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On the accretion of Uranus and Neptune

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ABSTRACT

In this paper, we discuss some problems concerning the formation of Uranus and Neptune. We find that the adoption of reduced Hill spheres as the region of close interaction between planetesimals introduces an enhancement of the gravitational cross-sections in previous numerical simulations. We also discuss a way to make possible the formation of Uranus and Neptune on time-scales shorter than the age of the Solar system.

Key words: accretion, accretion discs – planets and satellites: formation – planets and satellites: individual: Uranus – planets and satellites: individual: Neptune – Solar system: formation.

1 INTRODUCTION

In the 1960s, Safronov (1972) concluded that, owing to the low surface density of solid material in the outer solar nebula, the time-scale of formation of Uranus and Neptune would be longer than the age of the Solar system. This conclusion was based on calculations of planetesimal accretion in the particle-in-a-box approximation, where the modern concept of multi-planet accretion is not present. Moreover, the distant interactions between the planetesimals were not included. Thus the mass evolution of the system was that of orderly growth of the bodies. When a more accurate model was produced (see, for example, Wetherill & Stewart 1989 and Greenberg et al. 1996), it was noticed that the more massive body would decouple rapidly from the distribution, exhibiting a runaway growth. This effect is mainly due to the dynamical friction experienced by the biggest body. Dynamical friction, which tends to produce equipartition of energy, reduces the relative velocities between the large objects and the smaller ones, producing an enhancement of the gravitational cross-section of the runaway body. The end-state of the runaway growth phase leaves most of the total mass of the system contained in a few big planetesimals (or protoplanets), which orbit embedded in a swarm of smaller objects. This stage is usually taken as the initial condition for numerical simulations of the formation of the outer planets by multi-planet accretion (Fernández & Ip 1984, 1996; Brunini & Fernández 1999). Indeed, a number of features in the outer Solar system have been called upon as observational evidence of the existence of such a stage (Stern 1991).

Numerical simulations of the macro-accretion process of Uranus and Neptune were first carried out by Fernández & Ip (1984, 1996). Their simulations incorporated in a natural fashion the multi-planet accretion process. However, it was based on Öpik's formalism for planetesimal–planetesimal interactions (Öpik 1951) (thus keeping Safronov's particle-in-a-box approximation).

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A serious limitation of Öpik's formalism is that it allows bodies to collide if they are on mutual crossing orbits *only*. This is not a serious limitation whenever relative velocities are high, so radial excursions may be large. In this case planetary embryos grow at a moderate rate, because at large relative velocities the gravitational focusing factor is small. However, the most important regime of accretion, the runaway growth, occurs during the first stages of the process, when relative velocities are small enough to make dynamical friction operative (Wetherill & Stewart 1989; Ida 1990; Ida & Makino 1991; Greenberg et al. 1996).

In view of the previously mentioned limitations of the Öpik formalism to explain relevant features of the accretion of Uranus and Neptune, Brunini & Fernández (1999) investigated this problem by means of a more complete numerical modelling of the accretion process, based on the numerical integration of the equations of motion that naturally overcomes the limitations mentioned above. The main results of Fernández & Ip (1984) were confirmed. As a consequence of the runaway growth, in these new simulations the time-scale of formation of Uranus and Neptune was reduced to only some 10^7 yr. It was also found that the orbital evolution of the outer planetary system was governed by the exchange of orbital angular momentum between the protoplanets and the planetesimals, and the resonant coupling between the protoplanetary system.

2 PROBLEMS

In Fernández & Ip (1984) as in Brunini & Fernández (1999), close encounters were treated neglecting the central gravitational attraction of the Sun. This approximation tends to overestimate the gravitational cross-sections during low relative velocity encounters by a factor of up to 4 (Ida 1990). The fact that Fernández & Ip (1984) have considered cross-sections multiplied by a factor of up to 4, in an attempt to simulate the presence of primordial atmospheres surrounding the protoplanets, does not mean that the factor of multiplication of the gravitational cross-section was $4 \times 4 = 16$, because in those simulations all the encounters were at a relative velocity regime where the two-body approximation is good.

However, as a way to simulate the effects of the presence of a primordial solar nebula on the heliocentric dynamics of the planetesimal, Fernández & Ip (1984) and Brunini & Fernández (1999) have considered the interaction of the planetesimals only where the interacting bodies were within a certain *fraction* of the mutual Hill radius. It seemed to be a simple way to damp the gravitational warming rate of the system, which would be a natural consequence of the presence of a gaseous medium. This strategy introduces a large and non-systematic overestimation of the rate of accretion.

To understand this fact, we must consider first that the planetesimal–planetesimal interaction was accounted for only within this reduced sphere of action. Therefore, from the border of the mutual Hill sphere up to this reduced sphere of action, the acceleration of the planetesimals due to the gravity of the planet was neglected. This tends to underestimate the relative velocity of the encounters and hence to overestimate the gravitational cross-sections.

