominal Curvature method, and this is also true for the three types of sections. This fact is more accentuated in cases 3 and 4 due to the previously discussed β parameter (see 6.3.2). This comparison must be done with care, since the same column heights have been tested in all cases but for cases 3 and 4, which have lower effective lengths and lower slenderness ratios, not so high values of M_2 are represented in the graphs (if results for higher slenderness ratios for cases 3 and 4 were displayed, the graphs would look more similar).

When comparing the three types of sections, the rectangular section with uniform reinforcement (s_1) is the most affected by second order, followed by the circular section with uniform reinforcement (s_2) and the rectangular section with opposite reinforcement (s_0). This is true for both methods even though in the Nominal Curvature method the values for s_0 may look higher than for s_2. This is only like this because the circular section is more slender than the rectangular ones, but when comparing all the dots that represent equal column lengths the values for s_2 are higher than for s_0.

6.4- Influence of A_s and A_c on second order effects

In order to study the effect of a variation in the area of reinforcement and concrete in the second order moment, a particular example with its calculations are presented in this chapter.

Parting from the column previously presented as case 1, with no constraints at the top and totally constrained at the bottom, with a rectangular section and reinforcement distributed in opposite sides with and an applied axial load of N_{Ed}, the following initial characteristics are given:

- L = 7.5 m
- A = 0.25 m^2
- A_c = 0.2428 m^2
- A_s = 0.0072 m^2

The effect of adding more area of reinforcement or more area of concrete is limited by the reinforcement ratio A_s/A_c. Some constraints to this parameter are given for columns in chapter *9.5.2* in Eurocode 2, giving a minimum amount for longitudinal reinforcement $A_{s,min}$ in *9.5.2(2)* and a maximum amount that should not be exceeded $A_{s,max}$ in *9.5.2(3)*.

$$A_{s,min} = \max(\frac{0.10 \cdot N_{Ed}}{f_{yd}}; 0.002 \cdot A_c)$$

$$A_{s,max} = 0.04 \cdot A_c$$

The maximum amount of reinforcement that can be added whilst still complying with the Eurocode-2 is calculated the following way:

$$\frac{A_s}{A_c} = \frac{A_s}{A - A_s} = 0.04$$

$$A_{s,max} = \frac{0.04}{1.04} \cdot A$$

This way the total cross sectional area remains constant, and the amount of reinforcement added is enough for the desired A_s/A_c ratio to be true even when A_c decreases.

The same reasoning can be applied to the minimum amount of reinforcement, because if A_s decreases then A_c increases while A remains a constant:

$$\frac{A_s}{A_c} = \frac{A_s}{A - A_s} = 0.002$$

$$A_{s,min} = \max(\frac{0.10 \cdot N_{Ed}}{f_{yd}}; \frac{0.002}{1.002} \cdot A)$$

This last equation won't be used, but it is left as a reference to take into account a situation where reinforcement would want to be subtracted.

For a load of N_{Ed} = 1670 kN, these were the results obtained:

	A_s (m^2)	A_c (m^2)	As/Ac ratio
Actual data	0.0072	0.2428	0.0297
Min A_s	0.0005	0.2495	0.0020
Max A_s	0.0096	0.2404	0.0400
Max A_c	0.0072	3.6000	0.0020

Table 6: Areas and ratios of reinforcement and concrete cross sections

n ($N_{Ed}/(A \cdot f_{cd})$)	0.4008
ω ($A_s \cdot f_{yd}/(A \cdot f_{cd})$)	0.6010
μ	0.3434

Table 7: Initial non-dimensional parameters

	λ	λ_{lim}	e_0 (m)	e_2 (m)	M_{Ed} (kN·m)	M_{0Ed} (kN·m)	M_2 (KN·m)
Nstiffness	103.923	24.369	0.027	-	715.417	315.873	399.543
Ncurvature	103.923	24.369	0.027	0.194	715.417	391.851	323.566

Table 8: Initial results

6.4.1- Influence of adding more reinforcement

First, the effect of adding more reinforcement (A_s) is going to be tested by adding the maximum amount permitted i.e, $A_{s,max}$ = (0.04/1.04)·A. This affects the reinforcement ratio ω, μ and the area of concrete A_c as the total cross sectional area A remains constant and it is not affected. The new column presents the following results:

A_s	0.0096 m^2
A_c	0.2404 m^2
A	0.2500 m^2
n	0.4008
ω	0.8027
μ	0.4222

Table 9: Areas and non-dimensional parameters

	λ	λ_{lim}	e_0 (m)	e_2 (m)	M_{Ed} (kN·m)	M_{0Ed} (kN·m)	M_2 (KN·m)
Nstiffness	103.923	26.475	0.027		879.583	495.898	383.685
Ncurvature	103.923	26.475	0.027	0.193	879.583	556.450	323.133

Table 10: Results

The following table compares the initial and final values:

	Nominal Stiffness			**Nominal Curvature**		
	Initial values	**Final values**	**Increase (%)**	**Initial values**	**Final values**	**Increase (%)**
M_{Ed} (kN·m)	715.417	879.583	22.947	715.417	879.583	22.947
M_{0Ed} (kN·m)	315.873	495.898	56.993	391.851	556.450	42.006
M_2 (KN·m)	399.543	383.685	-3.969	323.566	323.133	-0.134
λ	103.923	103.923	0.000	103.923	103.923	0.000
λ_{lim}	24.369	26.475	8.642	24.369	26.475	8.642
μ	0.343	0.422	23.090	0.343	0.422	23.090

Table 11: Comparison of results

Comparing the new results with the initial ones, the first interesting fact is the increase of resistance of the section due to the increase of the non-dimensional parameter μ. In addition to this, in the Nominal Stiffness method adding reinforcement results in a higher stiffness EI and in turn in a higher M_1 value.

The increase of the total moment that the section can resist and the decrease of the second order moment is absorbed by the first order moment M1. Taking into account that the resistance of the section is calculated the following way:

$$\omega = \frac{A_s \cdot f_{yd}}{b \cdot h \cdot f_{cd}}$$

And that:

$$M_{Ed} = \mu \cdot b \cdot h^2 \cdot f_{cd}$$

When adding more reinforcement ω increases and in turn μ which also causes M_{Ed} to increase. So A_s not only affects M_1 through the stiffness but also through M_{Ed}. In global, the relation between all these parameters is given in the following expression:

$$M_{Ed} = M_{0Ed}\left[\left(\frac{\beta}{\frac{N_B}{N_{Ed}} - 1}\right) + 1\right] = M_{0Ed} \cdot \left[\left(\frac{\beta}{\frac{\pi^2 \cdot EI/{l_0}^2}{N_{Ed}} - 1}\right) + 1\right]$$

Looking at the differences in the Nominal Curvature method, the same conclusion can be drawn for the increase of the resistance of the section M_{Ed} through the non-dimensional parameter μ. Since this method is based on the curvature, the increase of ω and a decrease of n ultimately results in a slight decrease of M_2.

$$M_2 = N_{Ed} \cdot e_2 = N_{Ed} \cdot \frac{1}{r} \cdot \frac{{l_0}^2}{C} = N_{Ed} \cdot K_r \cdot K_\varphi \cdot \frac{1}{r_0} \cdot \frac{{l_0}^2}{C}$$

$$K_r = \frac{u_n - n}{u_n - n_{bal}} = \frac{1 + \omega - n}{1 + \omega - n_{bal}}$$

Since $n > n_{bal}$, it can be derived from the previous expressions that if the values of ω and n are higher, K_r decreases and also M_2 as a direct consequence. Altogether, this decrease is not very important In comparison with the increase of the first order moment and in global, the slenderness limit doesn't increase enough to neglect second order effects.

