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On improvements of Double Beta Decay using FQTDA Model

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Abstract.

The Quasiparticle Tamm-Dancoff Approximation (QTDA) is applied to describe the nuclear double beta decay with two neutrinos. Several serious inconveniences found in the Quasiparticle Random Phase Approximation (QRPA) are not present in the QTDA, as such as the ambiguity in treating the intermediary states, and further approximations necessary for evaluation of the nuclear matrix elements (NMEs) or, the extreme sensitivity of NME with the ratio between the pn and $pp + nn$ pairings. Some years ago, the decay ^{48}Ca $\rightarrow^{48}$Ti was discussed within the particle-hole limit of QTDA. We found some mismatch in the numerical calculations when the full QTDA was being implemented, and a new performance in the particle-hole limit of QTDA is required to guarantee the fidelity of the approximation.

1. INTRODUCTION

At the end of the 20th century the neutrino physics was born. This new era in neutrino physics was opened by the experimental evidence of neutrino oscillations obtained from the results of atmospheric, solar, reactor, and accelerator neutrino experiments. Nevertheless, we do not until this moment which is the absolute scale mass, and whether the neutrino is a Majorana or Dirac particle. The atomic nuclei are the detectors of the evasive neutrinos and the key of this puzzle is the neutrinoless double beta decay. The next generation of experiment for many different nuclei are searching for this rare decay mode, including ^{48}Ca, ^{76}Ge, ^{100}Mo, ^{116}Cd,^{130}Te, ^{124,126,134}Xe, ^{136}Ce, ^{150}Nd, and ^{160}Gd. A summary of the experiments with the above nuclei is well explained in recent review works as such as in Barabash [1], or Tosi [2]. In particular ^{48}Ca is studied at NEMO-III [3], CANDLES [4] and CARVEL [5], because the $Q_{\beta\beta}$-value lies well above the energy of naturally occurring background, a good signal-to-noise ratio is ensured, while the large phase-space factor enhances the $0\nu\beta\beta$-decay rate as it is remarked in Ref. [6].

The single β-decay is energetically forbidden from about 50 nuclear systems in nature, then the $\beta\beta$-decay turns out to be the only possible mode of disintegration. This kind of "anomaly" is due to the nuclear pairing force, by making the odd-odd isobar, within the isobaric triplet (N, Z), $(N-1, Z+1)$, $(N-2, Z+2)$, to have a higher mass than the even-even neighbors. The usual modes of $\beta\beta$ disintegrations are: (i) the two-neutrino double beta ($2\nu\beta\beta$) decay, that can occur by two successive β decays, passing through the intermediate virtual states of the

$(N-1, Z+1)$ nucleus, and (ii) the neutrinoless $\beta\beta$ ($0\nu\beta\beta$) decay, where there are not neutrinos in the final state. The half-lives of these decays are written as:

$$T_{2\nu}^{-1} = \mathcal{G}_{2\nu}\mathcal{M}_{2\nu}^2, \quad T_{0\nu}^{-1} = \mathcal{G}_{0\nu}\mathcal{M}_{0\nu}^2\langle m_\nu\rangle^2, \tag{1}$$

respectively, where $\mathcal{G}'s$ are geometrical phase space factors, and $\mathcal{M}'s$ are nuclear matrix elements (NME's) which present many similar features. There is consensus in the scientific community that we shall not understand the $0\nu\beta\beta$-decay unless we understand the $2\nu\beta\beta$-decay. So that, our goal will be describe in a consistent way the $2\nu\beta\beta$-decay.

Independently of the nuclear model which is used, and when only the allowed transitions are considered, the $2\nu\beta\beta$ matrix element for the $|0_f^+\rangle$ final state [8] reads

$$\mathcal{M}_{2\nu}(f) = \sum_{\lambda=0,1}(-)^\lambda \sum_\alpha \left[\frac{\langle 0_f^+||\mathcal{O}_\lambda^{\beta^-}||\lambda_\alpha^+\rangle\langle\lambda_\alpha^+||\mathcal{O}_\lambda^{\beta^-}||0^+\rangle}{\mathcal{D}_{\lambda_\alpha^+,f}}\right] \equiv \mathcal{M}_{2\nu}^F(f) + \mathcal{M}_{2\nu}^{GT}(f), \tag{2}$$

where the summation goes over all intermediate virtual states $||\lambda_\alpha^+\rangle$, and

$$\mathcal{O}_\lambda^{\beta^-} = (2\lambda+1)^{-1/2}\sum_{pn}\langle p||\mathrm{O}_\lambda||n\rangle\left(c_p^\dagger c_{\bar{n}}\right)_\lambda, \text{ where } \begin{cases} \mathrm{O}_0 = 1 & \text{, for F} \\ \mathrm{O}_1 = \sigma & \text{, for GT} \end{cases}, \tag{3}$$