When including the solar perturbations during close encounters, not predicting the accretion events by means of a two-body approximation, and considering the radius of the Hill sphere as the distance of close interactions, we noted that the accretion process was much less efficient. In fact, during some preliminary runs (using the same initial conditions as in Brunini & Fernández 1999), planets as massive as Uranus and Neptune were never formed (Melita & Brunini 1999). The rate of pairwise collisions was reduced by a factor of 6–10. Therefore the dynamical excitation of the system soon precludes the start of runaway growth of big planetesimals.

3 THE STANDARD MODEL

There is now a generalized consensus that the mainly gaseous Jupiter and Saturn had to form before the dispersal of hydrogen and helium from the primordial solar nebula, which is thought to have occurred before 10 Myr. This is supported by recent radio CO observations (Zuckerman, Forveille & Kastner 1995) and the observation of ‘naked T Tauri’ stars with ages of approximately 1 Myr (Walter et al. 1988), suggesting that the molecular gas surrounding young solar-type stars tends to dissipate rapidly, over time-scales no longer than a few Myr. Because of the possible presence of a non-negligible amount of hydrogen and helium in Uranus and Neptune, of the order of 1.6 and 1.1 $M_{\oplus}$ respectively, Pollack et al. (1996) suggested that their time-scales of formation could not have been much longer than the time-scale of dissipation of the gaseous component of the nebula, perhaps no longer than a few $\times 10^7$ yr. This conclusion was based on Mizuno's core instability theory of the accretion of gas by giant planets (Mizuno 1980). In this theory, once solid cores of the order of 10–20 $M_{\oplus}$ have accreted (Bodenheimer & Pollack 1986; Pollack et al. 1996), the primordial nebular gas enclosed within the Hill sphere of the planet is compressed on to the surface of the planetary core. The region left by this gas is occupied by fresh gas falling from the surrounding nebula. In addition, the Hill sphere is continuously expanding because of the increment in the planetary mass as a consequence of the rapid accretion of gas. This process generates a self-sustained hydrodynamical instability by which the planet is able to accrete large amounts of gas on very short time-scales.

Our preliminary results, briefly described in the previous section, suggest that, in the outer Solar system, the formation of solid cores of those magnitudes is unlikely on any time-scale, at least without the inclusion of the effect of the primordial nebular gas. Radial migration of the planetary embryos embedded in the protoplanetary gaseous nebula, as well as the damping in the relative velocities of planetesimals due to the presence of the primordial gas may favour accretion at large distances from the central star (Papaloizou & Larwood 2000). Nevertheless, these are problems which are beyond the scope of the present paper.

4 STEVENSON'S PLANETOIDS

An alternative scenario was proposed by Stevenson (1982, 1984). He argues that, in the outer Solar system, solid bodies were rich in condensed ices. The gravitational energy released in a collision of planetesimal pairs is enough to vaporize the ices. Although the low temperatures at this distances from the Sun favour a rapid re-condensation, a fraction of these vapours could mix with the primordial nebula, substantially enhancing its mean molecular density. A dense gas can be much more easily trapped by a low-mass planetesimal. Some simplifying arguments show that a Mars-sized planetoid is able to acquire a large and dense envelope (Lissauer et al. 1994) of densities as high as 10^{-5} g cm^{-3} at the distance of Neptune.

In this scenario, Uranus and Neptune accreted through successive pairwise collisions between those large planetoids. It is difficult to anticipate the fraction of gas that is retained during an energetic event such as a collision between two of those objects. Nevertheless, the sum of the gas present in all of the planetoid envelopes would be orders of magnitude larger than the combined gas in the actual atmospheres of Uranus and Neptune, so there is a certain tolerance for gas loss.

Another very interesting feature of this scenario is the fact that the time-scales of formation of Uranus and Neptune could be longer than the time-scale of complete dispersal of the primordial gas in the solar nebula, because the gaseous component is carried by the protoplanets.

We have performed a series of numerical experiments to determine the effects of these extended atmospheres in the dynamics of close encounters between these gaseous planetoids and planetesimals of different sizes.

We have adopted the model of a planetoid described by Lissauer et al. (1994). A solid core of mean density 1 g cm^{-3} and mass 10^{26} g is considered, orbiting the Sun at the distance of Uranus (an intermediate distance between Saturn and Neptune) on a circular orbit. We have assumed that these planetoids were surrounded by a dense, uniform and compact atmosphere, extended up to 10 planetary radii. Beyond this distance, the mean density reaches nebular conditions. We have explored gaseous densities for the compact atmospheres ranging from 10^{-5} to 10^{-7} g cm^{-3} [this last value is two orders of magnitude less than the densities considered by Lissauer et al. (1994)]. Planetesimals of radius 100–1000 km were launched from the border of the Hill sphere, with a relative velocity (shown in Fig. 1 as a fraction of the escape velocity, given by $V_e = \sqrt{2G M_{\oplus}/R_H}$, R_H being the radius of the Hills sphere of the planet) such that during the first pericentric passage they pass through the planetary atmosphere. For each relative velocity V_0 the impact parameter b was computed as

$$b = r_{\min}\sqrt{1 + \left(\frac{V_e}{V_0}\right)^2\left(\frac{R_H}{R_{\min}} - 1\right)}. \quad (1)$$

