A Decomposition Heuristics based on Multi-Bottleneck Machines for Large-Scale Job Shop Scheduling Problems

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Abstract:

Purpose: A decomposition heuristics based on multi-bottleneck machines for large-scale job shop scheduling problems (JSP) is proposed.

Design/methodology/approach: In the algorithm, a number of sub-problems are constructed by iteratively decomposing the large-scale JSP according to the process route of each job. And then the solution of the large-scale JSP can be obtained by iteratively solving the sub-problems. In order to improve the sub-problems' solving efficiency and the solution quality, a detection method for multi-bottleneck machines based on critical path is proposed. Therewith the unscheduled operations can be decomposed into bottleneck operations and non-bottleneck operations. According to the principle of "Bottleneck leads the performance of the whole manufacturing system" in TOC (Theory Of Constraints), the bottleneck operations are scheduled by genetic algorithm for high solution quality, and the non-bottleneck operations are scheduled by dispatching rules for the improvement of the solving efficiency.

Findings: In the process of the sub-problems' construction, partial operations in the previous scheduled sub-problem are divided into the successive sub-problem for re-optimization. This strategy can improve the solution quality of the algorithm. In the process of solving the sub-

problems, the strategy that evaluating the chromosome's fitness by predicting the global scheduling objective value can improve the solution quality.

Research limitations/implications: In this research, there are some assumptions which reduce the complexity of the large-scale scheduling problem. They are as follows: The processing route of each job is predetermined, and the processing time of each operation is fixed. There is no machine breakdown, and no preemption of the operations is allowed. The assumptions should be considered if the algorithm is used in the actual job shop.

Originality/value: The research provides an efficient scheduling method for the large-scale job shops, and will be helpful for the discrete manufacturing industry for improving the production efficiency and effectiveness.

Keywords: decomposition heuristics; multi-bottleneck; job shop scheduling; critical path

1. Introduction

The job shop scheduling problem (JSP) is a well-known NP-hard problem (Pinedo, 2008; Qingdaoerji, Wang & Wang, 2013). There are a lot of effective methods for solving the small-scale JSP. However, there are relatively fewer studies on the large-scale JSP. Some research (Chen & Luh, 2003; Haoxun, Chengbin & Proth, 1998) proposed a lagrangian relaxation (LR) approach for the large-scale JSP. In the approach, machine capacity constraints or operation precedence constraints are relaxed, and the relaxed problem is decomposed into single machine or single job scheduling sub-problems. These sub-problems are approximately solved by using fast heuristic algorithms. Shifting bottleneck procedure (SB) (Mönch, Schabacke, Pabst & Fowler, 2007; Scholz-Reiter, Hildebrandt & Tan, 2013; Braune, Zäpfel & Affenzeller, 2012) and constraint scheduling algorithm (CSA) (Zuo, Gu & Xi, 2008; Dalfard & Mohammadi, 2012) decomposes the JSP into a number of single machine scheduling sub-problems. In each sub-problem, a critical or bottleneck machine is identified and scheduled, with scheduling decisions at subsequent iteration being subordinated to those scheduled earlier. Some research (Bassett, Pekny & Reklaitis, 1996; Sourirajan & Uzsoy, 2007; Lin & Liao, 2012) proposed a rolling horizon heuristic that decomposes the JSP into smaller sub-problems that can be solved sequentially over time.

Most of the existing methods are proposed for the JSP in which the number of jobs is large. While, the JSP with a large number of machines has got fewer attentions. According to Zhang, Li, Rao and Guan (2008), it is relatively easy for solving the JSP when the number of jobs is larger than the number of machines. Therefore for the problems with the same scale, it is more difficult to solve the problem with a large number of machines than the problem with a

large number of jobs. In this paper, a decomposition heuristics based on multi-bottleneck machines (DH-MB) is proposed for the JSP with a large number of machines. The algorithm solves the problem by iteratively decomposing the original problem into a series of sub-problems. It adopts critical path method to detect the multi-bottleneck machines, and uses the characteristics of bottleneck machines for solving the sub-problems. The final solution is obtained by the iterative construction and solving of the sub-problems.

The paper is organized as follows: The large-scale JSP problem is formulated in section 2. Section 3 presents the detection method of the multi-bottleneck machines, the decomposition approach, the solving process, and some strategies in DH-MB. The simulation results are provided in Section 4. Finally, some conclusions are given in Section 5.

