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Singularity resolution in gauged supergravity and conifold unification

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Abstract

We obtain a unified picture for the conifold singularity resolution. We propose that gauged supergravity, through a novel prescription for the twisting, provides an appropriate framework to smooth out singularities in the context of gravity duals of supersymmetric gauge theories.

1. Introduction

String theory compactifications in a Calabi–Yau threefold have been the focus of a countless number of papers due to the fact that they provide effective four-dimensional vacua with $\mathcal{N}=1$ supersymmetry. Particular emphasis has been given to the study of singularities on these manifolds—in particular, conical singularities—as long as non-trivial phenomena take place on them such as gauge symmetry enhancement or the appearance of new massless particles. The archetype of these is the well-known conifold. It is a complex three-manifold which is a cone over the homogeneous space $T^{1,1}=\frac{SU(2)\times SU(2)}{U(1)}$. The conical singularity can be resolved in two different ways according to whether an S^2 or an S^3 is blown up at the singular point. The former is known as the *resolved* (or Kähler deformed) conifold, while the latter is the (complex) *deformed* conifold. Both regular manifolds depend on a single parameter (namely, the *resolution* a and the *deformation* μ), are non-compact and asymptotically behave as the singular conifold. That is, the three solutions display the same UV behavior for the associated gauge theories. Supersymmetry and matching holonomy conditions in the context of string theory [1] ensure that there must exist manifolds with G_2 holonomy metrics whose Gromov–Hausdorff limits are precisely the Ricci flat Kähler metrics on the resolved and deformed conifolds. These were explicitly found in [2]. It was shown afterwards that these G_2 manifolds arise as solutions of the same system of first-order equations, this providing a nicely unified picture of the resolved and deformed conifolds from the perspective of M-theory [3].

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In a different approach based on lower-dimensional gauged supergravity, it was recently shown that the resolved conifold comes out when studying the gravity dual of D6-branes wrapping an holomorphic S^2 in a $K3$ manifold [4]. The low-energy dynamics is governed, when the size of the cycle is taken to zero, by a five-dimensional supersymmetric gauge theory with eight supercharges. If the manifold is large enough and smooth, the dual description is given in terms of a purely gravitational configuration of eleven-dimensional supergravity which is the direct product of Minkowski five-dimensional spacetime and the resolved conifold. The general solution to this system was later shown [5] to be given by the *generalized* resolved conifold [6–9]. This is a one-parameter (say, b) generalization of the resolved conifold. There is an analogous extension metric both for the deformed conifold—though it is not regular—and the singular conifold. We will call the latter *regularized* conifold, following [9], because b smoothens the curvature singularity and the metric is regular upon imposing a $\mathbb{Z}_2$ identification of the $U(1)$ fiber. It is an ALE space that asymptotically approaches $T^{1,1}/\mathbb{Z}_2$.

Lower-dimensional gauged supergravities provide an explicit arena to impose the twisting conditions required to wrap a D-brane in a supersymmetric cycle [10,11]. (See [12] for a recent review.) Loosely speaking, in the conventional twist, the gauge connection has to be identified with the spin connection. This notion of the twist can be generalized, as shown in [13], in a way that involves non-trivially the scalar fields that arise in lower-dimensional gauged supergravity from the external components of the metric. The solutions obtained by these means usually correspond to the near horizon limit of wrapped D-branes [14]. However, in a recent paper [15], we have shown that the twist can be further generalized so that it encompasses much more general solutions either corresponding to wrapped D-branes or to special holonomy manifolds with certain RR fluxes turned on. On the one hand, the new twisting condition can be thought of as a non-trivial embedding of the world-volume in the lower-dimensional spacetime. More interestingly, as we will show in this Letter through an archetypical example, it can also be understood as a singularity resolution mechanism:[1] the ordinary twisting imposes the value of the gauge fields at infinity, while the lower-dimensional gauged supergravity governs the non-trivial dynamics in the bulk. This mechanism resembles that used in the Maldacena–Núñez solution [16], where the singularity is solved by turning on a non-Abelian gauge field that asymptotically approaches the Abelian one that twists the gauge theory.