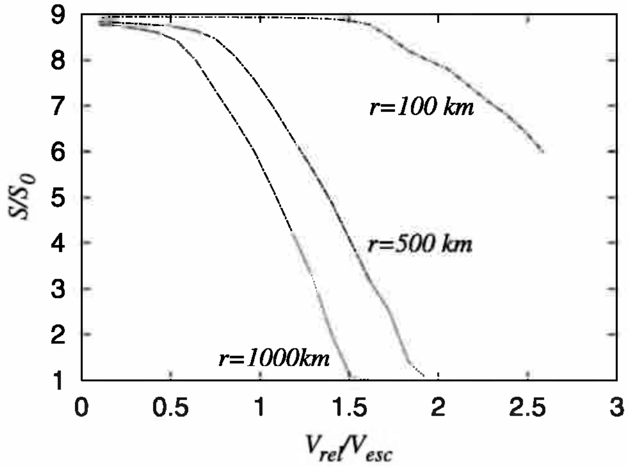

Figure 1. Relative enhancement of the cross-section owing to the presence of an extended envelope of density $\rho = 10^{-5}\,\mathrm{g\,cm^{-3}}$ for planetesimals of different radii. S_0 is the gravitational cross-section in the absence of gas. The solid lines correspond to orbits with apocentre distance less than the Hill radius of the planet after the first passage through the atmosphere. Dotted lines correspond to direct collisions on the planet.

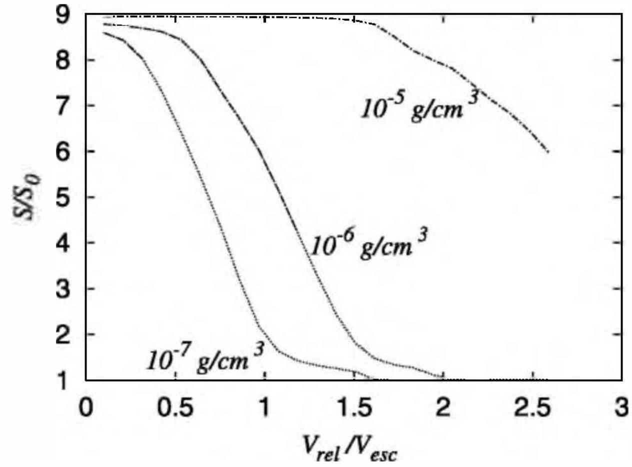

Figure 2. Relative enhancement of the cross-section due to the presence of an extended envelope, for a planetesimal of 100 km radius and different values of the density of the gas. The solid lines correspond to orbits with apocentre distance less than the Hill radius of the planet after the first passage through the atmosphere. Dotted lines correspond to direct collisions on the planet.

where $R_{\min}$ is a distance ranging from the geometrical radius of the planet to a certain fraction of the radius of the extended atmosphere (we used 9 planetary radii as the upper limit). In addition to a direct collision on to the planetary surface, we have also considered accretion when, after the first passage through the planetary atmosphere, the planetesimal becomes permanently bonded to the planet (i.e. when its apocentric distance becomes less than the Hill radius). Each encounter was treated as a two-body problem, but gas drag was included. The code is described in full detail by Brunini, Giordano & Orellana (1996).

Fig. 1 displays the factor of enhancement of the effective cross-section of the planet for a dense atmosphere and different planetesimal radii. It can be observed that, in this model, the cross-sections are strongly enhanced even for the less favourable case of $r = 1000\,\mathrm{km}$.

Fig. 2 shows the same as in Fig. 1, but for the same planetesimal radii ($r = 100\,\mathrm{km}$) and different atmospheric densities. We can see that small planetesimals are efficiently trapped by the effect of gas, even in the low-density case.

For very low relative velocities, all the curves shrink around $S/S_0 \sim 9$ (S is the gravitational cross-section in the presence of gas, whereas S_0 is the same quantity when gas is not present). This is easily explained as follows. Gas drag is effective in reducing the apocentric distance. If after passing through the atmosphere the apocentric distance is smaller than the Hill radius, the planetesimal becomes bonded to the planet. So, below a sufficiently small relative velocity, all the planetesimals are trapped. This means that below a certain relative velocity all the planetesimals with impact parameter given by equation (1), with $r_{\min} = 9$ planetary radii (the upper limit that we used), were considered as potential impactors. Using equation (1) to compare with the impact parameter for collision without gas (i.e. with $r_{\min} = 1$ planetary radius), we get $S/S_0 \sim 9$.

As, during the phase of runaway growth in the presence of a gaseous medium, the encounters are at very low relative velocities, the presence of compact atmospheres surrounding the biggest planetesimals could in principle provide a way to enhance the gravitational cross-sections by a factor high enough to solve the problem of the formation of Uranus and Neptune on time-scales shorter than the age of the Solar system.

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