2. Problem formulation

In a job shop scheduling problem, a set of jobs are to be processed on a set of machines. The number of jobs is n and the number of machines is m. Each job $J_i(i = 1 \ldots n)$ contains m operations which have the operation precedence constraints and must be processed on each machine $M_k(k = 1 \ldots m)$ only once. The task of the scheduling is to determine the processing order for each machine and the starting time for each operation while satisfying some objectives. The basic assumptions are as follows:

- The processing route of each job is predetermined, and the processing time of each operation is fixed.

- There is no machine breakdown, and no preemption of the operations is allowed.

- Each machine can process only one job at a time, and each job can be processed by only one machine at a time.

JSP can also be described by a disjunctive graph model $G(N, A, E)$, in which $N = \{0, 1, \ldots m \cdot n, *\}$ denotes the set of nodes. Each node corresponds to an operation (0 and * represent the dummy operations "start" and "finish"); A denotes the set of conjunctive arcs which connect the operations of each job. The conjunctive arcs have fixed directions according to the processing route of each job; $E = \bigcup_{k \in M} E_k$ is the set of disjunctive arcs, and E_k denotes the set of pairs of operations to be performed on machine k. A selection S_k in E_k contains exactly one member of each disjunctive arc pair of E_k . Actually, determining an acyclic selection on S_k is equivalent to sequencing the operations on machine k. Replacing the disjunctive arc set E by the conjunctive arc set S, gives rise to the directed graph $D(N, A \cup S)$ which corresponds to a

feasible solution for JSP. Figure 1 illustrates a disjunctive graph for a job shop scheduling problem with 3 jobs and 3 machines.
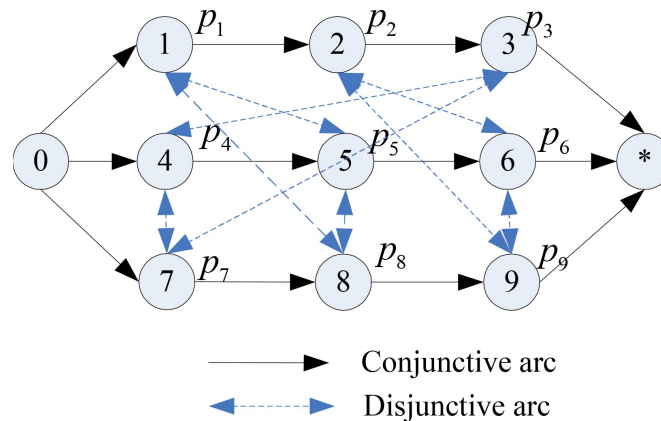

Figure 1. An illustration of a disjunctive graph for a job shop
scheduling problem with 3 jobs and 3 machines

The job shop problem can be formulated as follows:

$$\min WT = \sum_{j \in C} \max(w_j(t_j + p_j - d_j), 0) \tag{1}$$

$$t_j - t_i \geq d_i, \quad (i,j) \in A \tag{2}$$

$$t_i \geq 0, \quad i \in N \tag{3}$$

$$t_j - t_i \geq p_i \vee t_i - t_j \geq p_j, \quad (i,j) \in E_k, k \in M \tag{4}$$

where, C denotes the set of the last operation of each job; P_j and t_j are the processing time and starting time of operation j respectively; w_j and d_j are the weight and due-date of the job to which operation j belongs. Equation (1) is the scheduling objective of the total weighted tardiness; Equation (2) describes the operation precedence constraints for the operations of each job; Equation (3) ensures that no job can start in the past; Equation (4) describes the machine capacity constraints to make sure each machine can process only one job at a time.

3. The algorithm

In large-scale JSP, the commonly used algorithms hardly satisfy the requirements of solving time and solution quality simultaneously because the number of scheduling machines and jobs are very large. Therefore it is very important to reduce the scale and complexity of the large-scale JSP. A decomposition heuristics based on multi-bottleneck machines (DH-MB) is proposed for the large-scale JSP to improve the scheduling efficiency and effect. The method

focuses on two aspects: (1) The large-scale JSP is decomposed into a series of sub-problems to reduce the computing scale. And then each sub-problem corresponds to a small-scale JSP. The solution of the large-scale JSP can be obtained by iteratively solving the sub-problems. (2) According to the principle "Bottleneck leads the performance of the whole manufacturing system" in TOC (Watson, Blackstone & Gardiner, 2007; Costas, Ponte, Fuente, Pino & Puche, 2014), the bottleneck machines should get more attention than the non-bottleneck machines. Therefore in the solving process of the sub-problems, the bottleneck operations are scheduled by genetic algorithm for high solution quality, and the non-bottleneck operations are scheduled by dispatching rules for the improvement of the solving efficiency.

