Simulation of a pinhole test of an MX-80 bentonite

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An approach is proposed for simulating the coupled processes of unsaturated swelling and mechanical erosion of MX-80 bentonites. The rate of bentonite surficial mass removal per unit area $\dot{m}_B$ is estimated using the expression:

[1] $$\dot{m}_B = k_e \cdot \max(\tau_Q - \tau_C, 0)$$

where τ_Q is the hydraulic shear stress produced by the water flow, τ_C is the critical shear stress at which the mass removal starts, and the proportionality factor k_e is the coefficient of bentonite erosion. This equation is often used in Soil Science (Hanson and Simon, 2001; Sanford and Maa, 2001) and in the analysis of piping in embankment dams (see, among others, Wan and Fell, 2004; Wahl, 2010). If, as happens on the pinhole tests analysed (Sane *et al*., 2013), slaking plays an important role (see Lim, 2006, and Sane *et al*., 2013), a value of $\tau_C = 0$ may be adopted, assuming an "instantaneous" activation of the erosion in the preferential flow path (PFP) analysed.

If a quasi-steady laminar flow has fully developed in the PFP, τ_Q is given by the equation (Wahl *et al.*, 2008):

[2] $$\tau_Q = \frac{4 \cdot \mu_W \cdot Q}{\pi \cdot r_t^3}$$

where μ_W is the dynamic viscosity of the water, Q is the water flow rate (0.1 l/min in the pinhole tests), and r_t is the characteristic radius of the PFP at each time t.

Therefore, the total erosion rate per unit length of pinhole, $\dot{M}_B$, can be calculated as:

[3] $$\dot{M}_B = 2 \cdot \pi \cdot r_t \cdot \dot{m}_B$$

which, using material coordinates, can be rewritten as

[4] $$\dot{M}_B = 2 \cdot \pi \cdot R_t \cdot \rho_{dO} \cdot \dot{R}_t$$

where ρ_{dO} is the initial dry density of the bentonite, and R_t is the value of the material radial coordinate R at the erosion front at each time t (the soil at $R < R_t$ has already been eroded at time t; see Figure 1 b). Therefore, once R_t and r_t are known, the erosion rate $\dot{R}_t$ can be calculated from Eqs. (1) through (4) using the expression:

[5] $$\dot{R}_t = \frac{4 \cdot \mu_W}{\pi \cdot \rho_{dO}} \frac{k_e \cdot Q}{R_t \cdot r_t^2}$$

This ordinary differential equation allows for updating of the material domain by defining the rate at which the boundary of the active zone moves. Its time integration defines a relationship between the boundary of the active (not eroded) material domain ("material frame") and a reference domain or "geometric frame", equal to the initial material domain. The same finite element mesh is always used in the geometric frame. This is not the case in the material frame, changed by the erosion. We propose to apply a smoothing method in this frame to perturb the positions of the nodes of the finite-element mesh at every computer time step in order to continually adapt the mesh to the active domain, as the "Deformed Geometry" module of Comsol Multiphysics (COMSOL, 2013) does. Using the values of previous steps, the prediction of the values of the state variables are extrapolated in time and interpolated in space for the new nodes. Since the mesh used is of the same type as that defined in the geometric frame (the same mesh settings are used), a one-to-one relationship (controlled by $\dot{R}_t$ and by

the smoothing method) can be defined between the new nodes of the material mesh and those of the geometric mesh, which is kept fixed. This one-to-one transform is used for assigning the information of the material mesh to the geometric mesh, being the latter the frame for all the calculations that are performed. Thus, using the geometric frame as the workspace, the material frame is continually updated. As it is done when a Lagrangian formulation is applied, the determinant and the deformation gradient tensor associated with the transform between reference and material frames have to be included in the weak formulation of the equilibrium and balance equations. This is automatically done by Comsol Multiphysics when the Deformed Geometry module is activated.

The accuracy of the prediction of the erosion rate $_t$ depends on the accuracy of r_t, that is, on the quality of the bentonite deformability model. Taking into account the usual bimodal pore-size distribution of active, a double-porosity model was used to describe their deformational behaviour (Alonso et al., 1999; Gens and Alonso, 1992; Musso et al., 2013). Using the Barcelona Expansive Model (Alonso et al., 1999; Gens and Alonso, 1992) as a reference, an additional volumetric strain was introduced to model the deformation induced in the macrostructure by the aggregate destructuration (Navarro et al., 2014). This deformation can be very high in the direction orthogonal to the PFP, where free deformation condition applies. Hence, a Lagrangian formulation was adopted to take into account that the deformation gradient tensor can be considerably different from the identity tensor.

