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M. Tech. in Communication Systems  
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**ENERGY EFFICIENT 5G NETWORKS**

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Wireless Networks Group (WNG)



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# ABSTRACT

For a greener tomorrow, an important step today will be the energy efficient designs in any aspect. The 5G networks which is most awaited by all, though proposes better data rates, but also speaks about the energy efficiency in its agenda.

A broad study of different techniques for energy efficiency reveals that beamforming plays a crucial role. Thus leading to this thesis mainly concentrating on the aspects of beamforming. Beamforming though has been into existence for more than over a decade, continuous improvements in the methodology keeps it ahead of many other technologies used for the common goal. This thesis work is done with the concept called multi-beam beamforming. An interesting concept of amplitude tapering is tailed to keep a check on the magnitude of power supplied at the antenna terminals. Using these, the thesis compares the gain values of both the desired and undesired users which will aid in estimating the amount of power required for covering a set of users using different tapering methods. This works also includes the effect of increasing number of antennas and the users and the effect on the gain values for both desired and undesired users. This develops a scope to introduce a new metric called “potential power improvement” for different tapering methods. Also, a framework has been developed to expand and evaluate the cases mentioned above to a multi-cell scenario in both general antenna configuration and Massive MIMO configuration.

**Keywords:** Beamforming, Multi-cell, Amplitude Tapering, Massive MIMO, Potential Power Improvement, multi-beam beamforming.



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Signature

(SAIKRISHA KARTHIK MOLLURU)



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# 1 Introduction

*This chapter introduces the expectations from 5G, the current happenings in energy efficient networks and gives an introduction to beamforming. The structure of this thesis is given in short at the end of the chapter.*

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- [1.2](#) Structure

## 1.1 Overview

5G networks is the most awaited mobile technology development in the next 5 years. A lot of talks, proposals and ideas have been put forward in many conferences, congresses and events across the globe regarding the requirements and deployment of this technology. For most of the mobile users, the main requirement from an improvised version of the mobile communication technology is the “data rate”. This has grabbed the attention of many researchers to work on the aspects improvising the data rate. The expected data rate is to download a 1 GB of data in less than 1 second. This being one facet, few of the other aspects are the energy efficiency, better coverage and internet of things.

Communication giants like NTT DoCoMo are working very intensively in realising the aforementioned expectations. A recent article [1] on the internet reads that NTT DoCoMo has already started its trails for 5G and it is planning to deploy it by the year 2020. Fujitsu is working on coordinated scheduling for “Super dense base stations” using RRH (Remote radio heads). The NEC Corporation of Japan is working on time domain beamforming with multiple antennas in 5 GHz frequency.

As the thesis title mentions, this work mainly concentrates on the methods to reduce the energy consumption at the base stations. Though the work on energy efficiency is being done many researchers in both industry and academia, the technology till date has mainly concentrated on the data rates. But, energy also becoming an important parameter for the human survival, plays a vital role in the enhancement of technology. Realizing this, industries are working on moving to renewable energy sources like solar or wind for powering the base stations. The results are promising and seem to reduce the stress on non-renewable.

In the telecom sector, there are two aspects that concern the mobile operators, the CAPEX and the OPEX. The CAPEX (Capital Expenditure) is the initial price that is required for setting up the whole network, the installation costs. While the OPEX (Operation Expenditure) is the amount of price that the operators have to pay for operating the networks, after installation. For the case of the renewable energy resource being used for powering the network operations, the CAPEX is high, but the OPEX is very minimal except the servicing of the components periodically. The other way round, if the non-renewable resources are used, the CAPEX is comparatively low, but the price for the OPEX is very high as the power required will be heavy and with the increasing power rates across the globe, it becomes very difficult for the mobile operators to keep the networks functioning while offering the services at low prices to customers.

Most consider, it is a trade-off between two, but once effectively planned and implemented, both of them can be reduced to a great extent. A simple scenario in support of this statement is that if the power consumption at the base station (at hardware level) is reduced by 10 times, then the investment needed for the CAPEX will also reduce for example, the amount of solar panels required (in the case of solar power) will be greatly reduced. This is one such example, where the CAPEX and OPEX can be effectively, kept under check. 5G though has a lot to offer for the end users, also needs to offer for the service providers for them provide the services to the customers at an affordable price.

When CAPEX tells about the installation costs, it deals about the installation costs for the base stations, so a greater number of base stations will highly increase the CAPEX. But an effective network planning like the usage of small cells, can create a huge difference and has a high impact on the energy efficiency too. This was the topic of interest in the recent Mobile World Congress, 2014 that happened in Barcelona, Spain. The small cells are of the greatest interest to the industry currently. Plans have been proposed to replace the micro cells with small cells and then work them in sync with the macro cells. There has been an extensive research on this and still there is a wide scope for the improvement when the network planning is done along with the small cells.

After an effective research on the areas of the maximum energy consumption, the base station is consuming nearly 57 % of the power in the complete telecom network operation [2]. The urge for efficient algorithms at the base stations for operations is much needed at the hour. The widely used method for saving power at the base stations is the Beamforming technique. This is effective because, the amount of power radiated can be concentrated in a required direction thereby reducing the amount of power being radiated in the undesired direction.

Massive MIMO [3] is another area where a lot of research is being concentrated on, wherein the number of antennas are very large when compared to the number of users that the base station is serving. As the antennas increase, the amount of power consumed will greatly reduce. It is definitely true that the initial cost of installation of Massive MIMO will be very expensive as the hardware requirement is very high when compared to the current number of antennas that the telecom industry is using. But the energy savings by using Massive MIMO yield great results in the long run. This effectiveness of this technique will be explained in further chapters of this thesis. The Massive MIMO also uses the beamforming technique to cover the users. Thus leading to main objective of this thesis to revolve around the concepts of beamforming and making it as much efficient as possible with different heuristic algorithms.

Beamforming, also called spatial filtering, is a technique used by sensor arrays for a directional signal transmission or reception. Beamforming can be implemented in many applications like SONAR, RADAR, seismology, biomedicine and radio communications which involves the separating/ spatially locating a required target. This technique can be used at either transmitting or receiving ends or both [4]. This can be visualized as a bulb transmitting light in all directions is covered by a piece of cloth which has holes on it. So, every hole has some quantum light that is coming out of it. Light coming out of these holes can be considered to be different beams.

The fundamental point about beamforming is, when multiple sensors/ transducers are present next to each other and they emit, an interference pattern is seen. It can be pictured to having many bulbs in a room and when they all emit at the same time, portions of light of one bulb overlaps with the others. Now, doing some small modifications like increasing the distance between each of the bulb, or change the delay of glow for each bulb, the interference pattern obtained can be used for an advantage. In particular, when the bulbs are placed intelligently at an angle, then a greater part of the energy goes out in that angular direction. This reduces the interferences and the required location gets appropriately illuminated. In radio communications, beamforming can be defined as follows “It is the combination of radio signals from a set of small omnidirectional antennas to give a larger directional antenna” [5].

There are different possible applications of the beamforming technique, with different impacts also in terms of achievable improvements in energy efficiency. The two most common options are namely “per-cell” or reconfigurable beamforming and “per-user” or adaptive beamforming [6].

The idea behind these two approaches, allow not only addressing beamforming on a “per-cell” or “per-user basis, but also at two different timeframes. The concept of the reconfigurable antenna systems addressed herein allows to adapt antenna parameters to traffic conditions in a timeframe of hours or longer, hence, taking care of the non-uniform spatial traffic distribution by forming appropriate cells. Adaptive beamforming allows to continuously weight antenna elements to form a beam into the direction of interest (a user or a group of users), therefore, improving the spatial filtering within each cell. Hence, the combination of these two techniques, with different time granularities, will allow to significantly reducing BS power consumption.

The principle objective of this thesis is to generate beamformers for the users, and ensure the beam is as thin as possible along with casting the beam to the exact distance of the user instead of generic beamformer in a particular direction. For this, the multi-beam beamforming is used, which casts beams to the users in parallel time along and ensuring the farthest user gets the maximum amplitude of the beam while the users nearby will be covered with a smaller amplitude thereby reducing the amount of power consumed. It is important to note that, this multi-beam beamforming is different from the multicast beamforming which already exists in the literature. An important consideration for the inclusion and testing with the case of Massive MIMO has also been carried out. The whole work manifests itself with changing the two parameters that is number of users and antennas.

## 1.2 Structure

This thesis is structured into 6 chapters. The second chapter gives a deeper insight into the technical concepts of beamforming like array factor, types of arrays like broadside and end fire arrays. It deals with a few more concepts like Broadside and end fire arrays, different tapering types and finally ends with angle of arrival estimation.

The third chapter is relatively smaller one with an overview to the optimization techniques with parameters of beamforming like tapering types, element spacing, and number of elements. It deals with state-of-the-art technologies involving the optimization of the beam parameters.

The fourth chapter starts with the beamforming implementation using MATLAB. It deals with the implementation with a toolbox called “Electromagnetic waves and antennas” toolbox. A sample program is also given to demonstrate a simple case of beamforming using MATLAB. After this, the beam steering is dealt which tells about turning or steering the beam towards a desired direction. The third concept introduced in this chapter is the multi beam beamforming, which is the main beamforming type used for all scenarios in this thesis.

The mobile user generation and plotting them accurately is very important for making the test equivalent to a real time scenario. For this in the fifth chapter, the QUADRIGA tool was used to generate the mobile user and the base station positions accurately marking the distances in the units of Kilo meters. The second and third part of fifth chapter elaborates a sample simulation setup, and a simple demonstration of the multi cell scenario of mobile users and base stations.

And finally, the sixth chapter gives the results of the simulations. A wide variety of cases have been dealt, where different number of antennas and number of users are evaluated for different tapering types.

## 2 Basic aspects on Beamforming

*This chapter takes through the deeper into array factor, Broadside and End fire arrays, and non-uniform amplitude Arrays*

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## 2.1 Signals and systems explanation for beamforming

Consider an array of antennas, which are Omni directional. As described earlier, beamforming is a technique by which a group of antennas act as a large antenna with better directional properties. Hence, from now on, every antenna will be called as an antenna element of the larger antenna and not an individual antenna itself. In the generic sense, the performance of any antenna array increases with the number of elements, but the drawbacks for a very large number of elements is the increased cost and size. The alignment of these antennas can be in any fashion (line, grid, etc.). It is initially assumed that all array elements (individual antennas) are identical. However, the excitation (both amplitude and phase) applied to each individual element may differ. The far field radiation from the array in a linear medium can be computed by the superposition of the EM fields generated by the array elements [7].

Consider an N element antenna array, where the input signals are given by  $X_1, X_2, X_3, X_4,$  and so on till  $X_n$ . Let the individual element weight be  $w_1, w_2, w_3, w_4 \dots w_n$  as shown in the following figure.

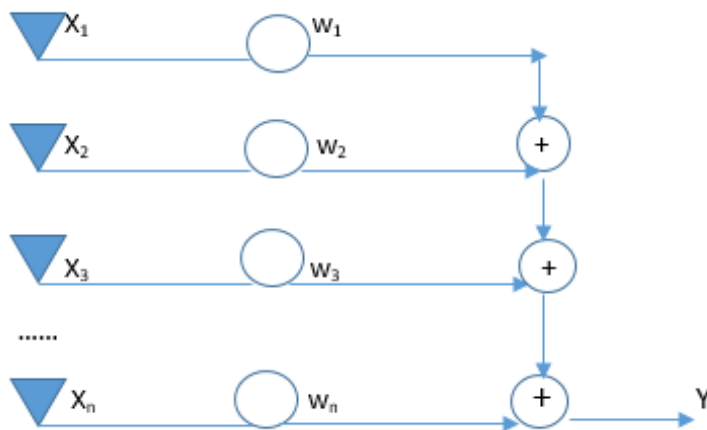


Figure 2.1 Received Signal and element weights

$$Y = \sum_{n=1}^N w_n X_n$$

Antenna elements, usually have  $(x, y) = (0, 0)$  and only the Z axis value changes and there of the distance  $\lambda/2$ . This means that the elements are half wave separated.

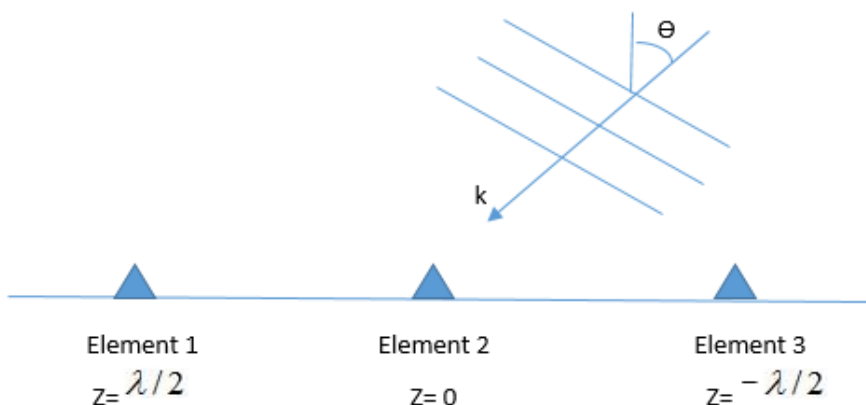


Figure 2.2 Parallel wave fronts of the infinite signal

This half wave separated antenna elements have an electric field (E) represented as following. It is assumed initially that the plane wave received is having the equal amplitude everywhere.

$$\begin{aligned} E(x, y, z) &= e^{-j(k_x x + k_y y + k_z z)} \\ &= e^{-j|k|(\sin\theta\cos\phi + \sin\theta\sin\phi + \cos\theta z)} \\ &= e^{-jkr} \end{aligned}$$

The k in the above equation represents the wave vector, which the variation of phase as a function of position, where  $|k| = \frac{2\pi}{\lambda}$

$$Y = e^{-j\frac{2\pi}{\lambda}\cos\theta z_1} + e^{-j\frac{2\pi}{\lambda}\cos\theta z_2} + e^{-j\frac{2\pi}{\lambda}\cos\theta z_3}$$

$$Y = \sum_{i=1}^3 e^{-j\frac{2\pi}{\lambda}\cos\theta z_i}$$

$$Y = \sum_{m=-1}^1 e^{-jm\pi\cos\theta}$$

The magnitude of the output (Y) as a function of the arrival angle for the antenna array will give the phased array output. This is for the case of a receiving array, the same will be applicable for a transmitting array as they are related by reciprocity. Therefore, by appropriately selecting the weights and adjusting the element positions, the phased array can be designed to receive power only from desired directions.

If the radiation pattern of each antenna element is given by  $R(\theta, \phi)$ . Then the output Y is a function of  $(\theta, \phi)$ . Hence, Y can be written as follows,

$$Y = R(\theta, \phi)w_1 e^{-jkr_1} + R(\theta, \phi)w_2 e^{-jkr_2} + \dots + R(\theta, \phi)w_N e^{-jkr_N}$$

where  $r = [x, y, z]$  of the antenna elements.

The above equation can be modified as

$$\begin{aligned} Y &= R(\theta, \phi) \sum_{i=1}^N w_i e^{-jkr_i} \\ &= R(\theta, \phi) AF \\ AF &= \sum_{i=1}^N w_i e^{-jkr_i} \end{aligned}$$

The AF is called as the **Array Factor**. The array factor is a function of the antenna element weights and their positions. The array factor is an important function, as the change in weights on this function, will allow it to change the direction of maximum transmission or reception accordingly.

Here the w, weights refer to the values, which divide the total input current coming to each antenna element by some specific ratio. This ratio of current division can be chosen based on the amount of required side lobe level suppression. More about the weights ratio will be discussed, when the array is excited with non-uniform amplitude excitation.

## 2.2 Array Factor particularised

A deeper insight into the Array factor is given as follows [8]. The field of an isotropic radiator located at the origin may be written as

$$E_{\theta} = I_0 \frac{e^{-jkr}}{4\pi r}$$

An assumption here is that elements are uniformly spaced with a distance  $d$ .

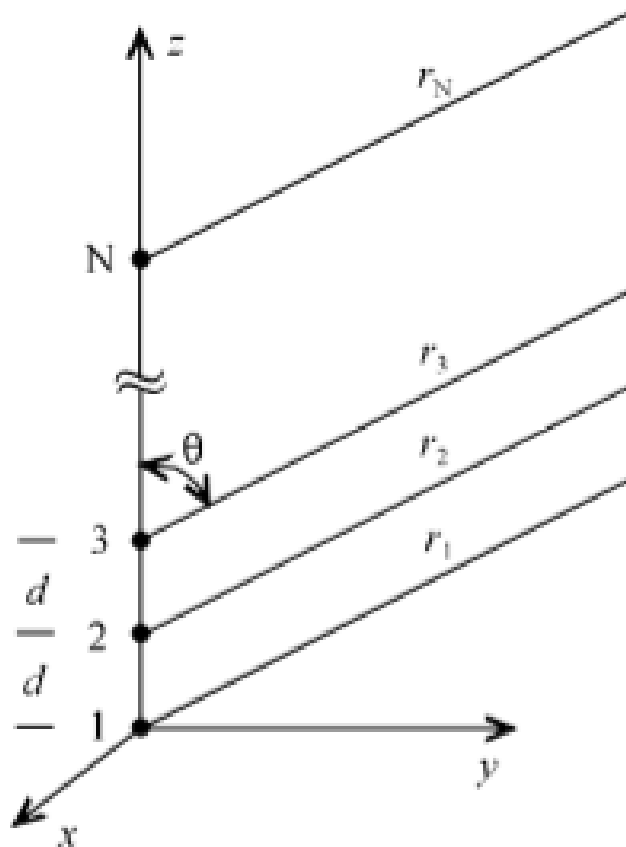


Figure 2.3 far field of the array with constant element spacing [8]

In the far field,

$$r_1 = r$$

$$r_2 \approx r - d \cos \theta$$

$$r_3 \approx r - 2d \cos \theta$$

$$r_N \approx r - (N - 1)d \cos \theta$$

The current magnitudes the array elements are assumed to be equal and the current on the array element located at the origin is used as the phase reference (zero phase) [8].

$$I_1 = I_0 \quad I_2 = I_0 e^{j\phi_2} \quad I_3 = I_0 e^{j\phi_3} \quad \dots \quad I_N = I_0 e^{j\phi_N}$$

Therefore the far fields of the individual array elements are

$$E_{\theta 1} \approx I_0 \frac{e^{-jkr}}{4\pi r} = E_0$$

$$E_{\theta_2} \approx I_o e^{j\phi_2} \frac{e^{-jk(r-d \cos \theta)}}{4\pi r} = E_o e^{j(\phi_2 + kd \cos \theta)}$$

$$E_{\theta_3} \approx I_o e^{j\phi_3} \frac{e^{-jk(r-2d \cos \theta)}}{4\pi r} = E_o e^{j(\phi_3 + 2kd \cos \theta)}$$

$$E_{\theta_N} \approx I_o e^{j\phi_N} \frac{e^{-jk(r-(N-1)d \cos \theta)}}{4\pi r} = E_o e^{j(\phi_N + (N-1)kd \cos \theta)}$$

The overall array far field is found using superposition as is given as

$$E_\theta = E_{\theta_1} + E_{\theta_2} + E_{\theta_3} + \dots + E_{\theta_N}$$

$$E_\theta = E_o [1 + e^{j(\phi_2 + 2kd \cos \theta)} + e^{j(\phi_3 + 2kd \cos \theta)} + \dots + e^{j(\phi_N + (N-1)kd \cos \theta)}]$$

$$E_\theta = E_o [AF]$$

$$\text{Where, Array Factor } AF = [1 + e^{j(\phi_2 + 2kd \cos \theta)} + e^{j(\phi_3 + 2kd \cos \theta)} + \dots + e^{j(\phi_N + (N-1)kd \cos \theta)}]$$

Let us now consider, a uniform N linear antenna array, where the spacing between elements is uniform, uniform amplitude and a linear phase progression.

$$\phi_1 = 0 \quad \phi_2 = \alpha \quad \phi_3 = 2\alpha \quad \dots \quad \phi_N = (N-1)\alpha$$

Inserting this linear phase progression into the formula for the general N- element array gives

$$AF = [1 + e^{j(\phi_2 + 2kd \cos \theta)} + e^{j(\phi_3 + 2kd \cos \theta)} + \dots + e^{j(\phi_N + (N-1)kd \cos \theta)}]$$

$$= 1 + e^{j\Psi} + e^{j2\Psi} + \dots + e^{j(N-1)\Psi} \quad (\Psi = \alpha + kd \cos \theta)$$

$$= \sum_{n=1}^N e^{j(n-1)\Psi}$$

The function is defined as the array phase function and is a function of the element spacing, phase shift, frequency and elevation angle. If the array factor is multiplied by e<sup>j</sup>, the result is

$$(AF)e^{j\Psi} = e^{j\Psi} + e^{j2\Psi} + \dots + e^{jN\Psi}$$

Subtracting the array factor from the equation above gives

$$AF(e^{j\Psi} - 1) = (e^{jN\Psi} - 1)$$

$$AF = \frac{e^{jN\Psi} - 1}{e^{j\Psi} - 1} = \frac{e^{jN\frac{\Psi}{2}} e^{jN\frac{\Psi}{2}} - e^{-jN\frac{\Psi}{2}} e^{jN\frac{\Psi}{2}}}{e^{j\frac{\Psi}{2}} e^{j\frac{\Psi}{2}} - e^{-j\frac{\Psi}{2}} e^{j\frac{\Psi}{2}}} = e^{j(N-1)\frac{\Psi}{2}} \frac{\sin(\frac{N\Psi}{2})}{\sin(\frac{\Psi}{2})}$$

The complex exponential term in the last expression of the above equation represents the phase shift of the array phase centre relative to the origin. If the position of the array is shifted so that the centre of the array is located at the origin, this phase term goes away. The array factor then becomes,

$$AF = \frac{\sin\left(\frac{N\Psi}{2}\right)}{\sin\left(\frac{\Psi}{2}\right)}$$

Some general characteristics of the array factor AF with respect to  $\Psi$ :

- (1)  $[AF]_{\max} = N$  at  $\Psi = 0$  (main lobe).
- (2) Total number of lobes =  $N-1$  (one main lobe,  $N-2$  side lobes).
- (3) Main lobe width =  $4\pi/N$ , minor lobe width =  $2\pi/N$

The array factor may be normalized so that the maximum value for any value of  $N$  is unity. The normalized array factor.

$$(AF)_n = \frac{1}{N} \frac{\sin\left(\frac{N\Psi}{2}\right)}{\sin\left(\frac{\Psi}{2}\right)}$$

The nulls of the array function are found by determining the zeros of the numerator term where the denominator is not simultaneously zero.

$$\sin\left(\frac{N\Psi}{2}\right) = 0 \rightarrow \frac{N\Psi}{2} = \pm n\pi \rightarrow \alpha + kd \cos \theta_n = \pm \frac{2n\pi}{N}$$

$$\theta_n = \cos^{-1} \left[ \frac{\lambda}{2\pi d} \left( -\alpha \pm \frac{2n\pi}{N} \right) \right] \quad n = 1, 2, 3, \dots$$

$$n \neq 0, N, 2N, 3N, \dots$$

The peaks of the array function are found by determining the zeros of the numerator term where the denominator is simultaneously zero.

$$\theta_m = \cos^{-1} \left[ \frac{\lambda}{2\pi d} \left( -\alpha \pm \frac{2m\pi}{N} \right) \right] \quad m = 0, 1, 2, 3 \dots$$

The  $m=0$  term,

$$\theta_m = \cos^{-1} \left[ \frac{\lambda\alpha}{2\pi d} \right] \quad m = 0, 1, 2, 3 \dots$$

represents the angle which makes  $\Psi = 0$  (main lobe)

### 2.3 Broadside and End Fire Arrays

Before, the steering of the beam with the phase or amplitude, we discuss two special cases of uniform linear array, where the elements can be chosen such that array pattern lies along the array axis (end-fire array) or normal to the array axis (broadside array) [8].