3.1. The definition of the sub-problem

DH-MB decomposes the operations of the large-scale JSP to construct sub-problems one by one according to the process routes of the jobs. In a sub-problem of the large-scale JSP, a set of n jobs are to be processed on a set of m machines. Each job $J_i(i = 1 \ldots n)$ contains $m_i(m_i \leq m)$ operations which have the operation precedence constraints and consist with the assumptions of the original JSP. The model of the sub-problem can be formulated as follows:

$$\min WT = \sum_{j \in C_{sub}} \max(w_j(t_j + p_j - d_j), 0) \tag{5}$$

$$t_j - t_i \geq p_i, \quad (i, j) \in A_{sub} \tag{6}$$

$$s_i \geq f_i, \quad i \in N_{sub} \tag{7}$$

$$t_j - t_i \geq p_i \vee t_i - t_j \geq p_j, \quad (i, j) \in E_k, k \in M \tag{8}$$

where, C_{sub} is the set of the last operations of each job in the sub-problem; A_{sub} denotes the operation precedence constraints for the operations in the sub-problem; S_i is the arriving time of J_i; f_i is the finish time of J_i in the previous scheduled sub-problem; $N_{sub} = \{1, 2, \cdots, \sum_{i=1}^{n} m_i\}$ is the set of the operations in the sub-problem.

3.2. The sub-problem construction

In the process of the sub-problems' construction, if too many operations of one job were divided into one sub-problem, this job could occupy more resources and finish faster than other jobs. Therewith some jobs would delay severely, and the resource utilization would be out of balance.

In order to overcome the flaw in the sub-problem construction described above, DH-MB constructs the sub-problems by separating some operations along the processing route of each job. The decomposing process follows the principle of "Uniform Distribution Load" to make sure that the load of each job distributes uniformly in each sub-problem. Then each sub-problem contains some operations of each job to avoid the phenomenon that some jobs finish in advance and some jobs delay severely. The decomposing process is as follows:

Step 1: Input the number of sub-problems (P), and then the average load of each job in each sub-problem can be determined by $L_i = \sum_{j=1}^{m} p_{ij} / P, i = 1 \cdots n$.

Step 2: Initialize the sub-problem S_k, $k = 0$. Set $S_k = \varnothing$, and the load of each job in S_k is $l_i = 0$.

Step 3: Start decomposing the operations of the jobs. For each job, take the first un-separated operation φ into the constructing sub-problem along the processing route. Set $S_k = S_k \cup \varphi$ and $l_i = l_i + p_{i\varphi}$.

Step 4: If $l_i < L_i$ and $\varphi < m$, then return to Step 3; else go to Step 5.

Step 5: Fix $_c$. If all the operations of each job are decomposed, then stop; else make $k = k + 1$, return to Step 3.

Figure 2 illustrates the decomposition scheme for a JSP with 3 jobs and 4 machines. In the JSP, the number of sub-problems is 3, the total load of each job is (26, 12, 18), and then the average load of each job in one sub-problem is (9, 4, 6). According to the sub-problem construction method above, the number of operations of each job in S_1 is (2, 1, 1), and S_2 is (1, 2, 2), and S_3 is (1, 1, 1).
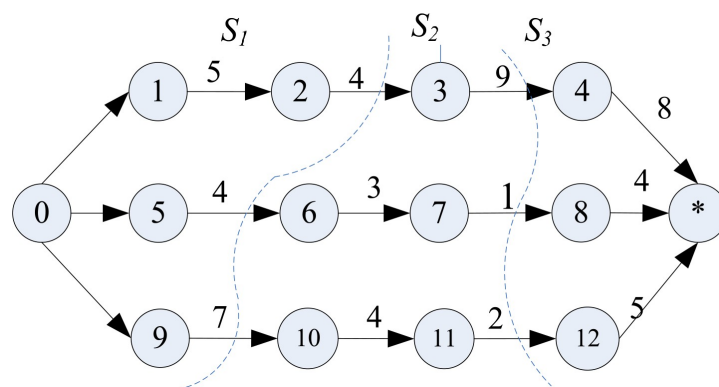

Figure 2. An illustration of the decomposition scheme for a JSP with 3 jobs and 4 machines

Theorem: By constructing the sub-problems sequentially, the solution of each sub-problem doesn't conflict with the processing route of each job in the original large-scale JSP, and the solution of the original problem can be easily obtained by solving the sub-problems.