We assume that Fermi (F) and Gamow-Teller (GT) operators for β^--decay, and $c^\dagger$ (c) are the particle creation (annihilation) operators. The corresponding β^+-decay operators are $\mathcal{O}_\lambda^{\beta^+} = \left(\mathcal{O}_\lambda^{\beta^-}\right)^\dagger$, and

$$\mathcal{D}_{\lambda_\alpha^+,f} = E_{\lambda_\alpha^+} - \frac{E_0 + E_{0_f^+}}{2} = E_{\lambda_\alpha^+} - E_0 - \frac{E_{0_f^+} - E_0}{2}, \tag{4}$$

is the energy denominator. E_0 and $E_{0_f^+}$ are, respectively, the energy of the initial state $|0^+\rangle$ and of the final states $|0_f^+\rangle$. There is a divergent issue about the existence of the Fermi transition in double beta decay. Some authors [9] claim that there is not Fermi transition in double beta decay due the isospin conservation. For other hand some authors this transition could be exist, assuming the non-isospin conservation and after a partial restoration of isospin and SU(4) symmetries [13]. We leave this discussion to a future work.

The $\beta\beta$ decays occur in medium-mass nuclei that are often far from closed shells, and as a consequence most of the recent attempts to evaluate $\mathcal{M}_{2\nu}$ and $\mathcal{M}_{0\nu}$ are: (i) the QRPA makes a large fraction of nucleons to take part in a large single-particle space, but within a modest configuration space, so it is by far simpler computationally, and (ii) the SM (Shell Model) deals with a small fraction of the nucleons in a limited single-particle space, but allows them to correlate in arbitrary ways within a large configuration space. Already it was noted in Ref. [8] that the kinds of correlations that these methods include are not the same. Also, various and different QRPA calculations have been solved, but that none of the amendments of the QRPA were able to change qualitatively the behavior of the amplitude $\mathcal{M}_{2\nu}$ (the model collapses in the physical region of the coupling constant for the particle-particle channel) unless we agree to assume the violation of the Ikeda's Sum Rule. The QRPA $\beta\beta$-decay amplitudes shown to be very sensitivity to the ratio between the pn and $pp+nn$ pairings. These amplitudes before goes to zero, meaning that the isospin and Wigner SU(4) (broken by the mean field) are restored by RPA correlations. After this, the lower QRPA energies become complex, and the amplitudes $\mathcal{M}_{2\nu}$ grow, developing a pole and the entire method breaks down [8].
In Ref. [8] the particle hole (ph-) limit of FQTDA was employed to describe the quenching

mechanism in the ^{48}Ca $\rightarrow^{48}$ Ti decay within the 1fp-shell. In a second work [12], the authors extended this study to a complete FQTDA, still within the 1fp-shell, but some mismatch in the numerical evaluations of the NME were performed. This fact requires a new analysis and the present work is developed to correct this issue and to extent the space $1fp$-shell to a two major shell (*sdfp*-shell consisting of *sd* and *pf* shells). On the other side, new efforts and advances were obtained in SM calculations with ^{48}Ca [7]. In that work, the authors developed a new fast algorithm and computing code to calculate the two-body matrix elements of the neutrinoless double-beta decay transition operator in the closure approximation reducing considerably the evaluation computer time.

2. FOUR QUASIPARTICLE TAMM-DANCOFF APPROXIMATION

A simple nuclear model for evaluating the $\beta\beta$ decay rates, based on the well-known Four Quasiparticle Tamm-Dancoff Approximation - FQTDA [10] is briefly sketched here. As in the QRPA, we conveniently express the total Hamiltonian as

$$H = H_p + H_n + H_{pn} + H_{pp} + H_{nn} \equiv H_0 + H_{res}, \tag{5}$$

where H_p and H_n are, respectively, the effective proton and neutron single-quasiparticle Hamiltonians (with eigenvalues ϵ_p and ϵ_n) , and H_{pn}, H_{pp}, and H_{nn} are the matching effective two-quasiparticle interaction Hamiltonians for the valence quasiparticles. Assuming that:

(1) the initial state is the BCS vacuum in the (N, Z) nucleus $|0^+\rangle = |BCS\rangle$,

