Quasiparticle-Rotor Model Description of Carbon Isotopes

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In this work we perform quasiparticle-rotor coupling model calculations within the usual BCS and the projected BCS for the carbon isotopes ^{15}C, ^{17}C and ^{19}C using ^{13}C as the building block. Owing to the pairing correlation, we find that ^{13}C as well as the cores of the other isotopes, namely ^{14}C, ^{16}C and ^{18}C acquire strong and varied deformations. The deformation parameter is large and negative for ^{12}C, very small (or zero) for ^{14}C and large and positive for ^{16}C and ^{18}C. This finding casts a doubt about the purity of the supposed simple one-neutron halo nature of ^{19}C.

Keywords: ^{13}C, ^{15}C,^{17}C, ^{19}C nuclei; Energy spectra; Particle projected BCS model; Core excitation; Quasiparticle-rotor model

I. INTRODUCTION

Many experimental and theoretical investigations for over two decades were concentrated upon study of light nuclei near neutron drip line (see references in recent reviews [1, 2]). Pairing correlations play an important role in the structure of these nuclei, because of the proximity of the Fermi surface to the single-particle continuum. This gives rise to many interesting phenomena and creates a challenge for conventional nuclear structure models. The experimental evidence for the $N = 8$ shell melting [3] and the appearance of the $1s_{1/2}$ intruder state in ^{11}Be are famous indications of the complicated structure of light nuclei. In Refs.[4, 5] it was shown that the increase in pairing correlations and the shallow single-particle potentials for nuclei close to the driplines may result in a more uniformly spaced spectrum of single particle states.

To mention a few recent works on this subject, a series of interesting articles appeared in the literature which study the pair correlation in spherical and deformed nuclei near the drip line using a simplified HFB model in coordinate representation with the correct asymptotic boundary conditions. In Ref.[6] the effects of continuum coupling have been studied, it was shown that for small binding energies, the occupation probability decreases considerably for neutrons with low orbital momentum. In Ref.[7] the weakly bound neutrons in $s_{1/2}$ state have been studied. The effective pair gap was found to be much reduced compared with that of neutrons with larger orbital momentum. In the presence of pair correlations, the large rms radius was obtained for neutrons close to the Fermi level, thus favouring the halo formation.

Nowadays it is generally accepted that nucleus tends to form a halo when the valence particles are loosely bound and the relative angular momentum is small. In this case, the last nucleons and the core are to a large extent separable and therefore these nuclei can be approximated as an inert core plus the halo formed by the valence particles. This supports the use of the cluster models for such systems. But for real nuclei, the admixtures between the core and valence particles have to be taken into account. One of the simple models that include core degrees of freedom is a *core + particle* cluster model for one-neutron halo systems including core excitation via deformation assuming a rotational model for the core structure [8, 9]. Although this model allowed successful description of the $1s_{1/2}$ intruder state in ^{11}Be, it has a serious drawback as it does not treat Pauli Principle correctly.

In this work we incorporate the residual interaction between nucleons moving in an overall deformed potential and perform a Bogoljubov-Valatin canonical transformations from particles to quasiparticles, thus modifying the band-head energies and the non-diagonal particle core matrix elements. The resulting model is known as the quasiparticle-rotor coupling model (QRCM). The inclusion of pairing effects allows a correct treatment of Pauli Principle. This model is then applied to the series of odd-mass carbon isotopes.

Heavy carbon isotopes have recently been studied extensively, and ^{17}C and especially ^{19}C are suggested to be candidates to possess neutron halo. To calculate the quasiparticle energies for ^{13}C and heavy carbon isotopes the BCS calculation with particle projection procedure was used (PBCS). It is known that in BCS formalism the number of particles is not conserved, which becomes a serious drawback for the case of light nuclei. It was shown in Refs.[12, 15], that the projection procedure is very important in such nuclei. The Fig. 1 shows the BCS and PBCS calculations confronted with experimental data, using the results obtained in [12], where we evaluate the ground states energies for the remaining odd-mass carbon isotopes, by only modifying the numbers of neutrons: $N = 8, 10$ and 12 for ^{15}C, ^{17}C and ^{19}C, respectively. One immediately sees that the pairing interactions account for the main nuclear structure features in these nuclei.