Prove: (1) According to the definition of sub-problem, the order of operations in each sub-problem consists with the operation precedence constraints of the jobs. Therefore the solution of each sub-problem cannot conflict with the processing route of each job in the original large-scale JSP. (2) If the original JSP is decomposed into p sub-problems $\{S_k\}_{k=1}^{p}$, the sub-problems have different priority levels which can be noted as $S_1 \prec S_2 \prec \ldots \prec S_p$ ($X \prec Y$ represents that the operations in X have higher scheduling priority than the operations in Y). For any two operations O_i and O_j, if $(O_i, O_j) \in A$ and $O_i \in S_1$, $O_j \in S_2$, then $S_1 \prec S_2$ (Zhang & Wu, 2010). Therefore the operations in adjacent sub-problems also satisfy the operation precedence constraints of the jobs.

From (1) (2), we can know that this decomposition method doesn't destroy the operation precedence constraints of each job. And then the solution of the original problem can be obtained by solving the sub-problems.

3.3. The sub-problem solving

3.3.1. Multi-bottleneck detection based on critical path method

By decomposing the original problem, the number of operations in each sub-problem is not very large, so it can be solved by genetic algorithm (GA) which is effective for small-scale JSPs. However in order to improve the solving efficiency, DH-MB adopts different scheduling strategies for different types of operations. The operations are divided into two types: the bottleneck operations which are processed on the multi-bottleneck machines and the non-bottleneck operations which are processed on the non-bottleneck machines. And a detection method based on critical path is proposed for the multi-bottleneck machines. The method is as follows：

In a feasible schedule, a critical path is the sequence of operations whose processing time adds up to the longest overall duration. Each operation on the critical path is a critical operation. Any delay of a critical operation will directly affect the complete date of the schedule. Shorten the length of the critical path can shorten the duration of the schedule. Therefore the critical path restricts the performance of a schedule. The machine on which more critical operations will be processed may have the greater influence on the performance of the schedule. According to Zhai, Sun, Wang and Niu (2011), bottleneck is the machine whose schedule alteration has the greatest effect on the objective of manufacturing system. So the machine on

which more critical operations will be processed may have the greater possibility to be a bottleneck machine.

For a job shop scheduling problem, different schedules may have different critical paths. Therefore for many different schedules, the machine statistically with larger average value and smaller fluctuation of critical operation number may have greater possibility to be a bottleneck. In this paper, a few different frequently used dispatching rules are adopted and randomly combined to generate many different feasible schedules (The dispatching rules are FCFS, FCLS, SPT, LPT, LWKR, MWKR, FOPNR, GOPNR, NINQ, WINQ, EDD, ODD, SL and OSL (Haupt, 1989)). For each schedule, the critical operation number of each machine is taken as the sample value. Then the multi-bottleneck machines can be detected by the bottleneck detection model which is as follows:

$$v_i = \rho \frac{\mu_i}{\sigma_i^2}, \qquad i = 1 \cdots m \tag{9}$$

$$\mu_i = \frac{\sum_{j=1}^{N} b_{ij}}{N} \tag{10}$$

$$\sigma_i^2 = \frac{\sum_{j=1}^{N} (b_{ij} - \mu_j)^2}{N-1} \tag{11}$$

Where, v_i is the bottleneck possibility of machine i; N is the sample number (It is 500 in the simulation of Section 4); μ_i is the average value of the critical operation number of machine i; b_{ij} is the critical operation number of machine i for the sample j; σ_i^2 is the variance value of the critical operation number of machine i, and reflects the fluctuation of the critical operation number of machine i; the constant ρ denotes the correction factor which avoids v_i too small to compare (It is 100 in the simulation of Section 4).

In the bottleneck detection model, both the average value and the fluctuation of the critical operation number are taken into account. The machine with larger v_i represents that the machine has larger average value and smaller fluctuation of the critical operation number. Correspondingly the machine has greater possibility to be a bottleneck machine. So according to the bottleneck detection model and the sample value, by sorting the v_i of each machine decreasingly, the higher the machine ranks, the greater possibility the machine will have to be a bottleneck machine. In the simulation of Section 4, we choose 30% of the machines as the multi-bottleneck machines whose v_i are larger than the average value of v_i.