In this Letter we present a unified scenario for conifold singularity resolutions. In a sense, we are providing the unified picture of the resolved and deformed conifolds from the perspective of M-theory advocated in [3]. The main difference being that we deal with conifolds in eleven dimensions instead of G_2 manifolds. A unique system encompass the generalized resolution and deformation of the conifold singularity, each of them emerging as the only two possible solutions of an algebraic constraint. Notice the difference with the G_2 case studied in [15], where the algebraic constraints are involved enough so as to admit several well distinct solutions. Here there are only two. Furthermore, we show that it is possible to impose at the same time both solutions of the algebraic constraint, this leading to the regularized conifold metric, which describes a complex line bundle over $S^2 \times S^2$.

2. D6-branes wrapped on S^2 revisited

The Lagrangian describing the dynamics of the sector of Salam and Sezgin's eight-dimensional gauged supergravity [17] on which we would like to focus (entirely coming from the eleven-dimensional metric), reads

$$e^{-1}\mathcal{L} = \frac{1}{4}R - \frac{1}{4}e^{2\phi}\left(F^i_{\mu\nu}\right)^2 - \frac{1}{4}(P_{\mu ij})^2 - \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{32}e^{-2\phi}\left(e^{-8\lambda} - 4e^{-2\lambda}\right), \tag{2.1}$$

[1] Notice that, in a sense, it is natural to expect that lower-dimensional gauged supergravity degrees of freedom cure singularities. For example, even when using the standard twisting conditions, the resolution of the conifold singularity has been shown to be driven by turning on a scalar field in gauged supergravity [4].

where ϕ is the dilaton, λ is a scalar in the coset $SL(3,\mathbb{R})/SO(3)$, A^i is an $SU(2)$ gauge potential, e is the determinant of the vierbein e^a, F^i is the Yang–Mills field strength and P_{ij} is a symmetric and traceless 1-form defined by

$$P_{ij} + Q_{ij} = \begin{pmatrix} d\lambda & -A^3 & A^2 e^{-3\lambda} \\ A^3 & d\lambda & -A^1 e^{-3\lambda} \\ -A^2 e^{3\lambda} & A^1 e^{3\lambda} & -2d\lambda \end{pmatrix}, \tag{2.2}$$

Q_{ij} being the antisymmetric counterpart. The supersymmetry transformations for the fermions are given by

$$\delta\psi_\gamma = \mathcal{D}_\gamma \epsilon + \frac{1}{24} e^{\phi} F^i_{\mu\nu} \widehat{\Gamma}_i \left(\Gamma_\gamma^{\mu\nu} - 10\delta_\gamma^\mu \Gamma^\nu\right)\epsilon - \frac{1}{288} e^{-\phi} \epsilon_{ijk} \widehat{\Gamma}^{ijk} \Gamma_\gamma T \epsilon, \tag{2.3}$$

$$\delta\chi_i = \frac{1}{2}\left(P_{\mu i j} + \frac{2}{3}\delta_{ij}\partial_\mu \phi\right)\widehat{\Gamma}^j \Gamma^\mu \epsilon - \frac{1}{4} e^{\phi} F_{\mu\nu i} \Gamma^{\mu\nu}\epsilon - \frac{1}{8} e^{-\phi}\left(T_{ij} - \frac{1}{2}\delta_{ij} T\right)\epsilon^{jkl}\widehat{\Gamma}_{kl}\epsilon, \tag{2.4}$$

where $T_{ij} = \mathrm{diag}(e^{2\lambda}, e^{2\lambda}, e^{-4\lambda})$, $T = \delta^{ij} T_{ij} = 2e^{2\lambda} + e^{-4\lambda}$, and the covariant derivative is

$$\mathcal{D}\epsilon = \left(\partial + \frac{1}{4}\omega^{ab}\Gamma_{ab} + \frac{1}{4} Q_{ij}\widehat{\Gamma}^{ij}\right)\epsilon. \tag{2.5}$$

We shall use the following representation for the Dirac matrices:

$$\Gamma^\mu = \gamma^\mu \otimes \mathbb{I}, \qquad \widehat{\Gamma}^i = \gamma_9 \otimes \tau^i, \tag{2.6}$$

where γ^μ are eight-dimensional Dirac matrices, $\gamma_9 = i\gamma^0\gamma^1\cdots\gamma^7$ ($\gamma_9^2 = 1$), τ^i are Pauli matrices and $\widehat{\Gamma}^i$ are the Dirac matrices along the $SU(2)$ group manifold, whereas $\Gamma_7 \equiv \Gamma_r$ corresponds to the radial direction.