End-fire array main lobe at  $\theta = 0^\circ$  or  $\theta = 180^\circ$

Broadside array main lobe at  $\theta = 90^\circ$

The maximum of array factor occurs when the array phase function is zero.

$$\Psi = \alpha + kd \cos \theta = 0$$

### 2.3.1 Broadside Array

For a broadside array, in order for the above equation to be satisfied with  $\Theta = 90^\circ$ , the phase angle  $\alpha$  must be zero [8]. In other words, all elements of the array must be driven with the same phase. With  $\alpha = 0^\circ$ , the normalized array factor reduces to

$$(AF)_n = \frac{1}{N} \frac{\sin\left(\frac{N\Psi}{2}\right)}{\sin\left(\frac{\Psi}{2}\right)}$$

From the previous discussion, it's understood that output radiation pattern is the product of individual element pattern and the array factor. Now, consider the normalized element field pattern for the infinitesimal dipole given by

$$\text{Individual Element Pattern } f(\theta, \phi) = \sin \theta$$

As an example, consider a broadside array ( $\alpha = 0^\circ$ ) of seven short vertical dipoles spaced  $0.5\lambda$  apart along the z-axis.

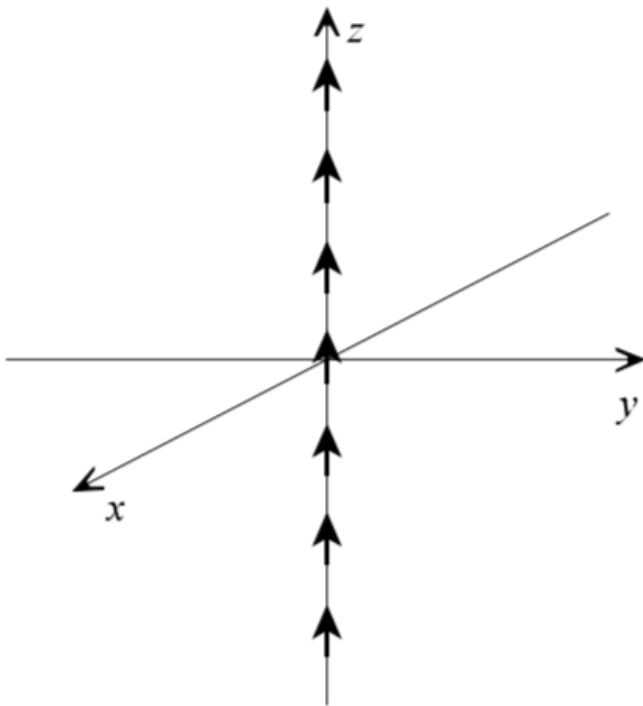


Figure 2.4 z-directed dipoles along z axis [8]

The **array factor** for the seven element array is given by

$$(AF)_n = \frac{1}{7} \frac{\sin\left(\frac{7\Psi}{2}\right)}{\sin\left(\frac{\Psi}{2}\right)}$$

The **overall normalized array pattern** of Broadside array is

$$F(\theta, \phi) = \frac{1}{7} \frac{\sin\left(\frac{7\Psi}{2}\right)}{\sin\left(\frac{\Psi}{2}\right)} \sin \theta$$

The plots for the above two terms, individual element pattern ( $\sin \theta$ ) and array factor and the array pattern is shown next

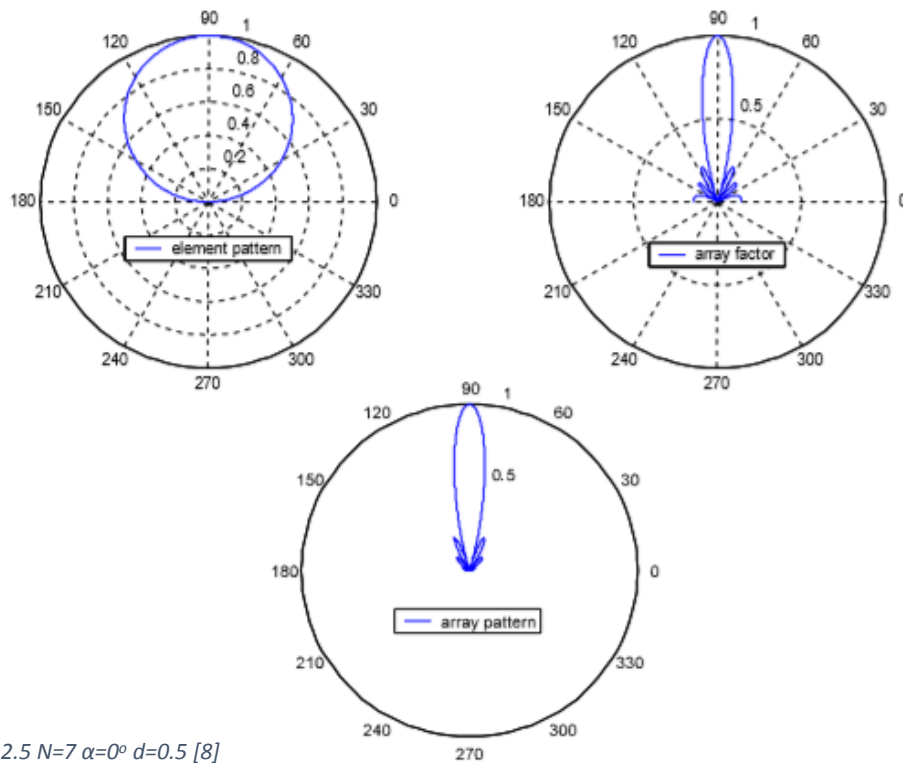


Figure 2.5  $N=7$   $\alpha=0^\circ$   $d=0.5$  [8]

For beam steering, taking the normalized general equation for broadside array given above and substituting  $d=0.25 \lambda$  and  $\alpha = 32^\circ$  in the  $\psi$  equation given above, gives the graphs as given below.

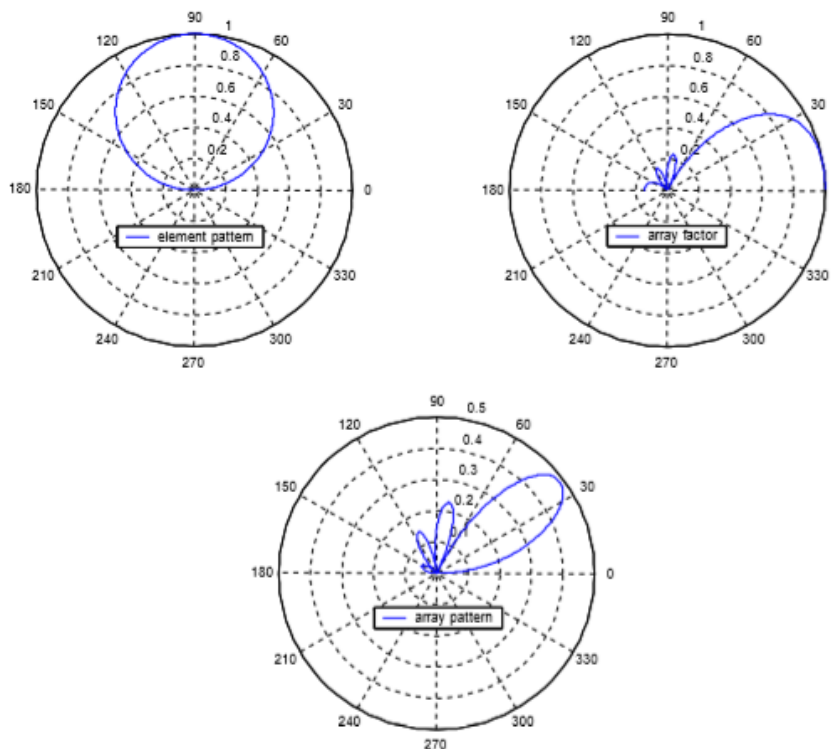


Figure 2.6  $N=7$   $\alpha=32^\circ$   $d=0.5$  [8]

### 2.3.2 End fire Array

If we consider the same array with horizontal (x-directed) short dipoles, the resulting normalized element field pattern is

$$f(\theta, \phi) = \sqrt{1 - \sin^2 \theta \cos^2 \theta}$$

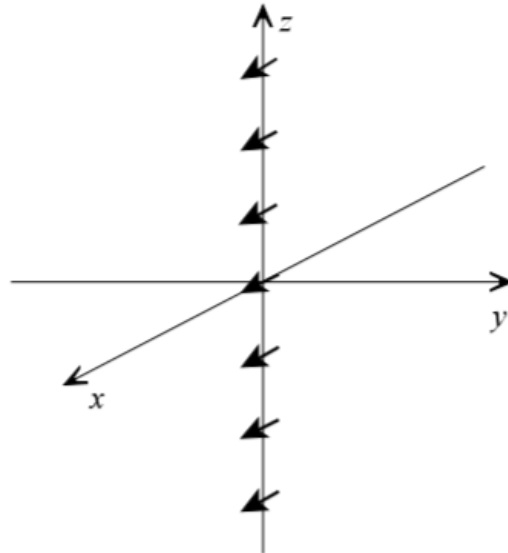


Figure 2.7 x-directed short dipoles [8]

From the above figure it is clear that, the main beam of the array of x-directed short dipoles lies along the y-axis. The nulls of the array element pattern along the x-axis prevent the array from radiating efficiently in that broadside direction. End-fire arrays may be designed to focus the main beam of the array factor along the array axis in either the  $\theta = 0^\circ$  or  $\theta = 180^\circ$  directions [8]. Given that the maximum of the array factor occurs when

$$\Psi = \alpha + kd \cos \theta = 0$$

In order for the above equation to be satisfied with  $\theta = 0^\circ$ , the phase angle  $\psi$  must be

$$\alpha = -kd$$

For  $\theta = 180^\circ$ , the phase angle  $\Psi$  must be

$$\alpha = +kd$$

Which gives

$$\Psi = kd \cos \theta \pm 1 \quad \theta = \{0^\circ \text{ or } 180^\circ\}$$

The normalized array factor for an end-fire array reduces to

$$(AF)_n = \frac{1}{N} \frac{\sin\left(\frac{Nkd}{2}(\cos \theta \pm 1)\right)}{\sin\left(\frac{kd}{2}(\cos \theta \pm 1)\right)}$$



The normalized array factor for the 7-element **end-fire array** is

$$(AF)_n = \frac{1}{7} \frac{\sin(\frac{7\pi}{4}(\cos\theta - 1))}{\sin(\frac{7\pi}{4}(\cos\theta - 1))}$$

The overall array field pattern is

$$F(\theta, \phi) = \frac{1}{7} \frac{\sin(\frac{7\pi}{4}(\cos\theta - 1))}{\sin(\frac{7\pi}{4}(\cos\theta - 1))} \sin\theta$$

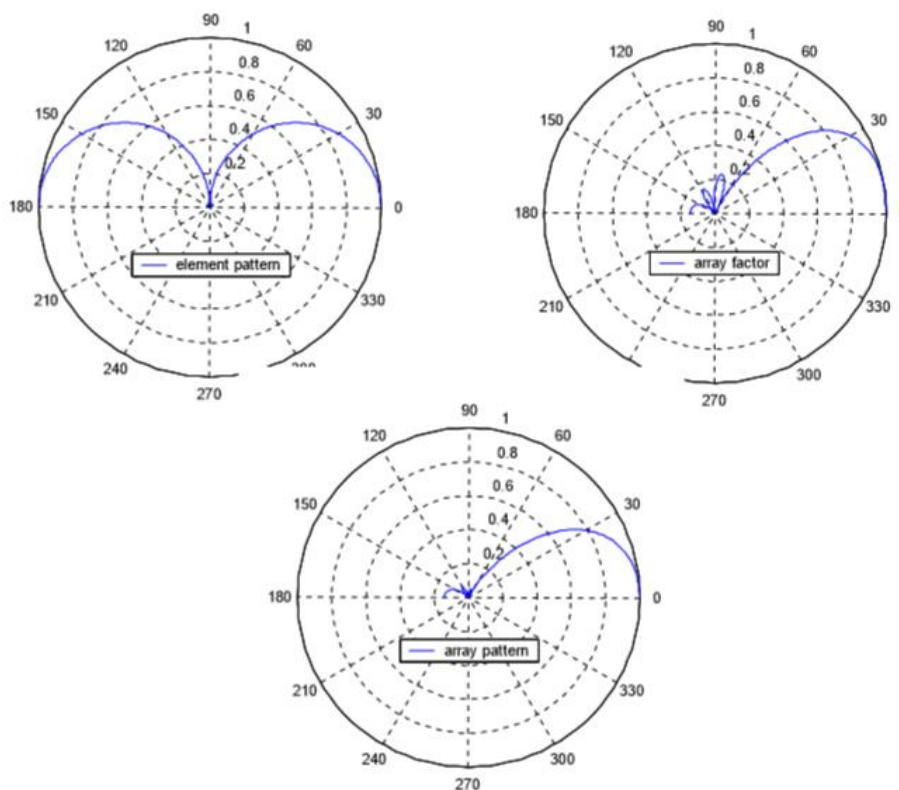


Figure 2.8 End fire array [8]

The above shown graphs are for the end fire array is generally used in case short horizontal dipoles while the broadside array is used in case of vertically aligned dipoles.

A special case of end fire array is the Hansen-Woodyard end-fire array designed for maximum directivity. The increase in the directivity of an end fire array is by approximate 1.79 times, 2.5dB but at the cost of higher side lobe levels.

We now consider the second case of non-Uniform linear arrays, where the elements are non-uniformly excited but are equally spaced arrays. For better explanation of non-uniform linear arrays, a brief introduction of different amplitude tapering methods will be dealt. It will then be followed by an elaborate explanation of the non-uniform linear arrays.

## 2.4 Amplitude Tapering Methods

Amplitude tapering (or) amplitude weighting is a technique used to produce the desirable side lobe levels. Amplitude weighting is simply a means of giving non-uniform amplitude levels to each of the antenna element. There are different methods to synthesize amplitude weights [9], they are:

1. Binomial Taper
2. Chebyshev Polynomial Taper
3. Taylor Taper
4. Bickmore - Spellmire Taper
5. Bayliss Taper

The elaborate understanding of these tapering types can be well visualized, when the amplitude weights of the antenna elements are excited non-uniformly.

### 2.4.1 Non-Uniform amplitude array

Given a two element array with equal current amplitudes and spacing [8]. This array factor is not the normalized array factor, it is the general array factor mentioned in array factor equation on page 7.

$$AF = 1 + e^{j\Psi}$$

For a broadside array with element spacing  $d$  is less than one half wavelength, the array factor has no side lobes. Now, if an array is formed with two such 2 array elements, it is given by the product of the array factors,

$$AF = (1 + e^{j\Psi})(1 + e^{j\Psi}) = 1 + 2e^{j\Psi} + e^{j2\Psi}$$

This being the square of an array factor which has no side lobes, results in this pattern which also has no side lobes. The array factor above represents a 3 element equally spaced array driven by current amplitudes with ratios of 1:2:1. Similarly, equivalent arrays with more elements may be formed as follows.

|               |  |
|---------------|--|
| 2 – element   | $AF = 1 + e^{j\Psi}$   |
| 3 – element   | $AF = (1 + e^{j\Psi})^2 = 1 + 2e^{j\Psi} + e^{j2\Psi}$               |
| 4 – element   | $AF = (1 + e^{j\Psi})^3 = 1 + 3e^{j\Psi} + 3e^{j2\Psi} + e^{j3\Psi}$ |
| $N$ – element | $AF = (1 + e^{j\Psi})^{(N-1)}$                                       |

The current coefficients of the resulting  $N$  element array take the form of a binomial series. The array is known as a binomial array.

$$AF = (1 + e^{j\Psi})^{(N-1)}$$

$$AF = 1 + (N-1)e^{j\Psi} + \frac{(N-1)(N-2)}{2!}e^{j2\Psi} + \frac{(N-1)(N-2)(N-3)}{3!}e^{j3\Psi} + \dots$$

The binomial array has the special property that the array factor has no side lobes for element spacing of  $\lambda/2$  or less. Side lobes are introduced for element spacing larger than  $\lambda/2$ .

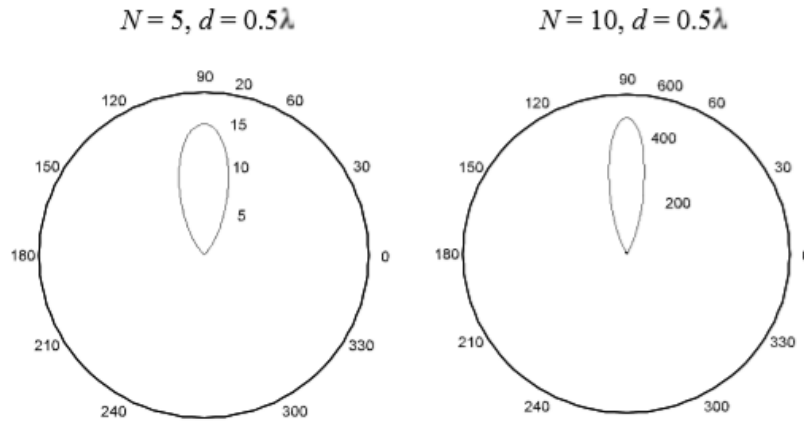


Figure 2.9 Decreasing beam width with increasing number of antenna elements [8]

### 2.4.2 Derivation of array factor in the non-uniform amplitude array

The previous section gave a little overview of the non-uniform amplitude array and how the binomial tapering is done.

Let's take an antenna array with P elements, the array factor now will be derived for a non-uniform current amplitudes. The amplitude distribution is assumed to be symmetric about the origin [8]. Consider a case where P is odd.

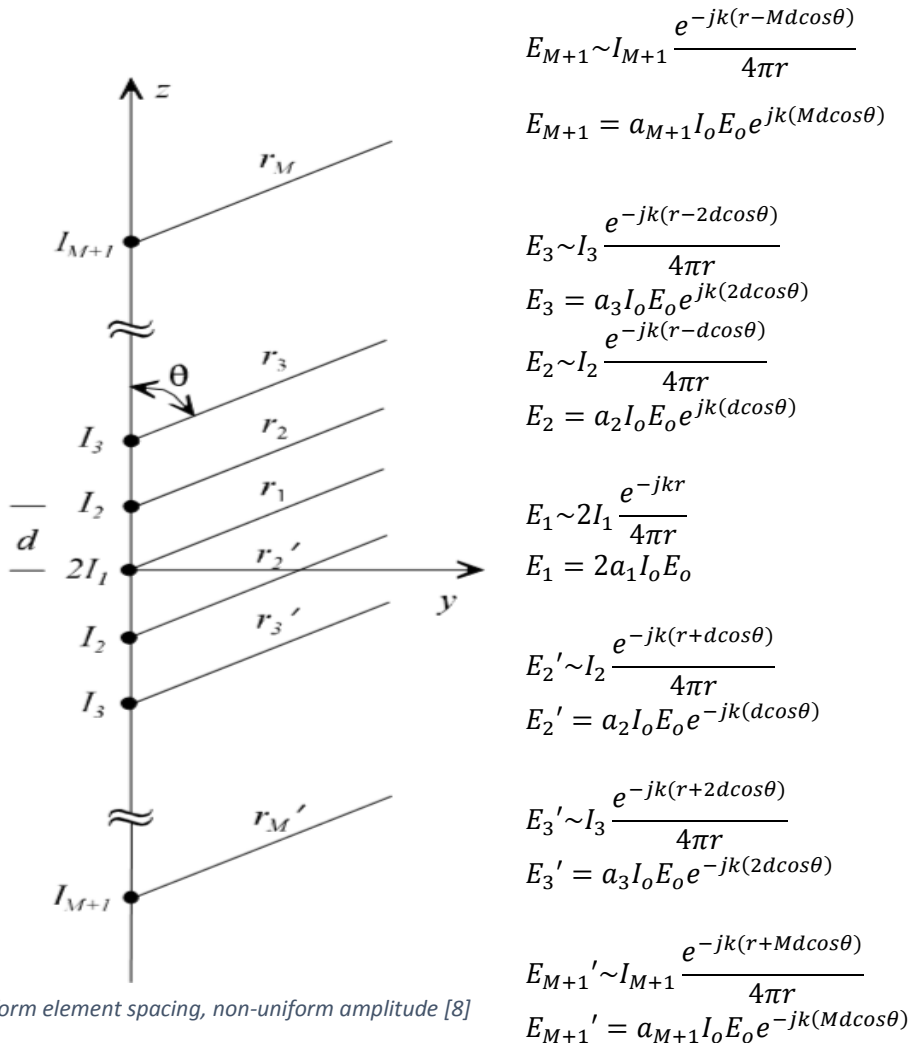


Figure 2.10 Uniform element spacing, non-uniform amplitude [8]

The total P element array, we have the electric field given by the following equation.

$$\begin{aligned}(E)_p &= E_{M+1} + \dots + E_3 + E_2 + E_1 + E_2' + E_3' + \dots + E_{M+1}' \\ &= 2I_0E_0\{a_1 + a_2 \cos(kd\cos\theta) + a_3 \cos(2kd\cos\theta) + \dots + a_{M+1} \cos(Mkd\cos\theta)\}\end{aligned}$$

The array factor now becomes,

$$(AF)_p = \sum_{n=1}^{M+1} a_n \cos \left[ 2(n-1) \frac{\pi d \cos\theta}{\lambda} \right] = \sum_{n=1}^{M+1} a_n \cos [2(n-1)u]$$

$$\text{Where } u = \frac{\pi d \cos\theta}{\lambda}$$

Note that the array factors are coefficients multiplied by cosines with arguments that are integer multiples of  $u$ . Using trigonometric identities, these cosine functions can be written as powers of  $u$ .

$$\cos 0u = 1$$

$$\cos 1u = \cos u$$

$$\cos 2u = 2\cos^2 u - 1$$

$$\cos 3u = 4\cos^3 u - 3\cos u$$

$$\cos 4u = 8\cos^4 u - 8\cos^2 u + 1$$

$$\cos 5u = 16\cos^5 u - 20\cos^3 u + 5\cos u$$

$$\cos 6u = 32\cos^6 u - 48\cos^4 u + 18\cos^2 u - 1$$

$$\cos 7u = 64\cos^7 u - 112\cos^5 u + 56\cos^3 u - 7\cos u$$

$$\cos 8u = 128\cos^8 u - 256\cos^6 u + 160\cos^4 u - 32\cos^2 u + 1$$

$$\cos 9u = 256\cos^9 u - 576\cos^7 u + 432\cos^5 u - 120\cos^3 u + 9\cos u$$

Through the transformation of  $x = \cos u$ , the terms may be written as a set of polynomials [Chebyshev polynomials –  $T_n(x)$ ]

$$\cos 0u = 1 = T_0(x)$$

$$\cos 1u = x = T_1(x)$$

$$\cos 2u = 2x^2 - 1 = T_2(x)$$

$$\cos 3u = 4x^3 - 3x = T_3(x)$$

$$\cos 4u = 8x^4 - 8x^2 + 1 = T_4(x)$$

$$\cos 5u = 16x^5 - 20x^3 + 5x = T_5(x)$$

$$\cos 6u = 32x^6 - 48x^4 + 18x^2 - 1 = T_6(x)$$

$$\cos 7u = 64x^7 - 112x^5 + 56x^3 - 7x = T_7(x)$$

$$\cos 8u = 128x^8 - 256x^6 + 160x^4 - 32x^2 + 1 = T_8(x)$$

$$\cos 9u = 256x^9 - 576x^7 + 432x^5 - 120x^3 + 9x = T_9(x)$$

Using properties of the Chebyshev polynomials, we may design arrays with specific side lobe characteristics. Namely, we may design arrays with all side lobes at some prescribed level.