3.3.2. The solving process for the sub-problem

DH-MB adopts different scheduling strategies for the bottleneck machines and non-bottleneck machines. For the bottleneck machines which have great impact on the performance of the system, GA is adopted for an optimal and effective schedule. For the non-bottleneck machines, a dispatching rule which selects the job with the earliest modified due-date (MOD) is adopted to get a schedule efficiently.

If the bottleneck machines and non-bottleneck machines are scheduled independently, there may be conflictions between the schedules of the bottleneck machines and non-bottleneck machines. In DH-MB, the scheduling of the non-bottleneck machines is integrated in the decoding process of the scheduling of the bottleneck machines, thus the bottleneck operations and non-bottleneck operations can be scheduled parallelly to avoid the coordination between the schedules of the bottleneck machines and non-bottleneck machines, and the efficient of DH-MB can be improved obviously.

The GA in DH-MB uses operation-based coding method which can easily decode the chromosome into an active schedule. The LOX operator and SWAP operator (Wang, 2003) are adopted to generate new chromosomes for maintaining the population diversity. In the decoding phrase, the operations in the previous scheduled sub-problem already have fixed start and finish time, so the chromosome is decoded based on the schedule result of the previous scheduled sub-problem. The decoding process is as follows:

Let PS denote the set of the operations which are already scheduled; S denotes the set of operations to be scheduled currently; σ_i denotes the earliest start time of operations i in S, ϕ_i denotes the earliest predicted completion time of operations i in S; C denotes the set of conflicting operations which satisfy the scheduling condition.

Step 1: Let $PS = \varnothing$, S be the set of operations which are the first unscheduled operations on the processing route of each job.

Step 2: Get $\phi^* = \min_{i \in S} \{\phi_i\}$ and m^* on which the corresponding operation of ϕ^* will be processed. If there is more than one machine, choose one machine randomly.

Step 3: Establish the conflicting operation set C with the operations which are processed on m^* and $\sigma_i < \phi^*(i \in S)$.

Step 4: Select one operation s to schedule.

I f m^* is a bottleneck machine, then s is the first unscheduled operation in the decoding chromosome. If m^* is a non-bottleneck machine, then according to the dispatching rule MOD, $s =$

arg $\min_{i \in C} d_i^m$, $d_i^m = \max_{i \in C} (\phi_i, d_i)$; if there are more than one operation, select one operation randomly.

Step 5: Compute and fix the start time and finish time for the operation s.

Step 6: Let $PS = PS \cup s$. Update S. If $S = \varnothing$, then the decoding process stops; Otherwise go to step 2.

Since the operations in the current scheduling sub-problem is partial operations of the original JSP, the optimal solution in the sub-problem may not have the same optimal performance for the original JSP. So in order to improve the global optimality of the DH-MB, we propose a strategy that is evaluating the chromosome's fitness in the sub-problem by predicting the global scheduling objective (EF-PGSO) of the original JSP. Specifically, in the decoding process of each chromosome in the sub-problem, we try to obtain a complete schedule by scheduling the remaining unscheduled operations which are not in the current scheduling sub-problem using the dispatching rule of MOD, and the objective value of the complete schedule is used as the evaluation value of the chromosome in the sub-problem.

3.4. The connection of the adjacent sub-problem

In DH-MB, the solving process of each sub-problem corresponds to a time window in the whole scheduling time domain. At the end of each time window, not all the jobs can complete at the same time. Figure 3 shows a schedule of one sub-problem S_k (k is not the last serial number of the sub-problems). At time t_1, all the decomposed operations of job J_1 in S_k are scheduled, and there are still unscheduled decomposed operations of job J_2 and J_3 left. Therefore after time t_1, job J_2 and job J_3 are scheduled without considering the resource utilization of the rest undecomposed operations of job J_1 in S_{k+1}. If the sub-problem S_{k+1} is directly constructed following on the scheduling result of S_k without any process, then the earliest start time of the first unscheduled operation of job J_1 in S_{k+1} will be delayed because job J_2 and job J_3 have used the resource in advance in S_k. So the global optimal performance of DH-MB will be weakened along with the solving sequence of the sub-problems.
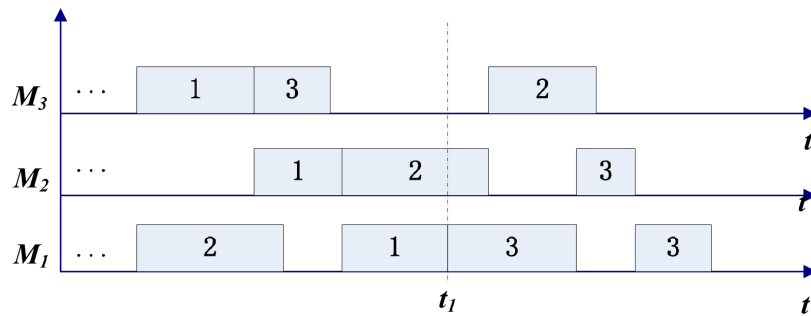
Figure 3. An illustration of the schedule of one sub-problem S_k