We shall consider the following ansatz for the eight-dimensional metric

$$ds_8^2 = e^{2f}\, dx_{1,4}^2 + e^{2h}\, d\Omega_2^2 + dr^2, \tag{2.7}$$

where $d\Omega_2^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$ is the metric of the unit S^2. The ansatz for the gauge field is better presented in terms of the triplet of Maurer–Cartan 1-forms on S^2

$$\sigma^1 = d\theta, \qquad \sigma^2 = \sin\theta\, d\varphi, \qquad \sigma^3 = \cos\theta\, d\varphi, \tag{2.8}$$

that obey the conditions $d\sigma^i = -\frac{1}{2}\epsilon_{ijk}\sigma^j \wedge \sigma^j$. It is:

$$A^1 = g(r)\sigma^1, \qquad A^2 = g(r)\sigma^2, \qquad A^3 = \sigma^3. \tag{2.9}$$

Notice that the twisting in [4] corresponds to $g(r) = 0$. We will check at the end that $g(r) \to 0$ asymptotically so, from the point of view of the dual gauge theory, the twisting is not modified. The field strength, $F^i = dA^i + \frac{1}{2}\epsilon_{ijk} A^j \wedge A^k$, reads:

$$F^1 = g'\, dr \wedge \sigma^1, \qquad F^2 = g'\, dr \wedge \sigma^2, \qquad F^3 = \left(g^2 - 1\right)\sigma^1 \wedge \sigma^2. \tag{2.10}$$

When uplifted to eleven dimensions, the unwrapped part of the metric should correspond to flat five-dimensional Minkowski spacetime. This condition determines the relation $f = \phi/3$ that we impose from now on. Actually, it is not difficult to write down the form of the eleven-dimensional metric for the ansatz we are adopting. Let w^i for $i = 1, 2, 3$ be a set of $SU(2)$ left invariant one forms of the external three sphere satisfying $dw^i = \frac{1}{2}\epsilon_{ijk} w^j \wedge w^k$. Then, the uplifted eleven-dimensional metric is:

$$\begin{aligned} ds_{11}^2 &= dx_{1,4}^2 + e^{2h - \frac{2\phi}{3}}\, d\Omega_2^2 + e^{-\frac{2\phi}{3}}\, dr^2 + 4e^{\frac{4\phi}{3} + 2\lambda}\left(w^1 + g\sigma^1\right)^2 \\ &\quad + 4e^{\frac{4\phi}{3} + 2\lambda}\left(w^2 + g\sigma^2\right)^2 + 4e^{\frac{4\phi}{3} - 4\lambda}\left(w^3 + \sigma^3\right)^2. \end{aligned} \tag{2.11}$$

In order to seek for supersymmetric solutions to the system, we start by subjecting the spinor to the following angular projection

$$\Gamma_{\theta\varphi}\epsilon = -\widehat{\Gamma}_{12}\epsilon, \tag{2.12}$$

which is imposed by the Kähler structure of the ambient $K3$ manifold in which the two-cycle lives. The equations $\delta\chi_1 = \delta\chi_2 = 0$ give:

$$\left(\lambda' + \frac{2}{3}\phi'\right)\epsilon = g e^{-h}\sinh 3\lambda \widehat{\Gamma}_1 \Gamma_\theta \Gamma_r \widehat{\Gamma}_{123}\epsilon - e^{\phi+\lambda-h} g' \widehat{\Gamma}_1 \Gamma_\theta \epsilon - \frac{1}{4} e^{-\phi-4\lambda}\Gamma_r \widehat{\Gamma}_{123}\epsilon, \tag{2.13}$$

while $\delta\chi_3 = 0$ reads:

$$\left(2\lambda' - \frac{2}{3}\phi'\right)\epsilon = \left[e^{\phi-2\lambda-2h}\left(g^2-1\right) - \frac{1}{4}e^{-\phi}\left(e^{-4\lambda} - 2e^{2\lambda}\right)\right]\Gamma_r\widehat{\Gamma}_{123}\epsilon + 2ge^{-h}\sinh 3\lambda \widehat{\Gamma}_1\Gamma_\theta\Gamma_r\widehat{\Gamma}_{123}\epsilon. \tag{2.14}$$