In conclusion, in very slender columns increasing the reinforcement doesn't permit to neglect second order effects, but for columns just over the slenderness limit it could prove as a good option.

6.4.1- Influence of adding more area of concrete

The effect of increasing the area of concrete (A_c) is studied taking into account the lower limit on the A_s/A_c ratio. A_s is kept constant, so the new A_s/A_c ratio cannot be smaller than what Eurocode 2 permits. With this in mind, A_c is incremented within its limits by increasing the total cross-section A to an equivalent square section of b·h= 1.3 x 1.3 m^2, giving the following results:

A_s	0.0072 m^2
A_c	1.6828 m^2
A	1.6900 m^2
n	0.0593
ω	0.0889
μ	0.0444

Table 12: Areas and non-dimensional parameters

	λ	λ_{lim}	e_0 (m)	e_2 (m)	M_{Ed} (kN·m)	M_{0Ed} (kN·m)	M_2 (KN·m)
Nstiffness	39.970	47.876	0.043	-	1625.780	1480.988	144.792
Ncurvature	39.970	47.876	0.043	0.099	1625.780	1460.556	165.224

Table 13: Results

The following table compares the initial and final values:

	Nominal Stiffness			Nominal Curvature		
	Initial values	Final values	Increase (%)	Initial values	Final values	Increase (%)
M_{Ed} (kN·m)	715.417	1625.780	127.249	715.417	1625.780	127.249
M_{0Ed} (kN·m)	315.873	1480.988	368.856	391.851	1460.556	272.732
M_2 (KN·m)	399.543	144.792	-63.761	323.566	165.224	-48.937
λ	103.923	39.970	-61.539	103.923	39.970	-61.539
λ_{lim}	24.369	47.876	96.463	24.369	47.876	96.463
μ	0.343	0.044	-87.055	0.343	0.044	-87.055

Table 14: Comparison of results

In this case, μ decreases since the non-dimensional parameters n and ω decrease drastically. However, the big increase in the dimensions of the section (b·h) compensates for this fact and the resistance of the section M_{Ed} suffers an overall improvement. This can easily be seen through the expression:

$$M_{Ed} = \mu \cdot b \cdot h^2 \cdot f_{cd}$$

As it happened before in the Nominal Stiffness method, the rigidity EI increases a great amount because the second moment of the concrete area is much higher (I_c). The same reasoning than for the previous case when A_s increased can be made.

In the Nominal Curvature method, the second order moment decreases mainly due to the increase of the effective depth d, which makes $1/r_0$ decrease greatly and in turn 1/r resulting finally in an M_2 decrease.

$$M_2 = N_{Ed} \cdot e_2 = N_{Ed} \cdot \frac{1}{r} \cdot \frac{{l_0}^2}{C} = Ned \cdot K_r \cdot K_\varphi \cdot \frac{1}{r_0} \cdot \frac{{l_0}^2}{C}$$

$$\frac{1}{r_0} = \varepsilon_{yd}/(0.45 \cdot d)$$

Creep also has an influence on the reduction of second order effects. The effective depth parameter φ_{ef} reduces from 1.689 to 1.505, and as it has been seen in chapter 6.3 of this report creep induces higher second order effects.

In this case, it is the combination of the slenderness limit almost doubling and the slenderness decreasing that permits neglecting second order effects. For cases in which the slenderness is too high to achieve its lower limit, increasing the total cross sectional area A through the area of concrete A_c presents itself as a good option.

7- Finite element simulation

In order to assess the results provided by the spreadsheets and obtained following both simplified methods from Eurocode-2, the behavior of the column previously seen as “case 1” (see chapter 6.1) is compared with various models created by finite elements through Diana.

The objective is to simulate the increase of the second order moment M_2 as the slenderness ratio λ also increases, and to see if the curves obtained in the Nominal Stiffness and Nominal Curvature behave in a realistic way.

Five different models have been created through Diana to create five different columns with ranging lengths of 1, 2.5, 5, 7.5 and 9 meters. The geometrical properties are identical to the ones of the column studied in the spreadsheets.

7.1- Creation of the finite element model

- Creation of the geometrical model

First the concrete part is created by entering the dimensions of the column. Since the model to create is two dimensional, a rectangle of 500mm by 9000mm is created, making this rectangle a “face”.

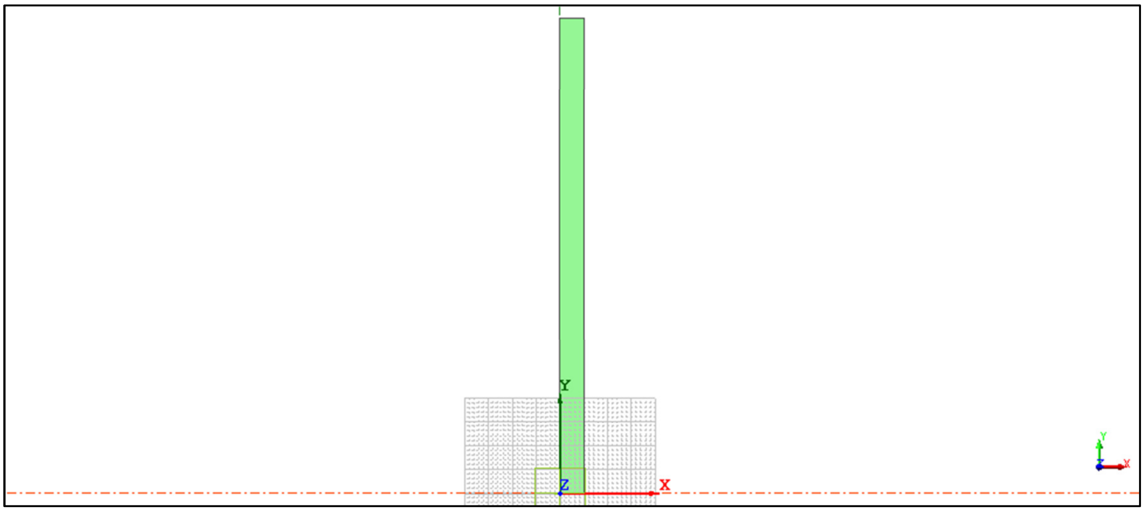


Figure 15: Creation of the geometry of the concrete

Next, the reinforcement is created. This reinforcement is drawn as two lines (the reinforcement is placed symmetrically in both sides of the column). It is important that the reinforcement surpasses the area of the concrete, this way it is certain that the whole area of concrete is reinforced.

