Risky Banking: Optimal Loan Quantity and Portfolio Quality Choices

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Abstract

In this paper we construct a model of a "risky bank." The bank faces excess demand in the loan market, can sort loan applicants by an observable measure of quality, and faces a small but positive probability of default on its loan portfolio. The bank uses two policies to allocate credit:

- Tighten restrictions on loan quality
- Limit the number of loans of a given quality

We show that the level of default risk and other structural conditions have important effects on the market for loanable funds and the bank's optimal policies (loan rates, deposit rates, and lending standards). The structural conditions that we examine are monitoring costs, returns on alternative investments, firms' minimum funding requirements, and the level of the reserve requirement. The model provides insight into several stylized facts observed in loan markets, especially in developing countries.

JEL Classification Numbers:
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1 Introduction

Banks are the dominant financial institution for channelling funds from savers to entrepreneurs in most "emerging financial markets." Many countries, especially developing economies, report the following problems (cf., Beim and Calomiris (2000)):

- Costly banking crises

- Large spreads between deposit and loan rates

- Reports of "credit crunches" (i.e., excess demand) in loan markets

We construct a model of a "risky bank" that can account for these stylized facts. The bank arises endogenously to accept deposits from investors and make loans to entrepreneurs with risky projects that can be sorted by an observable measure of project quality. The bank faces a small but positive probability of default. This friction in the bank's loan portfolio causes depositors to consider the risky bank's profitability. Specifically, depositors require a risky bank to be more profitable than a riskless bank because they must be compensated for the expected cost of recovering funds when default occurs.

We analyze the problem of a bank that chooses a deposit rate, loan rate, and a minimum loan quality standard when there is excess demand for loans. The bank must satisfy a reserve requirement, but there is no deposit insurance. The bank manages the excess demand by rationing loans in two ways. First, because the bank chooses the quality of its loan portfolio, the bank can tighten the minimum quality requirement for loan applicants.[1] Second, the bank can restrict the quantity of loans it grants to borrowers of a given quality level. Rationing by loan quantity was proposed by Williamson (1986) for banks that are not subject to default risk. To our knowledge, rationing by loan quality has not been studied previously in equilibrium models,[2] yet an important role of banks is to screen loan applicants based on measures of project quality. We assume that the quality of individual applicants is observable by the bank, and focus on the implication of loan portfolio risk for loan rates, deposit rates, and lending standards.

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[1] For example, quality can be measured by a parameter that indexes a mean preserving change in the variance of the distribution of a bank's portfolio of loan returns.

[2] There is a literature on loan portfolio diversification, but it is aimed at operational ways to measure and control a bank's credit risk exposure. Our focus is on the implications of a given level of default risk for the macroeconomic problems enumerated at the outset.

When banks are risky, the level of default risk and other structural conditions have important effects on the market for loanable funds and the bank's equilibrium decisions. We show how this default risk is "priced out," and that the default premium can affect the deposit rate, the loan rate, and the quality cutoff. We characterize four distinct equilibrium outcomes:

(i) Rationing by loan quality: The default premium is borne entirely by the loan rate. Neither the extent of credit rationing nor the deposit rate change. The change in the interest rate spread is larger than the change in the default premium, a type of multiplier effect.

(ii) Rationing by loan quantity: When the bank's expected return for a given quality level is insufficient to compensate depositors, increases in the default premium increase rationing by loan quantity and decrease the deposit rate. The decrease in the deposit rate causes dis-intermediation.

(iii) Both types of rationing can occur if the default risk is sufficiently high.

(iv) No banking equilibrium: This case corresponds to the costly banking crises observed worldwide.[3]

2   The Model

Consider a model with two types of risk neutral agents, α lenders and $1 - \alpha$ entrepreneurs. There is an initial planning period, and a subsequent consumption/production period. Each entrepreneur is endowed with a project with a random return y_i but no input. Hence entrepreneurs wish to borrow. Each lender is endowed with one unit of input but no project. All projects have a common scale $q > 1$. Agents are asymmetrically informed. Borrowers privately and costlessly observe their return, but lenders do not unless a state verification cost is paid. If a lender chooses to incur cost $c_b > 0$ to verify a project return, this cost is paid in output to an exogenous verification authority. Deadweight loss c_b "disappears" from the economy. The true project realization y_i is *privately* revealed only to the lender who pays the cost.

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[3]The IMF estimates that the cummulative output loss due to banking crises as a percentage of GDP is 10.2 % among industrial countries and 12.1 % among developing countries (cf., IMF (1998), Table 15, p. 79). Our results suggest that differences in the default premium and structural differences may account for some of this.

Williamson (1987) established that in this costly state verification model a bank emerges endogenously from among the set of investors. The bank writes deposit contracts with investors and loan contracts with entrepreneurs. In this standard setting individual project returns are identically and independently distributed with a common distribution function $G(y_i)$. We add two features that affect the distribution of average returns from the bank's loan portfolio $G(y, \theta; s)$. First, we introduce two states, $s = l, h$; the bank defaults in the low state and is solvent in the high state. Second, we introduce an index θ that measures quality of a project.

Assume that $G(y, \theta; s)$ is defined over the range of possible returns $(0, \bar{y})$ and has density function $g(y, \theta; s)$.

- As θ changes, $G_\theta(y, \theta) \geq 0$ for all y and $G_\theta(y, \theta; s) > 0$ for some y.[4] E.g., this captures the situation of a bank that faces a distribution of entrepreneur projects with returns that have the same mean but different variances. Quality parameter θ has a distribution $H(\theta)$ over a range $[\theta_{min}, \theta_{max}]$ with density $h(\theta)$.

- States $s = l, h$ affect distribution $G(y, \theta; s)$ in the sense of Second Order Stochastic Dominance: $G_l(y, \theta; s) \geq G_h(y, \theta; s) > 0$ for all y.

Assume that θ and s are independent. Let $\bar{s}$ denote no default risk and p_s be the probability of state s. Assume that s does not affect the expected return

$$p_l \int_0^{\bar{y}} y \, dG(y, \theta; s = l) + p_h \int_0^{\bar{y}} y \, dG(y, \theta; s = h) = \int_0^{\bar{y}} y \, dG(y, \theta; \bar{s}) = \tilde{y}$$

Lenders, who have an endowment of input but no project, inelastically supply labor when young to earn wage $w > 0$, and have access to two investment opportunities. First, they may lend to entrepreneurs under terms governed by a contract. Second, they may invest in an outside option that yields $x_i > 0$ for each unit invested. Return x is costlessly observable and does not require verification.[5] Prior to its realization, x is uncertain and

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[4]As θ increases the distribution is more risky in the sense of Second Order Stochastic Dominance. When agents are risk neutral, a mean-variance selection rule is appropriate for a normal distribution of returns, cf., Bawa (1975). We show that an increase in the variance of the distribution of loan returns decreases the "quality" of loan applicants, decreasing the bank's expected return.

[5]This introduces an upward sloping supply curve for saving deposits. The outside option can be motivated as a government bond with a publicly known return. In contrast, the returns on private projects are costly to verify.

has a distribution $I(x)$, with $i(x) = I'(x) > 0$ and $x\epsilon[0, \bar{x}]$, where $\bar{x}$ is the maximum return on the outside opportunity.