II. THE QRCM MODEL

Most applications of the particle-rotor coupling model (PRCM) use the simplest picture of the last odd nucleon moving outside the even-even core, and any kind of residual in-

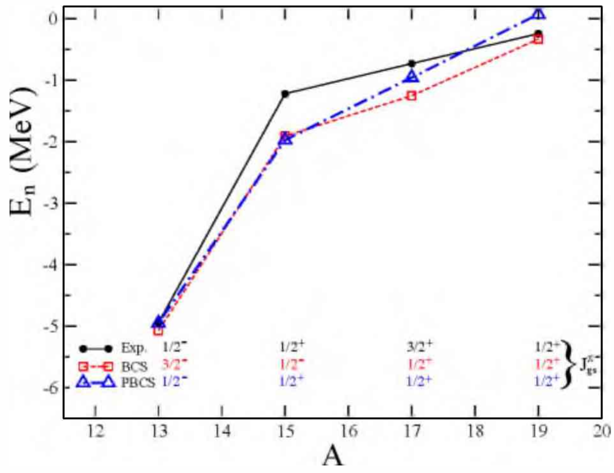

FIG. 1: Comparison between the experimental and the calculated ground state energies in BCS and PBCS in odd mass carbon isotopes. The experimental data were taken from Ref. [10] for ^{13}C, ^{15}C, and Ref. [11] for ^{17}C and ^{19}C.

teraction between nucleons is neglected (see, for example, Refs.[9, 13, 14]). The core is assumed to be an axially symmetric rigid rotor. The Hamiltonian of the core-neutron system can be written as

$$H_{p-rot} = H_{sp} + H_{rot} + H_{int}, \tag{1}$$

where $H_{sp} = T + U_0$ is single-particle (shell-model) Hamiltonian, that is the sum of central Woods-Saxon potential and standard spin orbit interaction, whose eigenvalues e_j are specified by the particle angular momentum $\mathbf{j}$. We consider here the quadrupole deformation only, specified by the deformation parameter β. In this case, the core-neutron coupling hamiltonian

$$H_{int} = -\beta k(r) Y_{20}(\theta', 0); \quad k(r) \equiv r\frac{dU_0(r)}{dr}, \tag{2}$$

where $k(r)$ is the radial part of the interaction.

The total angular momentum is $\boldsymbol{I} = \boldsymbol{j} + \boldsymbol{R}$ (in this work we consider only $I = 0, 2$) and the coupling between $\boldsymbol{j}$ and $\boldsymbol{R}$ is evidenced in the weak-coupling representation of the total wave function, which reads

$$|jR; IM\rangle = \sum_{M_R, m} (jmRM_R|IM)|jm\rangle|RM_R\rangle, \tag{3}$$

where $|RM_R\rangle$ is the rotor wave function. The matrix elements of the Hamiltonian (1) are:

$$\langle j'R'; IM|H_{p-rot}|jR; IM\rangle = \left[\frac{\hbar^2 R(R+1)}{2\mathcal{J}} + e_j\right]\delta_{RR'}\delta_{jj'}$$
$$-\beta(-)^{j'+R+I}\sqrt{2R+1}(R020|R'0)\left\{\begin{matrix} I & R' & j' \\ 2 & j & R \end{matrix}\right\}\langle j'||kY_2||j\rangle. \tag{4}$$

The resulting eigenfunctions are of the form:

$$|I_n\rangle = \sum_{jR} C^I_{jR}|jR; I\rangle, \tag{5}$$

with energies $\mathcal{E}_{I_n}$, and where the C^I_{jR} are constants that result from the solution of the coupled channel system of equations obtained by substituting the total wave function into the Schöedinger equation and projecting out the basis wave functions.

We now incorporate the residual interaction, V_{res}, between nucleons moving in an overall deformed potential. The total Hamiltonian becomes $H_{qp-rot} = H_{p-rot} + V_{res}$. After carrying out a Bogoljubov-Valatin canonical transformation from particle to quasiparticle operators, one gets that the matrix elements of H_{qp-rot} for odd-mass systems continue being of the form (4), except for (i) the band-head energies for a particle state are modified as $e_j \to E_j$ in BCS, and $e_j \to \varepsilon^K_j$ in PBCS, where E_j is the quasiparticle BCS energy, and $\varepsilon^K_j = \frac{R^K_0(j)+R^K_1(jj)}{I^K(j)} - \frac{R^K_0}{I^K}$, where quantities R^K and I^K, for $K = Z, N$, are defined in Ref.[15]; and (ii) the non-diagonal matrix elements are renormalised by the following overlap factors

$$\mathcal{I}_{j'j} = u_{j'}u_j - v_{j'}v_j \text{ (BCS)}$$
or
$$= \frac{u_{j'}u_j I^K(j'j) - v_{j'}v_j I^{K-2}(j'j)}{[I^K(j')I^K(j)]^{1/2}} \; (PBCS), \tag{6}$$

where u_j and v_j are the occupational numbers for the state j. $I^K(j'j)$, $I^{K-2}(j'j)$) and $I^K(j)$ are the PBCS number projection integrals (see Refs.[12, 16]).