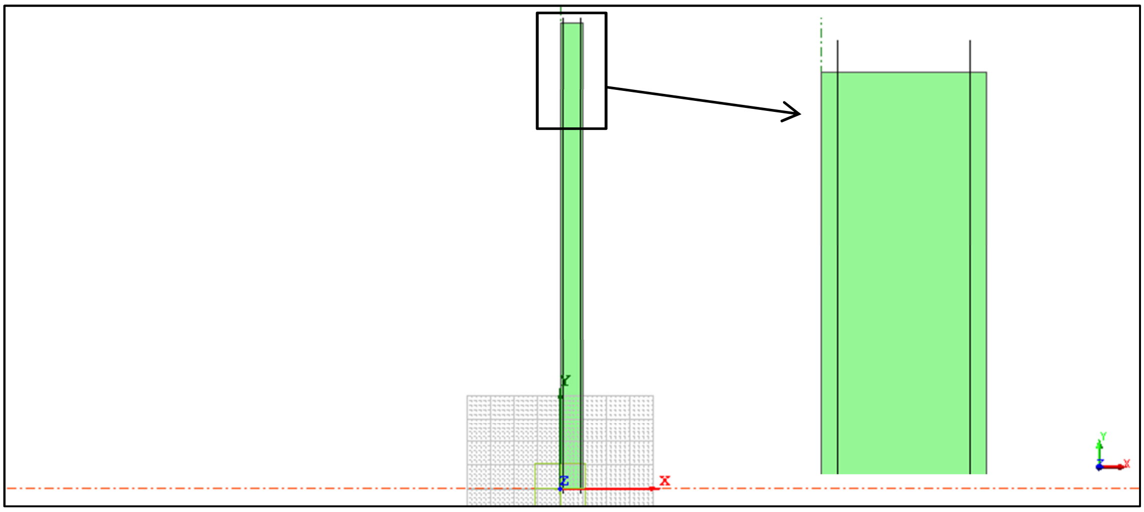


Figure 16: Creation of the reinforcement

- Meshing

First the materials must be defined. There are three types of materials and these are created with "Dummy" properties that will later be defined in the ".dat" file, which can be seen in Annex A. The properties defined in the generation of the model are geometric ones:

- Linear concrete
  - Thickness: 500mm
- Non-linear concrete
  - Thickness: 500mm
- Reinforcing steel
  - Cross sectional area: 3600 mm^2

When setting the properties of the mesh grid, the elements created measure 40mm wide and 50mm tall. Also, in the generation of these elements mid side nodes are created, to have a quadratic element that can undertake non-linear analysis. The meshed concrete can be seen in the following figure.

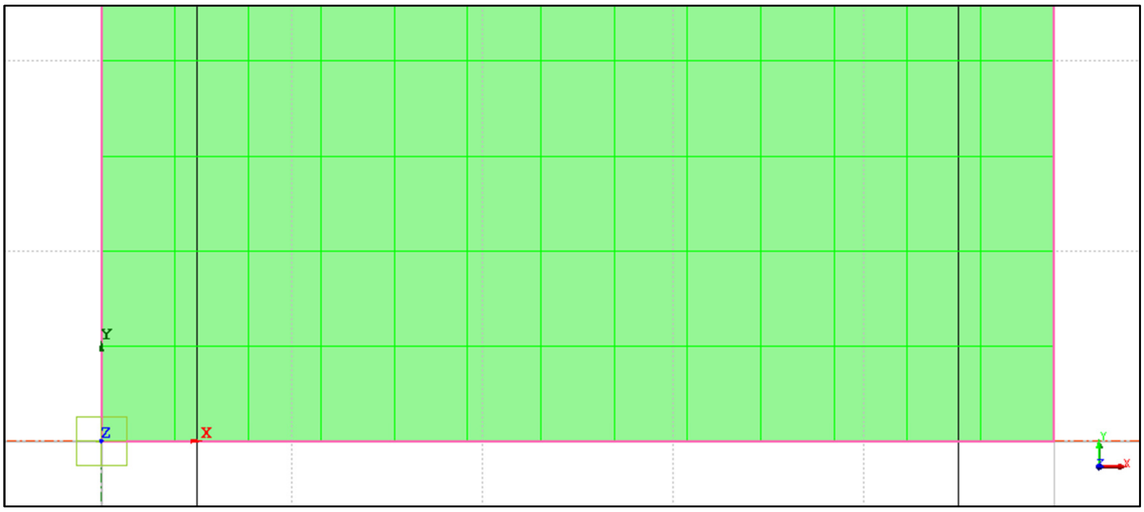


Figure 17: Meshing of the concrete

Next, the reinforcement is also meshed, as it can be seen in the following image.

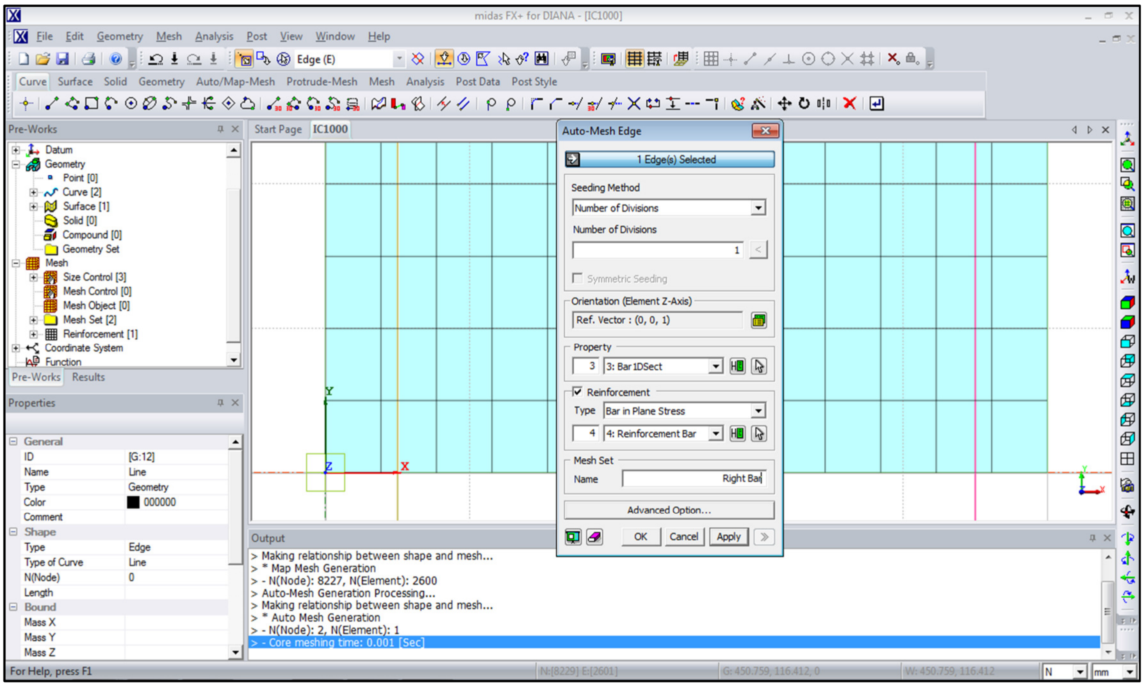


Figure 18: Meshing of the reinforcement

The last step is to change the material properties of the top elements, to linear instead of non-linear. This is done because the loads are applied directly at these elements and it ensures that there are no problems.