Finally, information is crucial in the economy. We assume that

- Ex-ante agents know $G(y, \theta; s)$, $I(x)$, $H(\theta)$, θ, p_l

- Ex-post entrepreneurs privately observe return y_i, and investors do not unless costly verification occurs. Return x is costlessly observed by all.

2.1 Distribution of the Bank's Loan Portfolio

We now derive the relationship among y, θ, s, and the probability of default, p_l. We begin by distinguishing between the bank's income from an individual borrower and the average income from its loan portfolio. The bank's income from entrepreneur $i = 1, \ldots, m$ is

$$L_i(x_i) = L_i(G(y_i, \theta_i; s))$$

Average income per borrower from the loan portfolio under contract $L(.)$ is

$$\frac{1}{m} \sum_{i=1}^{m} L_i(G(y_i, \theta_i; s)) \to E[L(G(y, \theta; s)|s)]$$

Then $G(\cdot)$ is the distribution of returns from loan portfolio $L(G(y, \theta; s)|s)$. Assume that $G(\cdot)$ takes two values given by

- $G_l(\cdot)$: The distribution of returns from the loan portfolio if $s = l$

- $G_h(\cdot)$: The distribution of returns from the loan portfolio if $s = h$

Krasa and Villamil (1992, p. 203) shows that the probability of bank failure, p_l, converges to the probability that the return from the bank's asset portfolio is less than return that the bank must pay the depositors, face value $\bar{D}$

$$P(\{\frac{1}{m} \sum_{i=1}^{m} L_i(G(y_i, \theta_i; s)) < \bar{D}\}) \to P(\{E[L(G(y, \theta; s)|s)] < \bar{D}\})$$

We assume that the bank defaults in the low state, with $p_l > 0$ but small

$$P(\{\frac{1}{m} \sum_{i=1}^{m} L_i(G(y_i, \theta_i; s = l)) < \bar{D}\}) = p_l$$

2.2 Riskless Banking

When the bank faces no default risk, Williamson (1986) showed that (i) the optimal contract is simple debt, (ii) banks arise endogenously to eliminate duplicative monitoring, and (iii) equilibrium credit rationing by loan quantity may arise. We briefly review these results in the Appendix since they provide a benchmark to which the bank's problem with default risk is compared. The Appendix shows that when the bank faces no default risk, i.e., $s = \bar{s}$, the expected return function for a bank that contracts with an infinite number of entrepreneurs is

$$\Pi(L(y,\theta),\theta;\bar{s}) = \int_{B_b}(L(y,\theta) - \frac{c_b}{q})dG(y,\theta;\bar{s}) + \int_{B_b'}\bar{L}dG(y,\theta;\bar{s}) \qquad (1)$$

The first term on the right hand side is the bank's expected return from loan contract $L(y,\theta)$, net of per project monitoring costs, $\frac{c_b}{q}$, in default states $y \in B_b$. The second term is the bank's expected return when loans are fully repaid at face value $\bar{L}$ in non-default states $y \in B_b'$.

In a perfectly competitive market, a riskless bank equates the expected return function with the interest rate on deposits, $\bar{D}$. Williamson showed that the depositors' expected cost of monitoring the bank goes to zero as portfolio size goes to infinity because the portfolio earns $\bar{L}$ with probability one. The bank can then pay depositors reservation value $\bar{D}$ with certainty. The bank never defaults and the cost of delegation is nil. Williamson also showed that the bank's expected return function $\Pi(\cdot)$ is concave in loan rate $\bar{L}$, thus it has an interior maximum at some $\bar{L}^*$. This can lead to equilibrium credit rationing by loan quantity at $\bar{L}^*$. Even if a rationed borrower offered to pay a loan rate higher than $\bar{L}^*$, the bank would refuse because $\bar{L}^*$ maximizes the bank's expected return.[6]

2.3 Risky Banking

When a bank may default in some states, Krasa and Villamil (1992) showed the following. (i) Banking remains optimal if monitoring costs are bounded and the probability of default is sufficiently small. (ii) The optimal contract is two-sided debt, where $(L(y,\theta), B_b)$ is the loan contract between the bank and entrepreneurs, and $(D(y), B_d)$ is the deposit contract between the bank

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[6]When failure is costly to the lender, an increase in the loan rate may decrease the bank's expected return because it raises the probability of borrower default.

and lenders. As before, the bank funds m entrepreneurs using deposits from $mq-1$ lenders. However, on set B_b some projects default and the bank incurs monitoring cost c_b. On B_d the bank defaults and the $mq-1$ depositors incur monitoring cost c_d.

When banking is risky and default occurs in state $s = l$, the bank's incentive constraint, which insures that it requests costly state verification of entrepreneurs in bankruptcy states, depends on:

(i) Bank assets: revenue from loan portfolio $\pi(\cdot) = q \sum_{i=1}^{m} min(L(y_i, \theta_i), \bar{L}_i)$

(ii) Bank liabilities: the bank owes depositors $D(\pi_b(L, \theta; s))$

(iii) Bank costs to monitor the y_i that default: $C = c_b N(s)^7$

A risky bank's ability to repay depositors (i.e., its liabilities) depends on its asset portfolio. Assume that the bank's total revenue is homogeneous. Then the *bank's incentive constraint* is

$$\sum_{s=l,h} p_s \left[\pi(L, \theta; s) - D(\pi_b(L, \theta; s)) - C(s) \right] = \frac{\bar{D}}{q} \tag{2}$$

Because the bank arises endogenously (i.e., investors delegate monitoring to one investor), the bank must earn the same expected return per project as the remaining investors, $\bar{D}/q$.[8]

The depositor's incentive constraint, which insures that depositors request costly state verification of the bank in bankruptcy states, is derived as follows. Depositors must monitor whenever $D(\pi_b(L, \theta; s))$ is less than $\bar{D}$, incurring cost $C_d = c_d M(s)(mq - 1)$, where $M(s)$ is a binary variable that equals one if the bank defaults and the $mq - 1$ depositors monitor and zero otherwise. Thus, the *depositor's incentive constraint* is given by

$$\sum_{s=l,h} p_s \left[D(\pi_b(L, \theta; s)) - C_d(s) \right] = \frac{\bar{D}}{q}(mq - 1) \tag{3}$$

As the number of loans goes to infinity, the bank can eliminate idiosyncratic risk but not default risk. Thus, the income from its loan portfolio may not be sufficient to fully repay depositors in some states, and the depositors

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[7] $N(s)$ is the number of projects that default.

[8] See Williamson (1986) or Krasa and Villamil (1992) for proofs of the optimality of delegated monitoring relative to direct investment without and with risk, respectively.

will monitor the bank. We assume that the bank defaults in state $s = l$. The risky bank's expected return function, which must be non-negative, is

$$\sum_{s=l,h} p_s \left[\int_{B_b} (L(\cdot) - \frac{c_b}{q}) dG(\cdot) + \int_{B_b'} \bar{L} dG(\cdot) - D(\pi_b(L, \theta; s)) \right] \qquad (4)$$

2.4   Comparison of Riskless vs. Risky Banking

In Section 2.2, (1) established that

$$\Pi(\cdot) = \bar{D}$$

In the Appendix we show that because a risky bank sometimes defaults, depositors' expected monitoring costs raise the effective reservation return to

$$\Pi(\cdot) = \bar{D} + \rho$$

The term $\rho = p_l q c_d$ reflects the cost of default. This risk premium depends on the size of depositor monitoring cost c_d, project scale q, and the probability that the low state will occur, p_l.