### 2.4.3 Dolph-Chebyshev Array Design Procedure

1. Select the appropriate AF for the total number of elements ( $P$ ) [8].

$$(AF)_p = \sum_{n=1}^M a_n \cos[(2n-1)u] \quad P = 2M \text{ (even)}$$

$$(AF)_p = \sum_{n=1}^M a_n \cos[(2n-1)u] \quad P = 2M + 1 \text{ (odd)}$$

2. Replace each  $\cos(\mu)$  term in the array factor by its expansion in terms of powers of  $\cos(u)$ .
3. For the required main lobe to side lobe ratio ( $R_o$ ), find  $x_o$  such that

$$R_o = T_{P-1}(x_o) = \cosh[(P-1) \cosh^{-1} x_o]$$

$$x_o = \cosh \left[ \frac{\cosh^{-1} R_o}{P-1} \right]$$

4. Substitute  $\cos(u) = x/x_o$  into the array factor of step 2. This substitution normalizes the array factor side lobes to a peak of unity.
5. Equate the array factor of step 4 to  $T_{P-1}(x)$  and determine the array coefficients.

Here, the array coefficients are nothing but the weights, which are again nothing but the roots of the equation in  $T_{P-1}(x)$ . The interesting part of these roots is that, the value of these roots is what divides the current into the elements accordingly. A few observations on these roots are as follows.

### 2.4.4 Roots of polynomial equation in $z$ domain

The equations mentioned above which are in the terms cosine can be better understood when converted to a  $z$  domain and plotted on the unit circle. This method of representation by plotting the unit circle and array factor makes the reader better visualize. Take a uniform array with the array factor equation as follows, here  $z = e^{j\Psi}$

$$z^3 + z^2 + z + 1 = (z+1)(z+j)(z-j)$$

The roots of this lie on the unit circle, and the array factor is shown next to it.

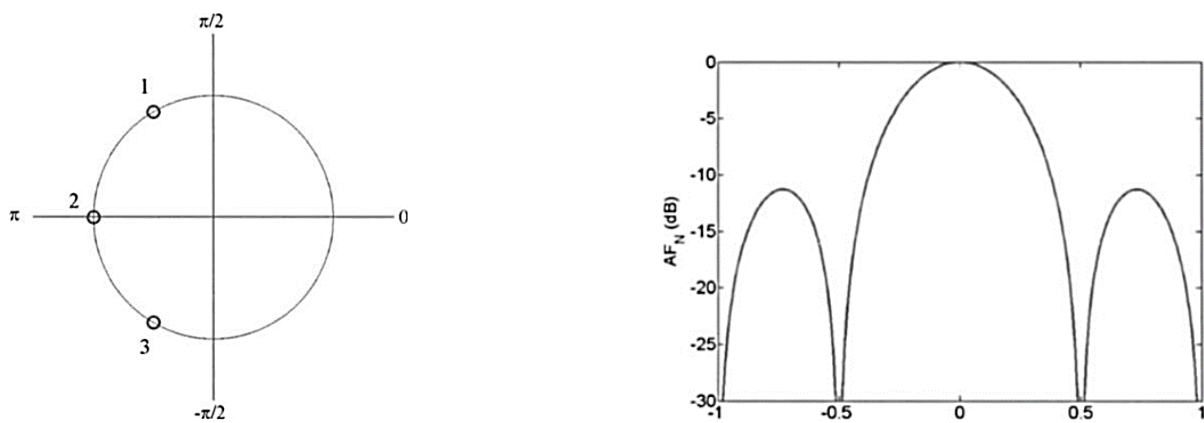


Figure 2.11 Unit Circle representation of a 4 element uniform array [9] Figure 2.12 AF of a 4 element uniform array with  $d=0.5$  [9]

The magnitude of the array factor between zeros relates to the angular separation of the zeros. Closely spaced zeros have small lobes between them, while widely spaced zeros have large lobes between them. The uniform array example has zeros 1 and 2 and zeros 2 and 3 closely spaced, while zeros 1 and 3 are widely spaced. The array factor between the closely spaced zeros are side lobes, and the array factor between the widely spaced zeros is the main beam. Thus, the side lobe levels of the uniform array may be lowered by moving the zeros on the unit circle [9].

For reducing the side lobes, Consider an array factor of this form with null locations at  $\Psi = \pm 120^\circ$  instead of  $\Psi = \pm 90^\circ$  result in the following equation for array factor

$$AF = (z + 1)(z - e^{j0.67\pi})(z - e^{-j0.67\pi}) = z^3 + 2z^2 + 2z + 1$$

The amplitude weights for the array elements are 1, 2, 2, and 1. These weights produce the normalized low-side lobe array factor which is evident from the below plot.

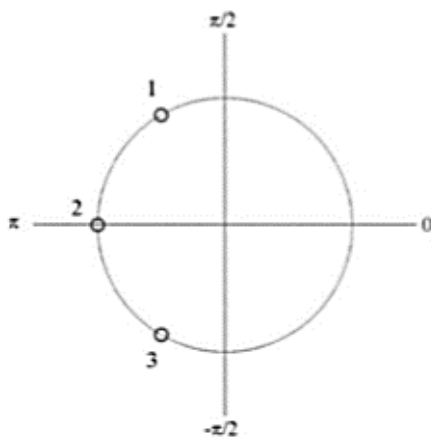


Figure 2.13 Zeros moved closer on unit circle [9]

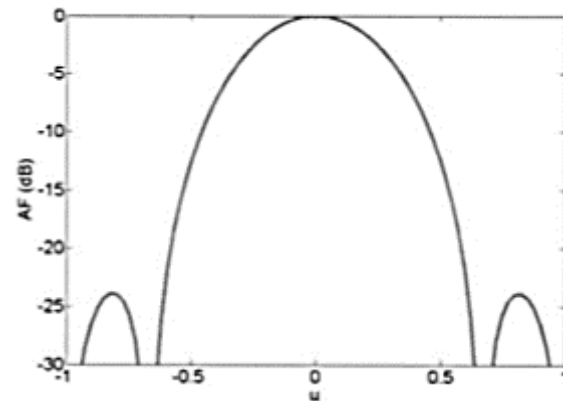


Figure 2.14 Expanding of the main beam by moving of zeros [9]

The examples so far only considered zeros lying on the unit circle. What happens to the array factor when the zeros move off the unit circle. The four element array has roots at  $z = -1$  and  $\pm j$  as indicated by the zeros labeled with a 1 in the below figure. Changing the roots to  $z = -1.2$  and  $\pm 1.2j$  moves them a radial distance of 0.2 outside of the unit circle as shown by the zeros labeled with a 2 in the figure below. The corresponding array weights are real since the roots are complex conjugate pairs and not symmetric:  $w = [0.5787 \ 0.6944 \ 0.8333 \ 1.0]$ . Moving the roots off the unit circle by an additional 0.2 results in roots at  $z = -1.4$  and  $\pm 1.4j$  which are labeled by a 3 in the figure below. The corresponding array weights are real since the roots are complex conjugate pairs and not symmetric:  $w = [0.3644 \ 0.5102 \ 0.7143 \ 1.0]$  [9]. The effects of moving the zeros is apparent on the array factor shown in Figure 2.44 and include

- Decreased directivity and efficiency
- Increased relative sidelobe levels
- Filled-in null

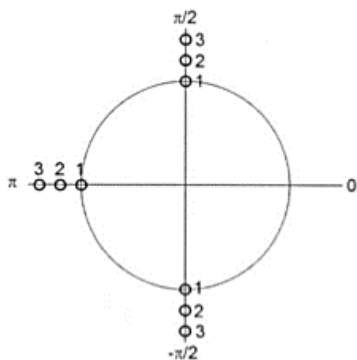


Figure 2.15 Zeros moved off the unit circle [9]

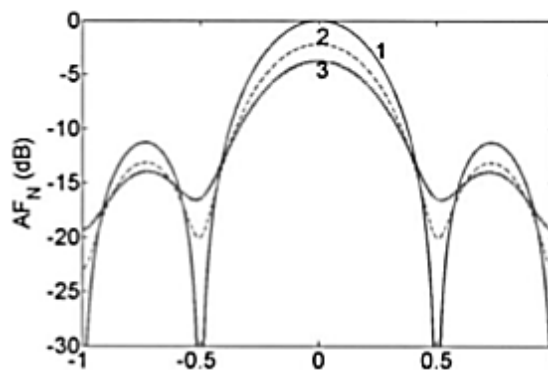


Figure 2.16 AF corresponding to when zeros moved off the unit circle [9]

## 2.5 Comparison of different tapering methods

For concise reason, this thesis mainly concentrates on four tapering types, namely, Uniform, Dolph-Chebyshev, Taylor-Kaiser (Taylor 1 parameter) and the Binomial method.

The table below lists the designed array weights and the corresponding 3-dB angular widths (in degrees).

Table 1 Comparison of element weights and the beam width for different tapering methods [10]

| Uniform | Dolph- Chebyshev | Taylor-Kaiser | Binomial |
|---------|------------------|---------------|----------|
| 1       | 1.0000           | 1.0000        | 1        |
| 1       | 1.2764           | 1.8998        | 6        |
| 1       | 1.6837           | 2.6057        | 15       |
| 1       | 1.8387           | 2.8728        | 20       |
| 1       | 1.6837           | 2.6057        | 15       |
| 1       | 1.2764           | 1.8998        | 6        |
| 1       | 1.0000           | 1.0000        | 1        |
| 14.5°   | 16.4°            | 16.8°         | 24.6°    |

It is observed from the above table that Uniform has the lowest beam width, while Binomial has the largest main beam. The Dolph-Chebyshev and Taylor 1 parameter have a very slight difference in the beam width. The difference between these tapering methods can be better noticed when considered along with the side lobes that are produced along with the main beam.

The following are the plots are achieved with the following parameters [10]

1. Number of elements = 7
2.  $d = 0.5 * \text{Lambda}$
3. All the beams are steered towards  $90^\circ$ , basically broadside array beams.
4. The Dolph-Chebyshev and Taylor Kaiser arrays, designed with a relative side lobe level of  $R = 20$  dB

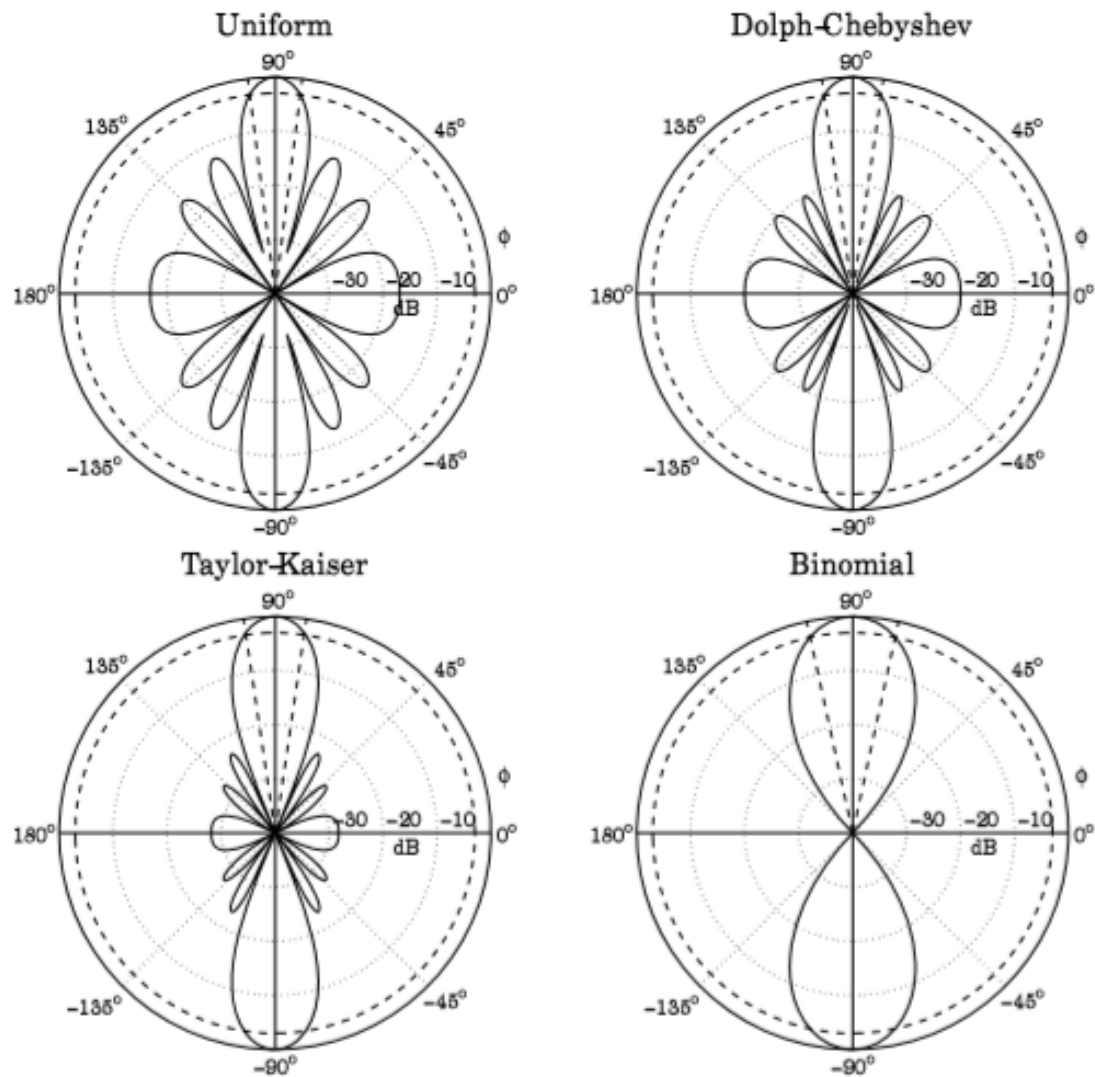


Figure 2.17 Amplitude tapering methods comparison [10]

The uniform array has the narrowest main lobe but also the highest sidelobes. The Dolph-Chebyshev is optimum in the sense that, for the given sidelobe level of 20dB, it has the narrowest width. The Taylor-Kaiser is somewhat wider than the Dolph-Chebyshev, but it exhibits better sidelobe behaviour. The binomial array has the widest main lobe but no sidelobes at all.

By amplitude tapering, the relative sidelobe levels are decided and then the array weights are accordingly adjusted. For making the antenna arrays smarter, an estimation of angle of arrival is very important.



## 2.6 Angle of arrival estimation

Direction finding with linear arrays is limited to either the 0 or  $\emptyset$  directions. In order to direction find in both azimuth and elevation directions, a planar array is needed. Circular arrays are also commonly used for direction finding [9].

For steering the beam to particular direction, the estimation of the angle of arrival plays a very important role. This is done by digital signal processing techniques, a few techniques are mentioned below [9]

1. Periodogram
2. Capon's Minimum Variance
3. MUSIC Algorithm
4. Pisarenko Harmonic Decomposition
5. Maximum entropy method
6. ESPRIT - Estimation of Signal Parameters via .Rotational /invariance Techniques

### 2.6.1 Periodogram

One way to determine the signals present in the vicinity of an array is to scan the beam over the region of interest and plot the output power as a function of angle. A plot of the output power versus angle is known as a periodogram. Resolving closely spaced signals is limited by the array beamwidth.

### 2.6.2 Capon's Minimum Variance

The periodogram basically uses the main beam of the array to determine signal locations. Since the main beam is wide, especially for small arrays, the ability to separate multiple signals or accurately locate a signal is not very good. Using nulls to locate signals is much more desirable, because the nulls have a narrow angular extent. Capon's method is the maximum likelihood estimate of the power arriving from a desired direction while all the other sources are considered interference. Thus, the goal is to minimize the output power while forcing the desired signal to remain constant.

### 2.6.3 MUSIC Algorithm

MUSIC is an acronym for Multiple Signal Classification. MUSIC assumes the noise is uncorrelated and the signals are either uncorrelated or mildly correlated. When the array calibration is perfect and the signals uncorrelated, then the MUSIC algorithm can accurately estimate the number of signals, the angle of arrivals, and the signal strengths.

The MUSIC algorithm is not very robust, so many improvements have been proposed. One popular modification, called root-MUSIC, accurately locates the direction of arrival by finding the roots of the array polynomial.

### 2.6.4 Maximum Entropy Method

The maximum entropy method (MEM) is also called the all-poles model or the autoregressive model. MEM is based on a rational function model of the spectrum having all poles and no zeros. Hence, it can accurately reproduce sharp resonances in the spectrum.

### 2.6.5 Pisarenko Harmonic Decomposition

The smallest eigenvector ( $\lambda_N$ ) minimizes the mean squared error of the array output with the constraint that the norm of the weight vector equals one.

### 2.6.6 ESPRIT

ESPRIT is an acronym for Estimation of Signal Parameters via .Rotational Invariance Techniques. It is based upon breaking an N-element uniform linear array into two overlapping sub arrays with N-1

elements. One sub array starts at the left end of the array, and the other starts at the right end of the array. The  $N - 2$  shared elements in the middle are called matched pairs. ESPRIT makes use of the phase displacement between the two sub arrays to calculate the angle of arrivals for the signals.

This chapter dealt in the detail about the different parameters that involved in the antenna array and the methods of finding them. The direction finding algorithms can assist in better finding the signal arrival direction while the amplitude tapering aids in maintaining a specific amount of sidelobe levels. On the other side, the number of antenna array elements helps in increasing the directivity.

## 3 Optimisation of the beam parameters

*This chapter deals with the optimising of the parameters mentioned in the chapter two. It explains the different types of optimization methods, and details about MMSE beamforming.*

### **Contents**

---

[3.1 State-of-the-art optimization techniques](#)

[3.2 MMSE Beamforming](#)

### 3.1 State-of-the-art optimization techniques

By now, it's clear that the output of the antenna array, can be controlled with primarily four factors

1. Amplitude
2. Phase
3. Number of elements
4. Distance between elements.

The changing of the distance between the arrays elements would be a technical challenge as it requires a special hardware that has to be installed which includes equipment like controller, motors for moving the array elements. This will be a burden for the mobile operators. Due to this reason, the varying of the number of elements is attracting the interest of many of the operators.

For a specific requirement of array factor, the above variables can be modified or optimised over some set of iterations and the required array factor value can be got. There are many algorithms present in the literature for the process of optimisation.

Checking all combinations of values of the array variables is not realistic unless the number of variables is small. Individual optimizing of the variables at a time is as good as the gradient vector downhill approach. The steepest descent method which was invented in about the 1800s, is extensively used even today and is based on the same concept. While the Hessian Matrix form is used by the Newton's method which uses the second-derivative to find the minimum. Although effective and powerful than steepest method, the main drawback about the newton method is the cost function [9].

For this reason, in 1965, Nelder and Mead thought away from the derivatives and introduced the downhill simplex method. It's stability in the approach has attracted many powerful computing software like MATLAB to make it an integral part of its packages. A simplex contains  $n + 1$  sides in an  $n$ -dimensional space. A new vertex of the simplex is generated for every iteration. Updating of the vertex with better values is done. Over a finite number of iterations, the simplex value gets small and accurate [9].

In the 1960s methods like Successive Line minimization were brought into force. The algorithm chooses a random point and proceeds in a pre-set method until the cost function increases. Once an increase is encountered, the algorithm starts with a new direction. Here the conjugate direction does not interfere with the minimization of the prior direction. Powell then proposed a method by which changes to the gradient of the cost function remains perpendicular to the previous conjugate directions [9].

The BFGS algorithm again considers the Hessian matrix (Matrix obtained from the second derivatives) for calculation of the next search point. As this method uses the similar concept of Newton's method it's also called as "Quasi Newton" method, though the hessian matrix that is used is not the same. The next type is where the cost function is assumed to be quadratic and the other constraints are linear. It is built on the basis of Lagrange Multipliers which again require the derivatives or approximation principles. This type is called as the Quadratic Programming [9].

For desired patterns with combinations of weights, tapers, non- uniform element spacing numerical optimization is one widely adopted method. A few examples of non-uniform synthesis are Nelder Mead downhill simplex algorithm, steepest descent, simulated annealing and dynamic programming. These methods were used with constraints of the side lobe levels and other parameters in order to better shape the main beam [9].

For finding the phase tapering that exploited the array directivity numerical optimization was incorporated, and to find the optimum phase tapering to diminish side lobe levels, the steepest descent algorithm was used.

A characteristic of an antenna array that are to be minimized is returned by the cost function that is associated with it. For example, in the case of a 6 element array to find the minimum and maximum side lobe level by varying the amplitude and phase weights that lies along the x axis. The parameters like elements spacing, amplitude and phase weights are symmetric with respect to centre of the array. Therefore, the specification for one half of the array should be sufficient. For the conception of cost surface, two variables will suffice. They can be any combination of amplitude, phase and distance between the elements. But a point to be noticed is that, these are local optimizations and don't offer good efficiency as the start is in a random point. For this reason, global optimizations like Simulated Annealing or Genetic Algorithms. The gist of genetic algorithm is given below [9].

The genetic algorithm begins with a random set of configurations (the matrix rows called as population) which consist of the variables such as element spacing, phase and amplitude. Every such configuration is assessed by the cost function that returns a value like maximum side lobe level. The configurations can have binary or continuous values. The high cost configurations are discarded while low costs are placed in the mating pool. From the formed pool, two parents are randomly selected and the selection is such that it is inversely proportional to the cost. As a result of the combination of parents, offspring is formed. These newly offspring replaces the array configurations that were discarded. This process continues for all the population, mutated or modified randomly. This finally gives a new array configuration and their costs are evaluated.

Based on the application requirements of efficient power consumption or better user directivity, the required format of steering or weighting can be used and the required level of optimisation can be incorporated for any the applications.