Figure 19: Different material properties

- Boundary conditions and loading

Once the column is properly meshed, the last step is to specify the boundary conditions and the loads. The bottom part of the column is totally constrained ant the top part of the column is free. In the next image, corresponding to the lower part of the column, the middle nodes previously created can also be seen as constrained.

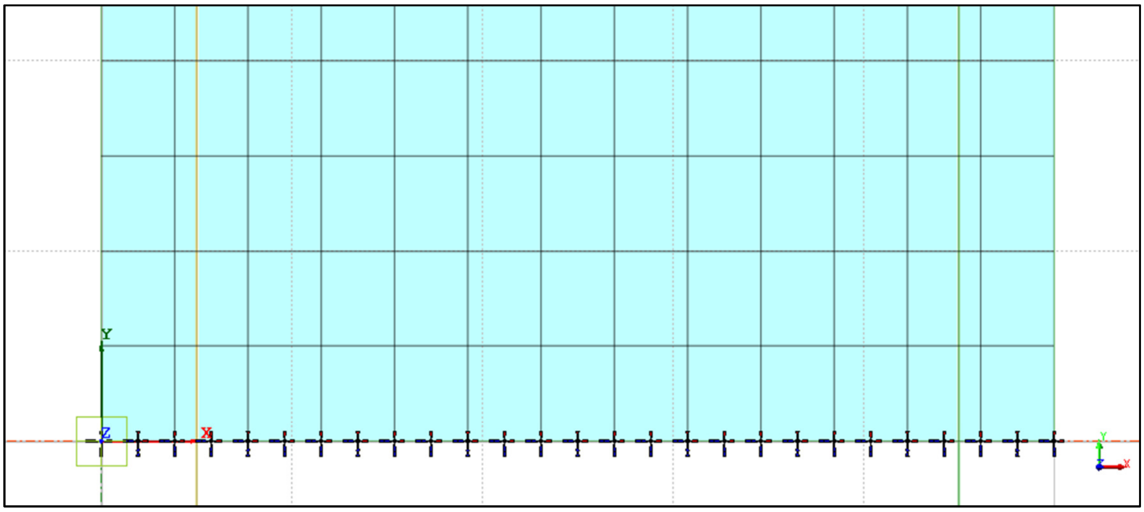


Figure 20: Bottom boundary conditions

The top load is set as a pressure of value 2N/mm in the top edge of the column. Since the width of the column is of 500mm, this gives a unitary load of 1kN that is very useful in order to translate the results as this load increases with every increment or step. The same applies to the horizontal load, which is set directly as a force of 1kN at the top left node. The objective of this load is to take the column to a state of maximum cross sectional resistance, and thus insuring that the column is at its maximum capacity as the columns calculated through the simplified methods of Eurocode-2 are.

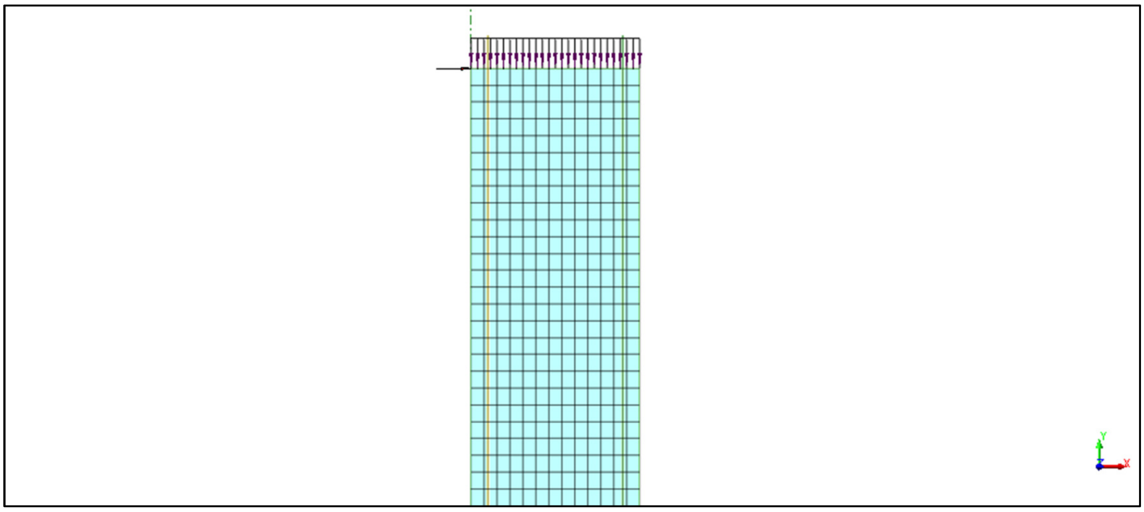


Figure 21: Top loads

After this last step, the column is finalized. The next image shows all the modeled columns in descending order of 1, 2.5, 5, 7.5 and 9 meters.