The bank's expected return function $\Pi(L(y, \theta), \theta; s)$, given by (16) in the Appendix, has two important properties. The proofs are in the Appendix.

Proposition 1: Assume $c_b g(0, \theta) < q$ and $\frac{c_b}{q} g_x(x, \theta) + g(x, \theta) > 0$.[9]
(a) $\Pi(L(y, \theta), \theta; s)$ is concave in L, given θ.
(b) $\Pi(L(y, \theta), \theta; s)$ is decreasing in θ, for $\bar{L} = \bar{L}^*$ and given $\bar{D}$.

Property (a) is Williamson's credit rationing result for a fixed portfolio quality level, θ. Williamson (1986) showed that in the costly state verification model with no risk of bank default, credit rationing by loan quantity can occur because the bank's expected return function is concave. Concavity follows from the fact that an increase in the loan rate has two effects on $\Pi(\cdot)$: Revenue increases as $\bar{L}$ increases, but expected monitoring costs also increase. The second effect occurs because raising $\bar{L}$ raises the probability that bankruptcy will occur. The second effect may dominate the first for sufficiently high loan rates. Concavity implies that there is an optimal loan value $\bar{L}^*$. When credit rationing by loan quantity occurs, some borrowers

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[9] These assumptions are standard. For example, see Boyd and Smith (1997).

are fully funded while other observationally identical borrowers are not.[10] A rationed entrepreneur will not get additional credit even if the agent is willing to pay $\bar{L} > \bar{L}^*$ because this would reduce bank profit. Property (b) states that the expected return function is decreasing in θ.

Insert Figure 1

Consider Figure 1, which shows the bank's expected return funtion $\Pi(\cdot)$. The interest rate on loans, $\bar{L}$, is measured on the x-axis and the bank's expected revenue is measured on the y axis. Quality is measured as a mean preserving change in the variance of the distribution of loan returns, where an increase in θ decreases the "quality" of loan applicants. The Figure shows that $\Pi(\cdot)$ is concave and reaches a maximum at $\bar{L}*$. Further as θ increases, with $\theta^B > \theta^A$, the expected return function $\Pi(\cdot)$) shifts down.

Proposition 2 establishes that there is an optimal quality cutoff level, θ^A.

Proposition 2: Assume $c_b g(0, \theta) < q$ and $\frac{c_b}{q} g_x(x, \theta) + g(x, \theta) > 0$. Then there is an optimal quality threshold level θ^A such that

- If $\theta_i \cdot \ \theta^A$: the entrepreneur is financed

- If $\theta_i > \theta^A$: the entrepreneur is rationed

Proposition 2 indicates that banks sort loan applicants based on quality using critical value θ^A. All θ above the threshold (i.e., high variance or low quality applicants) are rationed. Threshold quality level θ^A is an additional form of credit rationing that to our knowledge has not been considered in the economic literature previously.

In the remainder of the paper we first analyze the effect of default risk on these two forms of credit rationing, loan quantity and loan quality. Second, we analyze the qualitative effect of other structural changes in the economic environment on the bank's problem. Finally, we consider a parametric example to illustrate the quantitative significance of the results.

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[10]Credit rationing by loan quantity operates as follows. Suppose that loan demand is $(1 - \alpha)q$ and loan supply is α with $w = 1$. If at $\bar{L}^*$ there is excess demand in the loan market, then $(1-\alpha)q > \alpha$. In order to ration this excess demand α borrowers are randomly selected from the $(1 - \alpha)q$ potential borrowers. Those α borrowers are fully funded at q units each. The other observationally identical borrowers gets zero.

3 The Loan Market

Equilibrium in the loan market results from the equality of demand by borrowers and supply by lenders. Each borrower demands q units of credit to invest in the fixed scale project. Total loan demand is thus $(1 - \alpha)q$. Propositions 1 and 2 show that credit rationing can occur for two distinct reasons, thus we model the loan market as follows. Let $u \cdot 1$ be the fraction of entrepreneurs that receive credit for a given quality level θ^A.

(i) Proposition 1 shows that credit rationing by loan quantity, $u < 1$, is due to the concavity of the bank's expected profit function.

(ii) Proposition 2 shows that banks also ration credit by adjusting quality cutoff θ^A. Since $H(\theta^A)$ is the distribution of project quality, by varying θ^A the bank adjusts portfolio quality to clear the market.

The demand for bank loans by entrepreneurs is $(1 - \alpha)quH(\theta)$. The total supply of funds is αw. Because the α lenders have an outside investment opportunity with return x, they will divert funds away from banks if x exceeds the deposit interest rate $\bar{D}$. Then the supply of funds by depositors to banks is $\alpha w H(\bar{D})$. Assume that the economy has excess credit demand, $(1 - \alpha)q > \alpha w$. Then the loan market equilibrium is given by

$$(1 - \alpha)uqH(\theta) \geq \alpha w H(\bar{D})$$

Finally, banks must satisfy a reserve requirement $\bar{\delta}$ that constrains the amount the bank can lend. A reserve requirement has two effects

(i) Banks face an additional constraint, $\delta(\theta) \geq \bar{\delta}$, where $\delta(\theta) = (1 - H(\theta)) - k$. This specification of $\delta(\theta)$ captures the idea that banks choose the optimal θ^A given the reserve requirement. Constant k takes into account that banks choose portfolio quality even if $\bar{\delta}$ is zero.

(ii) Banks must keep a proportion of deposits on hand to satisfy the reserve requirement. This further reduces the supply of credit to

$$(1 - \alpha)uqH(\theta) \geq \alpha w H(\bar{D})(1 - \bar{\delta})$$

4　Credit Rationing

Assume perfect competition. We now state the bank's problem, and analyze it with and without default risk. Let $\rho = p_l q c_d$ denote default premium. When $\rho = 0$ there is no default risk and when $\rho > 0$ default risk exists.

The Bank's Problem. Choose $\bar{L}$, $\bar{D}$, and θ to maximize

$$\Pi(L, \theta) \geq \bar{D} + \rho \tag{5}$$

Subject to:

$$(1 - \alpha) u q H(\theta) \cdot \quad \alpha w H(\bar{D})(1 - \bar{\delta}) \tag{6}$$

$$(1 - H(\theta)) - k \geq \bar{\delta} \tag{7}$$

The bank chooses loan and deposit rates and a portfolio quality threshold to maximize its expected return.[11] Depositor incentive compatibility, (5), requires a risky bank's expected return to be at least as great as the risk augmented depositor reservation level, $\bar{D} + \rho$. The bank is also constrained by loan market equilibrium condition (6), which acts as a feasibility constraint, and the reserve requirement (7).

Our goals are two-fold. First, we analyze the factors that affect the two types of credit rationing. We pay particular attention to portfolio quality selection (i.e., the bank's choice of threshold θ^A) since this is a core operational function of a bank, is intrinsically related to default risk, has not been studied previously. Portfolio quality selection is irrelevant for a riskless bank, but it is crucial for "risky banks." Second, we will show both analytically and quantitatively that default risk interacts with both types of credit rationing. This can cause large interest rate spreads, "credit crunches," and a banking equilibrium to fail.