(2) the intermediate states are two quasiparticle excitations on this vacuum:

$$|\lambda^+_\alpha\rangle = \sum_{pn} X_{pn;\lambda^+_\alpha} |pn;\lambda^+\rangle, \ with \ |pn;\lambda^+\rangle = [a^\dagger_p a^\dagger_n]_{\lambda^+}|BCS\rangle, \tag{6}$$

(3) the final states are four quasiparticle excitations on this vacuum:

$$|0^+_f\rangle = \sum_{p_1p_2n_1n_2J} Y_{p_1p_2n_1n_2J;0^+_f} |p_1p_2, n_1n_2; J\rangle, \tag{7}$$

with

$$|p_1p_2, n_1n_2; J\rangle = N(p_1p_2)N(n_1n_2)\{[a^\dagger_{p_1}a^\dagger_{p_2}]_J[a^\dagger_{n_1}a^\dagger_{n_2}]_J\}_0|BCS\rangle, \tag{8}$$

and $N(ab) = (1+\delta_{ab})^{-1/2}$. Here $a^\dagger$ (a) is the quasiparticle creation (annihilation) operator relative to the BCS vacuum.

The energies in the denominator in (4) are

$$E_{\lambda^+_\alpha} = E_0 + \omega_{\lambda^+_\alpha} + \lambda_p - \lambda_n, \quad E_{0^+_f} = E_0 + \omega_{0^+_f} + 2\lambda_p - 2\lambda_n, \tag{9}$$

and the denominator energy is written as

$$\mathcal{D}_{\lambda^+_\alpha,f} = \omega_{\lambda^+_\alpha} - \frac{\omega_{0^+_f}}{2}, \tag{10}$$

where $\omega_{\lambda^+_\alpha}$ and $\omega_{0^+_f}$ are the eigenvalues of the Hamiltonian for the intermediate states $|\lambda^+_\alpha\rangle$ and of the final states $|0^+_f\rangle$, respectively, and λ_p and λ_n are the chemical potentials. A residual interaction kind δ-force: $V = -4\pi(v_sP_s + v_tP_t)\delta(r)$ is employed in the calculations as in the previous work [8].

3. NUMERICAL RESULTS

The choice of the suitable effective single-particle energies is a delicate issue. We used three sets of s.p.e labeled as:

SET 1 - s.p.e of neutrons and protons (corrected by the Coulomb interaction) of ^{40}Ca employed to describe ^{48}Ca using a core of ^{40}Ca . The $\epsilon^0_{j_n}$ are from the experimental SPE for ^{40}Ca, taken from Ref. [8], namely: $\epsilon^0_{f_{7/2}} = 0$, $\epsilon^0_{f_{5/2}} = 6.5$, $\epsilon^0_{p_{3/2}} = 2.1$, and $\epsilon^0_{p_{1/2}} = 4.1$, in units of MeV. They were corrected using the self-energies that account for 8 neutrons in the $f_{7/2}$ shell, and using the Coulomb displacement energy in ^{48}Ca for the protons. This set was used in [8].

SET 2 - experimental s.p.e of neutrons and protons ^{48}Ca. They were used in the previous QRPA calculations of Refs. [11, 13]).

SET 3 - modified s.p.e from usual harmonic oscillator using the inverse gap equations procedure [15] to reproduce the experimental s.p.e in ^{48}Ca.

These s.p.e. SETS are shown in Table 1. The underlined s.p.e. of Set 1 were employed to make the *ph*-limit of QTDA previously performed in Refs. [8, 12] for ^{48}Ca. For *ph*-limit we understand that the occupation probabilities satisfy the conditions: $v_p \to 0$ and $v_n \to 1$. An extension of the underlined s.p.e was necessary to complete an available space where the BCS equations must be consistently solved. As it was done in Ref.[8], these additional s.p.e are taken from experimental energies for neutrons in ^{40}Ca and for protons these s.p.e are corrected by the Coulomb energy. The resulting energies complete the Set 1.

Table 1. Single-particle energies (s.p.e) e_j (in Mev) for the SET 1, SET 2 and SET 3 of s.p.e. used for ^{48}Ca according the text. The pairing strength v_s^{pair} (dimensionless) for neutrons and proton within the BCS for $N = 28$ and $P = 20$ are also sketched. The underlined s.p.e. are the energies used in the preliminary *ph*-limit of QTDA.