The energies to be confronted with the experimental data are

	BCS	PBCS	State
$\varepsilon^{(+)}_{I_n}$	$\lambda + \varepsilon_{I_n}$	$\varepsilon^N_{I_n}$	particle
$\varepsilon^{(-)}_{I_n}$	$\lambda - \varepsilon_{I_n}$	$-\varepsilon^{N-2}_{I_n}$	hole

where λ is the BCS chemical potential. Of course, in the resulting quasiparticle-rotor coupling model (QRCM) the state $|jm\rangle$ is now a quasiparticle state. The QRCM differs in several important aspects from the PRCM. First, the BCS (PBCS) energies $\varepsilon_j^{\pm}$ are quite different from the single-particle energies e_j. Second, the factors $\mathcal{F}_{jj'}$ correctly take into account the Pauli principle. In addition, the particle-like states do not couple to the hole-like states. As a brief test, in Fig. 2 we show a comparison of the energy levels for ^{13}C and ^{11}Be in the Pure Rotor Model with those in the QRCM: Rotor+BCS and Rotor+PBCS as a function of the parameter of deformation β for the single particles taken from the Vinh-Mau's work [17] for ^{13}C and ^{11}Be. From a careful scrutinising of the energy levels in Fig. 2, one can easily convince oneself that, none of our particle-rotor coupling calculations is able to reproduce the experimentally observed spin sequence $1/2^+ - 1/2^- - 5/2^+$ in ^{11}Be, for such a value of β neither positive nor negative. The constant of pairing for ^{13}C is $v_s = 37.80$ in both BCS and PBCS, while that in ^{11}Be they are $v_s = 21.02$ in BCS and $v_s = 24.25$ in PBCS.

III. RESULTS AND CONCLUSIONS

In our procedure we use the nucleus ^{13}C as a building block of the calculation, assuming core ^{12}C deformation $\beta = -0.6$. In our calculations we adjust the single particle state energies e_j (for $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$ and $2s_{1/2}$) so that the resulting quasiparticle energies including the core excitation give correct binding energies of four bound states in ^{13}C. The same set of single particle energies are then used in PBCS calculation to obtain quasiparticle energies of ^{15}C, ^{17}C and ^{19}C, occupation numbers for the states in these nuclei and the reduction factors of the matrix elements. Subsequently, the quasiparticle energies and reduction factors then enter the coupled channel system to include core excitation and give the binding energies of ^{15}C, ^{17}C and ^{19}C. For ^{15}C, ^{17}C and ^{19}C we adjust the deformation parameter β to obtain a correct one-neutron separation energy in each of the isotopes. Using this deformation parameter we calculate the energy of the first excited state. For simplicity, in the present calculation we use fixed values for averaged radial matrix elements between different states, that is, $\langle j'|k(r)|j\rangle = 25$, 30 and 35 MeV. These values provide a reasonable estimation of non-diagonal part of the interaction and correctly reproduce the spin-orbit splitting between initial single-particle $1p_{3/2}$ and $1p_{1/2}$ states. The energies obtained within the Rotor+PBCS model on different carbon isotopes for $\langle k(r)\rangle = 25$ MeV (all energies are in MeVs) are presented in the Table I.

The results for the odd-mass carbon isotopes can be summarised as:

^{15}C: In our calculation we obtain a zero deformation for the core ^{14}C in case of $\langle k(r)\rangle = 25$ MeV and a binding energy of 1.25 MeV. The calculation with larger average matrix element 30 and 35 MeV, gives an oblate shape with deformation parameter equal to -0.23 and -0.32 respectively. (In this calculation we introduced a lower 2^+ rotational energy in ^{14}C $\varepsilon_{2^+} = 3$ MeV). This confirms the idea that the core nucleus ^{14}C is nearly spherical and supports the possibility of halo formation for ^{15}C. Finally, our calculation also predicts a first excited state $\frac{5}{2}^+$ with the binding energy -936 keV, -991 keV and -867 keV for $\langle k(r)\rangle$ equal to 25 MeV, 30 MeV and 35 MeV, respectively. The binding of the first excited state is overestimated in our model.