To solve the bank's problem, consider two cases described by Figure 2:

- Case 1. loan quantity rationing, $u < 1$: Not all borrowers of a given θ who request a loan receive one. $\bar{L}$ is fixed at the maximum income level for a given θ, $\bar{L}^*(\theta)$.[12] Banks choose $\bar{D}$, θ and indirectly u.

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[11] Proposition 1 shows that (5) is $\Pi(L, \theta) = [\bar{L} - \frac{c_b}{q} G(L, \theta; s) - \int_0^L dG(y, \theta; s)]$.

[12] The condition for this type of credit rationing is given in Proposition 3 below.

- Case 2. portfolio quality rationing, $u = 1$: Since $\bar{L}$ is fixed, banks maximize with respect to $\bar{L}$, $\bar{D}$ and θ.

To simplify, assume that the distribution of returns on the outside alternative is uniform, thus $I(x) = \frac{x}{\bar{x}}$, where $x = \bar{D}$ in a competitive market.

Insert Figure 2

Figure 2 illustrates the two rationing cases in the credit market and in the bank's expected return function. The top graph shows credit supply (LS) and two different cases for credit demand$(LD_1$ and $LD_2)$. The bottom graph is the bank's expected return function.

- In case 1, there is excess demand in the credit market (loan demand LD_1 and loan supply LS do not intersect and ED is the amount of excess demand in the top Figure). If u were to equal 1, the bank could not obtain sufficient expected return from its loan portfolio to pay depositors the market clearing rate $\bar{D} + \rho'$. Thus, the bottom graph shows that the bank chooses θ^A and $u < 1$ to ration credit when $\bar{L} = \bar{L}^*$. The bank takes $\bar{D} + \rho'$ as given in a competitive market.

- In case 2, loan supply equals loan demand $(LS = LD_2)$ and $u = 1$. For a given $\bar{D} + \rho$, the bank chooses θ^A to ration credit and $\bar{L} < \bar{L}^*$.

This graph provides the intuition for the results. We will derive the results formally in the next two sections.

Before beginning the formal analysis, return to Figure 1 to see that default risk also has implications for quality selection. When The expected return function evaluated at the optimal loan rate $\bar{L}^*(\theta)$ is: [13]

$$\Pi(\bar{L}^*(\theta), \theta) = \psi(\theta)$$

Proposition 2 shows that for a given $\bar{D} + \rho$, there is an optimal θ^A (see A and $\Pi(\theta^A)$ in Figure 1). The Figure shows that depositors at a risky bank must be compensated for the expected monitoring costs they bear by higher bank expected return. For the same loan rate $\bar{L}^*$, bank expected return must be higher at a risky bank $(\Pi(\theta^A))$ than at a riskless bank $(\Pi(\theta^B))$. The risky

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[13]The interest rate on loans is endogenous, and depends on the distribution of project returns. It can decrease, increase or remain constant when θ changes. In Figure 1 we assume that it remains constant.

bank chooses a tighter quality cutoff θ^A, which implies the higher $\Pi(\theta^A)$. Ceteris paribus, default risk increases quality rationing: Entrepreneurs with qualities between $\theta^A > \theta > \theta^B$ are rationed now.

4.1 Case 1. Rationing by Loan Quantity: $u < 1$

When rationing by loan quantity occurs, the fraction of entrepreneurs of a given quality that receive loans is less than one (i.e., $u < 1$) and $\bar{L}$ is fixed at the maximum income level for a given θ, $\bar{L} = \bar{L}^*(\theta)$. Banks choose $\bar{D}$, θ and indirectly u (i.e., the fraction of loan requests to grant). Consider the equations in the bank's problem, (5), (6) and (7).

Bank expected return is $\Pi(\bar{L}^*(\theta), \theta) = \psi(\theta)$. From (5), for a riskless bank $\Pi(\bar{L}^*(\theta), \theta) = \bar{D}$, and for a risky bank $\Pi(\bar{L}^*(\theta), \theta) = \bar{D} + \rho$. Since $u < 1$, loan market equilibrium condition (6) is

$$u = \frac{\alpha w(1-\bar{\delta})\bar{D}}{(1-\alpha)qG(\theta^A)\bar{x}} < 1 \qquad (8)$$

There is no change in the reserve requirement (7).

A riskless bank's expected revenue equals the deposit rate, $\psi(\theta) = \bar{D}$. Solving (8) for $\bar{D}$ and imposing $\psi(\theta) = \bar{D}$ gives

$$\psi(\theta) < \frac{(1-\alpha)qG(\theta^A)\bar{x}}{\alpha w(1-\bar{\delta})} \qquad (9)$$

This condition means that the bank cannot obtain sufficient expected return from its loan portfolio to pay depositors the market clearing deposit rate. As a consequence, rationing by loan quantity arises (see Case 1 in Figure 2); the bank can not finance all applicants. This situation arises whenever the return from borrowers is not sufficient to cover the deposit rate. Williamson showed that this type of rationing can occur for riskless banks, but we will now show that default risk deepens the problem.

When there is default risk and $u < 1$, $\bar{L}$ is fixed at the maximum income level for a given θ. Comparative static results in Claim 4.11 show the following. First, default risk does not affect the quality cutoff, θ^A. Second, quantity rationing increases as default risk increases.[14] Third, the deposit rate goes down by the same amount as the increase in the default premium,

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[14]A numerical example in Section 6 shows that an increase in ρ leads to a more than 13 fold decrease in u.

due to a decrease in u (i.e., an increase in credit rationing by loan quantity). Thus, the deposit rate and u adjust to restore an equilibrium.

Claim 4.11. When banks ration credit by loan quantity (i.e., $\bar{L}(\theta) = \bar{L}^*(\theta)$, (9) is satisfied and $\rho > 0$), then as the default premium increases
  (i) $\frac{d\theta}{d\rho} = 0$: There is no effect on portfolio quality.
  (ii) $\frac{du}{d\rho} = -\frac{\alpha w(1-\bar{\delta})}{\bar{x}(1-\alpha)qH(\theta)} < 0$: Quantity rationing increases (u declines).
  (iii) $\frac{d\bar{D}}{d\rho} = -1$: There is a one-for-one decrease in the deposit rate.

To understand the intuition for Claim 4.11, recall that $\bar{D} + \rho = \psi(\theta)$. Thus, if it were the case that $u = 1$, then (9) implies that for a risky bank

$$\psi(\theta) - \rho < \frac{(1-\alpha)qH(\theta)\bar{x}}{\alpha w(1-\bar{\delta})} \tag{10}$$

This equation means that if the risky bank granted all loan requests at the given quality level ($u = 1$), it would not obtain an expected return sufficient to pay the market clearing rate on deposits. Thus the bank cannot finance all applicants, because default risk causes the expected return on the bank's portfolio to decrease. Equation (10) shows that this credit rationing by loan size is more likely to occur as default risk increases.

We summarize this result formally in Proposition 3.

Proposition 3. When $\bar{L} = \bar{L}^*$, (10) is satisfied, and $\rho > 0$, then credit rationing by loan size occurs, i.e., $u < 1$.