	Neutrons			Protons		
Shell	$SET1$	$SET2$	$SET3$	$SET1$	$SET2$	$SET3$
$1f_{5/2}$	6.50	−1.54	−2.05	<u>12.89</u>	−5.80	−6.27
$2p_{1/2}$	4.10	−3.11	−3.79	<u>10.49</u>	−4.58	−4.86
$2p_{3/2}$	2.10	−5.14	−8.18	<u>8.49</u>	−6.93	−7.39
$1f_{7/2}$	<u>0.00</u>	−9.94	−8.44	<u>6.39</u>	−9.62	−10.87
$1d_{5/2}$	−12.73	−12.52	−11.71	−6.34	−15.69	−14.10
$2s_{1/2}$	−10.50	−12.55	−11.68	−4.11	−15.30	−12.48
$1d_{3/2}$	−7.24	−14.60	−14.05	−0.85	−26.40	−18.58
v_s^{pair}	31.00	32.00	23.87	41.90	34.43	24.40

Figure 1 shows the energy dependence of double GT strengths $S^{\beta\beta}_{GT}$, measured from the ground state of ^{48}Ca. The unperturbed strength (residual interaction off) is compared with the next perturbed results (residual interaction on). The perturbed results are obtained with the *ph*-parameters $v_s = 35, v_t = 65$ and *pp*-parameters $s = \frac{v_s^{pp}}{v_s^{pair}} = 1$ and $t = \frac{v_t^{pp}}{v_s^{pair}} = 0$, where $v_s^{pair} = (v_s^{pair}(p) + v_s^{pair}(n))/2$ [13]. According the choice set of s.p.e of Table 1, these results are separated in: (a) SET 2, (b) SET 3 and, (c) SET 1. In (a),(b) and (c) the BCS equations were solved consistently in a complete space with the v_s^{pair} shown in Table 1. It means that these calculations are a complete FQTDA calculations. We can observe that the unperturbed calculation for double GT strength, $S^{\beta\beta}_{GT}$, has been focused in three energy regions. On the other hand, the perturbed cases (a) SET 2 and (b) SET 3, lead a shift of strength to the left, instead

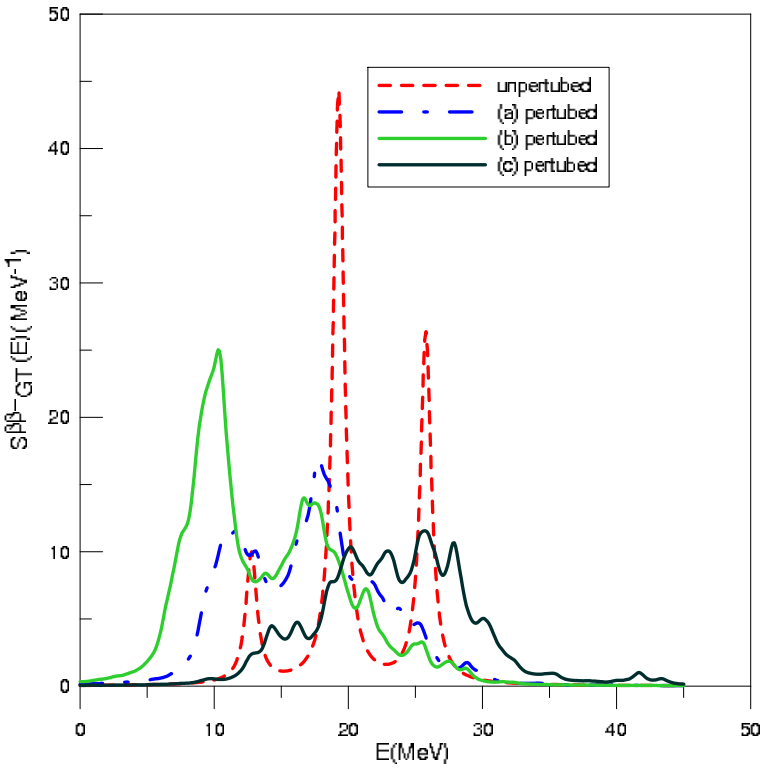

Figure 1. Double GT strength, $S_{GT}^{\beta\beta}$, for ^{48}Ca as function of the energy of daughter nucleus. The unperturbed case (red dashed line) is compared with the perturbed cases (a) SET 2 in blue dash-dot line, (b) SET 3 in green solid line, and (c) SET 1 in black solid line.

that the case (c) SET 1 shown the shift to the other side. This is an effect of the choice of the suitable effective single-particle energies. We see below how the choice of the suitable effective s.p.e. affect in the same way the behavior of the NME, because the $S_{GT}^{\beta\beta}$ are the numerators in Eq. 2.