The processes listed above tells about how the signal be directed to desired user with the best of its transmission abilities like reducing side lobe levels, better directivity and others. One of the most promising signal processing method when both angle of arrival and signal strength is known at the base station is the MMSE beamforming.

Though there are a several algorithms as mentioned to implement optimal weighting, differing fundamentally on the speed of convergence. The MMSE beamforming is one such method. The accuracy and the speed of convergence of MMSE beamforming has attracted many of the researcher's interest. The MMSE beamforming is explained in the following section.

## 3.2 MMSE Beamforming

Minimum Mean square error beamforming is a signal processing approach for finding the optimal weighting. Consider a uniformly spaced linear array, with elements spaced at  $d$ :

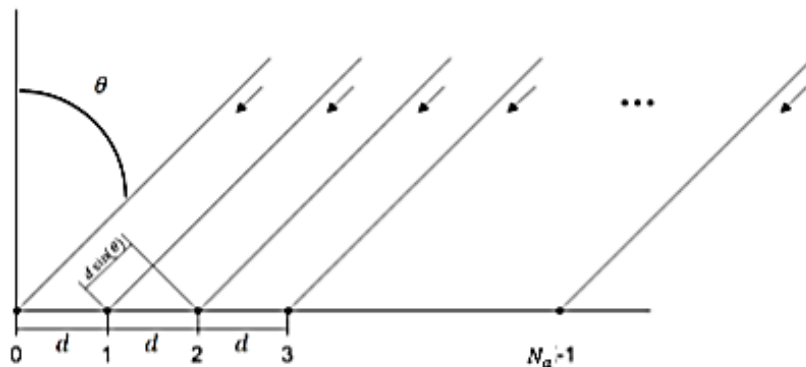


Figure 3.1 Uniformly spaced linear array [11]

The desire signal,  $u_1(t)$ , arrives at the array with a spatial angle  $\theta$ . At the same time, there are interfering signals. Each one arrives at the array with a spatial angle  $\theta_i$ . The output of the array is:

$$x(t) = \sum_{i=1}^{N_u} u_i(t) \mathbf{v}_i$$

$\mathbf{v}_i$  is the array propagation vector for each of the interfering signals and it is represented by:

$$\mathbf{v}_i = [1, e^{-jk d \sin(\theta_i)}, \dots, e^{-jk(k-1)d \sin(\theta_i)}]^T$$

The array's output is then multiplied by the weighting vector, which is choose to minimise the mean-square error between the array's output  $w^H x(t)$  and a reference signal  $u^*(t)$ , a signal known by both sides of the transmission and used to set the beam former's weighting vector. The index H above the vector denotes the conjugate transpose and it is used for matrix compatibility. The reference signal can be obtained from some UL control channel. The mean-square error is, in this way, expressed by:

$$\overline{\varepsilon^2(t)} = [u^*(t) - w^H x(t)]^2$$

The reference signal is expressed in its complex conjugate only for mathematical convenience. Taking the expected value of both sides of (2.20), and using some algebraic manipulations, the expected value for the mean-square error is given by:

$$E\{\varepsilon^2(t)\} = E\{d^2(t)\} - 2w^H E\{u^*(t)x(t)\} + w^H E\{x(t)x^H(t)\}w$$

Defining  $E\{x(t)x^H(t)\}$  as the covariance matrix  $R$  and representing  $E\{u^*(t)x(t)\}$  as  $r$ , the minimum mean-square error between the beam former's output and the reference signal is given by the null of the gradient vector of the above equation with respect to  $w$ , which is show below:

$$w(E\{\overline{\varepsilon^2(t)}\}) = 0 = -2r + 2Rw$$

From the above equation, it is possible to obtain the optimum weighting vector that minimises the MSE:

$$w_{opt} = R^{-1}r = (E\{x(t)x^H(t)\})^{-1} \cdot E\{u^*(t)x(t)\}$$

For a visual understanding of the minimum mean-square error criterion, Figure 3.1 represents the involved signals over the beamformer layout.

Although the beamforming technique is applied at the BS on DL, the algorithm for optimum weighting is based on a UL reference signal that is used to track the user's location and set up the weighting vector.

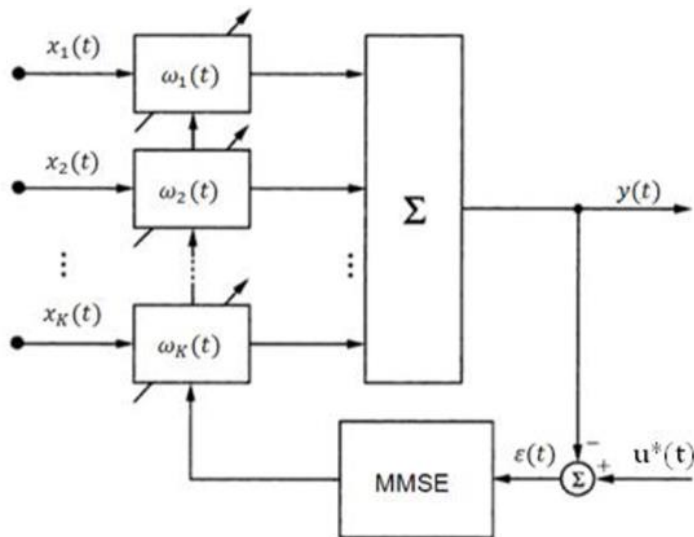


Figure 3.2 MMSE Signal Flow [11]

## 4 Beamforming Implementation using MATLAB

*This chapter gives an insight into the implementation of the basic beamforming using MATLAB. These implementations were into a simulation framework which could be used for simulating/modelling most of the real-time mobile communication scenario in MATLAB. The novel work of this thesis is that we are trying to experiment on the multi-beam beamforming technique in mobile networks.*

### **Contents**

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- [4.1 Beamforming Implementation in MATLAB](#)
- [4.2 Beam Steering](#)
- [4.3 Multi-beam Beamforming](#)



## 4.1 Beamforming Implementation in MATLAB

The Electromagnetic waves and Antennas toolbox which is available for a free download from Math works MATLAB support most of the required beamforming operations

A detailed textbook using the same toolbox is available again in open source [10]. As the basics of antenna arrays are clear, we will now move to a few demonstrations of beamforming in many scenarios.

Let's look at a basic example of antenna array

```
d=1;
N=8;
a=uniform(d,90,N); %steered uniform weights
[g,phi]=gain1d(d,a,400); % calculate normalized gain g(phi)
A=sqrt(g);
psi=2*pi*d*cos(phi); %phi to psi transformation
plot(psi/pi,A); %plot in psi space
figure(2);
dbz(phi,g,45,20); % azimuthal gain plot in dB
```

The figure below shows  $A(\psi)$  evaluated only over its visible region for an 8-element ( $N=8$ ) array, for the following three choices of the element spacing:  $d=0.25\lambda$ ,  $d=0.5\lambda$ , and  $d=\lambda$ .

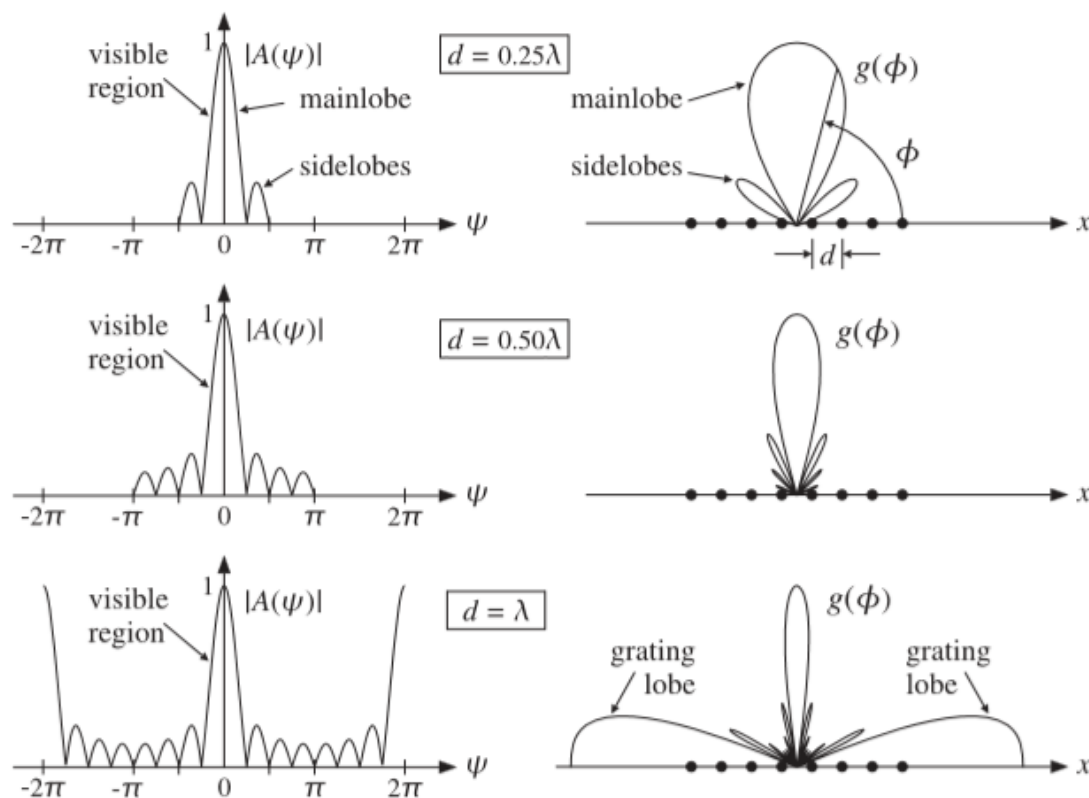


Figure 4.1 variation of beam width based on element spacing [10]

In real time, it's not technically feasible to change the antenna element distance  $d$ . So the beam width reduction by changing  $d$  is generally not implemented. An effective way would be by changing the number of antenna elements.

This above graph shows us for covering one user, this antenna array can be done. Now, consider the case where there are multiple (very large number) users, so an effective beamforming technique as per EARTH project is if users are close to each other then cover them in a multicast beam and if they are far away, then a single beam is cast to every user. This will effectively reduce the amount of power being transmitted when covering a very large number of users.

For this, an investigation on the concepts of multi beam beamforming will bring in a clear picture of multicast and single cast based on the user position (both distance and angle). The multi beam beamforming function requires two values, one being the amplitude of the beam and other the angle for which the beam has to be steered to.

## 4.2 Beam Steering

Before elaborating on multi-beam in depth, first consider the case of only steering the beam to the required direction of the user. This is because, the multi-beam has one of its input parameters are the desired angle, for which the understanding of beam steering is essential.

```
d=0.5;
N=11;
ph0=60;
a=uniform(d,ph0,N);    %steered uniform weights
[g,phi]=gain1d(d,a,400); % calculate normalized gain g(φ)
psi=2*pi*d*cos(phi);   %φ to ψ transformation
figure;
plot(psi/pi,sqrt(g));   %plot in ψ space
figure;
dbz(phi,g,30,20);      % azimuthal gain plot in dB
```

The code [10] above discusses a case of antenna array having 11 elements and the separation distance is  $0.5\lambda$ . The beam is steered from broadside to  $60^\circ$ . This is one of the most desirable property of beamforming.

The steering of a beam to the desired users will help in better locating the users and thereby aiding in the smoother casting of the beam. At the same time, to reduce the power radiation in the undesired direction. The example graphs for the above code snippet is given as follows.

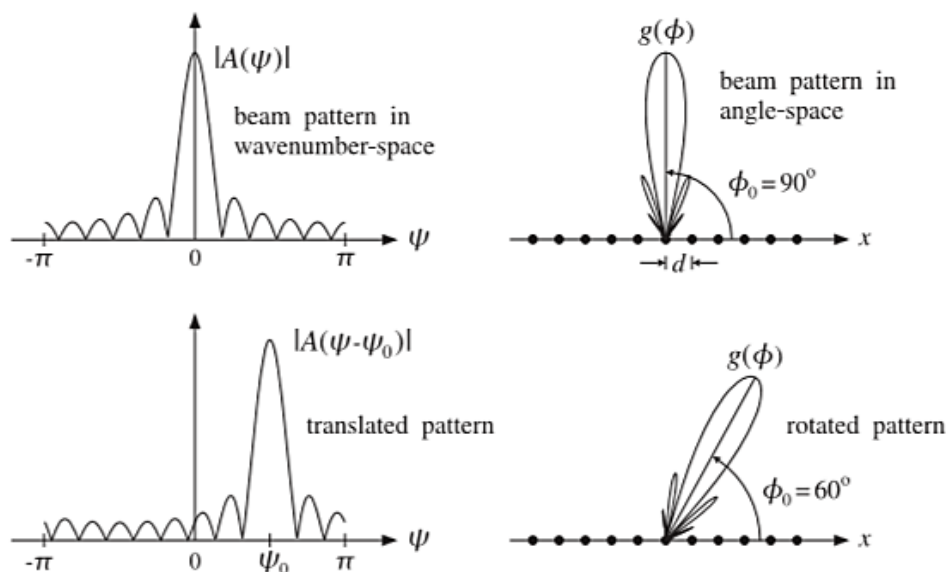


Figure 4.2 Beam steering [10]

### 4.3 Multi-beam Beamforming

Moving on to multi beam beamforming with arrays. As explained, it is used to cast the beams with a certain amplitude and in a certain direction. The following code does the multi beam beamforming [10]

```

w=taylor1p(0.5,90,21,30);           % unsteered weights
a=multibeam(0.5,w,[1,1,1],[45,90,120]); %equal-amplitude beams
[g,ph]=array(d,a,400);             % compute gain
dbz(ph,g);                          % plot gain in dB
addray(45); addray(-45);           % add ± 45° grid rays

```

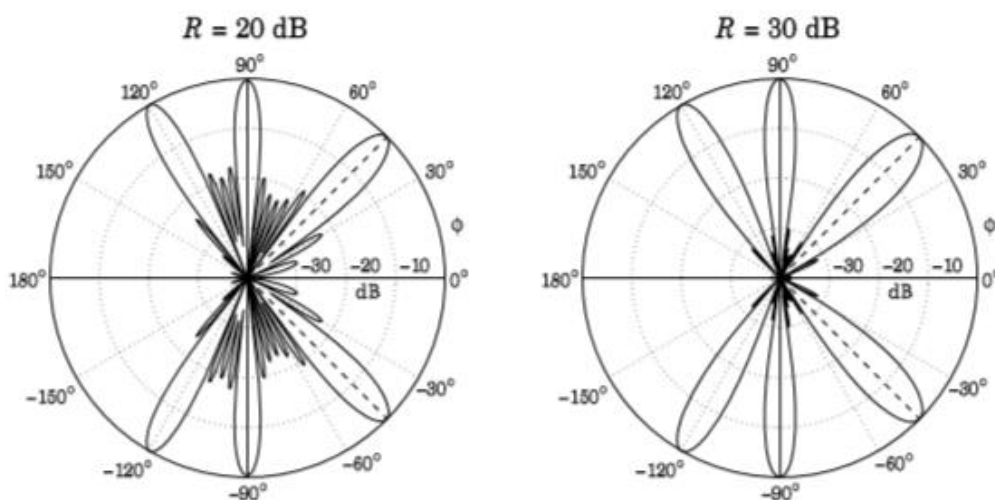


Figure 4.3 Multi beam beamforming with Relative Side lobe level at 20 dB and 30 dB respectively [10]

The above the multi beams for amplitudes of 1, angles as  $45^\circ$ ,  $90^\circ$ , and  $120^\circ$  respectively. Now, there are 2 cases above, the relative side lobe level R of 20 dB and 30 dB

We note the broadening of the beam widths of the larger beam angles. The left array has narrower main lobes than the right one because its side lobe attenuation is less. But, it also exhibits more constructive interference between main lobes causing somewhat smaller side lobe attenuations than the desired one of 20 dB

Now examine the case with varying a few parameters but with the same beam amplitudes [10].

#### Case 1:

```
a=multibeam(0.5,w,[0.2,0.5,1],[45,90,120]);
```

#### Case 2:

```
a=multibeam(0.5,w,[0.2,0.5,1],[90,90,120]);
```

The multi beam function is now fed with different amplitude levels and the second case with equal same angles for 2 of the beams. The plots are as shown below.

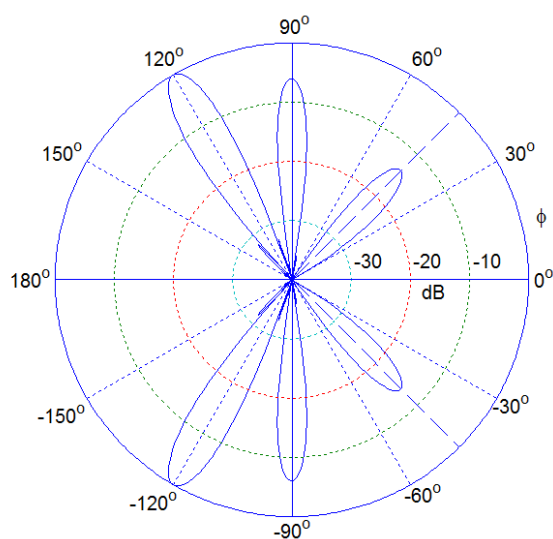


Figure 4.4 Case 1, 3 different beams

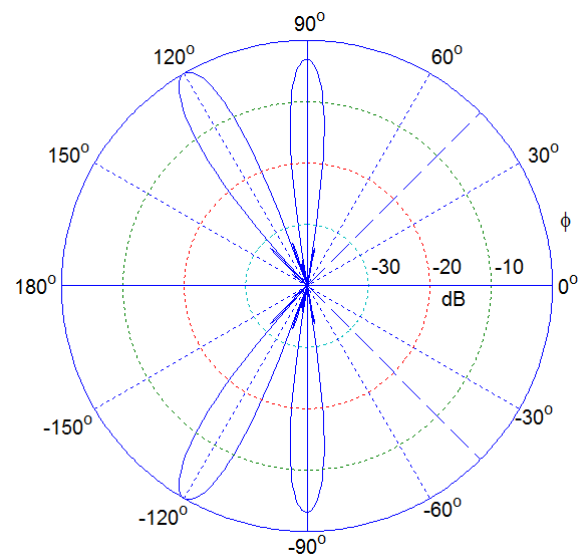


Figure 4.5 Case 2, 2 beams overlapped

From the above figure it is evident that the beam is formed based on the amplitude and angles. The case where two users are in the same direction, then the beam amplitude is set for the user who is farthest from the base station. In this process, both the users are reached with just one beam. This is a very noticeable advantage of the multi beam beamforming technique.

This ends the brief introduction to the implementation of the beamforming. Now, to test this beamforming in real time, the mobile users have to be generated. The generation of the mobile user is a research field by itself. The detailed method of user generation is described in the next chapter.

## 5 Real-time setup of mobile users and base stations

*This chapter details about the real-time setup of the mobile users, base station based on realistic observations collected from various cities. This thesis has a framework that is developed to model with the real-time mobile user positions in a multi-cell scenario. The framework integrates the basic beamforming implementation in MATLAB with the complex concepts of mobile user and base station location estimations in real-time.*

### **Contents**

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- 5.1 Aspects on positioning of mobile users and base stations*
- 5.2 Simulation Parameters/Setup*
- 5.3 Multi cell Scenario – Demonstration*

## 5.1 Aspects on positioning of mobile users and base stations

The simulation of mobile users in MATLAB is by itself a research field for the fact that, there are many factors that come into consideration. A few of the factors are:

1. The height of the user.
2. The position of the user
3. The next possible path of the user
4. The height of the base station
5. The distance accuracy between user and base station

The above mentioned are a few of the myriad parameters that are actually considered while simulating. The values have to be accurate to real time which involves observing the user patterns over a long period of time and generating the paths.

The researcher's from Fraunhofer Heinrich Hertz Institute have developed a tool named QuaDRIGA (QUAsi Deterministic RadIo channel GenerAtor) to enable the modelling of MIMO radio channels for specific network configurations.