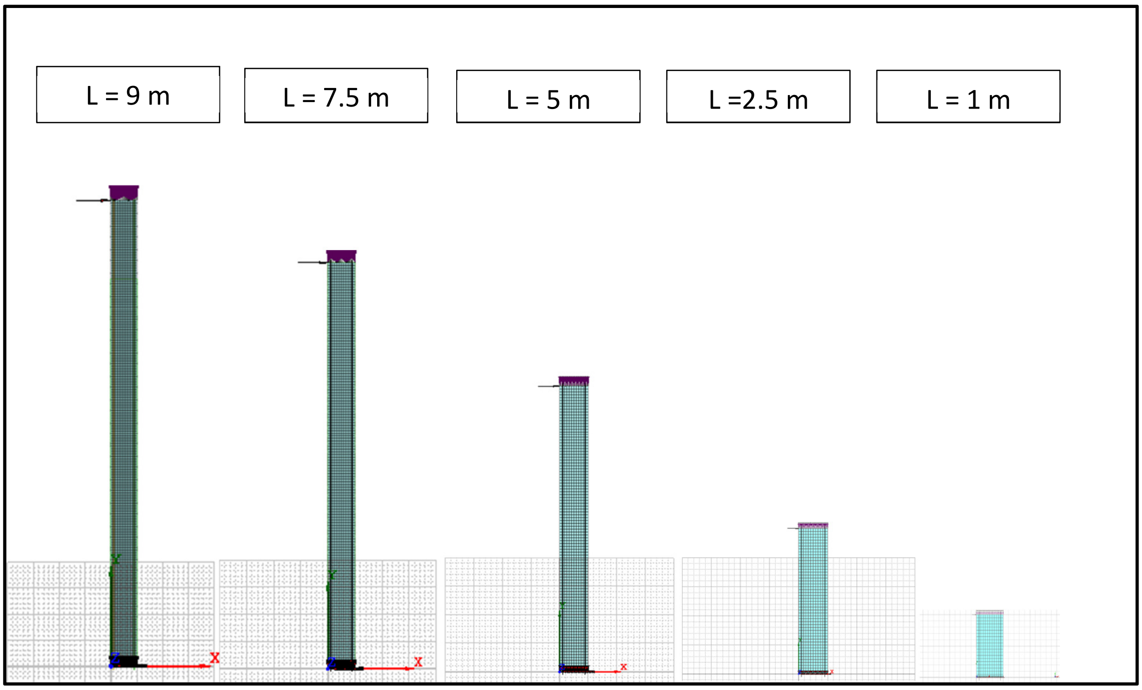


Figure 22: Columns of ranging lengths modeled for the analysis

7.2- Analysis

The analysis process is exemplified for the largest column of all, with a height of 9 meters.

The first buckling mode can be seen in figure 23. As it can be observed, the maximum displacement is unitary so when the loading starts the initial imperfection will be entered as a factor multiplying this buckling mode i.e, $l_0/400$, which is the recommended value in Eurocode-2 under paragraph *5.2(7)*. This way, the effect of imperfections is taken into account.

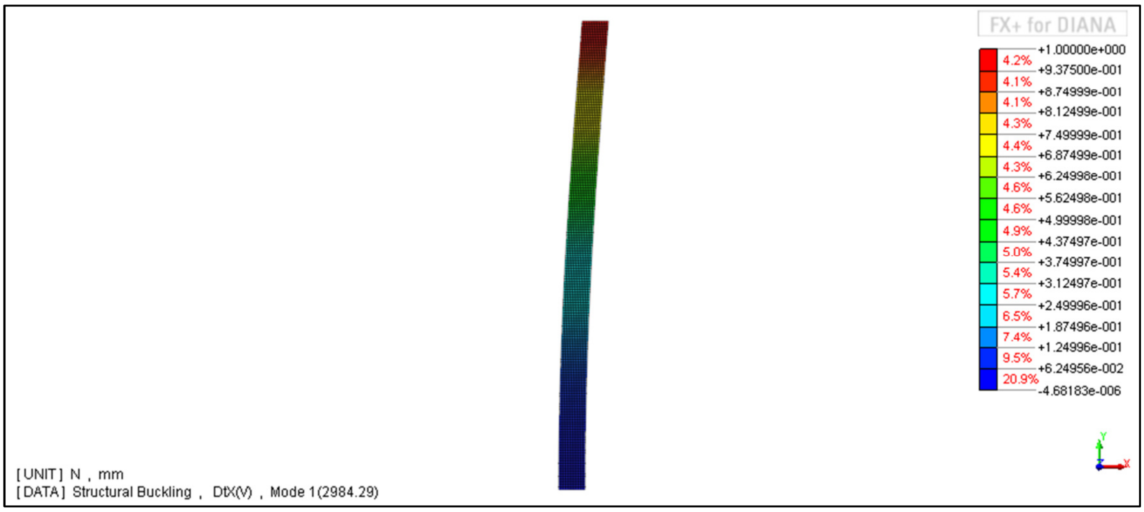


Figure 23: First Buckling mode

The loads are applied gradually with every load step, which first multiplies the vertical load set as 1kN by every load step factor until this factor reaches 1670. Then the full 1670·1kN are being applied.

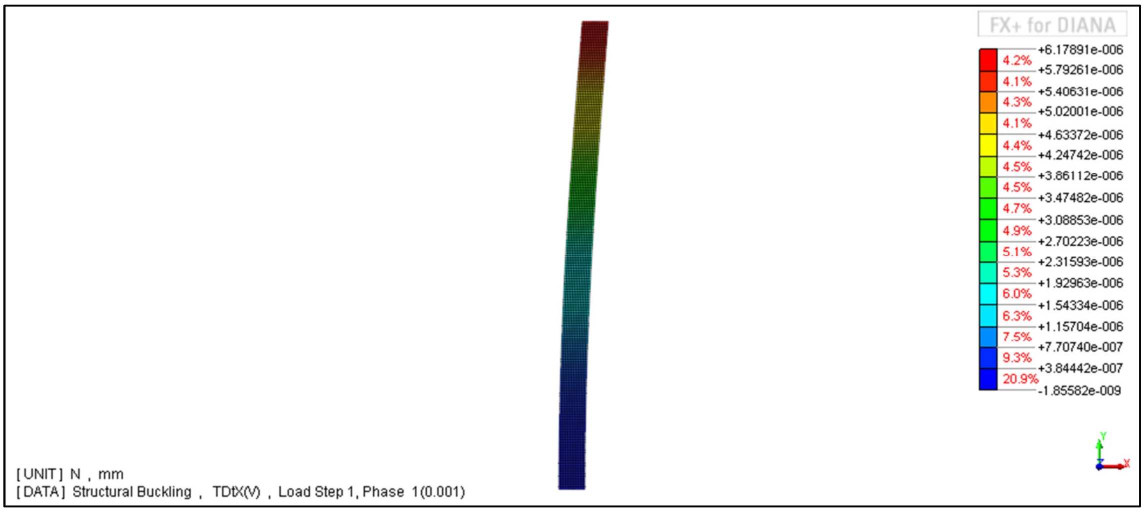


Figure 24: Load step 1 (vertical load)

Next, the horizontal force is applied, but this time the load step factors keep increasing without a known limit. This way the column is submitted to the maximum force it can withstand before it starts to fail and force starts decreasing again. This maximum load step factor is registered together with the total horizontal displacement. The following images represent the evolution of the total horizontal displacements (tdx) until the maximum load that can be applied is observed.