One of the improvements on FQTDA in this work was to separate the *pp* and *ph*-channels in the residual interaction. In this way, we shown in Figure 2 the $\mathcal{M}_{2\nu}^{GT}$ as function of t, *pp*-strength parameter, with three different values of (v_s, v_t) *ph*-parameters. For SET 1, the results in the *ph*-limit of FQTDA (dashed lines), are compared with those obtained in the "reduced" FQTDA (solid lines). "Reduced" FQTDA means that the BCS equations are solved in the complete space of SET 1, and after a reduction to the *ph*-space of 1n-4p is employed. Here, we noted the effect of the probability occupation numbers v^2 have over the NME. We note in both cases that the behavior of $\mathcal{M}_{2\nu}^{GT}$ as function of t is continuous until to very high values of t parameter (values may be without physical meaning) and there is not a collapse of the FQTDA. A brief discussion of this issue is given below when we will analyze the behavior of NME. The effect of inclusion of pairing is notable moving from constant values of $\mathcal{M}_{2\nu}^{GT}$ at each *ph* parametrization to smooth decreasing curve as function of t. This effect appears in the "reduced" and single *ph*-space and it could be washed when the complete space will be used.

The results of Figures 1 and 2 were obtained with different strengths in the *pp* and *ph*-channels of δ-residual interaction. But in previous calculations of Ref. [8], only one pair of parameters was adopted in the residual interaction :$v_s^{PH} = v_s^{PP} = v_s$ and $v_t^{PH} = v_t^{PP} = v_t$. This adoption, required for simplicity, have been implemented for two main reasons: (i) to avoid a fine fitting of the residual interaction and, (ii) to show how the QTDA quenching-mechanism works. Then

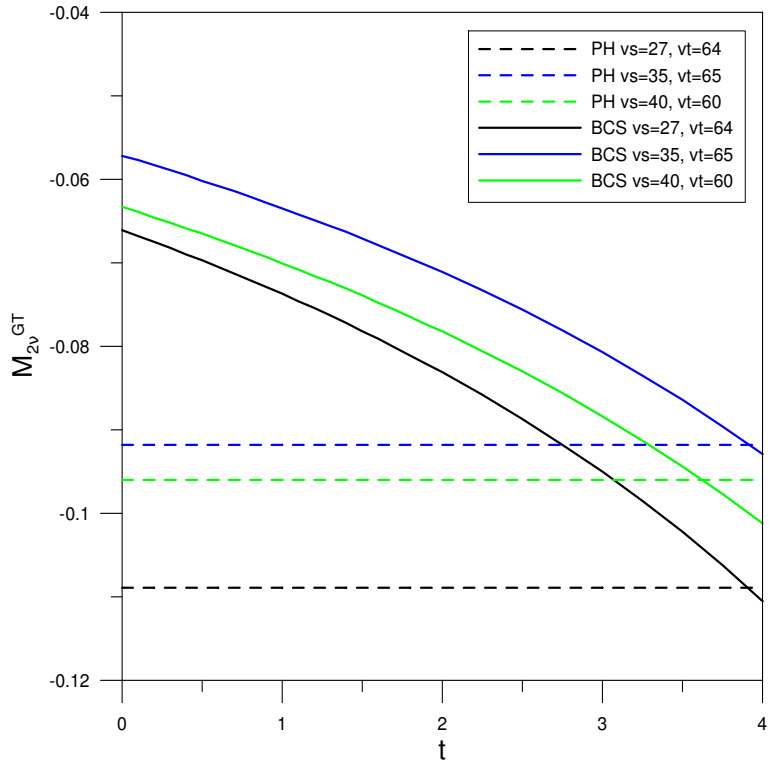

Figure 2. $\mathcal{M}_{2\nu}^{GT}$ as function of t for the three different ph parameters with SET 1. The ph-limit is shown with dashed lines, while that those from "reduced" calculations are shown with solid lines.

we can learn, using this recipe, which is the effect when the BCS is on, without worrying of the parametrization in the residual interaction. Figure 3 shows a comparison of $\mathcal{M}_{2\nu}^{GT}$ as function of v_t for different v_s values with the SET 1. We noted the ph-limit in the complete space of SET 1 with dashed lines. When the BCS calculations are included in a complete FQTDA (solid lines) the $\mathcal{M}_{2\nu}^{GT}$ reduces its values, like a quenching effect in the NME. It is necessary to remark that the ph-limit is obtained when $v_p \to 0, v_n \to 1$, and with BCS-on means that the $v_{p(n)}^2$, merged from BCS equations, give the occupation probability for the quasiparticle in the p (n) level.