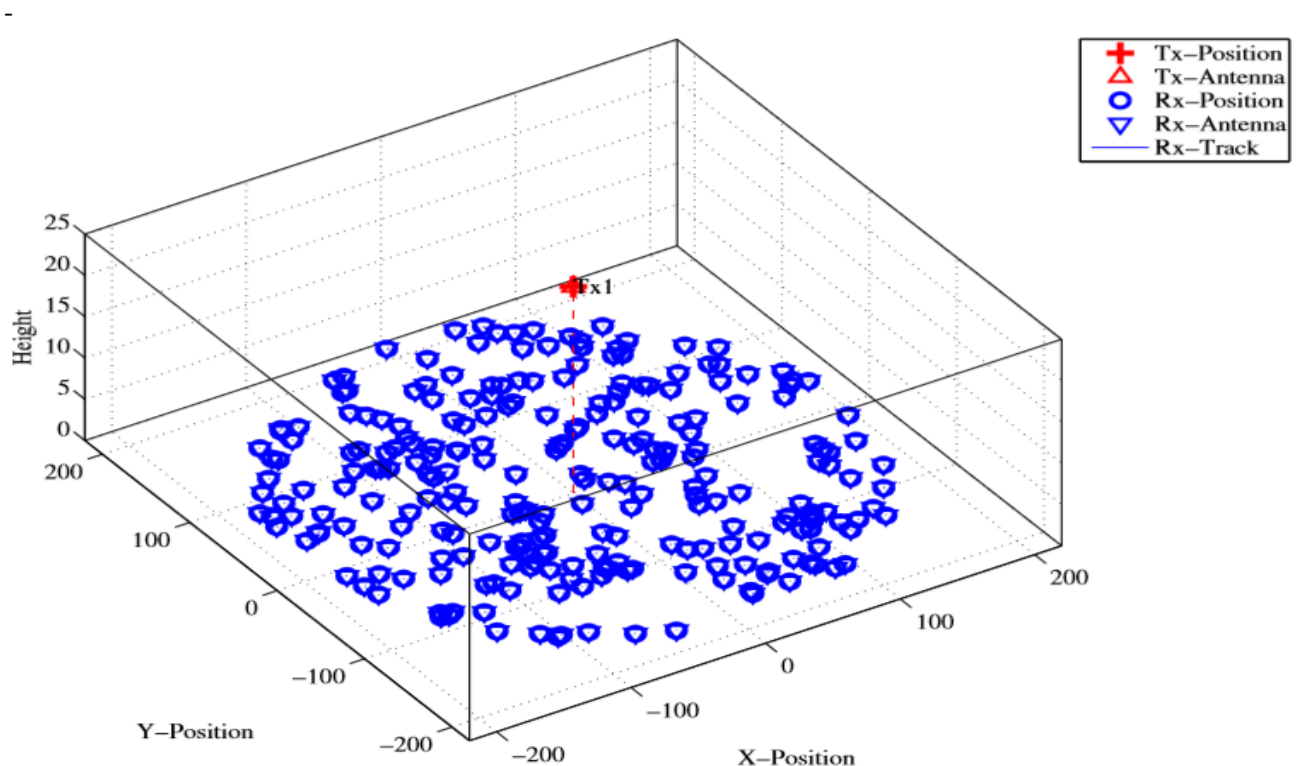
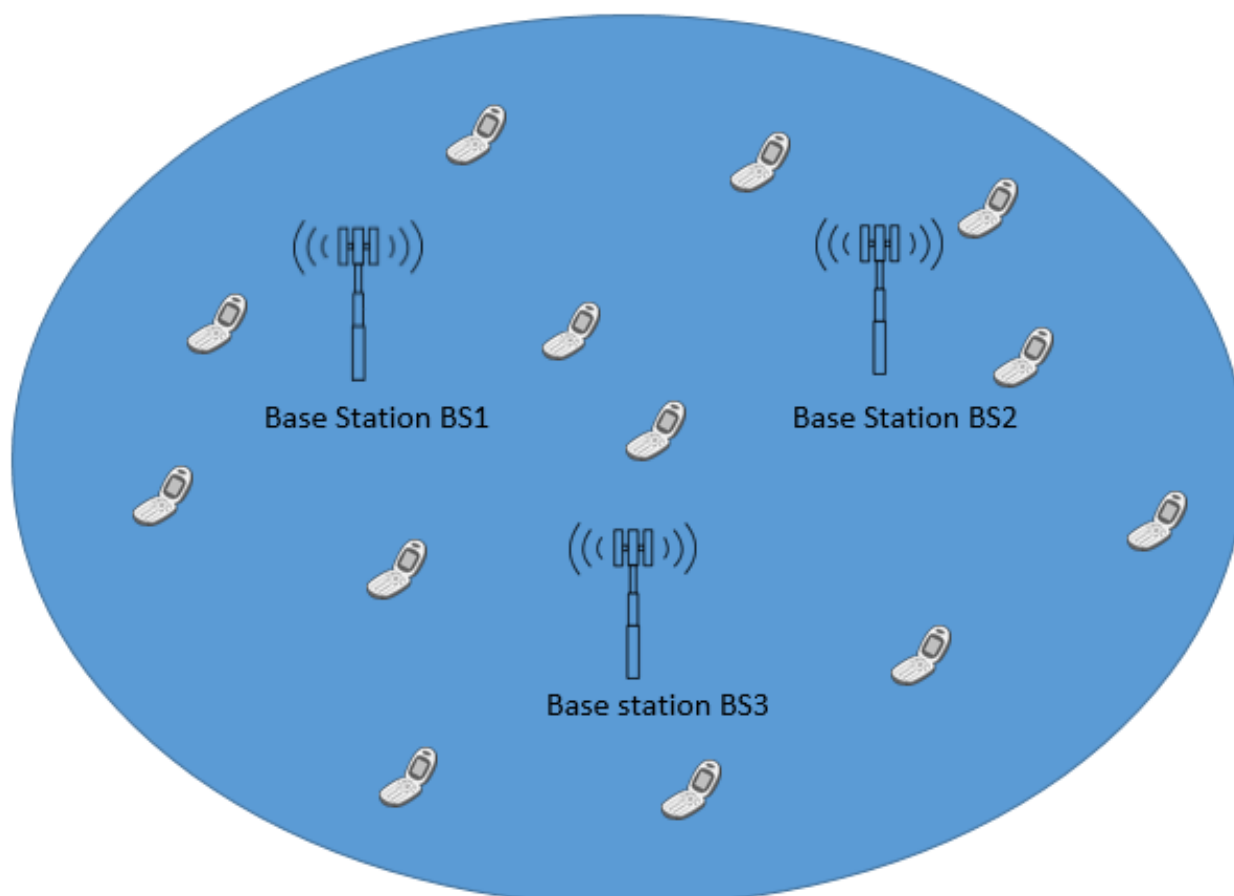


Figure 5.1 General distribution of users and Base stations

The above figure shows the users being distributed. The parameters considered here are the height of the user, base station and their positions. As the configuration of an antennas for beamforming is a little tedious in QUADRIGA tool, we have used the tool only for the user and base station position generation. Though the users and base stations can be generated manually in MATLAB without this tool, the principle purpose of using this tool is that it has a realistic user distribution for cities like BERLIN etc. This will help in getting the simulation closer to a real time scenario.

The primary parameter under consideration for the whole thesis is the Multi Cell scenario. There have been many works on beamforming in single cell and one of the objective of this thesis to check the

performance in the case of a multi cell scenario. The model of multi cell scenario is given in the below figure.



*Figure 5.2 Illustration of the multi cell scenario. The Base station here has N antennas and the mobile is assumed to have a single antenna*

Each base station having N antennas have the capability to beamforming. Now, when the users are spread out in random places. The distance between each user and base station is computed and then the users are grouped to a base station based on the distance. So the distance is the deciding factor as to which Base station covers a mobile.

Apart from the distance, the angle of arrival of the beam is also calculated. It is assumed here in this thesis that both the mobile user distance from the base station and the angle subtended by the base station to the mobile user is perfectly known at the base station.

The next section will describe in the detail the parameters that were setup during the simulation and execution of the complete scenario mentioned above.

## 5.2 Simulation Parameters/Setup

This thesis uses tools from various latest sources for getting accurate simulation setup. The following table describes the complete setup scenario.

Table 2 Functional tools and its names

| S. No | Required Functionality  | Tool/Toolbox Name                                       |
|-------|-------------------------|---|
| 1.    | Beamforming             | EWA (Electromagnetic waves and Antennas Toolbox) MATLAB |
| 2.    | User/Base station Setup | QUADRIGA Tool   |
| 3.    | Channel Model           | 3GPP Channel Model as mentioned in [9]                  |

The simulation Parameters for the multi cell scenario are as follows:

Table 3 Multi cell scenario parameters description [12]

| S. No | Parameter                         | Value                       |
|-------|-----------------------------------|-----------------------------|
| 1.    | Number of Base stations           | 3                           |
| 2.    | BS to BS distance                 | 2.8 Km                      |
| 3.    | Frequency Reuse                   | 1                           |
| 4.    | Base station Transmit Antenna No. | 100                         |
| 5.    | Number of mobile Users            | 40                          |
| 6.    | Distance Dependent Path Loss      | $128.1 + 37.6 \log_{10}(d)$ |

## 5.3 Multi cell Scenario – Demonstration

This section will show in detail the simulation environment and beamforming plots for the users.

Below is the image MATLAB simulation of the scenario with the following parameters as input

1. No of mobile users – 40
2. No of antenna elements – 100

The default input parameters taken are:

1. Height of each base station is 25 Metres
2. Height of user changes from 1 to 2 Metres
3. Scenario is BERLIN\_UMa\_NLOS
4. The user spread out radius is 5 KMs.
5. Power input at every base station is 8W (or) 39 dBm

The output parameters:

1. Total radiated power at the base station
2. Half Power Beam Width
3. Peak Gain
4. Side lobe Level
5. Mean SINR (Signal to interference plus noise ratio)



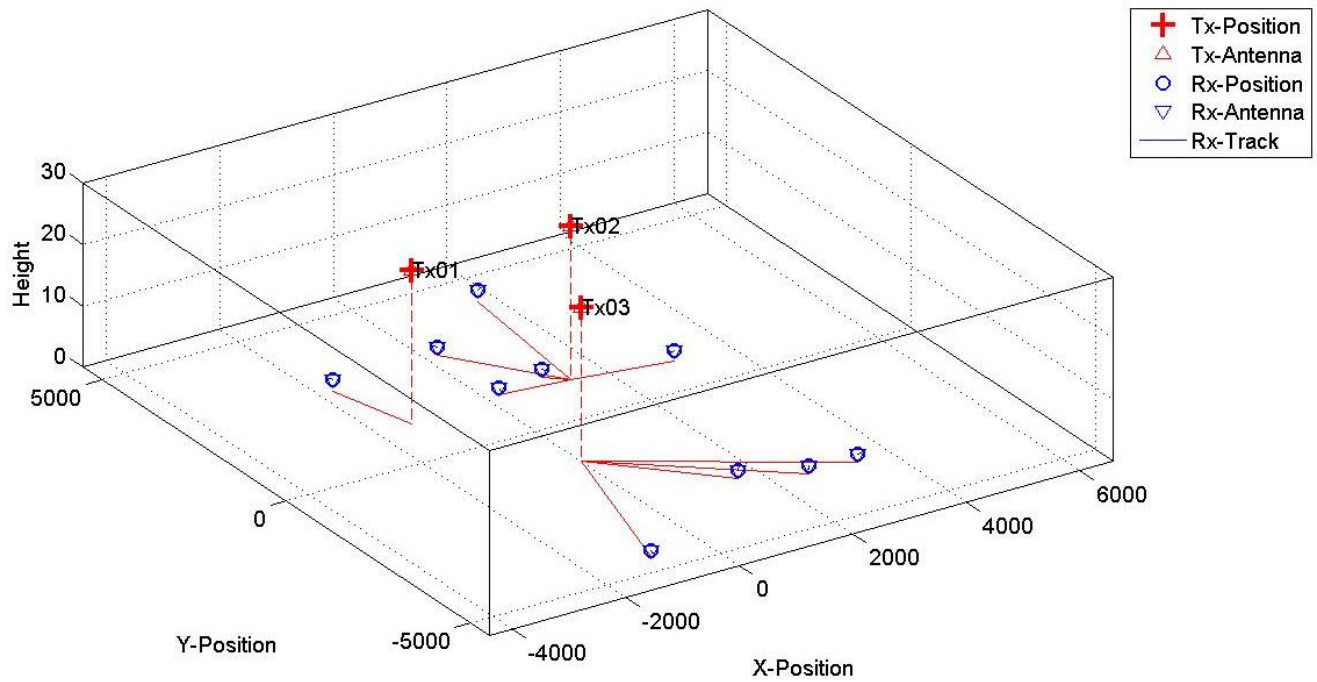


Figure 5.3 Distribution of users and Base stations

It is seen from the above figure that there is a line connecting from the base station to the user. These are the lines to show which user will serve a specific user. It would be worth to note again that inter base station distance is 2.8 KMs and the users are spread out in a radius of 5 KMs.

The beamforming plots are shown below

### Base station 1

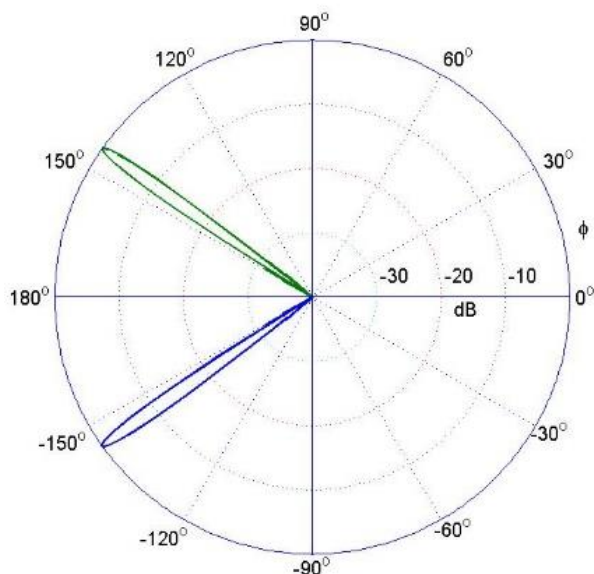


Figure 5.4 Base station 1 beams

Base station 1 beamforming plots show about the coverage of 1 user. The similar case of beams being formed for the users covered by base station 2 and 3 are shown below.

## Base station 2

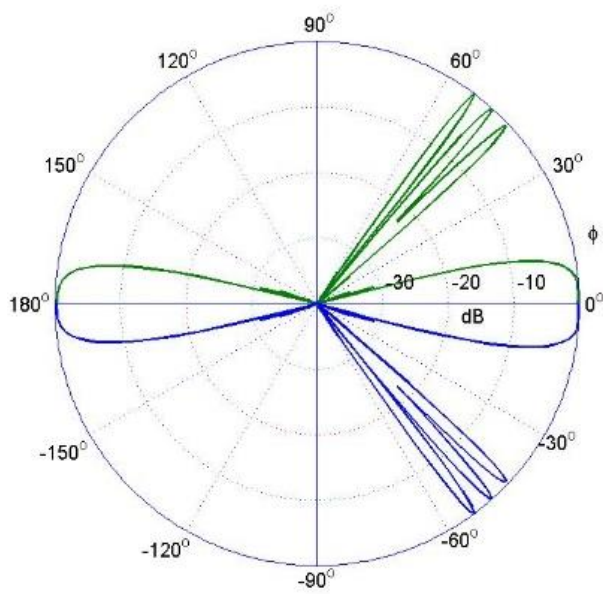


Figure 5.5 Base station 2 beams

## Base station 3

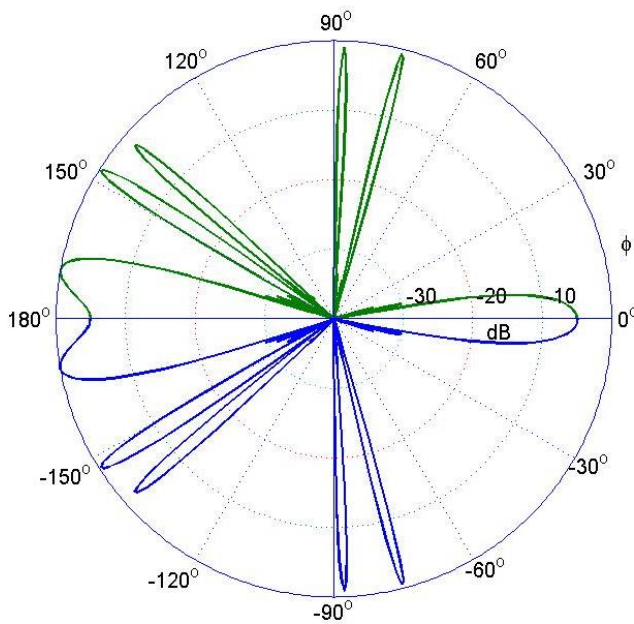


Figure 5.6 Base station 3 beams

The above plots show that the beams are cast to the users based on the distance from the base station and the angle subtended. The end fire beam that is seen in all the three base station resembles that angle subtended by the user and base station is  $0^\circ$ . The interesting observation is the beam amplitude being changed based on the distance of the user.

## 6 Model Description

*This chapter presents in detail the complete setup algorithm and the parameters chosen for the simulation. The second, third and fourth part of the chapter deals with the observations and inferences from the results obtained with the setup.*

### **Contents**

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*6.1 Simulation setup elaborated*

*6.2 Observations and Inferences*

### 6.1 Simulation setup elaborated

Firstly, a list of the complete setup scenario is given below, the elaborating is done further on.

1. Generation of Users and Base stations locations in Kilo meters scale.
2. Computing the distance between the base station and mobile user

3. Categorizing the mobile users to a specific base station based on the distances calculated in the step 2
4. The angles subtended are calculated for the set of mobile users categorized to that mobile station.
5. Casting of the multi beam in parallel time to all the users
6. Obtaining the beamforming vectors for covering the particular position of the users.
7. In the case of a single cell, step 3 can be ignored.
8. Computation of the nearest angle to the user angle from the azimuthal angles generated by the array gain function
9. Extracting the gain values based on the nearest angle
10. Compute individual user gain values by dividing with the sum of gain values at all angles.
11. Plotting the box plots.

### Step 1:

The users and base stations are generated base on the QUADRIGA tool. The X and Y axis can be easily got. These values can now be used to calculate the distances between the base station and the angle subtended by the base stations to the mobile users.

### Steps 2, 3:

In this thesis, the mobile users and base stations are first classified, i.e., the mobile user's are grouped to a particular base station based on the distance. So the minimum distance between the mobile user and base station is the grouping parameter. This computing of the minimum of the distance between the user and base station is done with the help of **delaunayn** and categorizing by the **dsearchn** functions of MATLAB. If the scenario is considered for a single cell environment, then the step 3 of grouping the user to a base station can be ignored.

### Step 4:

For the calculation of angle between base station and mobile user, the basic formula of

$$\theta = \tan^{-1} \frac{\Delta y}{\Delta x}$$

Supposing base station coordinates are (X, Y) and the mobile user coordinates are (A,B) then

$$\begin{aligned} \Delta y &= Y - B \\ \Delta x &= X - A \end{aligned}$$

As the antennas cannot cover the users behind them, the angles if negative are converted to positive angles for further calculations

### Step 5:

Once the angles and distances have been calculated, they can be directly fed into the multi beam function. A point of notice here is that, the beam amplitude has to be mentioned which is in the order of dB. The distance is then converted to dB with the distance dependant path loss. The path loss values are fed into the multi beam beamforming function in the place of beam amplitudes. As the values are relative values, a value of more than 1 will not affect the pattern. For this simulation, to get a quicker convergence, a simulation is done by placing all the users at a constant distance from the base station.

**Step 6:**

The multi-beam beamforming generates the beamforming vectors for the given condition of users and their respective angles. This functionality can be got by using the “multibeam.m” function in the EWA toolbox.

**Step 7:**

The power gain is computed for all the beamforming vectors over all the azimuthal angles. This can be achieved by the function “array.m” which is present in EWA toolbox

**Step 8:**

The user angles being known, these angles can be used to compute the nearest angle with respect to the azimuthal angles got after the array function. The matrices of gain and azimuthal being the same in terms of dimension, will get us the equivalent gain value for the particular angle.

**Step 9:**

The associated gain values for the user angles are extracted for both the desired and undesired users. For the undesired users, even the smallest side lobe in that particular direction is considered. As the smallest value of side lobe can also affect the user.

**Step 10:**

The gain values are obtained for each user or gain at the particular angle is divided by the total gain values of the radiation pattern. This will give the gain of the user at that particular direction. This obtained value will help compute the power in that particular direction.

**Step 11:**

The box plot for the case of 10 users and increasing number of antennas are plotted.

For a quicker convergence and making the system real time, the angles of users has been converted to positive angles, between 0 and 180 as in the case of Massive MIMO antenna will not have an access to the users behind it.

The following are the polar plots of the four tapering types. The polar plots have the markings of both the desired and the undesired users. They are marked with square and circle respectively. The multi-beam beamforming can observed along with the sidelobes produced with the help of the simulation framework developed as a part of this thesis.

Uniform tapering

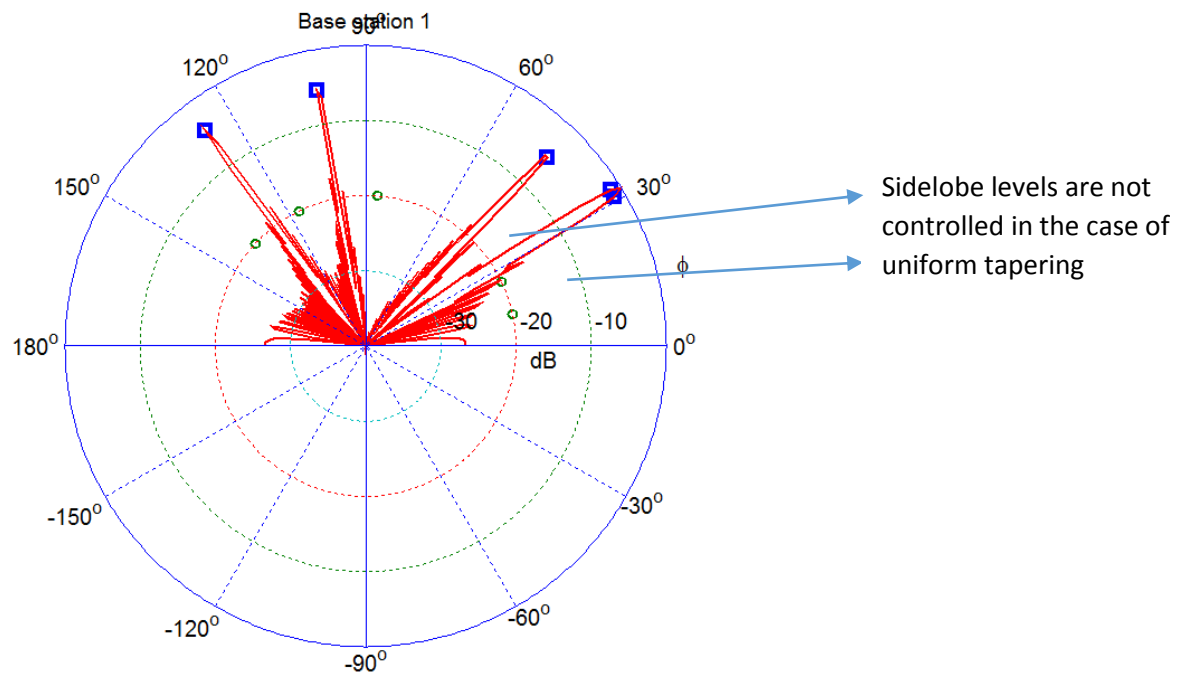


Figure 6.1 Uniform Tapering - 10 Users, 100 Antennas

### Binomial Tapering

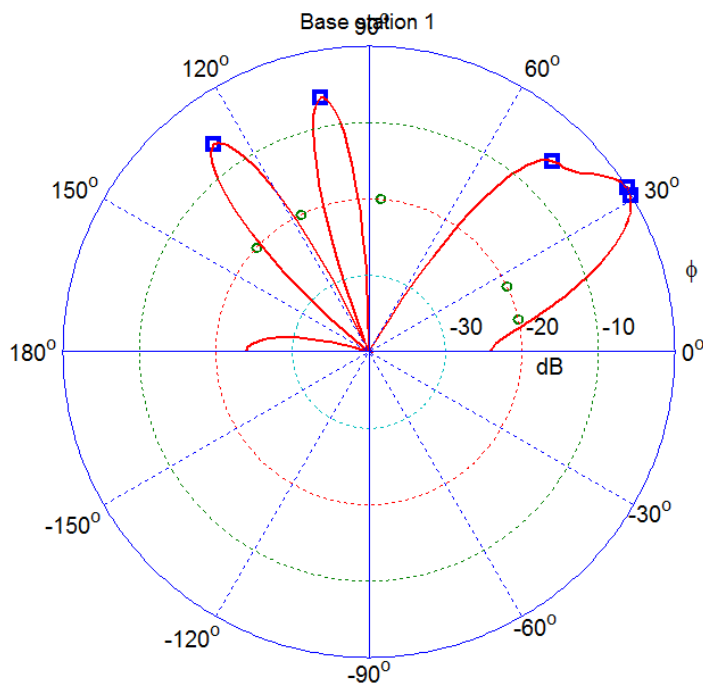


Figure 6.2 Binomial Tapering - 10 Users, 100 Antennas

Here Binomial tapering doesn't have any sidelobes, but the price paid for that is the larger beam width. In the uniform tapering shown in figure 6.1, clearly shows that there is no control on the sidelobe level. This could be an undesired case in two conditions, one being the interference and the other power constraint. This research in this thesis lies in exploring this underlying conditions.