One can combine these two equations to eliminate λ':

$$\phi'\epsilon + e^{\phi+\lambda-h}g'\widehat{\Gamma}_1\Gamma_\theta\epsilon + \left[\frac{1}{2}e^{\phi-2\lambda-2h}\left(g^2-1\right) + \frac{1}{8}e^{-\phi}\left(e^{-4\lambda}+2e^{2\lambda}\right)\right]\Gamma_r\widehat{\Gamma}_{123}\epsilon = 0. \tag{2.15}$$

From this last equation, it is clear that the supersymmetric parameter must satisfy a projection of the sort:

$$\Gamma_r\widehat{\Gamma}_{123}\epsilon = -\left(\beta + \tilde{\beta}\widehat{\Gamma}_1\Gamma_\theta\right)\epsilon, \tag{2.16}$$

where β and $\tilde{\beta}$ are (functions of the radial coordinate) proportional to the first derivatives of ϕ' and g':

$$\phi' = \left[\frac{1}{2}e^{\phi-2\lambda-2h}\left(g^2-1\right) + \frac{1}{8}e^{-\phi}\left(e^{-4\lambda}+2e^{2\lambda}\right)\right]\beta, \tag{2.17}$$

$$e^{\phi+\lambda-h}g' = \left[\frac{1}{2}e^{\phi-2\lambda-2h}\left(g^2-1\right) + \frac{1}{8}e^{-\phi}\left(e^{-4\lambda}+2e^{2\lambda}\right)\right]\tilde{\beta}. \tag{2.18}$$

This radial projection provides the generalized twist introduced in [15]. It encodes a non-trivial fibering of the two-sphere with the external three-sphere as will become clear below. Since $(\Gamma_r\widehat{\Gamma}_{123})^2\epsilon = \epsilon$ and $\{\Gamma_r\widehat{\Gamma}_{123}, \widehat{\Gamma}_1\Gamma_\theta\} = 0$, one must have $\beta^2 + \tilde{\beta}^2 = 1$. Thus, we can represent β and $\tilde{\beta}$ as

$$\beta = \cos\alpha, \qquad \tilde{\beta} = \sin\alpha. \tag{2.19}$$

Also, it is clear that both independent projections (2.12) and (2.16) leave unbroken eight supercharges as expected. Inserting the radial projection (2.16), as well as (2.17), in (2.14), we get an equation determining λ':

$$\lambda' = ge^{-h}\sinh 3\lambda\tilde{\beta} - \left[\frac{1}{3}e^{\phi-2\lambda-2h}\left(g^2-1\right) - \frac{1}{6}e^{-\phi}\left(e^{-4\lambda}-e^{2\lambda}\right)\right]\beta, \tag{2.20}$$

together with an algebraic constraint:

$$ge^{-h}\sinh 3\lambda\beta + \left[\frac{1}{2}e^{\phi-2\lambda-2h}\left(g^2-1\right) - \frac{1}{8}e^{-\phi}\left(e^{-4\lambda}-2e^{2\lambda}\right)\right]\tilde{\beta} = 0. \tag{2.21}$$

Let us now consider the equations obtained from the supersymmetric variation of the gravitino. From the components along the unwrapped directions one does not get anything new, while from the angular components we get:

$$\begin{aligned} h'\epsilon = {} & -ge^{-h}\cosh 3\lambda\widehat{\Gamma}_1\Gamma_\theta\Gamma_r\widehat{\Gamma}_{123}\epsilon + \frac{2}{3}e^{\phi+\lambda-h}g'\widehat{\Gamma}_1\Gamma_\theta\epsilon \\ & - \frac{1}{6}\left[-5e^{\phi-2\lambda-2h}\left(g^2-1\right) + \frac{1}{4}e^{-\phi}\left(2e^{2\lambda}+e^{-4\lambda}\right)\right]\Gamma_r\widehat{\Gamma}_{123}\epsilon. \end{aligned} \tag{2.22}$$