Among the simplicity introduced when one chooses the same parameters in pp and ph-channels of residual interaction, this could lead to non-physical values when both channels work separately. Already, we mentioned that one improvement was to separate these channels and then the parameters of δ-interaction must be selected. We adopted three different set of $(v_s^{PH}, v_s^{PH}) = \{(27, 64), (35, 65), (40, 60)\}$ (in units of MeV.fm^3) parameters adopted to reproduce reasonably well the experimental data of energy levels of ^{48}Sc (Figure 5, [12]). The pp-channel parameters are fixed on the basis of the SU(4) and isospin symmetry, as $v_s^{pp} = v_s^{pair}$, and $v_t^{pp} \gtrsim v_s^{pp}$ [11]. Then $s = 1$ and, t is variable to be responsible for the known collapse in QRPA calculations.

Figure 4 shows a comparison of $\mathcal{M}_{2\nu}^{GT}$ as a function of t, for several ph parameters in FQTDA calculations. In the left side are the results for complete SET 1, while that in the right side those for complete SET 2. We previously obtained that the behavior $\mathcal{M}_{2\nu}^{GT}$ as function of t in a "reduced" space yielded high values of t and this effect could be washed when a complete space is implemented. Nevertheless, when the complete SET 1 is used the $\mathcal{M}_{2\nu}^{GT}$ are still weakly dependent of t. On the right side we shown the $\mathcal{M}_{2\nu}^{GT}$ for complete SET 2. They are most

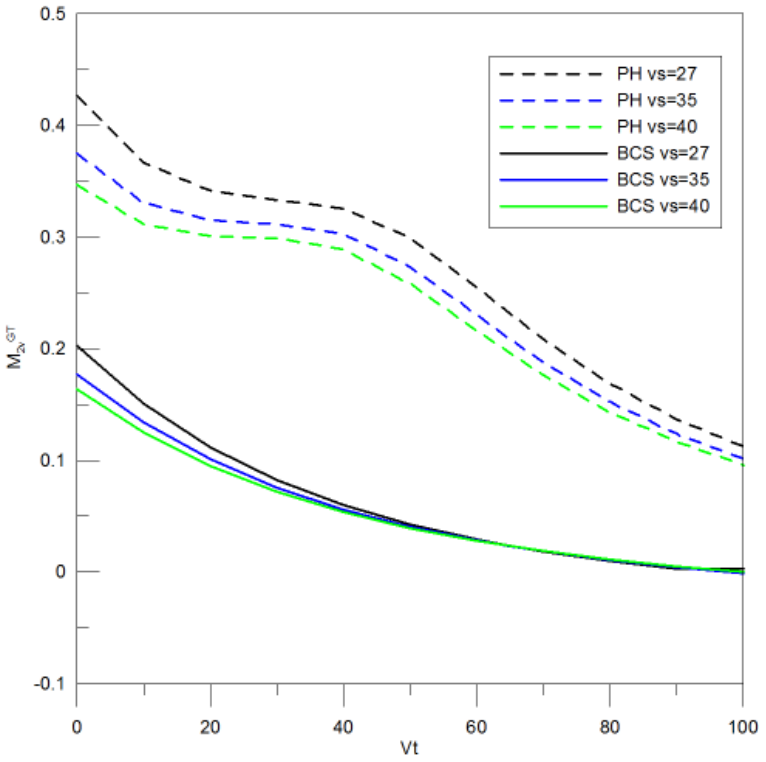

Figure 3. $\mathcal{M}_{2\nu}^{GT}$ as a function of v_t for complete SET 1: (dashed lines) in the *ph*-limit of FQTDA and, (solid lines) with complete FQTDA (BCS on).

sensitivity to the *ph*-parameters ($v_s = 27, v_t = 64$) doing that the FQTDA results in $t \approx 0.4$ assuming a discontinuity and , as well as with the other parameters ($v_s = 35, v_t = 65$) and ($v_s = 40, v_t = 60$) where the FQTDA collapses in $t \approx 0.54$ and $t \approx 0.6$ respectively. The main difference between the $\mathcal{M}_{2\nu}^{GT}$ was the choice os s.p.e.

We compared the $\mathcal{M}_{2\nu}^{GT}$ obtained with another set of s.p.e. In Figure 5 is shown that comparison. At right-hand, we can observe that the behavior of $\mathcal{M}_{2\nu}^{GT}$ of SET 3 is similar to SET 2, the values are decreasing when t increases with a smooth behavior. The spread relative to the different values of *ph*-strength is observed in $t = 0$. A similar spread was appeared with the another SET 2, but in SET 1 this phenomena does not appear. For the set 3, the $\mathcal{M}_{2\nu}^{GT}$ values according to the ($v_s = 27, v_t = 64$) *ph*-parameters are most sensitive at the t parameter up to $t \approx 1$ where appears a discontinuity. With ($v_s = 35, v_t = 65$) and ($v_s = 40, v_t = 60$) the t-values are extended up to $t \approx 1.3$. The irregularities presented in the NME for the SET 2 and SET 3 are not a collapse type such in QRPA calculations. There are not complex values in the QTDA equations and the discontinuities in the NME, according to the different sets of s.p.e, obey to the ratio between the of the numerator (product of single beta decay NME) and the denominator energies. In these discontinuities, the energy denominators are not zero to be a pole of $\mathcal{M}_{2\nu}^{GT}$.