### Dolph-Chebyshev tapering

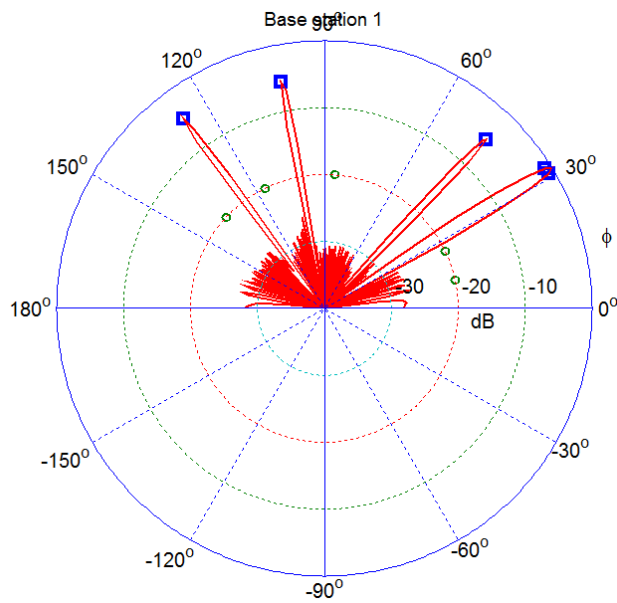


Figure 6.3 Dolph Chebyshev- 10 Users, 100 Antennas

### Taylor 1 parameter

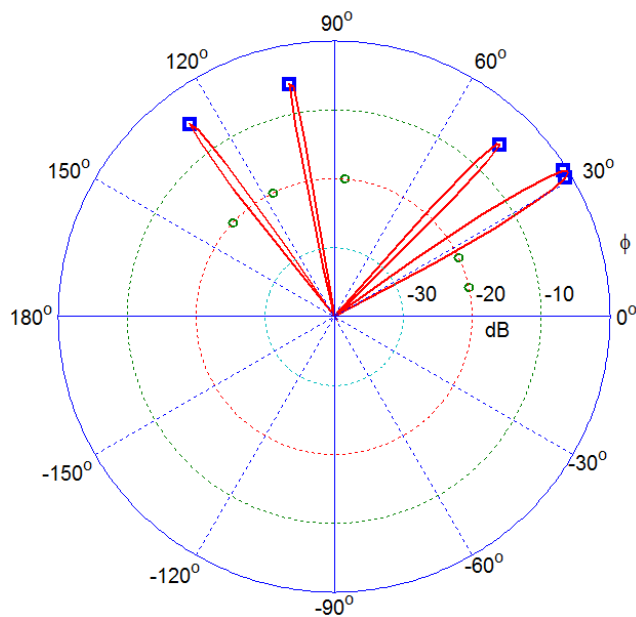


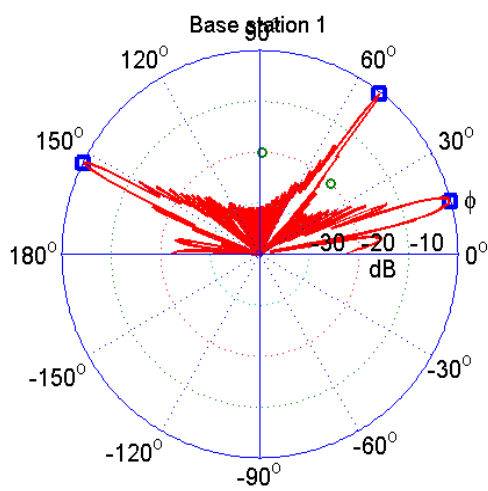
Figure 6.4 Taylor 1 Parameter method - 10 Users, 100 Antennas

Here the undesired users and desired users are kept at a constant distance from the base station for a quicker convergence. The circle represent the undesired users while the squares are the desired users. The gain value computation for the following box plots is dividing into two parts, one is the gain for the desired users and the other is the gain for the non-desired users.

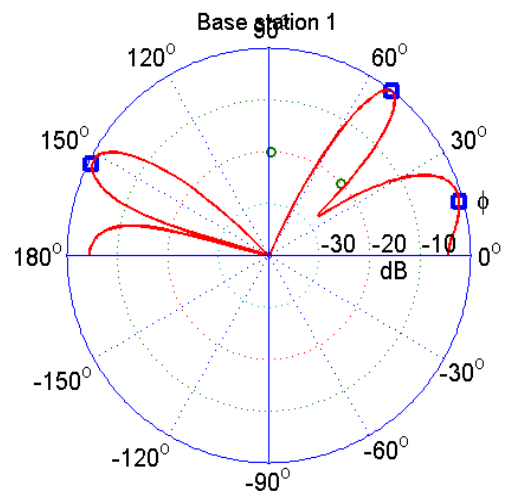
The mean values of the gain in each case (desired and non-desired users) is first computed. A point of notice is that individual users gain is normalized to unity maximum. The following definition explains the normalizing of gain to unity maximum.

The plots for different cases of antennas and users are in the following pages.

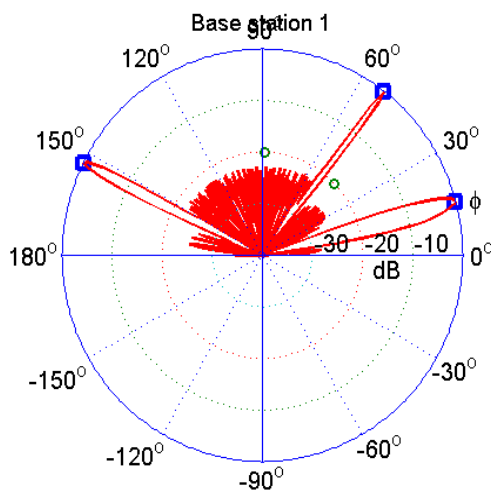
6.1.1 Radiation patterns for different cases of users and antennas



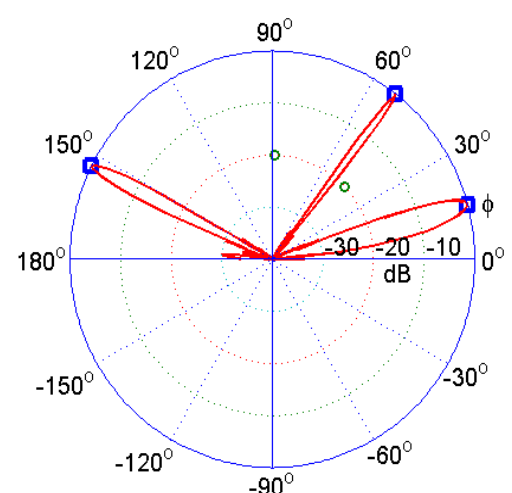
Uniform Tapering – 5 Users, 100 Antennas



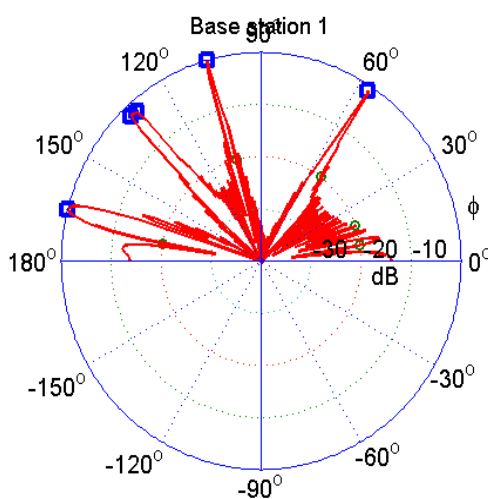
Binomial Tapering – 5 Users, 100 Antennas



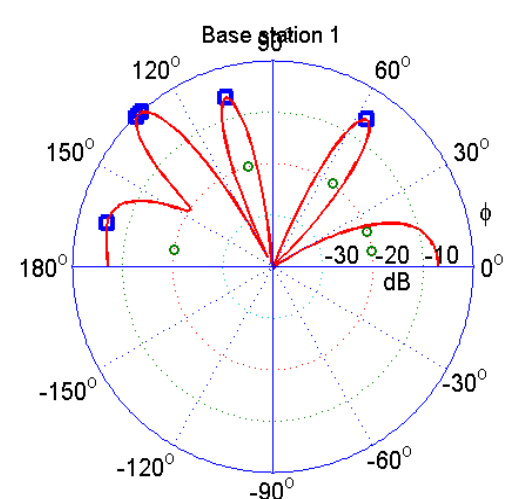
Dolph-Chebyshev Tapering – 5 Users, 100



Taylor 1 Parameter Tapering – 5 Users, 100 Antennas



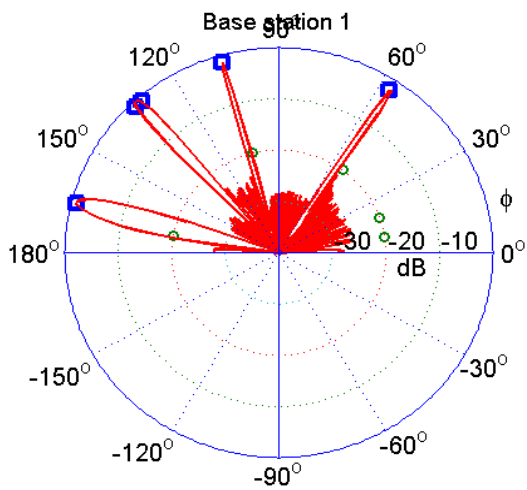
Uniform Tapering – 10 Users, 100 Antennas



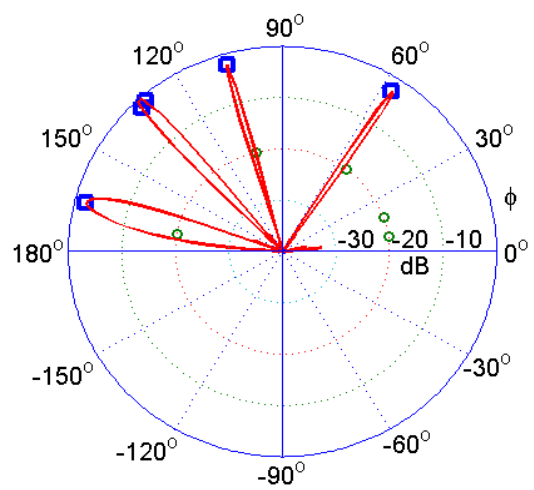
Binomial Tapering – 10 Users, 100 Antennas

Figure 6.5 Radiation Patterns with Multi-beam beamforming with increasing number of users for all 4 tapering types

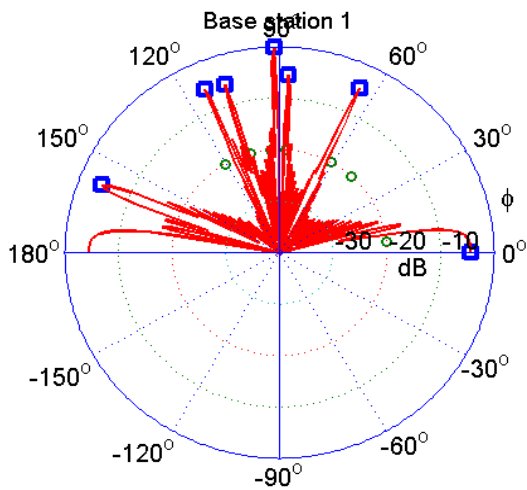




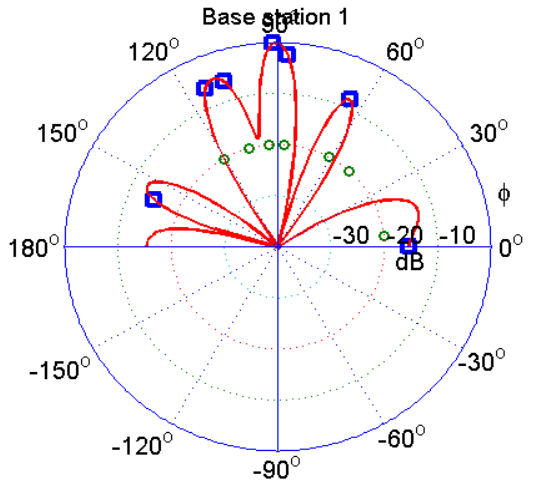
Dolph-Chebyshev Tapering – 10 Users, 100 Antennas



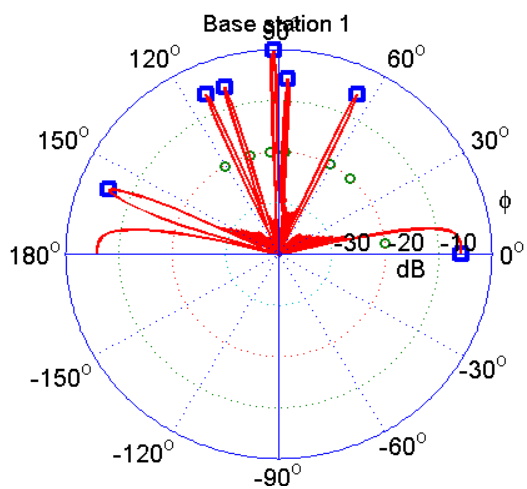
Taylor 1 parameter Tapering – 10 Users, 100 Antennas



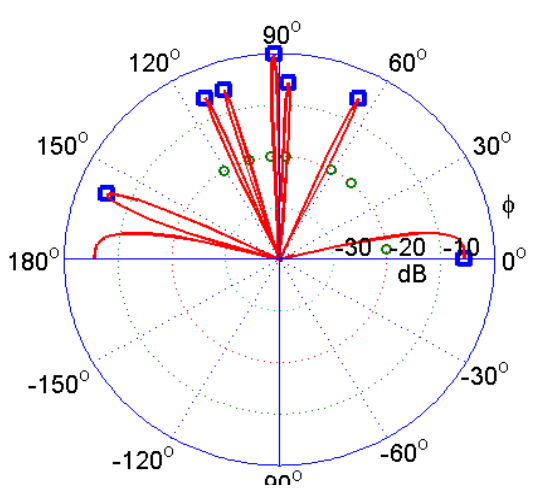
Uniform Tapering – 15 Users, 100 Antennas



Binomial Tapering – 15 Users, 100 Antennas

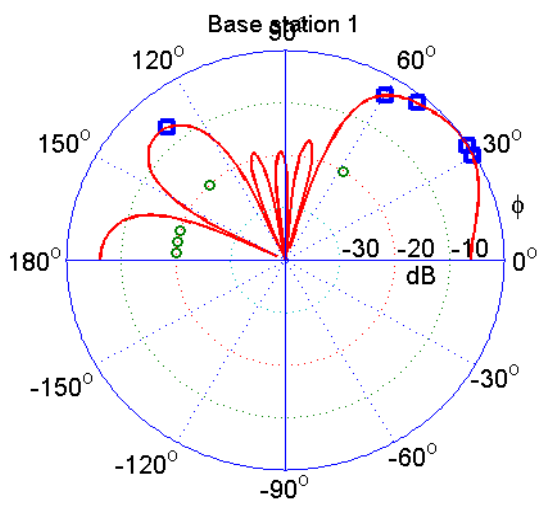


Dolph-Chebyshev Tapering – 15 Users, 100 Antennas

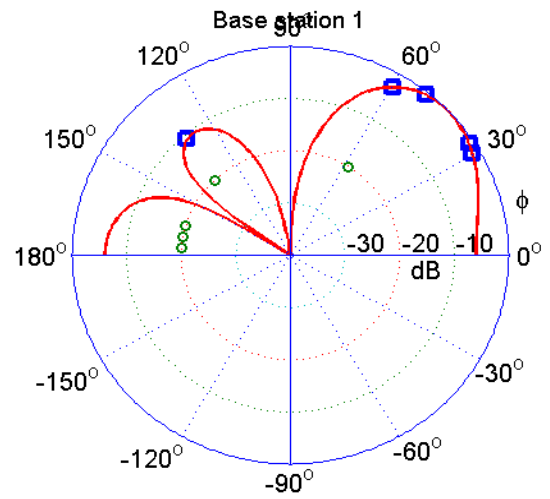


Taylor 1 parameter Tapering – 15 Users, 100 Antennas

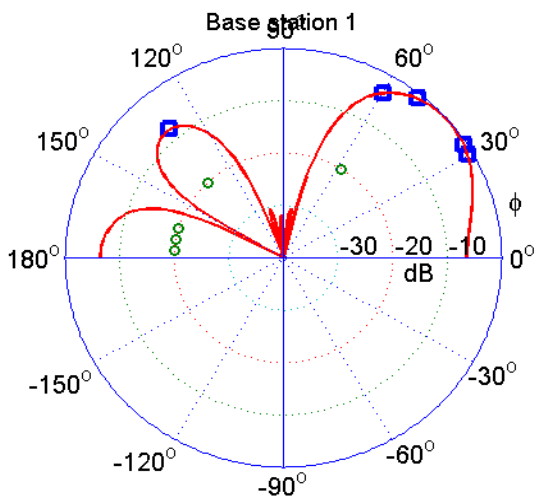
Figure 6.6 Radiation Patterns with Multi-beam beamforming with increasing number of users for all 4 tapering types



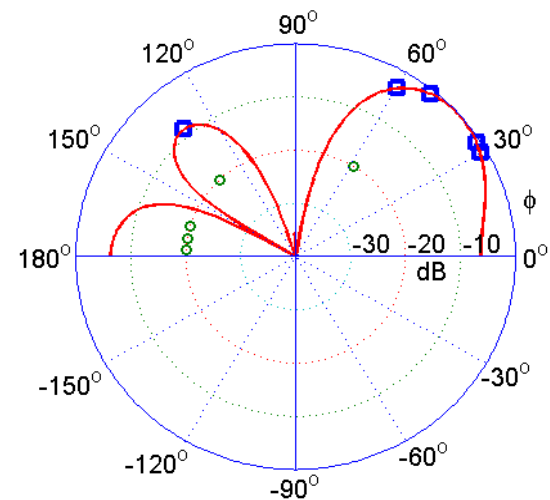
Uniform Tapering – 10 Users, 10 Antennas



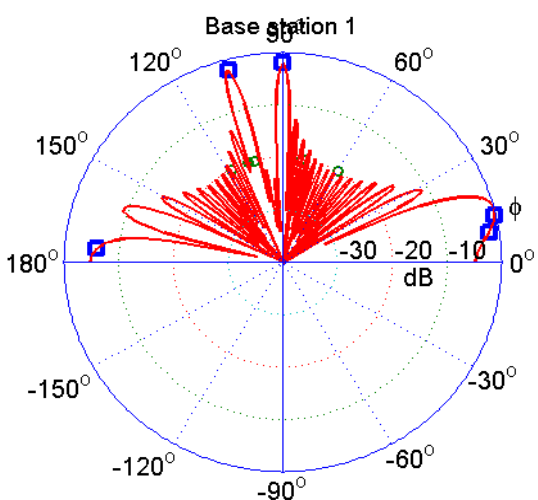
Binomial Tapering – 10 Users, 10 Antennas



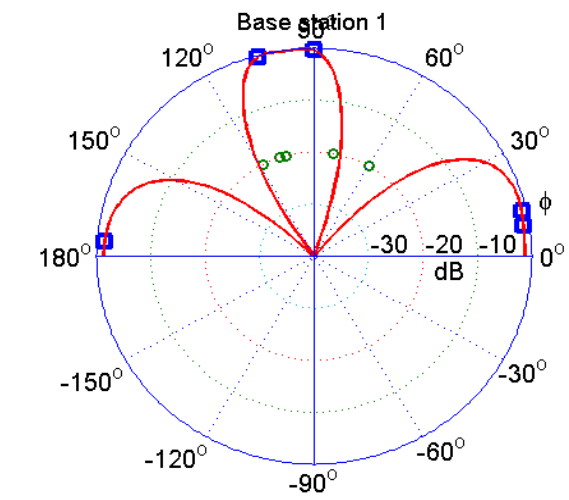
Dolph Chebyshev Tapering – 10 Users, 10 Antennas



Taylor 1 parameter Tapering – 10 Users, 10 Antennas

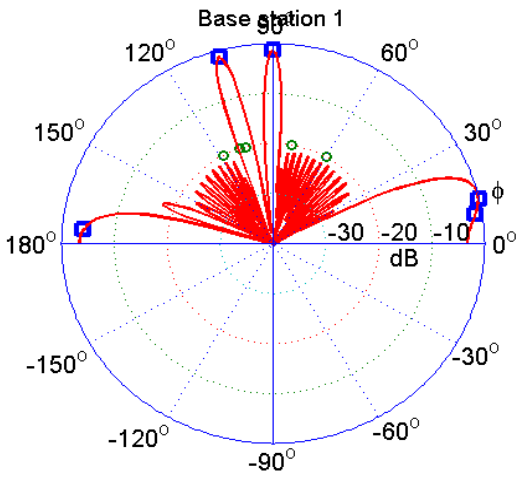


Uniform Tapering – 10 Users, 30 Antennas

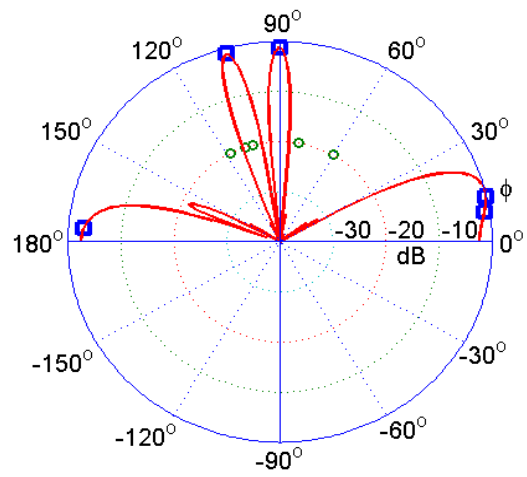


Binomial Tapering – 10 Users, 30 Antennas

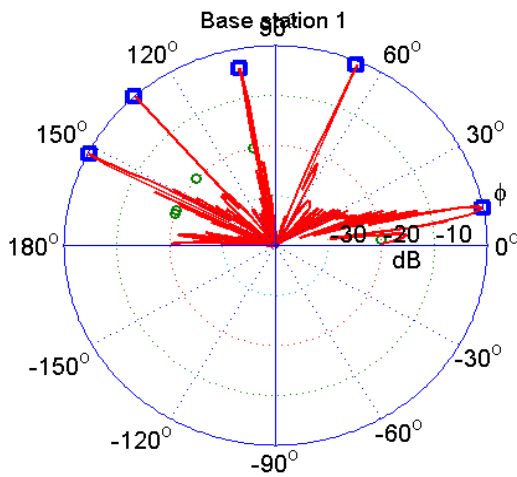
Figure 6.7 Radiation Patterns with Multi-beam beamforming with increasing number of antennas for all 4 tapering types



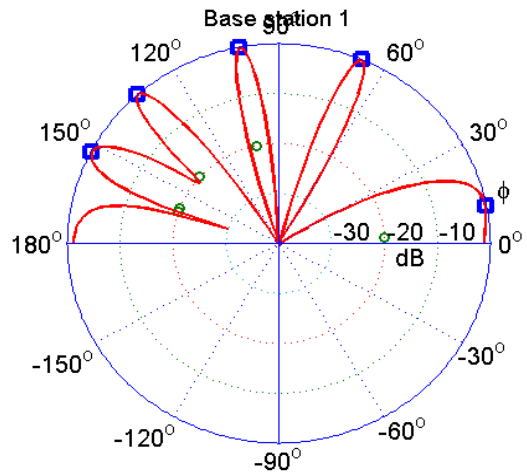
Dolph Chebyshev Tapering – 10 Users, 30 Antennas



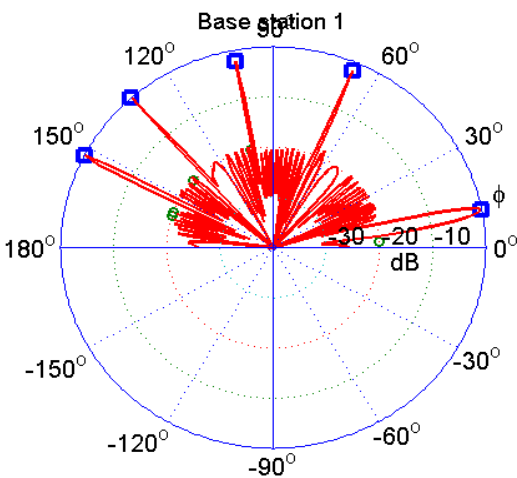
Taylor 1 Parameter Tapering – 10 Users, 30 Antennas



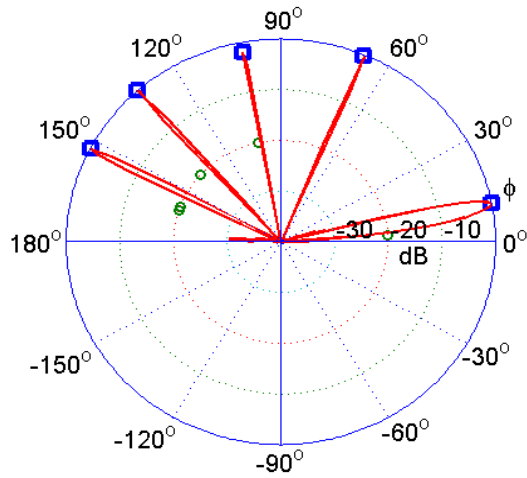
Uniform Tapering – 10 Users, 200 Antennas



Binomial Tapering – 10 Users, 200 Antennas



Dolph Chebyshev Tapering – 10 Users, 200 Antennas



Taylor 1 Parameter Tapering – 10 Users, 200 Antennas

Figure 6.8 Radiation Patterns with Multi-beam beamforming with increasing number of Antennas for all 4 tapering types

### 6.1.2 Normalised Gain

In describing the angular distribution of radiation, it proves convenient to consider it relative to its maximal value. Thus, we define the normalized power pattern, or normalized gain by

$$g_{norm}^i(\theta) = \frac{g^i(\theta)}{\text{Max}(g^i(\theta))}$$

With the above formula, the normalized gain of every user is computed. Once this is obtained, it is very important to calculate the amount of gain is offered in a particular direction when taken over the gain in all the directions in which the antenna is radiating. This is given by the following definition

### 6.1.3 Gain at a particular angle

The gain at a required angle is given is computed by taking the integral of all the gain values in the direction of radiation. Here in this thesis, the angles are divided into 1000 parts and the gain at each part is known. So, Instead of integral, the summation of the 1000 gain values.

$$g_i(\theta) = \frac{g_{norm}^i(\theta)}{\sum_{i=1}^N g_{norm}^i(\theta)}$$

Where N= Number of angles into which the radiation pattern is divided into.