^{17}C: Small separation energy of ^{17}C 0.728 MeV suggests a possible halo structure of this isotope. There is an uncertainty about ground state spin assignment for this nucleus. In our model we obtain $\frac{1}{2}^+$ ground state for ^{17}C, although the experimental value is $\frac{3}{2}^+$ [11]. A simple explanation for this experimental result could be found in the so called $J = j-1$ anomaly discussed by Bohr and Mottelson [18]. Core-particle calculations without pairing similar to those in Ref.[13] used deformation parameter $\beta = 0.55$. We obtain the β equal to 0.505, 0.583 and 0.609 for average radial matrix element $\langle k(r)\rangle$ equal to 25, 30 and 35 MeV, respectively. We see that the deformation changes slowly with the value of the matrix element and is rather large. The first excited state of ^{17}C in our model is $\frac{5}{2}^+$ state. The binding energy of this state is as follows: -0.616 for $\langle k(r)\rangle = 25$ MeV, -0.527 MeV for $\langle k(r)\rangle = 30$ MeV and -0.430 for $\langle k(r)\rangle = 35$ MeV. These energies are close to the experimental separation energy of -0.433 MeV.

^{19}C: Ground state with spin-parity $\frac{1}{2}^+$ and small one-neutron separation energy favours the formation of the halo in ^{19}C. In our analysis we adjust the deformation parameter in order to obtain the adopted separation energy of 0.58 MeV. Core-particle calculations without pairing used deformation parameter $\beta = 0.5$. For $\langle k(r)\rangle = 25, 30$ and 35 MeV we need, respectively, $\beta = 1.27, 1.51$ and 1.82. That is, we obtain very a large prolate deformation for the core ^{18}C. The first excited state we obtain to be $\frac{5}{2}^+$ state with the binding energy -0.21MeV ($\langle k(r)\rangle = 25$ MeV).

The Fig. 3 summarises the above results and compares them with the results of other models and experimental data.

Finally, in Fig. 4 we show the dependence of the deformation parameter on the number of neutrons in the core N for three values of $\langle k(r)\rangle$. It is seen that in each case, the deformation changes from negative values (for cores ^{12}C and ^{14}C) to large positive values (for cores ^{16}C and ^{18}C). The dependence is nearly linear and crosses the zero deformation in the region of the core ^{14}C.

To summarise, in this work we analyse the structure of heavy carbon isotopes ^{15}C, ^{17}C and ^{19}C in the quasiparticle-rotor model using the ^{13}C as the building block of the calculation. The quasiparticle energies and the reduction factors in the matrix elements are calculated in the projected BCS model. As it was mentioned in the introduction, we first explore the results of the pure BCS and PBCS models, that use the single-particle energies and pairing strengths of Ref. [12] fixed so that the experimental binding energies of ^{11}C and ^{13}C, together with the ^{13}C energy spectra, are reproduced by the calculations. With these parameters we evaluate next the low-lying states in ^{15}C, ^{17}C and ^{19}C, by augmenting correspondingly the number of neutrons only. We found that proceeding in this way both models are capable to explain fairly well the decrease of the binding energies in going from ^{13}C to ^{19}C. Second, the QRCM was implemented, and in this approximation we obtain ^{15}C, ^{17}C and ^{19}C with $\frac{1}{2}^+$ ground state and