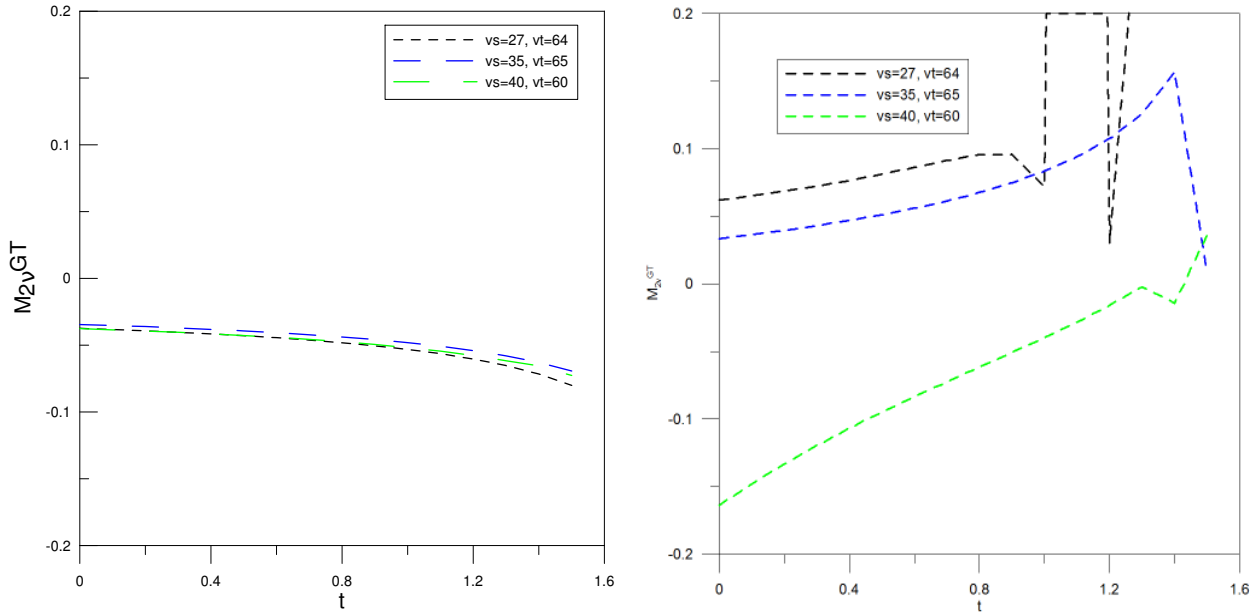

Figure 4. $\mathcal{M}^{GT}_{2\nu}$ as a function of t, *pp*-strength parameter and several *ph* parameters in a complete FQTDA. Left side: for SET 1. Right side: $-\mathcal{M}^{GT}_{2\nu}$ for SET 2.