By using the projection (2.16) we obtain an equation for h':

$$h' = -ge^{-h}\cosh 3\lambda\tilde{\beta} + \frac{1}{6}\left[-5e^{\phi-2\lambda-2h}\left(g^2-1\right) + \frac{1}{4}e^{-\phi}\left(2e^{2\lambda}+e^{-4\lambda}\right)\right]\beta, \tag{2.23}$$

together with a second algebraic constraint:

$$-ge^{-h}\cosh 3\lambda\beta + \left[\frac{1}{2}e^{\phi-2\lambda-2h}\left(g^2-1\right) - \frac{1}{8}e^{-\phi}\left(2e^{2\lambda}+e^{-4\lambda}\right)\right]\tilde{\beta} = 0. \tag{2.24}$$

Finally, from the radial component of the gravitino we get the functional dependence of the supersymmetric parameter ϵ:

$$\partial_r\epsilon = \frac{5}{6}e^{\phi+\lambda-h}g'\widehat{\Gamma}_1\Gamma_\theta\epsilon - \frac{1}{12}\left[e^{\phi-2\lambda-2h}\left(g^2-1\right) + \frac{1}{4}\left(2e^{2\lambda}+e^{-4\lambda}\right)\right]\Gamma_r\widehat{\Gamma}_{123}\epsilon. \tag{2.25}$$

The projection (2.16) gives the generalized twisting conditions first studied in [15]. Its interpretation follows from a similar reasoning as the one used in that reference: using the trigonometric parametrization (2.19), the generalized projection can be written as:

$$\Gamma_r\widehat{\Gamma}_{123}\epsilon = -e^{\alpha\widehat{\Gamma}_1\Gamma_\theta}\epsilon, \tag{2.26}$$

which can be solved as:

$$\epsilon = e^{-\frac{1}{2}\alpha\widehat{\Gamma}_1\Gamma_\theta}\epsilon_0, \qquad \Gamma_r\widehat{\Gamma}_{123}\epsilon_0 = -\epsilon_0. \tag{2.27}$$

We can determine ϵ by plugging (2.27) into the equation for the radial component of the gravitino (2.25). Using (2.26), we get two equations. The first one gives the characteristic radial dependence of ϵ_0 in terms of the eight-dimensional dilaton, namely:

$$\partial_r\epsilon_0 = \frac{\phi'}{6}\epsilon_0 \;\Rightarrow\; \epsilon_0 = e^{\frac{\phi}{6}}\eta, \tag{2.28}$$

with η being a constant spinor. The other equation determines the radial dependence of the phase α:

$$\alpha' = -2e^{\phi+\lambda-h}g'. \tag{2.29}$$

Thus, the spinor ϵ can be written as:

$$\epsilon = e^{\frac{\phi}{6}}e^{-\frac{1}{2}\alpha\widehat{\Gamma}_1\Gamma_\theta}\eta, \qquad \Gamma_r\widehat{\Gamma}_{123}\eta = -\eta, \qquad \Gamma_{\theta\varphi}\widehat{\Gamma}_{12}\eta = \eta. \tag{2.30}$$

The meaning of the phase α can be better understood by using the following Γ-matrices identity $\Gamma_{x^0\cdots x^4}\Gamma_{\theta\varphi}\Gamma_r \times \widehat{\Gamma}_{123} = -1$, so that

$$\Gamma_{x^0\cdots x^4}\left(\cos\alpha\Gamma_{\theta\varphi} - \sin\alpha\Gamma_\theta\widehat{\Gamma}_2\right)\epsilon = \epsilon, \tag{2.31}$$

which shows that the D6-brane is wrapping a two-cycle which is non-trivially embedded in the $K3$ manifold as seen from the uplifted perspective that is implied in (2.31). Indeed, the case $\alpha = 0$ corresponds to the D6-brane wrapping a two-sphere that is fully contained in the eight-dimensional spacetime where supergravity lives, studied in [4].