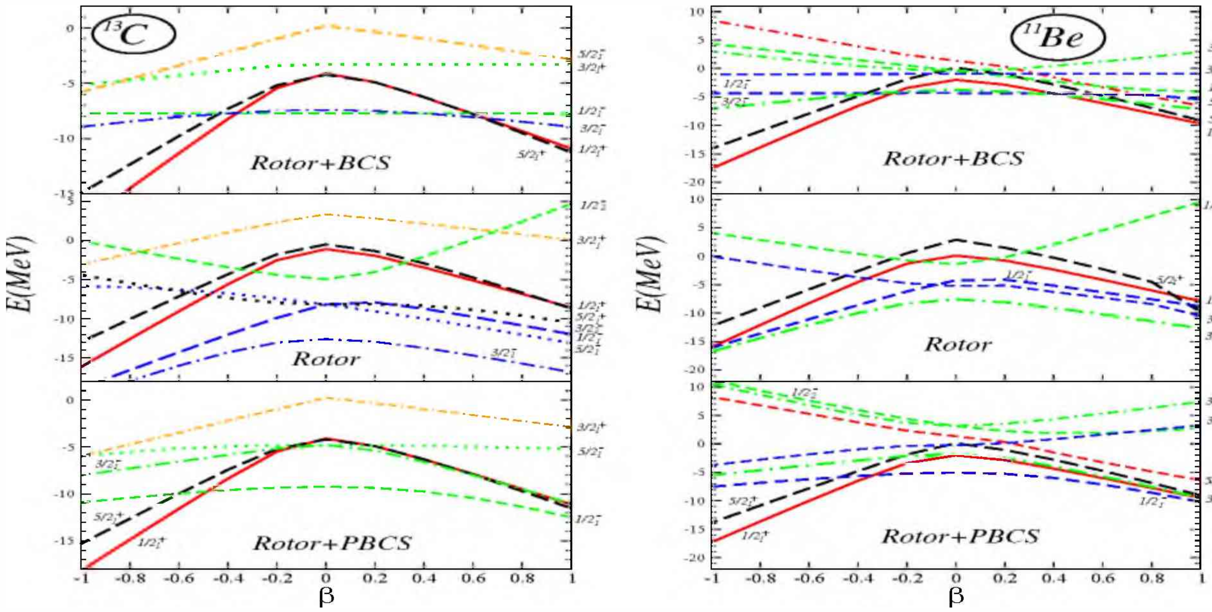

FIG. 2: (Color online) Energies of the lowest states in ^{13}C and ^{11}Be as functions of deformation parameter β calculated in the Pure Rotor model (middle panels), Rotor+BCS model(upper panels) and Rotor+PBCS(lower panels).

TABLE I: Energies obtained within the Rotor+PBCS model on different carbon isotopes for $\langle k(r)\rangle = 25$ MeV. All energies are in MeVs.

State	^{13}C	^{15}C	^{17}C	^{19}C
$1p_{3/2}$	0.63	6.95	9.66	11.86
$1p_{1/2}$	-4.75	1.92	4.38	6.32
$1d_{5/2}$	-0.10	-0.94	-0.30	0.32
$2s_{1/2}$	-0.75	-1.25	-0.36	0.45

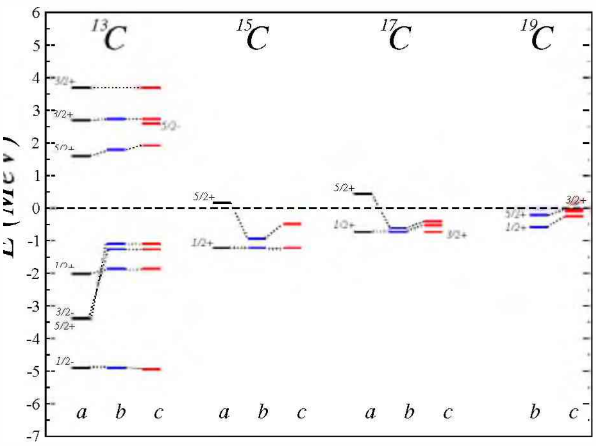

FIG. 3: Comparison between the calculated (a) core-particle, (b) QRCM, and (c) experimental level scheme for odd mass carbon.

$\frac{5}{2}^+$ first excited state. For ^{15}C we obtain zero or small oblate deformation. For ^{17}C large prolate deformation is given. Finally ^{19}C has a very large prolate deformation, indicating that the nature of this nucleus is more complicated than the simple one-neutron halo picture of this nucleus would suggest.

The further analysis of this model and the details of its application to describe the low-lying spectra of light nuclei model will be presented in our next work [19].

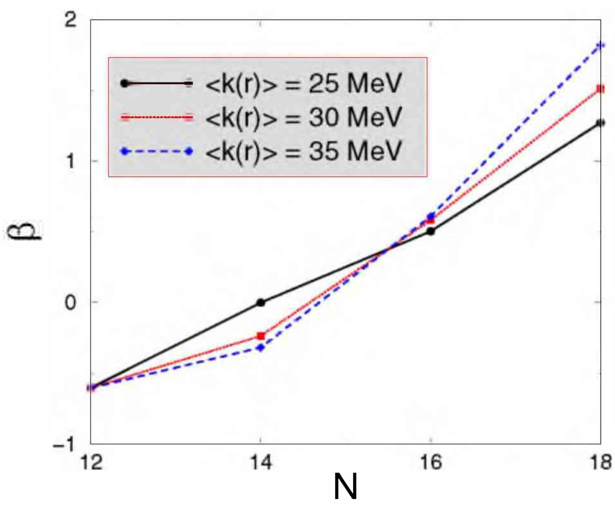

FIG. 4: The adjusted value of deformation parameter β of cores ^{12}C, ^{14}C, ^{16}C and ^{18}C for three values of $\langle k(r)\rangle$.

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