Proposition 3 establishes that in order for quantity rationing to occur, banks are already at the maximum expected revenue and $\bar{L}(\theta) = \bar{L}^*(\theta)$. Therefore an increase in default risk has no effect on the loan rate.[15] As the default premium (ρ) increases, banks become less profitable and attract less deposits. The outside opportunity becomes more attractive and banks lose their deposit base. As a result of this dis-intermediation, Claim 4.11 shows that quantity rationing increases since less funding is available for borrowers

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[15] An increase in ρ can trigger the transition from only quality to quality and quantity rationing. See Table 7 for a numerical example which shows that for a riskless bank $u = 1$, but when default risk ρ increases then $u < 1$ and rationing by loan quantity occurs.

and the interest rate spread increases. The increase in risk has no effect on the quality cutoff in this case.[16]

Williamson showed that credit rationing by loan quantity can arise even when banks are not risky (i.e., $\rho = 0$). Proposition 3 indicates that default risks deepens this type of rationing because (10) is more likely to be satisfied when $\rho > 0$. To illustrate this, we perform comparative statics on (10). Assume that equation (10) holds as an equality. Then:

Claim 4.12. As w, α, $\bar{x}$, $\bar{\delta}$, or q increase
(i) $\frac{d(\psi(\theta)-\rho)}{dw} = -\frac{(1-\alpha)qH(\theta)\bar{x}}{\alpha w^2(1-\delta)} < 0$ and $\frac{d(\psi(\theta)-\rho)}{d\alpha} = -\frac{qH(\theta)\bar{x}}{\alpha^2 w(1-\delta)^2} < 0$
(ii) $\frac{d(\psi(\theta)-\rho)}{d\bar{x}} = \frac{(1-\alpha)qH(\theta)}{\alpha w(1-\delta)} > 0$; $\frac{d(\psi(\theta)-\rho)}{d\delta} = \frac{(1-\alpha)qH(\theta)\bar{x}}{\alpha w(1-\delta)^2} > 0$; $\frac{d(\psi(\theta)-\rho)}{dq} = \frac{(1-\alpha)H(\theta)\bar{x}}{\alpha w(1-\delta)} > 0$

Part (i) indicates that credit rationing by loan quality is less likely if there is an increase in the supply of funds, due to either an increase in wages or an increase in lenders. Part (ii) indicates that credit rationing is more likely in two cases. First, if there is a decrease in the supply of funds, due to an increase in the return on the outside opportunity or reserve requirement. Second, if there is an increase in the demand for funds due to an increase in the minimum project scale.

4.2 Case 2. Rationing by Loan Quality: $u = 1$

Assume that there is no credit rationing by loan quantity, so $u = 1$. Banks choose $\bar{L}$, $\bar{D}$ and θ.[17] Given the reserve requirement, banks select θ so that

$$\delta(\theta^A, 1) = \bar{\delta}$$

Then solving equations (6) and (7) with $u = 1$, we get $\bar{L}_A$ and $\bar{D}$ such that

$$(1 - \alpha)qG(\theta^A) = \alpha w H(\bar{D})(1 - \delta(\theta^A, 1))$$

First consider the case for a riskless bank. The deposit rate equals

$$\bar{D} = \frac{(1 - \alpha)qG(\theta^A)\bar{x}}{\alpha w(1 - \bar{\delta})} \tag{11}$$

---

[16]Note that (7) fixes the cutoff θ^A. The bank attains the maximum expected revenue, but the supply of loanable funds is insufficient to finance all loan applicants (i.e., clear (6)). See Guzman (2001), footnote 6.

[17]Given the assumption that $H(\bar{D})$ has a uniform distribution and that $H(\bar{D}) < 1$, then $(1 - \alpha)q > \alpha w H(\bar{D})$ (there is excess demand in the loan market for projects).

The interest rate on loans is the $\bar{L}_A$ that solves

$$\pi(\bar{L}_A, \theta^A) = \frac{(1-\alpha)qG(\theta^A)\bar{x}}{\alpha w(1-\bar{\delta})}$$

This interest rate is lower than $\bar{L}^* = \eta(\theta^A)$ since there is no rationing by loan quantity $(u = 1)$.

Now consider the case for a risky bank. Equation (5) holds and depositors must be compensated for default risk $\rho = p_l q c_d$. The bank's expected revenue function is now given by

$$\Pi(\bar{L}, \theta) \geq \bar{D} + p_l q c_d$$

To make the results comparable, we assume that the deposit rate is the same as in the case with no default risk. Banks again maximize expected revenue subject to equations (6) and (7), selecting $\bar{D}$, $\bar{L}$ and θ and taking into account the default premium. From (7), $\theta = \theta^A$ and $\delta(\theta^A, 1) = \bar{\delta}$. Equation (10) holds with equality, and $\bar{D}$ is the same as in the case with no default risk. Since the bank must now compensate depositors for the expected recovery cost in case of bankruptcy, the interest rate on loans is higher than when there is no default risk. Then $\bar{L}_L > \bar{L}_A$. But this interest rate is still lower than $\bar{L}^* = \eta(\theta^A)$, since there is no rationing by loan quantity.[18]

Total differentiation of (5), (6) and (7) allows us to establish the following comparative static results about the effect of default premium on interest rates and the quality cutoff:

Claim 4.2. When banks ration credit by loan quality, then as the default premium increases
  (i) $\frac{d\bar{L}}{d\rho} = \frac{1}{\pi_{\bar{L}}} > 0$: The loan rate increases.
  (ii) $\frac{d\theta}{d\rho} = 0$: There is no effect on portfolio quality.
  (iii) $\frac{d\bar{D}}{d\rho} = 0$: There is no effect on the deposit rate.

Under quality rationing an increase in default premium generates an increase in the loan rate. However, there is no effect on the quality cutoff or on the deposit rate. Only the spread is affected. Since banks have not reached

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[18]As in Guzman (2000), we divide the analysis of the bank's problem into two cases: $u < 1$ and $u = 1$. As Proposition 3 indicates, for $u < 1$ to hold it must be the case that the bank is already at the maximum expected return level with $\bar{L} = \bar{L}^*(\theta)$. Otherwise, $u = 1$ and there is no rationing by loan quantity.

the maximum expected return, an increase in the loan rate can still increase expected return. Therefore banks transfer the increase in the default premium to borrowers by increasing the loan rate. It is not necessary for banks to tighten quality. See Section 6 for a numerical treatment of the case.

In summary, Figure 3 shows that a default premium generates an increase in the loan rate from L_0 to L_1 due to the additional risk that banks must compensate depositors for (i.e., $\bar{D}+\rho > \bar{D}$). In order to do this, banks charge a higher rate on loans. This increases the observed spread between deposit and loan rates, a fact observed in many developing countries. Note that in Figure 3, $L_0 < L_1 < \bar{L}^*$. Since $L < \bar{L}^*$, the conditions of Proposition 3 are not satisfied and rationing by loan quantity does not occur.

Insert Figure 3

5 Comparative Statics

We now analyze the comparative static properties of the model. ¿From an initial equilibrium, totally differentiate equations (5), (6) and (7) with respect to the endogenous policy variables, $\bar{L}$, $\bar{D}$, θ, and u, given changes in exogenous variables $\bar{\delta}$, q, $\bar{x}$, c_b and c_d. This allows us to analyze how rationing by portfolio quality (θ), quantity rationing (u), and loan and deposit rate spreads ($\bar{L} - \bar{D}$) react to changes in regulation $\bar{\delta}$, project scale q, the return on alternative assets $\bar{x}$, and monitoring costs c_b and c_d.