In Figure 1 experimental data and model results of mass loss per unit height are compared. A variety of pinhole tests on MX-80 samples with various configurations were analysed: sample diameters, sample heights and initial hole diameters: of (a) 100x100x6 mm, (b) 100x400x6 mm, (c) 100x100x12 mm. The trend observed in the experiments is correctly reproduced. Thus, although the approach could be further improved (introducing, e.g., the effect of water salinity), the obtained results indicate that this method is a useful tool for describing and quantifying the coupled erosion and swelling of MX-80 bentonites.
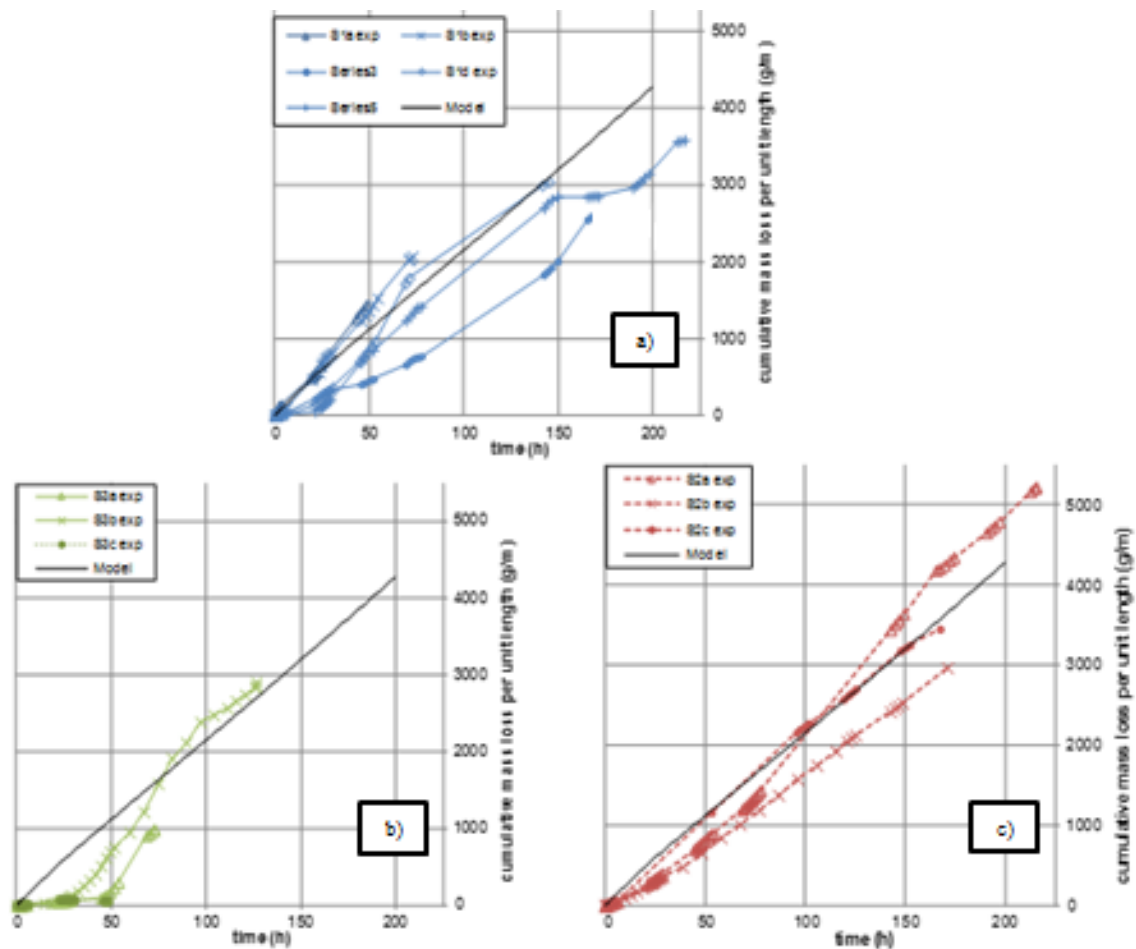

Figure 1

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