3. Solution of the algebraic constraints

In the previous section we derived two algebraic constraints (2.21) and (2.24) that the functions of our ansatz must obey. Let us presently solve them. By adding and subtracting the two equations, we get:

$$\tan\alpha \equiv \frac{\tilde{\beta}}{\beta} = -2ge^{\phi+\lambda-h} = \frac{ge^{-3\lambda-h}}{e^{\phi-2\lambda-2h}(g^2-1) - \frac{1}{4}e^{-\phi-4\lambda}}. \tag{3.1}$$

Whereas the first part of this equation allows us to write α in terms of the remaining functions, the last equality provides an algebraic constraint that restricts our ansatz. It is not hard to arrive at the following simple equation:

$$g\left[g^2 - 1 + \frac{1}{4}e^{-2\phi-2\lambda+2h}\right] = 0. \tag{3.2}$$

There are obviously two solutions. The first one is clearly $g = 0$, which corresponds to $\tilde{\beta} = 0$, $\beta = 1$, or $\alpha = 0$. One can check that this is a consistent truncation of the system of equations that actually reduce to the case studied in [4], whose integral is the generalized resolved conifold (see also [5]). Indeed, the resulting eleven-dimensional metric can be written as $ds_{11}^2 = dx_{1,4}^2 + ds_6^2$, where the six-dimensional metric ds_6^2 is:

$$\begin{aligned} ds_6^2 &= \left[\kappa(\rho)\right]^{-1} d\rho^2 + \frac{\rho^2}{9}\kappa(\rho)\left(d\psi + \sum_{a=1,2}\cos\theta_a\, d\phi_a\right)^2 \\ &\quad + \frac{1}{6}\left(\rho^2 + 6a^2\right)\left(d\theta_1^2 + \sin^2\theta_1\, d\phi_1^2\right) + \frac{1}{6}\rho^2\left(d\theta_2^2 + \sin^2\theta_2\, d\phi_2^2\right), \end{aligned} \tag{3.3}$$

with $\kappa(\rho)$ being:

$$\kappa(\rho) = \frac{\rho^6 + 9a^2\rho^4 - b^6}{\rho^6 + 6a^2\rho^4}. \tag{3.4}$$

In Eq. (3.3), ρ is a new radial variable, $\theta_1 \equiv \theta$, $\phi_1 \equiv \varphi$ and (θ_2, ϕ_2, ψ) parametrize the w^i's. The constants a and b provide the generalized resolution of the conifold singularity [7,9]. In the context of gauged supergravity, even when this solution corresponds to the conventional twist, a and b are non-zero when certain scalar fields are excited [4,5]. The case $a = 0$, $b \neq 0$ corresponds to the above mentioned regularized conifold [9].

The other solution to Eq. (3.2) gives a non-trivial relation between g and the remaining functions of the ansatz, namely:

$$g^2 = 1 - \frac{1}{4}e^{-2\phi-2\lambda+2h}. \tag{3.5}$$

It is not difficult to find the values of β and $\tilde{\beta}$ for this solution of the constraint:

$$\beta = \frac{1}{2}e^{-\phi-\lambda+h}, \qquad \tilde{\beta} = -g. \tag{3.6}$$

Notice that they satisfy $\beta^2 + \tilde{\beta}^2 = 1$ as a consequence of the relation (3.5). Moreover, one can verify that (3.5) is consistent with the first-order equations. Indeed, by differentiating Eq. (3.5) and using the first-order equations for ϕ, λ and h (Eqs. (2.17), (2.20) and (2.23)), we arrive precisely at the first-order equation for g written in (2.18). It can be also checked that Eq. (2.29) is identically satisfied for this solution of the constraint. Thus, one can eliminate g from the first-order equations arriving at the following system of equations for ϕ, λ and h:

$$\phi' = \frac{1}{8}e^{-2\phi+\lambda+h},$$
$$\lambda' = \frac{1}{24}e^{-2\phi+\lambda+h} - \frac{1}{2}e^{3\lambda-h} + \frac{1}{2}e^{-3\lambda-h},$$
$$h' = -\frac{1}{12}e^{-2\phi+\lambda+h} + \frac{1}{2}e^{3\lambda-h} + \frac{1}{2}e^{-3\lambda-h}. \tag{3.7}$$