5.1 Credit Rationing by Loan Quantity: $u < 1$

In this case when the conditions of Proposition 3 are satisfied, banks choose $u < 1$ to ration credit. Recall that $\bar{L} = \bar{L}^*(\theta)$ is fixed at the optimal level. In matrix notation, totally differentiating the system yields

$$
\begin{pmatrix}
\pi_\theta & 0 & -1 \\
(1-\alpha)qh(\theta)u & (1-\alpha)qH(\theta) & -\frac{\alpha w(1-\bar{\delta})}{\bar{x}} \\
-h(\theta) & 0 & 0
\end{pmatrix}
\begin{pmatrix}
d\theta \\
du \\
d\bar{D}
\end{pmatrix}
$$

$$= \begin{pmatrix} 0 & 1 & \frac{\rho}{q} & 0 & -\pi_{c_b} & p_l q \\ -\frac{\alpha w \bar{D}}{\bar{x}} & 0 & -(1-\alpha)H(\theta)u & -\frac{\alpha w \bar{D}(1-\bar{\delta})}{\bar{x}^2} & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} d\bar{\delta} \\ d\rho \\ dq \\ d\bar{x} \\ dc_b \\ dc_d \end{pmatrix}$$

The determinant is $\Delta = -(1-\alpha)qH(\theta)h(\theta) < 0$.

We summarize the comparative static exercises as a series of Claims.

Claim 5.11. When banks ration credit by loan quantity, and $\bar{L}(\theta) = \bar{L}^*(\theta)$, then as the reserve requirement increases

(i) $\frac{d\theta}{d\delta} = -\frac{1}{h(\theta)} < 0$: Portfolio quality increases.

(ii) $\frac{du}{d\delta} = \frac{-((1-\alpha)qu\bar{x} - \alpha w \bar{D})h(\theta) + \alpha w \bar{D}\pi_\theta}{\bar{x}\Delta} > 0$: Quantity rationing decreases.

(iii) $\frac{d\bar{D}}{d\delta} = -\frac{\pi_\theta}{h(\theta)} > 0$: The deposit rate increases.

An increase in the reserve requirement increases excess demand in the loan market. The bank reacts to this by tightening its loan quality standard. The increase in portfolio quality allows banks to pay a higher deposit rate since the bank's expected return increases as result of its better loan portfolio. This permits a decrease in rationing by loan quantity. The final effect on the deposit rate and quantity rationing depends on the marginal effect of quality on expected revenue. Banks pay a higher rate on deposits due to the increase in expected return that results from a better portfolio.

Claim 5.12. When banks ration credit by loan quantity, and $\bar{L}(\theta) = \bar{L}^*(\theta)$, then as the project scale increases

(i) $\frac{d\theta}{dq} = 0$: There is no effect on the quality threshold.

(ii) $\frac{du}{dq} = -[\frac{u}{q} + \frac{\alpha w(1-\bar{\delta})}{\bar{x}(1-\alpha)qH(\theta)}] < 0$: Quantity rationing increases.

(iii) $\frac{d\bar{D}}{dq} = -\frac{\rho}{q} < 0$: The deposit rate decreases.

Changes in project scale (q) generate an effect that is qualitatively similar to an increase in the default premium (cf., Claim 4.11). There is no effect on the quality cutoff. The increase in loan demand increases the default premium since it is more costly for depositors to monitor the bank. Therefore, quantity rationing increases and the deposit rate is reduced. Compared with an increase in the default premium, the effect of an increase in the project

scale is lower on the deposit rate but larger on credit rationing by loan quantity.

Claim 5.13. When banks ration credit by loan quantity, and $\bar{L}(\theta) = \bar{L}^*(\theta)$, then as the return on the outside asset increases

    (i) $\frac{d\theta}{d\bar{x}} = 0$: There is no effect on the quality threshold.

    (ii) $\frac{du}{d\bar{x}} = -\frac{\alpha w \bar{D}(1-\bar{\delta})}{\bar{x}(1-\alpha)qH(\theta)} < 0$: Quantity rationing increases.

    (iii) $\frac{d\bar{D}}{d\bar{x}} = 0$: There is no effect on the deposit rate.

As the return on the outside investment opportunity ($\bar{x}$) increases, banks react by rationing loan quantity. An increase in the outside opportunity generates a decrease in banks' deposit base, a disintermediation result. Therefore, quantity rationing increases and portfolio quality stays the same.

Claim 5.14. When banks ration credit by loan quantity, and $\bar{L}(\theta) = \bar{L}^*(\theta)$, then as the bank verification cost increases

    (i) $\frac{d\theta}{dc_b} = 0$: There is no effect on the quality threshold.

    (ii) $\frac{du}{dc_b} = \pi_{c_b}[\frac{\alpha w(1-\bar{\delta})}{\bar{x}(1-\alpha)qH(\theta)}] < 0$: Quantity rationing increases.

    (iii) $\frac{d\bar{D}}{dc_b} = \pi_{c_b} < 0$: The deposit rate decreases.

Claim 5.15. When banks ration credit by loan quantity, and $\bar{L}(\theta) = \bar{L}^*(\theta)$, then as the depositor verification cost increases

    (i) $\frac{d\theta}{dc_d} = 0$: There is no effect on the quality threshold.

    (ii) $\frac{du}{dc_d} = -[\frac{\alpha w(1-\bar{\delta})}{\bar{x}(1-\alpha)qH(\theta)}] < 0$: Quantity rationing increases.

    (iii) $\frac{d\bar{D}}{dc_d} = -p_l q < 0$: The deposit rate decreases.

The results are qualitatively similar for both monitoring costs. Monitoring costs have no effect on the quality threshold. Increases in verification costs exacerbate quantity rationing and drive the deposit rate down because when ρ increases both u and $\bar{D}$ adjust to clear the loan market.

The comparative static results for rationing by loan quantity reported in Claims 4.11 and 5.11 – 5.15 are summarized in the following Table.

Table 1: Quantity Rationing

Quantity Rationing	ρ	$\bar{\delta}$	q	$\bar{x}$	c_b	c_d
u	$(-)$	$(+)$	$(-)$	$(-)$	$(-)$	$(-)$
θ	(0)	$(-)$	(0)	(0)	(0)	(0)
$\bar{D}$	$(-)$	$(+)$	$(-)$	(0)	$(-)$	$(-)$

5.2 Quality Rationing: $u = 1$

Suppose there is quality rationing only (i.e., no quantity rationing $u = 1$). Then totally differentiating system in matrix notation yields

$$
\begin{pmatrix}
\pi_{\bar{L}} & \pi_\theta & -1 \\
0 & (1-\alpha)qh(\theta) & -\frac{\alpha w(1-\bar{\delta})}{\bar{x}} \\
0 & -h(\theta) & 0
\end{pmatrix}
\begin{pmatrix}
d\bar{L} \\
d\theta \\
d\bar{D}
\end{pmatrix}
$$

$$
=
\begin{pmatrix}
0 & 1 & \frac{\varrho}{q} & 0 & -\pi_{c_b} & p_l q \\
-\frac{\alpha w \bar{D}}{\bar{x}} & 0 & -(1-\alpha)H(\theta) & -\frac{\alpha w \bar{D}(1-\bar{\delta})}{\bar{x}} & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
d\bar{\delta} \\
d\rho \\
dq \\
d\bar{x} \\
dc_b \\
dc_d
\end{pmatrix}
$$

The determinant is given by $\Delta = \pi_{\bar{L}}\left(-\frac{h(\theta)\alpha w(1-\bar{\delta})}{\bar{x}}\right) < 0$ since $\pi_{\bar{L}} > 0$.