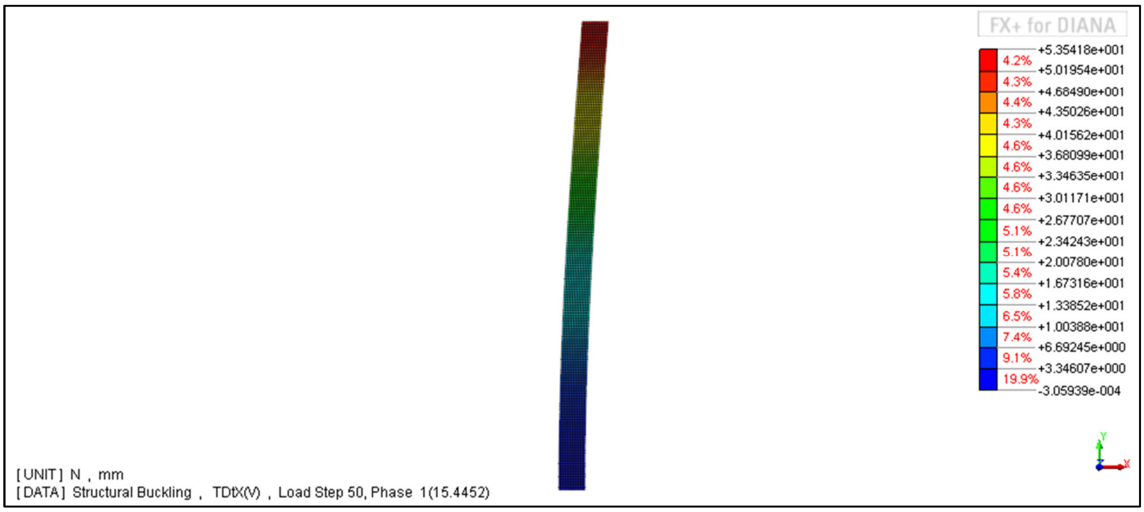


Figure 25: Load Step 50 (horizontal force)

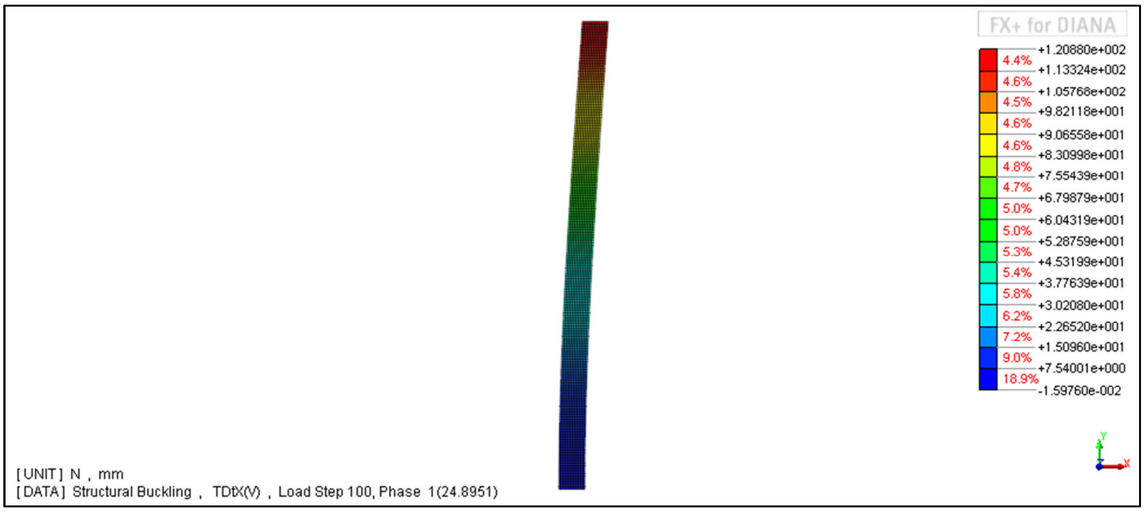


Figure 26: Load Step 100 (horizontal force)

The next figure is the one corresponding to the maximum horizontal force, its total displacement in x is registered in order to calculate the second order moment given by the vertical load (1670 kN).

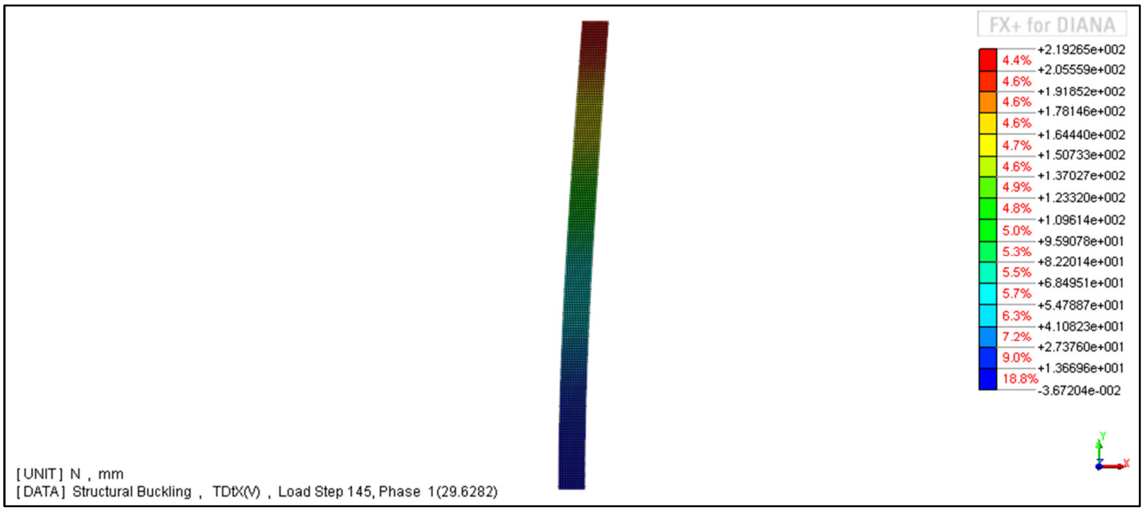


Figure 27: Load Step 145 corresponding to the maximum load step factor applied (horizontal force)

7.3- Results of Diana tests

The same analysis has been performed for all of the five columns, enabling to record the column behavior as the loads increase and the column starts to fail. As explained, the vertical load increases and the column starts to deflect with the shape of the first buckling mode, until a point where the total vertical load remains of 1670kN. At this point, the horizontal force starts pushing the column and adding to the first order moment so the capacity of the column to withstand a first and second order moment is tested.

It is important to remember that the methodology followed to calculate the second order effects following the simplified methods in Eurocode-2, consisted in calculating the maximum first order and second order moments possible given a certain resistance of the cross section. In the created model, the horizontal force tests the column until it fails whilst still applying the vertical load, so the first and second order moments grow until they equal the maximum moment that the section can withstand. Once the value of the horizontal force achieves its maximum, it starts to decrease as the column cannot withstand any more load increase due to its failure.

At the maximum point, the total horizontal displacement (tdx) gives the eccentricity to calculate the second order moment. This parameter is the equivalent to "e_2" in the Nominal Curvature method and equivalent parameter could also be calculated in the Nominal Stiffness method if the second order moment and the vertical load are known.

For the five different columns, that represent columns with various slenderness ratios as the height is different, the second order moment is calculated recording the displacement of the head of the column. Then the second order moment is calculated since the axial load N_{Ed} is known.

The result is a graph that can be compared to those obtained in the parameter study (see chapter 6), which represents the evolution of the second order moment as the slenderness ratio increases. The model created by finite elements is identical in geometry and properties to the previously studied "case 1" column, with a square section and reinforcement distributed in opposite sides. Since the inclusion of creep in certain models is still experimental in Diana, the results have been obtained without taking into account this effect and thus, are compared with the calculations of the Simplified Methods with an effective creep value φ_{ef} of 0.