4. The generalized deformed conifold

In order to integrate the system (3.7), let us define the function $z = \phi + \lambda - h$ and a new radial coordinate τ, $dr = 2e^{\phi-2\lambda}\,d\tau$. Then, if the dot denotes the derivative with respect to τ, it follows from (3.7) that z satisfies the equation:

$$\dot{z} = \frac{1}{2}e^{-z} - 2e^{z}. \tag{4.1}$$

This equation can be immediately integrated:

$$e^{z} = \frac{1}{2}\frac{\cosh(\tau+\tau_0)}{\sinh(\tau+\tau_0)}, \tag{4.2}$$

where τ_0 is an integration constant, which from now on we will absorb in a redefinition of the origin of τ. We can obtain ϕ by noticing that it satisfies the equation:

$$\dot{\phi} = \frac{1}{4}e^{-z}. \tag{4.3}$$

Since we know $z(\tau)$ explicitly, we can obtain immediately $\phi(\tau)$, namely:

$$e^{\phi} = \hat{\mu}(\cosh\tau)^{\frac{1}{2}}, \tag{4.4}$$

where $\hat{\mu}$ is a constant of integration. Finally, h satisfies the following differential equation:

$$\dot{h} = -\frac{1}{6}e^{-z} + e^{z} + e^{6\phi-5z-6h}. \tag{4.5}$$

If we define, $y = e^{6h}$ and use the expressions of z and ϕ as functions of τ, we get:

$$\dot{y} = \frac{\cosh^2\tau + 2}{\cosh\tau\sinh\tau}y + 192\hat{\mu}^6\frac{(\sinh\tau)^5}{(\cosh\tau)^2}, \tag{4.6}$$

which is also easily integrated by the method of variation of constants. In order to express the corresponding result, let us define the function:

$$K(\tau) \equiv \frac{(\sinh 2\tau - 2\tau + C)^{\frac{1}{3}}}{2^{\frac{1}{3}}\sinh\tau}, \tag{4.7}$$

where C is a new constant of integration. Then, h is given by:

$$e^{h} = 3^{\frac{1}{6}}2^{\frac{5}{6}}\hat{\mu}\frac{\sinh\tau}{(\cosh\tau)^{\frac{1}{3}}}\left[K(\tau)\right]^{\frac{1}{2}}. \tag{4.8}$$

As we know z, h and ϕ, we can obtain λ. The result is:

$$e^{\lambda} = \left(\frac{3}{2}\right)^{\frac{1}{6}}(\cosh\tau)^{\frac{1}{6}}\left[K(\tau)\right]^{\frac{1}{2}}. \tag{4.9}$$

Finally, we can get g from the solution of the constraint (Eq. (3.5)), namely:

$$g = \frac{1}{\cosh\tau}. \tag{4.10}$$

It follows immediately from (4.10) that $g \to 0$ as $\tau \to \infty$, as claimed above. Moreover, by using the explicit form of this solution we can find the value of the phase α:

$$\cos\alpha = \frac{\sinh\tau}{\cosh\tau}, \qquad \sin\alpha = -\frac{1}{\cosh\tau}. \tag{4.11}$$

Notice that $\alpha \to -\pi/2$ when $\tau \to 0$, whereas $\alpha \to 0$ for $\tau \to \infty$. In order to express neatly the form of the corresponding eleven-dimensional metric, let us define the following set of one-forms:

$$\begin{aligned} g^1 &= \frac{1}{\sqrt{2}}\left[\sigma^2 - w^2\right], \qquad g^2 = \frac{1}{\sqrt{2}}\left[\sigma^1 - w^1\right], \qquad g^3 = \frac{1}{\sqrt{2}}\left[\sigma^2 + w^2\right], \\ g^4 &= \frac{1}{\sqrt{2}}\left[\sigma^1 + w^1\right], \qquad g^5 = \frac{1}{\sqrt{2}}\left[\sigma^3 + w^3\right], \end{aligned} \tag{4.12}$$

and a new constant μ, related to $\hat{\mu}$ as $\mu = 2^{\frac{11}{4}} 3^{\frac{1}{4}} \hat{\mu}$. Then, by using the uplifting formula (2.11), the resulting eleven-dimensional metric ds^2_{11} can again be written as $ds^2_{11} = dx^2_{1,4} + ds^2_6$, where now the six-dimensional metric is:

$$ds^2_6 = \frac{1}{2}\mu^{\frac{4}{3}} K(\tau)\left[\frac{1}{3K(\tau)^3}\left(d\tau^2 + \left(g^5\right)^2\right) + \cosh^2\left(\frac{\tau}{2}\right)\left(\left(g^3\right)^2 + \left(g^4\right)^2\right) + \sinh^2\left(\frac{\tau}{2}\right)\left(\left(g^1\right)^2 + \left(g^2\right)^2\right)\right], \tag{4.13}$$

which, for $C = 0$ is nothing but the standard metric of the deformed conifold, with μ being the corresponding deformation parameter.