We obtain the following comparative static results.

Claim 5.21. When banks ration credit by loan quality, then as the reserve requirement increases
  (i) $\frac{d\bar{L}}{d\bar{\delta}} = -\frac{\bar{x}(1-\alpha)q - \alpha w \bar{D}}{\pi_{\bar{L}}\alpha w(1-\bar{\delta})} + \frac{\pi_\theta}{\pi_{\bar{L}}h(\theta)} < 0$: The loan rate decreases.
  (ii) $\frac{d\theta}{d\bar{\delta}} = -h(\theta) < 0$: Portfolio quality increases.
  (iii) $\frac{d\bar{D}}{d\bar{\delta}} = -\frac{\bar{x}(1-\alpha)q - \alpha w \bar{D}}{\alpha w(1-\bar{\delta})} < 0$: The deposit rate decreases.

A change in the reserve requirement leads to a similar change in the quality cutoff. This "flight to quality" allows banks to charge a lower interest rate on loans. On the other hand, an increase in the reserve requirement generates an additional cost to banks that lowers the interest rate on deposits.

Claim 5.22. When banks ration credit by loan quality, then as the project scale increases
  (i) $\frac{d\bar{L}}{dq} = \frac{\bar{x}(1-\alpha)H(\theta)}{\pi_{\bar{L}}\alpha w(1-\bar{\delta})} + \frac{\varrho}{q\pi_{\bar{L}}} > 0$: The loan rate increases.
  (ii) $\frac{d\theta}{dq} = 0$: There is no effect on portfolio quality.
  (iii) $\frac{d\bar{D}}{dq} = -\frac{(1-\alpha)H(\theta)\bar{x}}{(1-\delta)\alpha w} > 0$: The deposit rate increases.

A change in project scale generates an increase in loan demand, therefore the loan rate increases. A larger project scale increases the default premium since it increases the depositors' verification cost if default occurs. The loan

rate increases more than the increase in the deposit rate in order to compensate for the extra cost. The spread increases due to the effect of increased credit demand and increased risk. Note that $\frac{d\bar{L}}{dq} = \frac{1}{\pi_{\bar{L}}}(\frac{d\bar{D}}{dq} + \frac{\rho}{q})$, meaning that spread increases as the project scale increases, given the effect of q on the default premium.

Claim 5.23. When banks ration credit by loan quality, then as the return on the outside investment opportunity increases
  (i) $\frac{d\bar{L}}{d\bar{x}} = \frac{\bar{D}}{\bar{x}\pi_{\bar{L}}} > 0$: The loan rate increases.
  (ii) $\frac{d\theta}{d\bar{x}} = 0$: There is no effect on portfolio quality.
  (iii) $\frac{d\bar{D}}{d\bar{x}} = \frac{\bar{D}}{\bar{x}} > 0$: The deposit rate increases.

As the maximum return on the outside investment opportunity increases, banks increase the deposit rate. Quality does not change, and the loan rate increases to compensate for the extra cost that is necessary to retain funds.

Claim 5.24. For $\bar{L} \cdot \ \bar{L}^*(\theta)$, when banks ration credit by loan quality, then as the bank verification cost increases
  (i) $\frac{d\bar{L}}{dc_b} = -\frac{\pi_{c_b}}{\pi_{\bar{L}}} > 0$: The loan rate increases.
  (ii) $\frac{d\theta}{dc_b} = 0$: There is no effect on portfolio quality.
  (iii) $\frac{d\bar{D}}{dc_b} = 0$: There is no effect on the deposit rate.

Claim 5.25. For $\bar{L} \cdot \ \bar{L}^*(\theta)$, when banks ration credit by loan quality, then as the depositors' verification cost increases
  (i) $\frac{d\bar{L}}{dc_d} = \frac{p_1 q}{\pi_{\bar{L}}} > 0$: The loan rate increases.
  (ii) $\frac{d\theta}{dc_d} = 0$: There is no effect on portfolio quality.
  (iii) $\frac{d\bar{D}}{dc_d} = 0$: There is no effect on the deposit rate.

The results are qualitatively similar for both the bank's and the depositors' monitoring costs. We discuss this case in detail in Section 6.

The comparative static results are summarized in the Table below.

Table 2: Quality Rationing

Quality Rationing	ρ	$\bar{\delta}$	q	$\bar{x}$	dc_b	dc_d
$\bar{L}$	(+)	(−)	(+)	(+)	(+)	(+)
θ	(0)	(−)	(0)	(0)	(0)	(0)
$\bar{D}$	(0)	(−)	(+)	(+)	(0)	(0)

6 Numerical Example

We now solve a numerical example to illustrate credit rationing by loan quantity, credit rationing by loan quality, and the effect of the key parameters on the loan market. We assume that $H(\bar{D}) = \frac{\bar{D}}{\bar{x}}$ is uniformly distributed and that $H(\theta) = \theta$ is uniformly distributed between 0 and 1. Then the bank's problem is

$$\max \Pi(\bar{L}, \theta) \geq \bar{D} + \rho$$

$$\text{subject to:} \quad (1 - \alpha)qu\theta \cdot \quad \alpha w \frac{\bar{D}}{\bar{x}}(1 - \bar{\delta})$$
$$(1 - \theta) - k \geq \bar{\delta}$$

The system is solved using the following baseline parametric values:

Table 3: Baseline Parameter Values

Parameter	Symbol	Value
Project scale	q	1.5
Wage	w	0.75
Depositors	α	0.66
Entrepreneurs	$1 - \alpha$	0.34
Bank Recovery Cost	c_b	0.6
Depositor Recovery Cost	c_d	0.6
Reserve Requirement	$\bar{\delta}$	0.04
Outside Investment	$\bar{x}$	0.078
Liquidity	k	0.04
Default Probability	p_l	0 or 0.001
Aggregate Risk	ρ	0 or 0.0009

Under these values it is easy to verify that there is excess demand in the loan market, $(1 - \alpha)q > \alpha w$.

We start by assuming that there is no default risk and solve the bank's problem. We then introduce default risk and compare the results. Default risk is introduced by assuming that $G(y, \theta; s)$ has a normal distribution with mean 0.25 and a variance that changes between 0.09 and 0.12.[19] In accordance with Proposition 1, the expected revenue function is decreasing in quality parameter θ and has an interior maximum for $\bar{L}^*(\theta)$. Banks face a distribution of entrepreneurs with returns that have the same mean but

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[19]In the Appendix we show that the results are similar for the log normal distribution.

different variances. An increase in the variance decreases the loan applicant quality, and the bank's expected return decreases. The proof of Proposition 1 shows that the expected return function can be written

$$\pi(\bar{L}, \theta) = \bar{L} - \frac{c_b}{q} G(\bar{L}, \theta) - \int_0^{\bar{L}} dG(y, \theta; s)$$

Recall Figure 1. It plots values for a normal distribution with mean 0.25, alternative levels of the variance, and the parameters in Table 3. The expected return function shifts down as θ increases because quality decreases. $\Pi(\bar{L}, \theta)$ reaches a maximum for $\bar{L}^*(\theta) = 0.14$, and $\bar{L}^*(\theta)$ is constant for alternative values of θ. For $\theta^A = 0.82$ the maximum expected revenue is 0.0909. For $\theta^B = 0.92$, expected revenue reaches a maximum of 0.0778 as predicted by the model. Banks charge a maximum loan rate of $\bar{L}^*(\theta) = 0.14$. Expected revenue decreases for $L > \bar{L}^*(\theta)$, thus credit rationing by loan quantity exists whenever the deposit rate reaches that maximum level. Applicants do not get loans even if they are willing to pay a higher rate. Since the mean expected value for projects is 0.25, investors, if funded, can expect to get at least a $0.25 - 0.14 = 0.11$ net profit.