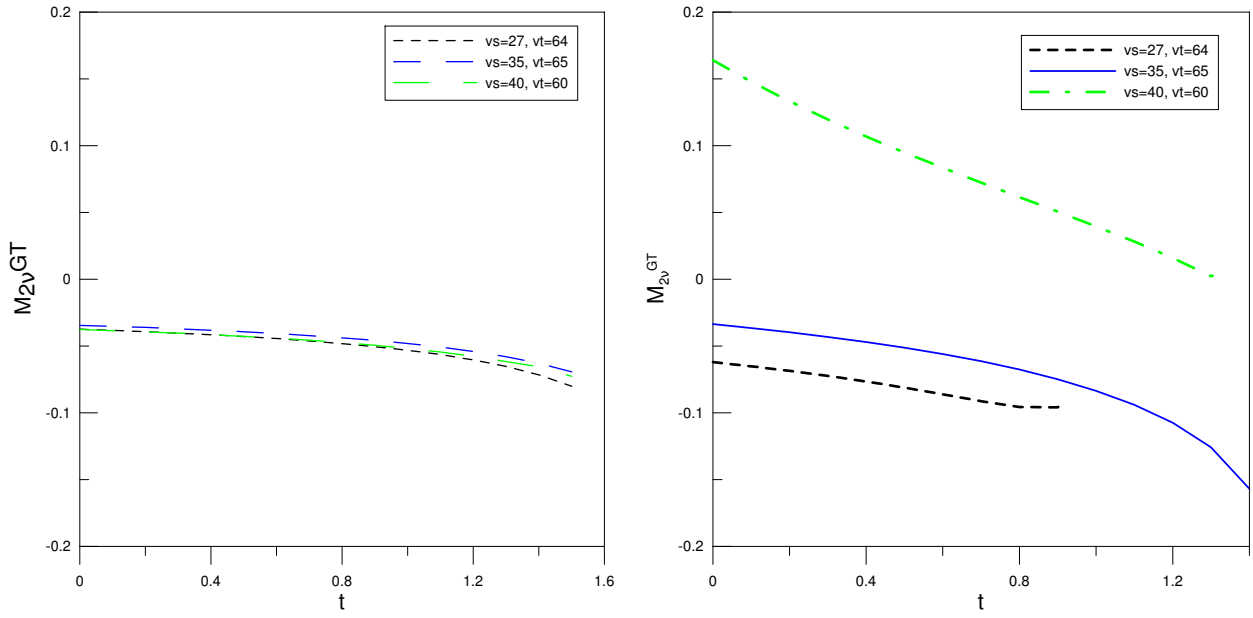

Figure 5. $\mathcal{M}^{GT}_{2\nu}$ as a function of t, *pp*-strength parameter and several *ph* parameters in a complete FQTDA. Left side: for SET 1. Right side: for SET 3.

4. DISCUSSION AND BRIEF CONCLUSIONS

The use of FQTDA model around the problem of collapse of QRPA, where the elements of the nuclear matrix $M_{2\nu}$ goes to zero and after to infinity for certain values of the interaction strength of *pp*. A combination of the method of FQTDA, treating the initial and final 0^+ states with the same BCS wave function as vacuum and as a four quasiparticle excitation on this vacuum, respectively, the intermediary states as two quasiparticle excitations on this vacuum, leads to

avoid the known collapse of QRPA. The collapse in QRPA is related with the appearance of complex eigenvalues in the QRPA equations at certain values of *pp* strength. This leads to obtain a non-physical wave function of the ground state with zero energy (or complex) values with a subsequent zero and latter one pole in the expression of the 2ν NMEs [13]. Also the collapse of the RPA equation indicates that the 1^+ component is ambiguous and thus also limits somewhat the reliability of the result on the neutrinoless decay [14]. We noted from Figures (4) and (5) the extreme sensitivity of $M_{2\nu}$ with interaction strength of *pp*, here represented by the values of t parameter. In particular for two different set of values (SET 2 and SET3) of s.p.e., this phenomena is reflected in a discontinuity of the $M_{2\nu}$ values when t is increased, more notable in SET2. At difference of the weakly decreasing behavior of $M_{2\nu}$ as function of t for SET1. We do not note that the indications of a kind 'collapse of QRPA' appears in FQTDA beyond the discontinuity appearing in $M_{2\nu}$ for some t-values with some set of s.p.e. The $M_{2\nu}$ increases significatively in this point. Neither the ground state energy of wave function of the 0^+ final state have been presented some kind of irregularity, either the intermediary 1^+ states. A detailed study of the collapse of QRPA was performed by Civitarese *et al.* [16], where the interpretation of the collapse was pointed out as a signature of a phase transition. This important issue in the FQTDA will be studied in detail in future work , anyway we enforced the idea that the choice of the suitable effective single-particle energies is a delicate issue, that leads to singularities in the NME, non interpreted at all with the known collapse of QRPA.

As final conclusions, with the experimental half-life $((4.3 \pm 2.3)10^{19}$yr [**?**] in Eq.**??** the NME for two-neutrino double-beta decay of ^{48}Ca is $M_{2\nu}^{GT} = 0.0560 \pm 0.0162$ MeV^{-1}. It is possible to reproduce this value with the parametrizations employed for the *ph*-chanel as well as for some physical parameters of the t parameter in *pp*-chanel. A fine-tune to choice the *ph*-parameters must be performed comparing the experimental strength of single β-decay with our theoretical results. After this procedure, one needs to fit the best t-value of *pp* channel to reproduce the experimental $M_{2\nu}^{GT}$.

Our future work is to expand the study of the model, testing the decay $2\nu\beta\beta$ in ^{76}Ge and ^{100}Mo to check the reliability of FQTDA evaluating double beta decay NME.

5. ACKNOWLEDGMENT

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