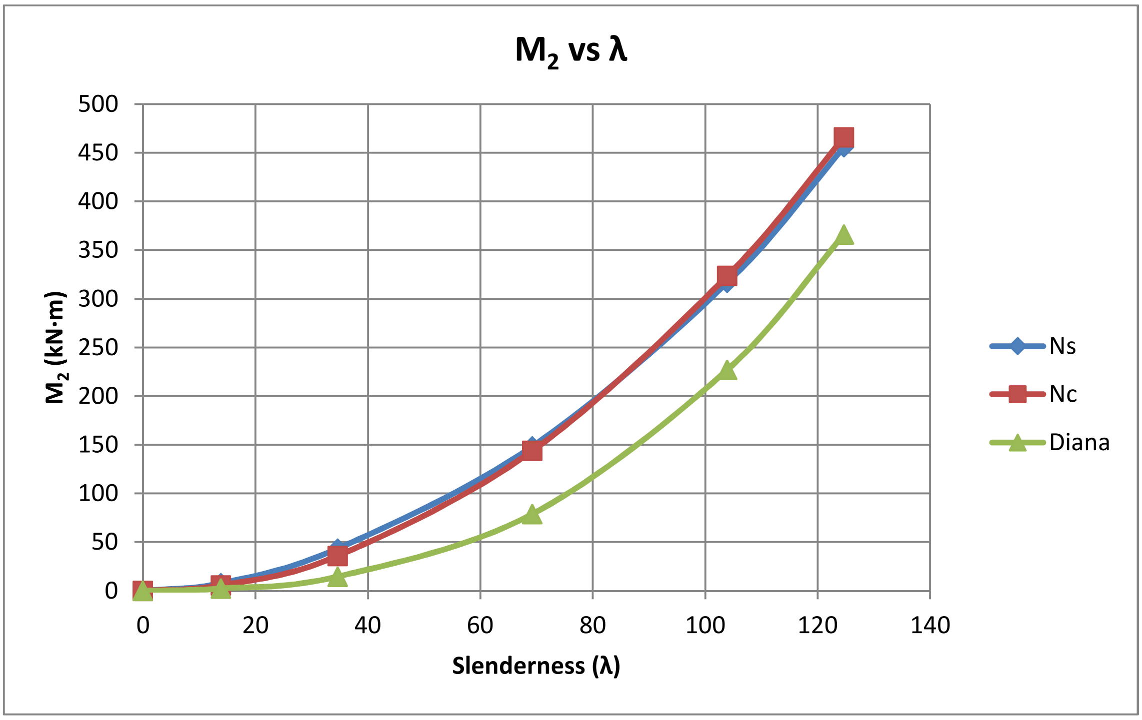


Figure 28: M_2 vs λ comparison between the Simplified Methods and the Diana simulations

It is interesting to observe how the tendency of the Diana curve is very similar to the ones observed in both Nominal Stiffness and Nominal Curvature Methods. The Diana model follows the same curved shape, with M_2 increasing exponentially giving a higher moment for the more slender columns.

The Nominal Stiffness and Nominal Curvature values are always higher throughout the whole graph, showing that the Simplified Methods are giving more conservative values on the safer side. All the safety factors and conservative approximations contained in Eurocode-2 methods are not included in the Diana models or analysis, so it is reasonable that the second order moments are lower than the calculated ones through the spreadsheets.

In conclusion, the second order effects calculated through the simplified methods give quite realistic values when comparing them with a finite element analysis.

8- Concluding remarks

The main focus of this report was to study the methodology found in Eurocode-2 for the second order effects in columns and, in particular, the simplified methods of Nominal Stiffness and Nominal curvature.

A study of the influence of creep (φ_{ef}) and slenderness (λ) in the second order moment has been done. For the Nominal Stiffness method, creep has an ascending influence as the slenderness ratio increases. For the Nominal Curvature method, it has been determined that creep has an influence under a certain threshold value of λ, which depends on the characteristic compressive strength f_{ck} and that this influence first increases and then decreases until this slenderness value is achieved.

Remembering that s_0 is the rectangular section with reinforcement laid in opposite sides, s_1 is the rectangular section with uniformly laid reinforcement and that s_2 is the circular section with uniformly laid reinforcement, it has also been determined that for the Nominal Stiffness method creep amplifies the second order moment more in s_1, followed closely by s_2 and at last by s_0 with a considerable difference. On the other hand, in the Nominal Curvature method creep amplifies more the second order moment for s_1 followed by s_0 and at last by s_2.

It can also be concluded that in general both simplified methods give similar results for second order moments, but when including creep in the calculations the Nominal Stiffness method gives slightly higher M_2 values for columns with higher slenderness parameters than the slenderness limit. This fact is accentuated in cases 3 and 4, corresponding to columns with pinned-fixed and fixed-fixed end constraints, due to effect of the parameter β that takes into account the distribution of the first and second order moments.

When comparing the evolution of the second order moment depending on the type of section, for all four studied types of columns with different constraints s_1 gives the higher values followed by s_2 and s_0.

The effect of adding more area of reinforcement was studied, with the conclusion that in very slender columns the added reinforcement doesn't allow to neglect second order effects, but for columns just over the slenderness limit it could be a valid option. On the other hand, adding more area of concrete reduces second order effects substantially. For cases in which the slenderness ratio is too high to achieve its lower limit by only adding reinforcement, increasing the cross sectional area of concrete proves to be a good option, both reducing the slenderness ratio and increasing the slenderness limit.

Great emphasis has been given to the creation of a tool for the calculation of the Simplified Methods. It was important that the spreadsheet enabled to input the initial values and obtain as outputs the first and second order moments, whilst taking into account in an automatic way all the different constraints and conditions mentioned in the Eurocode-2. This objective has been fulfilled, and the user can easily customize the calculation choosing from three types of section and from different types of constraints through the effective length, or even input their own.

In order to try and assess the behavior of the M_2 vs λ curves obtained through the simplified methods, five columns with ranging lengths of 1, 2.5, 5, 7.5 and 9 meters have been modeled and analyzed through the FEM program Diana. The curve followed by the simulations is similar to the ones obtained through the spreadsheet calculations and enables to contrast the validity of the spreadsheets calculations. The results show that the simplified methods give slightly higher second order moments than in the finite element simulations, which indicates that the simplified methods of Eurocode-2 tend to give more safe and conservative values for the second order effects.