In order to avoid the one-sidedness in the solving process of the sub-problems, we propose a strategy that divides partial operations in the previous scheduled sub-problem into the adjacent sub-problems for re-optimization (DPO-AS).

The re-optimization operations are:

$$i = \arg(\sigma_i > t), \quad t = \min(c_j), j = 1 \cdots n \tag{12}$$

Where, σ_i is the starting time of operation i; c_j is the completion time of job J_j.

The strategy can strength the connection between the solutions of the sub-problems, and the construction of the next sub-problem is closely linked to the solution of the previous scheduled sub-problem. All the sub-problems are dynamically constructed in the solving process of the sub-problems. Therefore all the jobs can be scheduled with equal resource competition, and the global optimization of DH-MB can be improved.

3.5. The specific steps of DH–MB

Step 1: Initialization. Input the sub-problem number P, then the average load of each job in each sub-problem is $L_i = \sum_{j=1}^{m} p_{ij} / P, i = 1 \cdots n$. Let $N^C = \varnothing$ denote the set of scheduled operations. Let $N^{NC} = N$ denote the set of unscheduled operations. Set the serial number of the current sub-problem k=1, the current sub-problem $S_k = \varnothing$, and the re-optimization operations set $N^J = \varnothing$.

Step 2: Multi-bottleneck detection. According to the multi-bottleneck detection method in section 3.3.1, determine the bottleneck machines in the large-scale JSP.

Step 3: Construct S_k according to the sub-problem construction method in Section 3.2 and the strategy DPO-AS in Section 3.4.

3a: Let $l_i = 0$, $i = 1 \ldots n$ denote the load of job i. Add the re-optimization operations in the previous scheduled sub-problem into S_k, and calculate the load of job i $l_i = \sum_{j \in N_i^J} p_{ij}, i = 1 \cdots n$, where N_i^J is the re-optimization operations of job i.

3b: According to the processing route of job i, take the first operation φ from the set of unscheduled operations. Let $S_k = S_k \cup \varphi$, then $l_i = l_i + p_{i\varphi}$.

3c: if $l_i < L_i$ and $\varphi < m$, go to 3b; otherwise go to Step4.

Step 4: Solve the sub-problem S_k according to scheduling method in Section 3.3.2.

Step 5: Determine the re-optimization operations N^J according to Section 3.4 and fix the schedule of the operations in S_k except N^J. Let $N^C = N^C \cup (S_k \backslash N^J)$, $N^{NC} = N^{NC} \backslash (S_k \backslash N^J)$.

Step 6: If $N^{NC} = \varnothing$, then the algorithm stops; Otherwise let k=k+1, and go to step 3.

4. Simulation results and analysis

4.1. The generation of the testing instances and DH-MB parameters

In order to analyze the performance of DH-MB, 12 JSP instances are generated for simulation. The instances contain 1 small-scale instance (S1), 2 medium-scale instances (M1 and M2) and 9 large-scale instances (L1~L9). In each instance, the processing route of each job is a random permutation of m machines, the (integral) processing time of each operation follows a uniform distribution U [1, 100], and the due-date of each job is set according to reference (Feng, Leung & Tang, 2005):

$$d_i = r \times \sum_{j=1}^{m} p_{ij}, i = 1 \cdots n \tag{13}$$

Where r=1.5.

In DH-MB, the parameters of GA are:

- The mutation probability $p_m = 0.1$;

- The population size $popsize = 50$;

- The number of generations $GN = 200$;

- The number of exiting iterations GN_exit = 20; (If the performance of the optimal solution is not improved in 20 iterations, then the algorithm exits and exports the optimal solution.)