The metric (4.13) for $C \neq 0$ was studied in Ref. [9], where it was shown to display a curvature singularity when $\mu \neq 0$. On the contrary, for $\mu = 0$ and $C \neq 0$ this metric is regular, after a $\mathbb{Z}_2$ identification of the $U(1)$ fiber, and reduces to the one written in (3.3) for $a = 0$ and $b \neq 0$ (the regularized conifold), the parameter b being related to the constant C [9]. It is not difficult to reobtain this result within our formalism. Notice, first of all, that both solutions of the constraint (3.2) are not incompatible, i.e., one can take $g = 0$ in Eq. (3.5) if $z = \phi + \lambda - h$ is fixed to the particular constant value $e^z = 1/2$. Notice that this is consistent with Eq. (4.1). Actually, this value of z can be obtained by taking $\tau_0 \to \infty$ in the general solution (4.2). Moreover, the differential equation (4.3) for ϕ in this $g = 0$ case reduces to $\dot{\phi} = 1/2$, which can be immediately integrated to give $e^{\phi} = Ae^{\tau/2}$, with A being a non-zero constant. Again, this solution can be obtained from the general expression (4.4) by first reintroducing

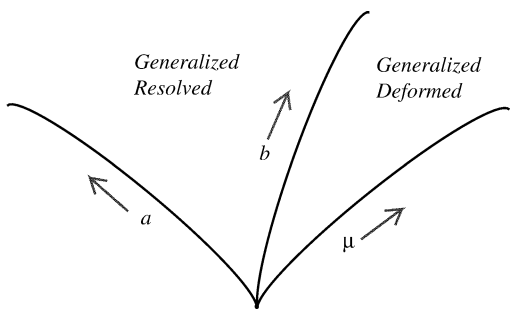

Fig. 1. Representation of the moduli space of generalized resolutions of the conifold singularity. The two regions depicted correspond to the two solutions of our constraint. The generalized deformed conifold metric is singular. A point on each of the three lines represents, from left to right, the resolved, regularized and deformed conifold. They meet at a single point, the singular conifold.

the τ_0 parameter (i.e., by changing $\tau \to \tau + \tau_0$) and then by taking $\tau_0 \to \infty$ and $\hat{\mu} \to 0$ in such a way that $\hat{\mu} e^{\tau_0/2} = \sqrt{2} A$. Notice that this corresponds to taking $\mu = 0$, as claimed. It follows from this discussion that the regularized conifold is a boundary in the moduli space separating the regions that correspond to the generalized deformed and resolved conifolds, as depicted in Fig. 1. Notice that we cannot continuously connect the deformed and resolved conifolds through a supersymmetric trajectory of non-singular metrics.

5. Summary and discussion

In this Letter, we have shown that lower-dimensional gauged supergravities are an appropriate framework to resolve singularities in the study of gravity duals of supersymmetric gauge theories arising in D-branes that wrap a supersymmetric cycle. The key ingredient is provided by the novel twist prescription recently introduced in [15]. The value of the gauge fields at infinity implied by the conventional twisting is preserved, the lower-dimensional gauged supergravity governing the non-trivial dynamics in the bulk. The singularity resolution takes place by switching on the appropriate fields of the gauged supergravity which correspond to the generalized twisting.

We have presented a unified scenario for conifold singularity resolutions from the perspective of M-theory: a single system encompassing both the generalized resolution and deformation of the conifold singularity, each of them emerging as the only two possible solutions of an algebraic constraint. It might be possible to relate this constraint to those appearing in the study of G_2 manifolds carried out in [15] by a reduction mechanism of the sort discussed in [18].

It would be interesting to understand the meaning of b on the dual five-dimensional gauge theory. In the regularized conifold it plays the rôle of a mass scale: If a stack of D3-branes and fractional branes is at the tip of the conifold [19], $b \neq 0$ breaks the otherwise conformal invariance associated to the AdS factor for small ρ. See, for example [20].

The mechanism presented in this Letter must be useful in studying other singularity resolutions. It would be also interesting to understand the appearance of cascading solutions with chiral symmetry breaking occurring in the IR—through the resolution of naked singularities—[19], in the framework of gauged supergravity. We hope to report on some of these issues in a near future.

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