9- Annex A: Diana *.dcf and *.dat files

-Dcf files: General Code (*By Ir.Geoffrey Decan, University of Gent, 7500mm Column*)

```
*FILOS
 INITIA
*INPUT
*PHASE
*EULER
  BEGIN EIGEN
   STABIL LOAD=1
   EXECUT NMODES=5                    *3
   IMPERF BUCKLI MODE=1 MAX=37.5
   BEGIN OUTPUT
     FXPLUS
     DISPLA
   END OUTPUT
  END EIGEN
*NONLIN
  MODEL OFF
  BEGIN TYPE
   PHYSIC ON
   GEOMET ON
  END TYPE
  BEGIN OUTPUT
   FXPLUS
   DISPLA  TOTAL  TRANSL  GLOBAL
   STRESS  TOTAL  CAUCHY
   STRAIN  TOTAL
   FORCE   REACTI
   STRAIN  CRACK GREEN
  END OUTPUT
:  BEGIN OUTPUT
:    TABULA
:    STRAIN TOTAL
:    STRESS TOTAL
:    DISPLA TOTAL
:  END OUTPUT
  BEGIN EXECUT
    BEGIN LOAD                                      *1
      STEPS EXPLIC SIZE 0.001 0.999  9 20(3) 50(8) 100(12)
: total step:                 0.001 1.000 10 70    470   1670
      LOADNR=1
    END LOAD
    BEGIN ITERAT
      METHOD NEWTON REGULA
      MAXITE=21
      BEGIN CONVER
        ENERGY CONTIN  TOLCON=1E-03
        FORCE CONTIN TOLCON=1E-02
        DISPLA OFF
      END CONVER
    END ITERAT
  END EXECUT
```

```
 BEGIN EXECUT
   BEGIN LOAD                              *2.1
     STEPS EXPLIC SIZE 0.01 0.09 0.1(9) 4 5(7)
: total step:              0.01 0.1   1        5 40
     LOADNR=2
   END LOAD
   BEGIN ITERAT
     METHOD NEWTON REGULA
     MAXITE=21
     BEGIN CONVER
       ENERGY CONTIN  TOLCON=1E-03
       FORCE CONTIN TOLCON=1E-02
       DISPLA OFF
     END CONVER
   END ITERAT
 END EXECUT
 BEGIN EXECUT
   BEGIN LOAD
    LOADNR 2
    BEGIN STEPS
      BEGIN ENERGY
        CONTIN        *2.2
        INISIZ 2
        MAXSIZ 5
        MINSIZ 0.1
        NSTEPS 300
      END ENERGY
    END STEPS
   END LOAD
   BEGIN ITERAT
     METHOD NEWTON REGULA
     MAXITE=21
     BEGIN CONVER
       ENERGY CONTIN  TOLCON=1E-03
       FORCE CONTIN TOLCON=1E-02
       DISPLA OFF
     END CONVER
   END ITERAT
 END EXECUT
*END
```

**1&*2.1,*2.2: Step sizes for axial and horizontal loads, respectively.*3: Initial Imperfection*

- Modifications in the Dcf files: particular step size code for columns of 1000, 2500, 5000 and 9000 mm hight, substituting *1, *2.1, *2.2 and *3.

L=1000mm

***1**

STEPS EXPLIC SIZE 0.001 0.999 9 20(3) 50(8) 100(12)

: total step: 0.001 1.000 10 70 470 1670

***2.1**

STEPS EXPLIC SIZE 0.01 0.09 0.1(9) 4 10(8)

: total step: 0.01 0.1 1 5 85

***2.2**

INISIZ 5

MAXSIZ 50

MINSIZ 0.5

NSTEPS 500

***3**

IMPERF BUCKLI MODE=1 MAX=5

L=2500mm

***1**

STEPS EXPLIC SIZE 0.001 0.999 9 20(3) 50(8) 100(12)

: total step: 0.001 1.000 10 70 470 1670

***2.1**

STEPS EXPLIC SIZE 0.01 0.09 0.1(9) 4 5(7) 8(10)

: total step: 0.01 0.1 1 5 40 120

***2.2**

INISIZ 5

MAXSIZ 50

MINSIZ 0.5

NSTEPS 500

***3**

IMPERF BUCKLI MODE=1 MAX=12.5

L=5000

***1**

STEPS EXPLIC SIZE 0.001 0.999 9 20(3) 50(8) 100(12)

: total step: 0.001 1.000 10 70 470 1670

***2.1**

STEPS EXPLIC SIZE 0.01 0.09 0.1(9) 4 5(7)

: total step: 0.01 0.1 1 5 40

***2.2**

INISIZ 5

MAXSIZ 5

MINSIZ 0.5

NSTEPS 500

***3**

IMPERF BUCKLI MODE=1 MAX=25

L=9000

***1**

STEPS EXPLIC SIZE 0.001 0.999 9 20(3) 50(8) 100(12)

: total step: 0.001 1.000 10 70 470 1670

***2.1**

STEPS EXPLIC SIZE 0.01 0.09 0.1(9) 1

: total step: 0.01 0.1 1 2

***2.2**

INISIZ 5

MAXSIZ 50

MINSIZ 0.5

NSTEPS 1000

***3**

IMPERF BUCKLI MODE=1 MAX=45

- Dat file: 'materi' heather with properties of the materials (*By Ir.Geoffrey Decan, University of Gent*)

```
'MATERI'
  1 NAME "Conc"                 <- Non-linear properties of concrete
    YOUNG   2.58330E+004
    POISON  1.50000E-001
    TOTCRK  ROTATE
    TENCRV  HORDYK
    TENSTR  1.80000E+000
    GF1     1.36979E-001
    CRACKB  100.00
    COMCRV  THOREN
    COMSTR  1.41700E+001
    BETA    1.00000E-002
  2 NAME "ConcLin"              <- Linear properties of concrete
    YOUNG   2.58333E+004
    POISON  1.50000E-001
  3 NAME "Steel"                <- Properties of the reinforcement
    YOUNG   2.05000E+005
    POISON  3.00000E-001
    YIELD   VMISES
    YLDVAL  3.47826E+002
```

10- Annex B: List of figures and tables

Figures

Tables

11- Annex C: Contents of the CD

1_Spreadsheet for the calculation of the simplified methods (Excel)

2_Results with tables and graphs for the parameter study

3_Results with tables and graphs for the finite element simulation

4_ Files for all columns: *.dcf, *.dat, *.dmb, *.dpb and *.fdb

5_Digital version of the Thesis

12- Bibliography

- *Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings*. EN 1992-1-1 (2004).
- *Second order effects, Background to chapters 5.8, 5.9 and Annex H in EN 1992-1-1*. Bo Westerberg. (May 2002).
- *Gewapend Beton Volume I*. Prof.ir.H.Lambotte, Dr.Ir.D.Van Nieuwenburg. Rijksuniversiteit Gent. (1990).
- *Hormigón armado, 14 edición*. Pedro Jiménez Montoya, Álvaro García Meseguer y Francisco Morán Cabré. Editorial Gustavo Gili, S.A, Barcelona. (2000).
- *The use of Diana/FX+/AutoFEM package for the (probabilistic) structural analysis of concrete elements*. Ir.Geoffrey Decan. Magnel Laboratory for Concrete Research, Department of structural Engineering, Universiteit Gent. (March 2011).
- *Hoofdstuk 11, Bezwijkgrenstoestanden beïnvloed door tweede-orde effecten (knik en kip)*. Course notes by Prof.dr.ir. Luc Taerwe. (2011).