We compare DH-MB with the following frequently-used scheduling methods:

- Dispatching Rules (DR): three dispatching rules (EDD/MDD/SL) are selected and the optimal result of the three dispatching rules is taken as the final result;

- Standard Genetic Algorithm (SGA): the parameters of SGA are the same to DH-MB;

- Constraint Scheduling Algorithm (CSA) (Zuo, Gu & Xi, 2008).

Considering the randomness of the GA, we use DH-MB and SGA to schedule each instance for 10 times, and the average of the objective values of the 10 schedules is selected as the final value, the average of the solving time in seconds of the 10 schedules is selected as the running time. Table 1 shows the final value and the running time of each instance using different scheduling methods. Table 2 shows the percentage of the value of DH-MB better than other methods which can be calculated as follows:

$$\delta(X, IS) = \frac{WT(X, IS) - WT(DH - MB, IS)}{WT(DH - MB, IS)} \times 100\%, \quad X \in (DR, SGA, CSA) \tag{14}$$

Where, $WT(X, IS)$ is the final value of instance IS calculated by method X.

From Table 1 and Table 2, we can see that the performance of DH-MB is slightly inferior to the other scheduling methods for the small-scale instance (S1). That is because DH-MB decomposes the original problem into a few sub-problems, and the schedules which is optimal for the sub-problems is not equal to be optimal for the original problem.

Instance	Scale ($n*m$)	DR		SGA		CSA		DH–MB		
		Value	Time	Value	Time	Value	Time	Value	Time	P
S1	10*10	617	0.06	575.5	1.82	542	0.55	623	2.18	4
M1	20*10	8685	0.13	9194.2	4.13	6459	9.23	6907	4.77	10
M2	30*10	20693	0.20	25855.7	13.75	18010	26.44	18556	6.36	7
L1	50*20	64433	0.69	79898	160.63	61238	676.70	56050.8	27.98	16
L2	50*30	58921	0.72	71087.9	73.59	125035	1833.08	46651.1	41.81	26
L3	50*50	43413	1.17	52480.4	536.59	170496	1089.06	32092.4	70.34	42
L4	80*20	156394	0.84	235787	515.87	235994	2130.08	140381.4	49.71	15
L5	80*30	162733	1.19	227059.2	437.49	*	*	140193.5	70.65	24
L6	80*50	154721	1.80	191417.3	301.13	*	*	128455.6	131.75	44
L7	100*20	261982	1.11	386866.5	624.86	*	*	235509.3	74.02	18
L8	100*30	260848	1.50	377001.4	783.05	*	*	232450.2	119.00	24
L9	100*50	253036	2.30	338438.4	770.14	*	*	214532.8	215.82	48

The note of * represents the feasible schedule cannot be obtained within 1 hour.
P is the number of sub-problems for DH-MB.

Table 1. The results of different scheduling methods

Instance *IS*	Scale *n*m*	DR	SGA	CSA
S1	10*10	-0.96%	-7.62%	-12.98%
M2	20*10	25.74%	33.11%	-6.49%
M3	30*10	11.52%	39.34%	-2.94%
L1	50*20	14.95%	42.55%	9.25%
L2	50*30	26.30%	52.38%	168.02%
L3	50*50	35.28%	63.53%	431.27%
L4	80*20	11.41%	67.96%	68.11%
L5	80*30	16.08%	61.96%	*
L6	80*50	20.45%	49.01%	*
L7	100*20	11.24%	64.27%	*
L8	100*30	12.22%	62.19%	*
L9	100*50	17.95%	57.76%	*

The note of * represents there is no feasible solution in 1 hour of computing.

Table 2. The percentage of the value of DH-MB better than other methods (δ)

However, with the scale of the instances increasing, the solution quality obtained by DH-MB is obviously better than DR and SGA, and the solving efficiency is also better than SGA. The reason can be analyzed as follows: For DR, the dispatching rules can get feasible schedules quickly, but cannot ensure the optimality of the schedules; For SGA, the increasing of the problem's scale will lead to the great expansion of the solving space, also the premature convergence and randomness are usually associated with SGA itself, so the possibility of obtaining the optimal solution will decrease. In addition, the time consumed by the chromosome decoding process in SGA increases greatly with the number of the operations increasing, therefore the solving efficiency of SGA is inferior to DH-MB.