From the above two definitions, we have an estimate of how much of gain is being offered in a particular direction which essentially means, the amount of gain that is being offered to a particular user. The user could be desired user or an undesired user. An important point to be noticed is that, the  $\text{Max}(g^i(\theta))$  term is cancelled while calculating the gain at a particular angle.

The box plots in the next section has the plot mean of the gain values for both the desired and undesired users. Gain there refers to the mean gain values of all the users (desired or undesired) considering the particular angles in which the users are distributed. The box plot that is plotted doesn't show the boxes based on the mean and variance but shows on the basis on medians and the number of outliers

## 6.2 Observations and Inferences

The above simulation setup is executed in MATLAB for the following test scenarios and the observations are shown below. There are two parameters of observations, number of users are taken as constant. The varying parameter here is the number of antennas. Simulations of each combination and plotted with the help of a box plot.

The first 8 plots have the gain of the desired user vs the number of antennas and the undesired user vs the number of antennas for the four types of tapering methods. The Tapering Type mentioned as 1, 2, 3 and 4 in the graphs denote Uniform, Binomial, Dolph-chebyshev and Taylor 1 parameter types respectively.

### 6.2.1 Gain of desired users (vs) varying number of antennas

No of Users = 10

No of Antennas = {10, 30, 100, 200}

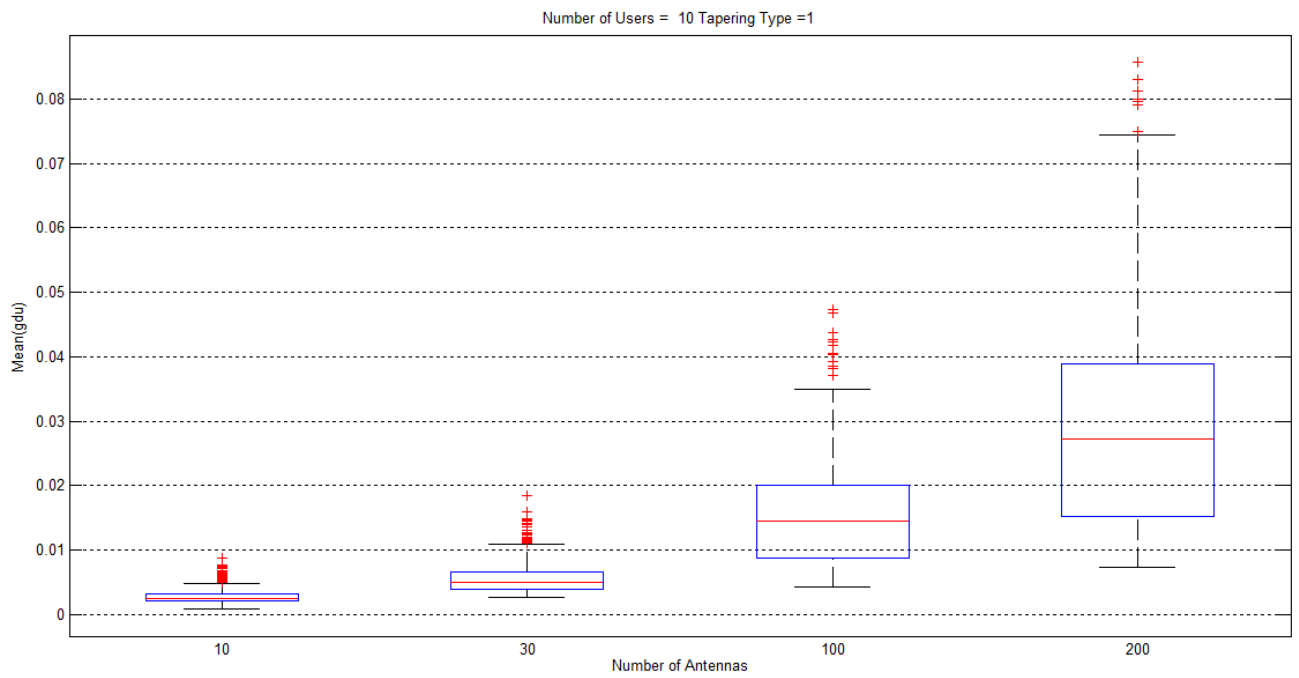


Figure 6.9 Gain values of desired users with, number of users=10, Tapering Type =1 and Increasing number of antennas

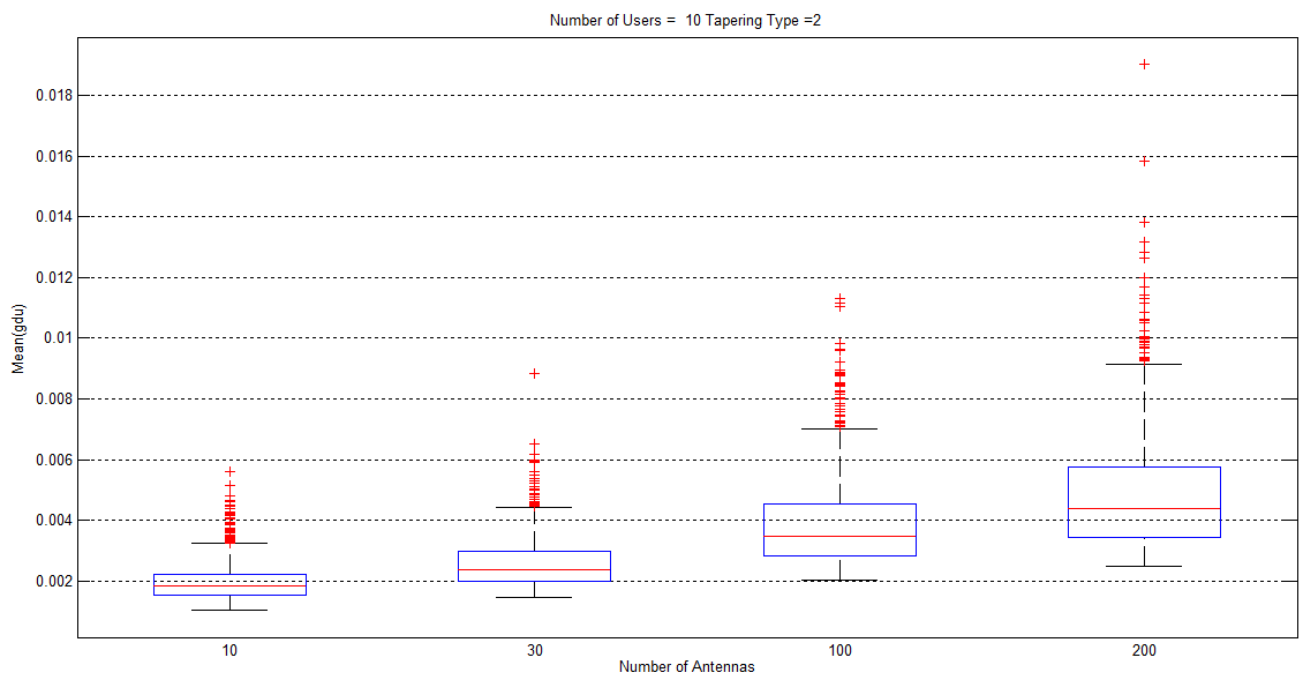


Figure 6.10 Gain values of desired users with number of users=10, Tapering Type =2 and Increasing number of antennas

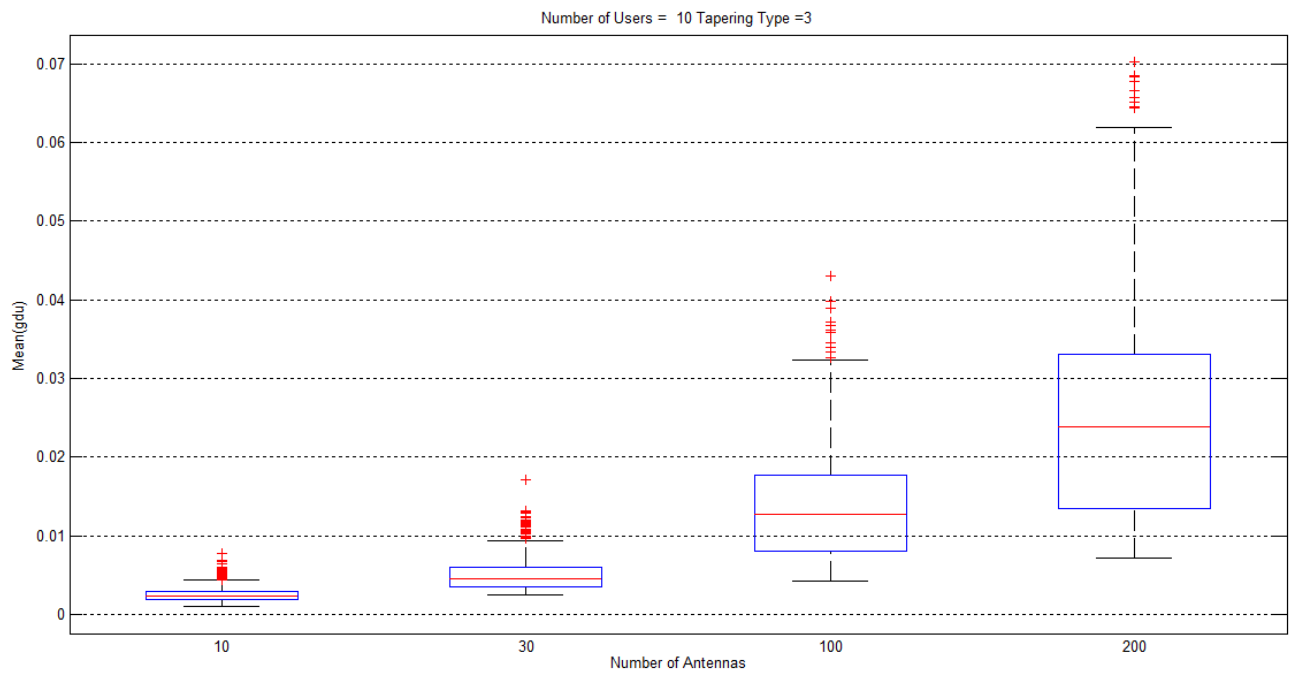


Figure 6.11 Gain values of desired users with number of users=10, Tapering Type =3 and Increasing number of antennas

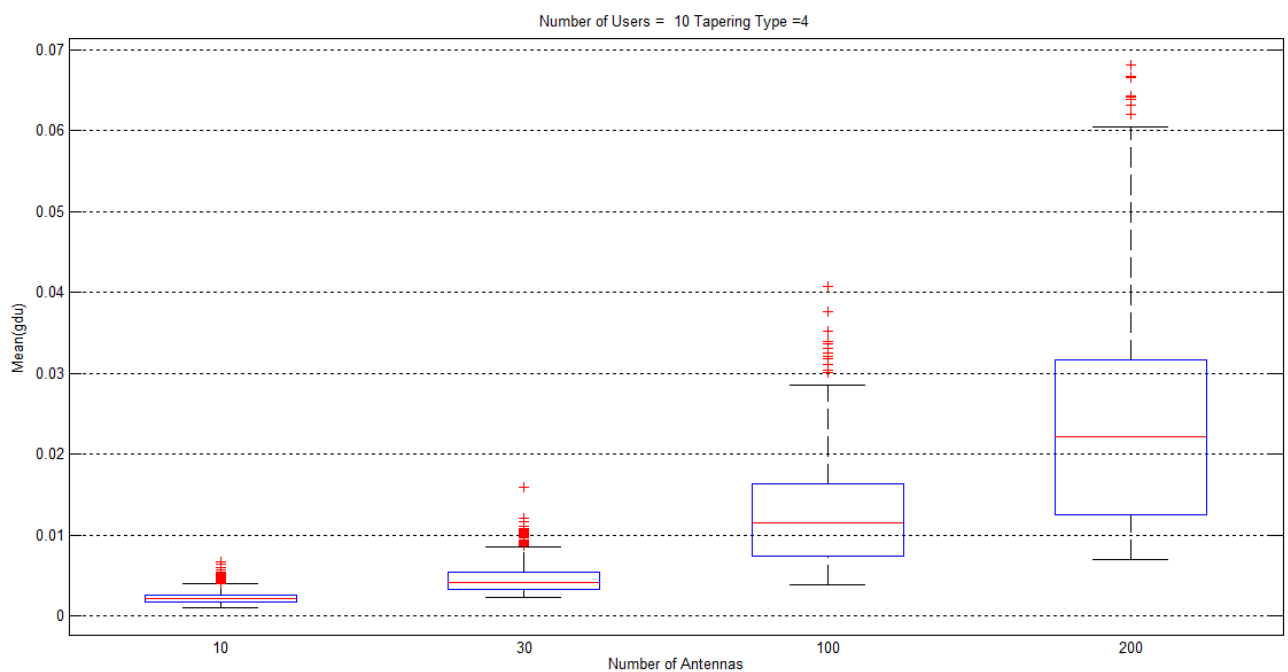


Figure 6.12 Gain values of desired users with number of users=10, Tapering Type =4 and increasing number of antennas

This graphs are generated for 1000 simulations. From the above 4 figures, its evident that as the number of antennas increase the mean value of the gain of the desired user is increasing. Another quick observation is that, the gain value is better in the case of uniform tapering when compared to binomial tapering. When compared to the Dolph and Taylor, Dolph has the best gain value to the desired user as the beamwidth in Dolph is lesser than the Taylor method. But a trade off here is that, though the difference in gain value of desired user between Dolph and Taylor is not very noticeable, the difference between gain values of the undesired users is a very important point to be observed. Based on the requirement, the apt method can be chosen. As the case of Massive MIMO get satisfied,

where the number of antennas are very large than the number of users, after a certain number of antennas, the gain values seems to not vary by a large difference.

### 6.2.2 Gain of undesired users (vs) varying number of antennas

No of Users = 10

No of Antennas = {10, 30, 100, 200}

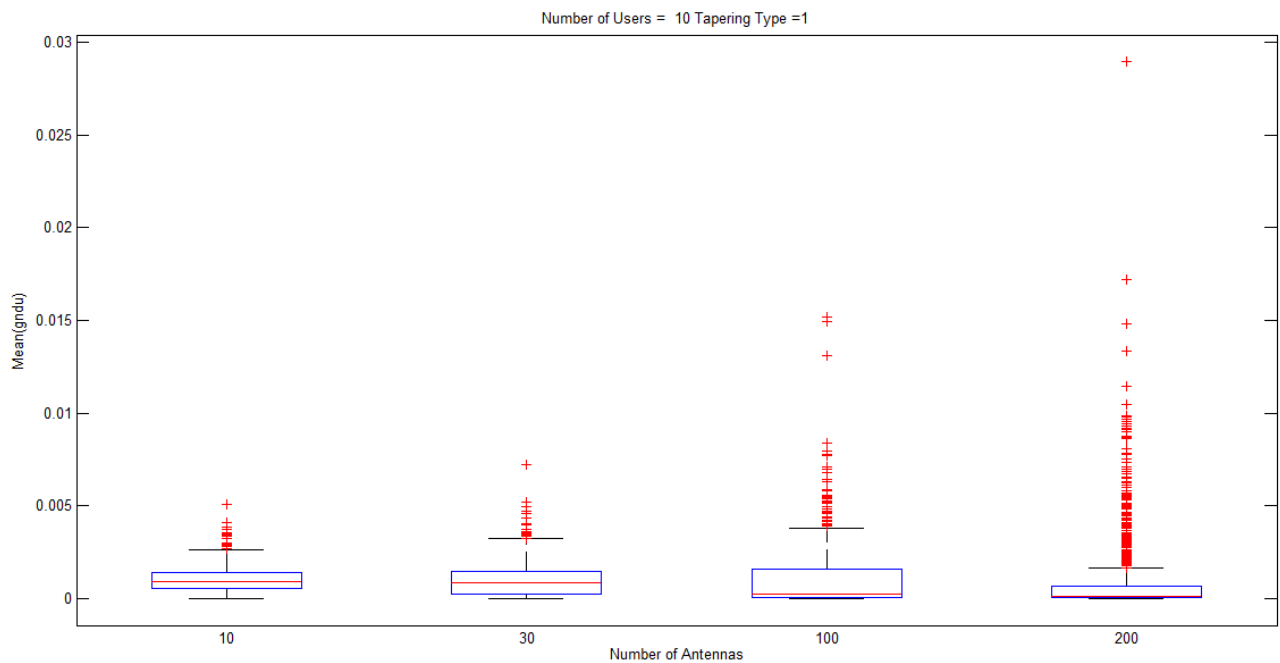


Figure 6.13 Gain values of undesired users with number of users=10, Tapering Type =1, Increasing number of antennas

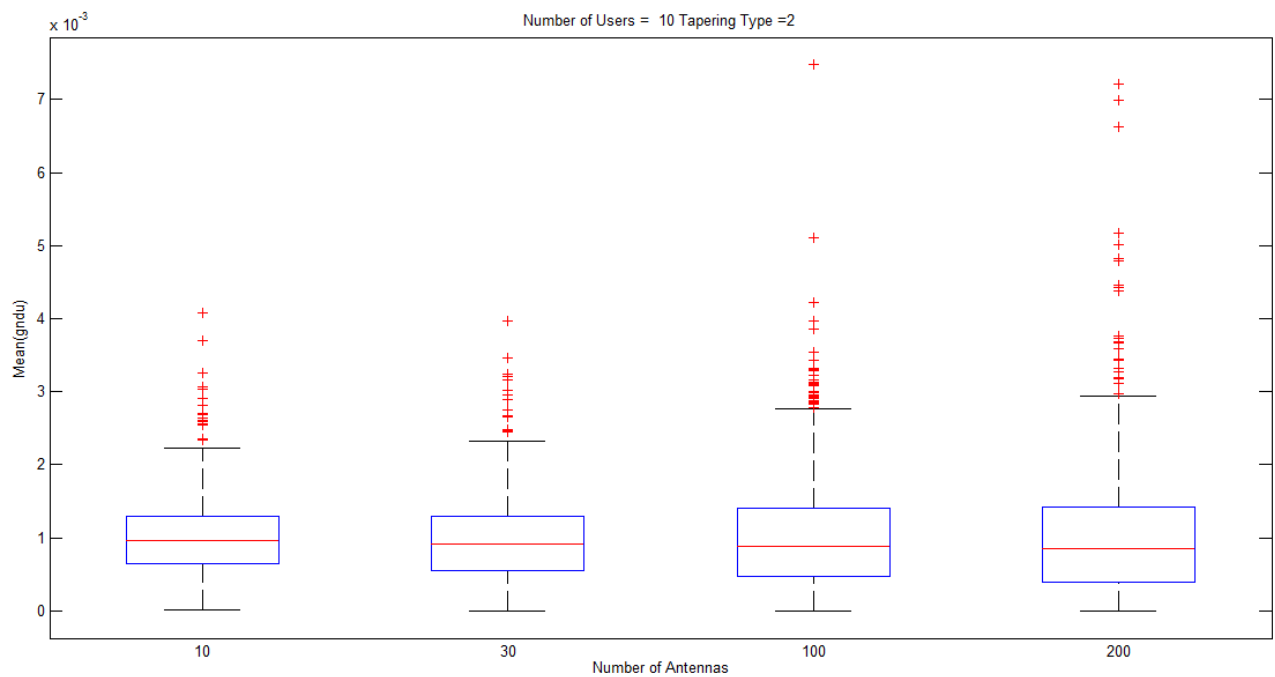


Figure 6.14 Gain values of undesired users with number of users=10, Tapering Type =2, Increasing number of antennas