From Table 1, we can also see that the performance of DH-MB is better than CSA for the large-scale instances. Although the performance of CSA is better than DH-MB for the small-scale and medium-scale instances (S1, M1 and M2), the solution quality of CSA deteriorates rapidly with the scale of the instances increasing. The reason can be analyzed as follows: CSA divides the job shop problem into a number of single-machine scheduling sub-problems, and schedules them sequentially. Once a machine is scheduling, the other scheduled machines will be re-optimized meanwhile. For small-scale JSP, the re-optimization of CSA can improve the solution quality. However, because the objective function for each sub-problem cannot exactly reflect the characteristics of the global objective function of the original JSP, the optimum schedules for the single-machine scheduling sub-problems may be quite far from the optimal solution of the original JSP. For DH-MB, it also divides the job shop problem into a number of sub-problems, and solves the sub-problems one by one. But it adopts the strategies of EF-PGSO and DPO-AS to improve the solution quality. So the performance of DH-MB is better than CSA for the large-scale JSPs.

4.2. The influence of the strategies of DH-MB for the performance

In order to analyze the influence of the strategies (EF-PGSO and DPO-AS), we compare the performance of DH-MB with and without the strategies to solve the JSP instances. We denote the DH-MB without the strategy of EF-PGSO as DH-MB-l, and the DH-MB without the strategy of DPO-AS as DH-MB-2. Table 3 shows the result of the calculation.

Instance IS	Scale *n*m*	DH-MB		DH-MA-1		DH-MA-2	
		Result	Time	Result	Time	Result	Time
S1	10*10	623	2.18	606	0.75	2081	2.01
M1	20*10	6907	4.77	6923.8	2.84	18869	3.87
M2	30*10	18556	6.36	19438	5.04	35851.4	5.89
L1	50*20	56050.8	27.98	56755.4	17.28	108128.4	22.61
L2	50*30	46651.1	41.81	51443.4	34.66	117249.2	38.76
L3	50*50	32092.4	70.34	37002.2	66.55	84840.8	68.03
L4	80*20	140381.4	49.71	152863.8	38.91	317588.6	44.75
L5	80*30	140193.5	70.65	156486.2	63.52	295859.8	68.12
L6	80*50	128455.6	131.75	140734.6	83.72	270814.8	98.35
L7	100*20	235509.3	74.02	250300.6	65.63	521989	69.24
L8	100*30	232450.2	119.00	254477.6	95.64	495349.8	107.31
L9	100*50	214532.8	215.82	229453.8	157.65	502296	183.35

Table 3. The influence of the strategies of DH-MB to the JSP instances

From Table 3, we can see that DH-MB-1 can save the solving time. The reason is that DH-MB-1 doesn't schedule the unscheduled operations which are not in the scheduling sub-problem. However, the solving process doesn't consider the global objective of the original JSP, and the optimal solution for the scheduling sub-problem may not be also optimal for the original JSP. So the solution of each sub-problem has local effect in DH-MB-1, and the solution quality of DH-MB-1 inferior to DH-MB. Therewith the strategies of EF-PGSO in DH-MB are beneficial to improve the solution quality of the original JSP.

From Table 3, we can also see that the efficient of DH-MB-2 is higher than DH-MB, but the solution quality of DH-MB-2 is inferior to DH-MB. That is because that the strategy of DPO-AS enhance the interconnection of the adjacent sub-problems which can guide the solving process to the global optimization. DH-MB-2 without the strategy of DPO-AS isolates the sub-problems with each other, and there is no coordination and interaction between the sub-problems. Therefore DH-MB-2 can save solving time at the cost of solution quality. So the strategies of DPO-AS in DH-MB is also beneficial to improve the solution quality of the original JSP.

5. Conclusion

In this paper, a decomposition heuristics based on multi-bottleneck machines is proposed for large-scale job shop scheduling problems. In the algorithm, the original problem is decomposed into a series of sub-problems to reduce the problem scale and solving complexity. The critical path method is adopted to detect the multi-bottleneck machines, and the characteristics of the bottleneck machine is used in the sub-problems' solving to improve the solving efficiency. The principle of "Uniform Distribution Load" and two strategies (DPO-AS and EF-PGSO) are proposed to improve the solution quality.

Simulation results show that, the performance of DH-MB is slightly inferior to other methods for the small-scale and medium-scale instances, but DH-MB has better performance for the large-scale instances. The algorithm can get satisfactory solutions within reasonable computational time for large-scale JSPs. In the end, we also analyze the influence of the two strategies (DPO-AS and EF-PGSO) to DH-MB, and the results verified their effectiveness on improving the solution quality.

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