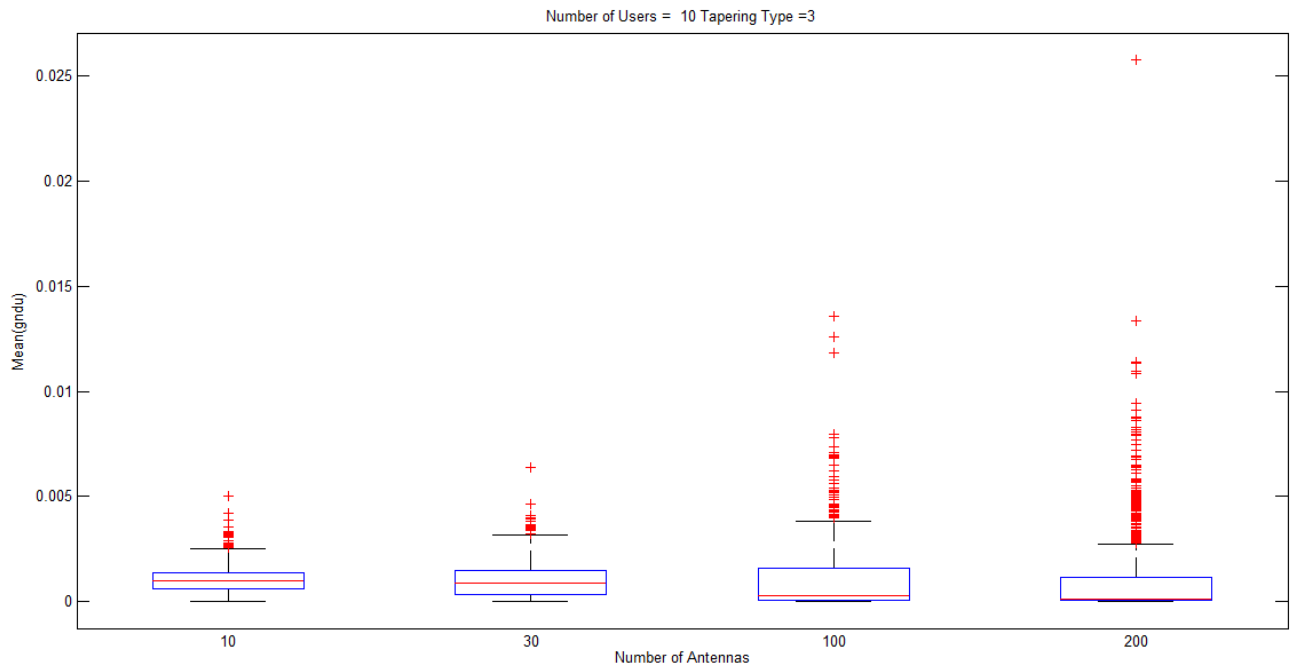


Figure 6.15 Gain values of undesired users with number of users=10, Tapering Type =3, Increasing number of antennas

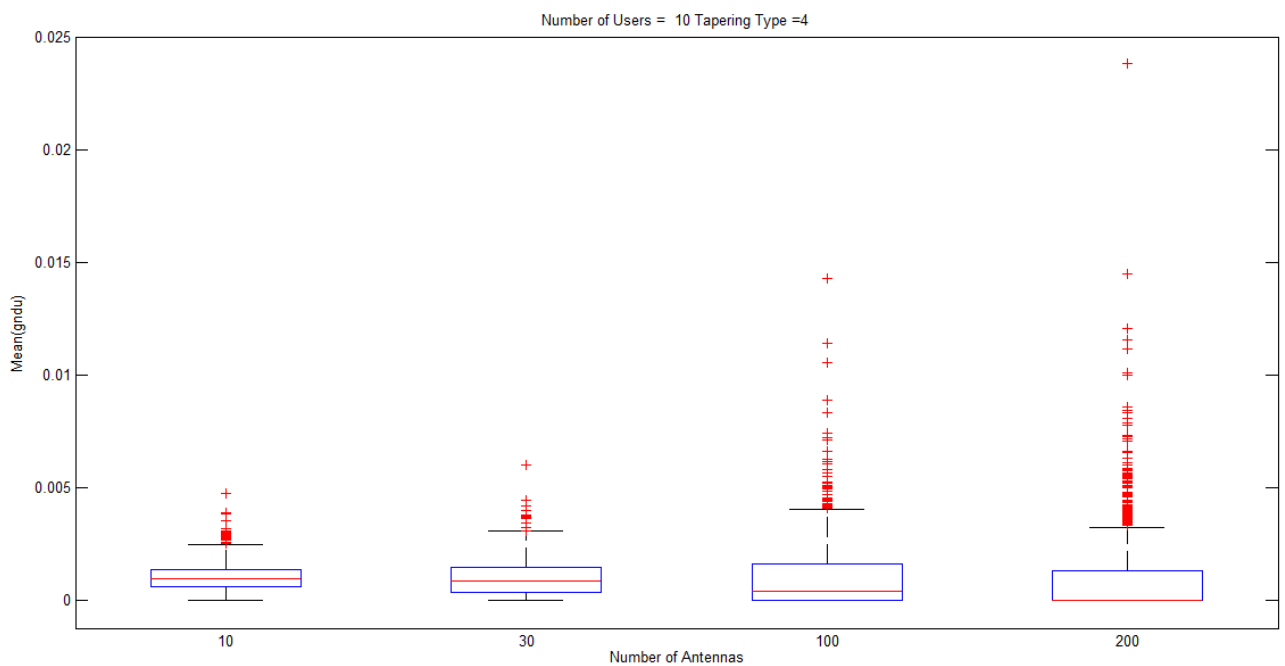


Figure 6.16 Gain values of undesired users with number of users=10, Tapering Type =4, Increasing number of antennas

The above plots are generated with 1000 Simulations. The observation here shows that, for the case of Binomial tapering, the minimum value for the case of 200 Antennas is extremely low, which is because of the absence of side lobes. But on a whole, as the number of antennas increase, for the comparison of other tapering types of 1, 3 and 4 the gain values of undesired users keeps decreasing with the increase in the number of antennas.

The above observations give a clear insight into a thought of adaptive tapering based on the requirement. For example, the median of gain values of undesired users in the case of 200 antennas, Taylor 1 parameter has a lower median value when compared to Dolph-chebyshev method, this is due to the fact that Taylor 1 parameter method has small side lobes when compared to Dolph method.



But a point to remember is that, for the gain values of desired users, Dolph performs marginally better when compared to Taylor because of the thinner beamwidth.

### 6.3 Mean Gain values

The gain values for the different cases of antennas and users have been computed. The previous section dealt with the median values and the plotting of the box plots to observe the nature of change in the values for the increasing number of antennas.

Median values are not accurate to consider for the calculation of the newly coined metric “Potential Power Improvement”. The mean of the gain values for all the 1000 simulations will now be used for the calculation of metric. Here, the mean terms should not be confused with the previous mean values. The previous mean values are the mean for the current set of users gain values at their specific angles. The mean value in the computation of this metric are the mean of all such cases of users for all the 1000 simulations. The values for the case with users as 10 and the number of antennas varying from 10, 30, 100, and 200 and the four tapering types. The results are given below:

#### 6.3.1 Number of users constant for different tapering types and increasing antenna number.

Number of Users = 10 Tapering Type =1 Antenna Number =10  
Undesired Users (gMean) =0.00101299913 Desired Users (gMean) =0.00275084873

Number of Users = 10 Tapering Type =1 Antenna Number =30  
Undesired Users (gMean) =0.00100149431 Desired Users (gMean) =0.00552009835

Number of Users = 10 Tapering Type =1 Antenna Number =100  
Undesired Users (gMean) =0.00103583391 Desired Users (gMean) =0.01535741639

Number of Users = 10 Tapering Type =1 Antenna Number =200  
Undesired Users (gMean) =0.00100346345 Desired Users (gMean) =0.02874884462

-----  
Number of Users = 10 Tapering Type =2 Antenna Number =10  
Undesired Users (gMean) =0.00101454497 Desired Users (gMean) =0.00199644599

Number of Users = 10 Tapering Type =2 Antenna Number =30  
Undesired Users (gMean) =0.00097760774 Desired Users (gMean) =0.00260317193

Number of Users = 10 Tapering Type =2 Antenna Number =100  
Undesired Users (gMean) =0.00102250774 Desired Users (gMean) =0.00389218566

Number of Users = 10 Tapering Type =2 Antenna Number =200  
Undesired Users (gMean) =0.00101615023 Desired Users (gMean) =0.00488583739

-----  
Number of Users = 10 Tapering Type =3 Antenna Number =10  
Undesired Users (gMean) =0.00101849254 Desired Users (gMean) =0.00251524477

Number of Users = 10 Tapering Type =3 Antenna Number =30  
 Undesired Users (gMean) =0.00098875151 Desired Users (gMean) =0.00500367306

Number of Users = 10 Tapering Type =3 Antenna Number =100  
 Undesired Users (gMean) =0.00103083428 Desired Users (gMean) =0.01354065014

Number of Users = 10 Tapering Type =3 Antenna Number =200  
 Undesired Users (gMean) =0.00100214956 Desired Users (gMean) =0.02461695442

-----  
 Number of Users = 10 Tapering Type =4 Antenna Number =10  
 Undesired Users (gMean) =0.00101712659 Desired Users (gMean) =0.00232988192

Number of Users = 10 Tapering Type =4 Antenna Number =30  
 Undesired Users (gMean) =0.00098688109 Desired Users (gMean) =0.00460873635

Number of Users = 10 Tapering Type =4 Antenna Number =100  
 Undesired Users (gMean) =0.00103586066 Desired Users (gMean) =0.01250013253

Number of Users = 10 Tapering Type =4 Antenna Number =200  
 Undesired Users (gMean) =0.00099575122 Desired Users (gMean) =0.02337380130

-----  
 From the above values, it's very evident that uniform is performing well in case of gain for the desired users. But with respect to the undesired users case, though it offers a stiff competition to both Dolph-chebyshev and Taylor 1 parameter.

### 6.3.2 Number of antennas is constant for different tapering types and increasing number of users.

Number of Users = 5 Tapering Type =1 Antenna Number =100  
 Undesired Users (gMean) =0.00084671122 Desired Users (gMean) =0.02677737736

Number of Users = 5 Tapering Type =2 Antenna Number =100  
 Undesired Users (gMean) =0.00097907739 Desired Users (gMean) =0.00559748927

Number of Users = 5 Tapering Type =3 Antenna Number =100  
 Undesired Users (gMean) =0.00087619670 Desired Users (gMean) =0.02339170494

Number of Users = 5 Tapering Type =4 Antenna Number =100  
 Undesired Users (gMean) =0.00088118799 Desired Users (gMean) =0.02153236305

-----  
 Number of Users = 10 Tapering Type =1 Antenna Number =100  
 Undesired Users (gMean) =0.00103583391 Desired Users (gMean) =0.01535741639

Number of Users = 10 Tapering Type =2 Antenna Number =100  
 Undesired Users (gMean) =0.00102250774 Desired Users (gMean) =0.00389218566

Number of Users = 10 Tapering Type =3 Antenna Number =100  
 Undesired Users (gMean) =0.00103083428 Desired Users (gMean) =0.01354065014

Number of Users = 10 Tapering Type =4 Antenna Number =100  
 Undesired Users (gMean) =0.00103586066 Desired Users (gMean) =0.01250013253

-----  
 Number of Users = 15 Tapering Type =1 Antenna Number =100  
 Undesired Users (gMean) =0.00100557426 Desired Users (gMean) =0.009444400764

Number of Users = 15 Tapering Type =2 Antenna Number =100  
 Undesired Users (gMean) =0.00101372987 Desired Users (gMean) =0.00290034144

Number of Users = 15 Tapering Type =3 Antenna Number =100  
 Undesired Users (gMean) =0.00101127348 Desired Users (gMean) =0.00846879791

Number of Users = 15 Tapering Type =4 Antenna Number =100  
 Undesired Users (gMean) =0.00100925162 Desired Users (gMean) =0.00787717170

-----  
 Number of Users = 20 Tapering Type =1 Antenna Number =100  
 Undesired Users (gMean) =0.00101074165 Desired Users (gMean) =0.00756442055

Number of Users = 20 Tapering Type =2 Antenna Number =100  
 Undesired Users (gMean) =0.00101283134 Desired Users (gMean) =0.00251193943

Number of Users = 20 Tapering Type =3 Antenna Number =100  
 Undesired Users (gMean) =0.00101811815 Desired Users (gMean) =0.00680595352

Number of Users = 20 Tapering Type =4 Antenna Number =100  
 Undesired Users (gMean) =0.00102192104 Desired Users (gMean) =0.00633480009

-----  
 In this case also, it's evident that uniform amplitude tapering has proved to be very effective in providing gain to the desired and the methods Dolph-chebyshev and Taylor 1 parameter are effective are reducing the side lobes and reduce the gain for the undesired users. But the difference between the uniform and the other two is very noticeable for them to consider as a replacement for the uniform.

#### 6.4 Potential Power Improvement

The power efficiency for a particular tapering type is the ratio of the difference of the power with two different tapering types to the power of the required comparison type.

For example, the efficiency is to be calculated for two tapering types of Binomial and Taylor 1 parameter method, then the following equation holds well

$$P_B^T = \frac{P^T - P^B}{P^B}$$

Where,

$P^T$  – Power value computed with mean gain values for all angles of desired users by the Taylor 1 Parameter method.

$P^B$  – Power value computed with mean gain values for all angles of desired users by the Binomial method.

#### Calculation:

Potential Power Improvement from binomial and Taylor 1 parameter.

$$P_B^T = \frac{0.02337380130 - 0.00488583739}{0.00488583739} = 3.783$$

So this shows that, the amount of feed power required in the case of Taylor 1 parameter for covering the desired users is 3.783 times lesser than the amount required in the case of binomial.

The following tables will give an overall picture of the metric “potential power improvement” in all the four cases of tapering with the change of both users and antennas keeping them constant alternatively. The calculation was done by the similar principle explained before.

#### 6.4.1 For a constant number of users as 10

| Desired Users, Mean gain values and Potential Power Improvement for 10 Users |          |          |          |          |          |          |          |  |
|--|----------|----------|----------|----------|----------|----------|----------|--|
|  | Uniform  | Binomial | Dolph    | Taylor   | Uniform  | Dolph    | Taylor   |  |
| 10   | 0.002751 | 0.001996 | 0.002515 | 0.00233  | 0.377873 | 0.259861 | 0.167015 |  |
| 30   | 0.00552  | 0.002603 | 0.005004 | 0.004609 | 1.120528 | 0.922145 | 0.770431 |  |
| 100  | 0.015357 | 0.003892 | 0.013541 | 0.0125   | 2.945705 | 2.478932 | 2.211597 |  |
| 200  | 0.028749 | 0.004886 | 0.024617 | 0.023374 | 4.884118 | 4.038431 | 3.783991 |  |

| Undesired Users, Mean gain values and Potential Power Improvement for 10 Users |          |          |          |          |          |          |          |  |
|--|----------|----------|----------|----------|----------|----------|----------|--|
|  | Uniform  | Binomial | Dolph    | Taylor   | Uniform  | Dolph    | Taylor   |  |
| 10   | 0.001013 | 0.001015 | 0.001018 | 0.001017 | -0.00152 | 0.003891 | 0.002545 |  |
| 30   | 0.001001 | 0.000978 | 0.000989 | 0.000987 | 0.024434 | 0.011399 | 0.009486 |  |
| 100  | 0.001036 | 0.001023 | 0.001031 | 0.001036 | 0.013033 | 0.008143 | 0.013059 |  |
| 200  | 0.001003 | 0.001016 | 0.001002 | 0.000996 | -0.01249 | -0.01378 | -0.02007 |  |

The last three columns in the above two excel sheets shows the Potential Power Improvement of the Uniform, Dolph-Chebyshev, Taylor 1 parameter method, over the Binomial method. Binomial method is used for reference as this has the worst performance when compared to the others.

## 6.4.2 For a constant number of antennas as 100

**Desired Users, Gain values and Potential Power Improvement for 100 Antennas**

|    | Uniform  | Binomial | Dolph    | Taylor   | Uniform  | Dolph    | Taylor   |
|----|----------|----------|----------|----------|----------|----------|----------|
| 5  | 0.026789 | 0.005599 | 0.023402 | 0.021542 | 3.784809 | 3.17984  | 2.847624 |
| 10 | 0.015357 | 0.003892 | 0.013541 | 0.0125   | 2.945705 | 2.478932 | 2.211597 |
| 15 | 0.009444 | 0.0029   | 0.008469 | 0.007877 | 2.256171 | 1.919931 | 1.715946 |
| 20 | 0.007564 | 0.002512 | 0.006806 | 0.006335 | 2.011387 | 1.709442 | 1.521876 |

**Undesired Users, Gain values and Potential Power Improvement for 100 antennas**

|    | Uniform  | Binomial | Dolph    | Taylor   | Uniform  | Dolph    | Taylor   |
|----|----------|----------|----------|----------|----------|----------|----------|
| 5  | 0.000846 | 0.000978 | 0.000875 | 0.00088  | -0.13519 | -0.10503 | -0.09998 |
| 10 | 0.001036 | 0.001023 | 0.001031 | 0.001036 | 0.013033 | 0.008143 | 0.013059 |
| 15 | 0.001006 | 0.001014 | 0.001011 | 0.001009 | -0.00805 | -0.00242 | -0.00442 |
| 20 | 0.001011 | 0.001013 | 0.001018 | 0.001022 | -0.00206 | 0.00522  | 0.008975 |

The last three columns in the above two tables shows the “Potential Power Improvement” of the Uniform, Dolph-Chebyshev, Taylor 1 parameter method, over the Binomial method. Binomial method is used for reference as this has the worst performance when compared to the others. In both the cases of varying number of users or antennas, uniform tapering method performs the best in the case of providing the best gain value for the desired user. While the Dolph and Taylor method compete to reduce the gain for the undesired users where the uniform still gives a tough competition.

From this a few inferences are that, tapering methods are very effective in reducing the side lobe levels and hence the interferences reduces. The principle problem with the case of uniform method, the sidelobe levels are very high. In the case of multi cell scenario, this will heavily effect in the computation of the SINR (Signal to interference plus noise ratio).

The SINR computation for multi-beam beamforming in the multi-cell scenario is not yet known to best of my search and knowledge. This thesis novelty also lies in trying to incorporate the formula available for the conventional beamforming and adapting that to the case defined in this thesis.

The SINR is the best metric to gauge the signal power received at the users. But, a formula has been adapted from another method which suites the purpose [13].

$$\gamma_{i,j} = \frac{P_{feed} |w_{i,j}^H h_{i,i,j}|^2}{P_{feed} \sum_{l \neq j} |w_{i,l}^H h_{i,i,j}|^2 + P_{feed} \sum_{m \neq i,n} |w_{m,n}^H h_{m,i,j}|^2 + \sigma^2}$$

Where,

$h_{l,i,j} \in \mathcal{C}^{N_t \times 1}$  – Is the channel vector from the base-station of the  $l^{\text{th}}$  cell to the  $j^{\text{th}}$  user in the  $i^{\text{th}}$  cell.

$w_{i,j}^H \in \mathbb{C}^{N_t \times 1}$  – Be its associated beamforming vector, for the  $j^{\text{th}}$  user in the  $i^{\text{th}}$  cell.

So in the case of the multi-cell, and multi-beam the first term in the denominator will be a one term with the beamforming vector and the channel. The denominator beamforming vector as indicated will be generated after the current user considered in the numerator.

The ongoing work is on defining the parameters and compute the SINR at every user using multi-beam beamforming. This will be first be computed in the case of single cell, where the SINR will be SNR. Post these results with single cell, the multi-cell scenario will be setup based on the setup parameters given in this thesis and the SINR at every user will be computed.

## 7 Conclusion and Future work

*This chapter finishes this thesis with the conclusion. The second part of this chapter deals with the future work in this field.*

## 7.1 Conclusion

The thesis is an initial research towards achieving energy efficient 5G networks. A detailed initial study was done on the potential places where there is maximum power consumption in whole mobile network. This research progresses with a focus to reduce both the CAPEX and the OPEX in the telecom sector with the incorporation of energy efficiency aspect.

The generation of mobile users based on the real-time maps of Berlin was an added advantage in testing the system in similar to real-time system. This thesis has also discussed a good insight of simulation in the multi-cell scenario. The importance for beamforming was discussed in the thesis and the concept of beamforming was served by using the multi-beam beamforming, which this work expects to perform better than the alternatives like conventional beamforming, and multicast beamforming.

The multi-beam beamforming system was initially tested with different amplitude tapering types. The results for over 1000 Simulations of each tapering type with different combination of users and antennas was done. The box plot for the results was plotted. An important assumption to be noted here is that, the as much as possible minimum gain values of sidelobes for the undesired users is also considered to affect the user. For example, in the Taylor 1 Parameter, the sidelobes are highly suppressed and the gain values at an undesired user are in the order of  $10^{-11}$  are also considered to affect to user and are included in the calculation. But in real-time, values of such low magnitude will not be of significance. The mean gain values for all the combinations of the users and antennas was done and was found that Uniform tapering performs the best, followed by Dolph-Chebyshev and Taylor 1 parameter method. In the process of gauging with users and antennas combinations, the case of Massive MIMO was also included as to verify the current methods compatibility with it. The binomial as expected would perform the worst, though it has the advantage of no side lobes, the main beam width causes a lot of power wastage and gains in the undesired directions.

Currently, for the gain values at each direction of the desired user and the undesired users are taken, and based on these values, the power required for both the case of users are calculated taking binomial method as a reference. Uniform again performed better in this aspect also. For a fair testing between the tapering methods, the next step would be thresholding of the gain values to a certain value. The reason being that, gain values of such low magnitude will not be of significance. The gain values after thresholding to a specific value which for its low magnitude is not going to affect the user in any form, should result in a better performance of the Dolph and Taylor method when compared to the uniform method.

Once the multi-beam beamforming clears this verification, the SINR which is proposed in this thesis, will then be calculated in the case of all the tapering methods and it is so expected that Dolph and Taylor method should be doing good. This is for the reason that uniform method doesn't provide the option of maintaining a relative side lobe level which the Dolph and Taylor offer. This varying of sidelobe levels in the uniform method has been marked in the figure 6.1



## 7.2 Future Work

The current work with the generation of mean gain values works on the assumption that even the slightest (order of  $10^{-11}$ ) of the side lobe level will affect with the user and hence the computations have been done. The future work, will include thresholding of the gain values for the undesired users and if the value is lower than threshold value, the gain towards the undesired user will be zero. In this case, we expect Taylor 1 parameter and Dolph chebyshev methods to perform better. This is because, the mean value in the case of Dolph and Taylor may be higher thereby resulting in the increase of the mean value for some cases and hence making uniform tapering method standing a better chance when compared to Taylor and Dolph method. After this thresholding, as the values will become zero, it is expected that the power consumption also reduces to a considerable amount when compared to uniform tapering case.

Also, the SINR in each case of tapering will be calculated based on the formulas mentioned in this thesis. The uniform case will be giving a lot of interference value to the users, because the side lobes are large and there is no control of the relative side lobe level as it is present in the case of Dolph and Taylor 1 parameter method.

In the situations where the mobile operator/researchers are required to choose between the interference and power requirement or both, this thesis and its future work will hope to serve as a contribution to choose the required tapering for the combinations of the antennas, users and the frequency allocation strategies. This includes working with Massive MIMO. All the above discussed cases, will also have to verify in the case of multi-cell scenario.

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