6.1 Credit Rationing by Loan Quality: $u = 1$

Assume there is no default risk ($\rho = 0$) and $u = 1$. The bank solves the system of equations (5), (6) and (7). Equation (7) gives $\theta = 0.92$ and from (6) the solution for the deposit rate is $\bar{D} = 0.076$. Given these values, equation (5) implies that $\bar{L} = 0.123$ is the optimal loan rate.

Now suppose there is default risk, with a premium of $\rho = 0.0009$. Solving the system gives the same deposit rate and quality cutoff, but now the loan rate is given by $\bar{L} = 0.137$, a twelve percent increase. Recall that $\rho = p_l q c_d$, $q = 1.5$, and that the depositor recovery cost is $c_d = 0.6$ (the same as the bank's recovery cost). The Table below shows that a very small increase in default risk generates a large increase in the loan rate, even with no change in the quality cutoff. Recall from Claim 4.2 that $\frac{d\bar{L}}{d\rho} = \frac{1}{\pi_{\bar{L}}} > 0$. Since $\Pi(\bar{L}, \theta) \geq \bar{D} + \theta = 0.0776$ in this example, $\frac{1}{\pi_{\bar{L}}}$ is a multiplier that is significantly greater than one (numerical calculations reported in Table 8 in the Appendix show that $\frac{d\bar{L}}{d\rho} = 12.69$ for the example). Thus, relatively small increases in default risk are amplified by the multipliers.

Table 4: The Effect of Default Risk: $c_d = 0.6$, $q = 1.5$

Variable	No Risk	Risk
$\bar{L}$	0.123	0.137
$\bar{D}$	0.076	0.076
$Spread$	0.047	0.061
θ	0.92	0.92
u	1	1
p_l	−	0.001
ρ	0	0.0009

Now suppose that the verification costs change. We analyze the quantitative effect of a change in c_b on interest rates when $u = 1$ but portfolio quality can be varied. Suppose that the bank's cost to verify entrepreneurs, c_b, increases. Then the expected revenue function will decrease. Consider a 10 and 12.5 percent change from an initial situation where $c_b = 0.6$, to $c_b = 0.606$ and $c_b = .675$. For a given deposit rate $\bar{D} = 0.076$, the loan rate increases up to $\bar{L} = 0.125$ for $c_b = 0.606$. When $c_b = 0.675$ no loan rate can generate a benefit equal to $\bar{D} = 0.076$. Therefore, the bank must increase quality rationing. A change in quality that moves the baseline quality level to $\theta = 0.88$ generates enough revenue to keep $\bar{L} = 0.121$, but now credit rationing has increased by 4 percent from $\theta = 0.92$ to $\theta = 0.88$. See the Table below.

Table 5: Increases in Bank Monitoring Costs

Verification Cost	Increase	Interest Rate on Loans	Increase
$c_b = 0.6$	$baseline$	$\bar{L} = 0.123$	0
$c_b = 0.606$	10	$\bar{L} = 0.125$	1%
$c_b = 0.675$	12.5	$no\ equilibrium$	−

Consider now an increase in the depositors' verification cost. This change only affects the default premium parameter ρ. ¿From the initial situation of $c_d = 0.6$, increase c_d by 10 and 12.5 percent. Then for $q = 1.5$ and $p_l = 0.001$, when $c_d = 0.606$ the loan rate increases to $\bar{L} = 0.136$ as Claim 5.25 predicts. When $c_d = 0.675$, the loan rate increases to 0.141 and credit rationing by quantity emerges. No loan rate exists that can generate enough expected revenue to cover an expected augmented deposit return of 0.078. Therefore, the bank must tighten the quantity cutoff to compensate depositors. Indeed, the

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quantity cutoff is tightened to $u = 0.95$, a five percent decrease to generate a new equilibrium. The results are summarized in the following table.

Table 6: Increases in Depositor Monitoring Costs

Verification Cost	Increase	Interest Rate on Loans	Increase
$c_d = 0.6$	*baseline*	$\bar{L} = 0.123$	0
$c_d = 0.606$	10	$\bar{L} = 0.136$	9.7%
$c_d = 0.675$	12.5	$\bar{L} = 0.141$	14.6%

6.2 Quality and Quantity Rationing: $u < 1$

Credit rationing by both loan quality and quantity can arise if the default premium increases up to a value $\rho \geq 0.0009$ in this example[20]. This occurs because the bank's expected return for a given quality ($\theta = 0.92$) does not cover the opportunity cost of funds ($\bar{D} = 0.0778$).[21] When there is default risk and $p_l = 0.01$, the maximum expected return from loan rate $\bar{L}^* = 0.141$ does not generate enough revenue to cover the interest rate on deposits. Therefore, equilibrium is reached by increasing credit rationing by loan quantity. Hence, $u = 0.89$ and $\bar{D} = 0.0686$ are the equilibrium values now. This example illustrates the case where, when there is no default risk, rationing by quality only occurs. This follows from the fact that $\bar{L} < \bar{L}^*$, thus the conditions of Proposition 3 are not satisfied. As a consequence $u^* = 1$, and it is optimal to ration only by quality. However, the introduction of default risk drives $\bar{L}$ up to $\bar{L}^*$, and $u^* < 1$ is required to clear the loan market. Thus, default risk deepens credit rationing by causing a market with rationing by quality only to have rationing by both quality and quantity. The results are summarized below.

Table 7: Increase in Default Risk: $c_d = 0.6$, $q = 1.5$

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[20]From $\bar{D} = 0.076$ an increase of $\rho \geq 0.0009$ leads to quantity rationing

[21]Default risk is not a necessary or sufficient condition for credit rationing by loan quantity, as Williamson showed. This type of credit rationing can arise without default risk if the expected return for a given quality level is low enough. However, Table 7 shows that default risk exacerbates the problem, significantly widening interest rate spreads.

p_l	0	0.01	0.02	0.03	0.05	0.06	0.07
L	0.137	0.141	0.141	0.141	0.141	0.141	0.141
$\bar{D}$	0.0776	0.0686	0.0596	0.0506	0.0326	0.0236	0.0146
$Spread$	0.05	0.0724	0.0814	0.0904	0.1084	0.1174	0.1264
θ	0.92	0.92	0.92	0.92	0.92	0.92	0.92
u	1	0.89	0.78	0.66	0.42	0.3	0.2
ρ	0	0.009	0.018	0.027	0.045	0.054	0.063

6.3   Discussion of the Numerical Results

The parametric example illustrates the quantitative significance of the comparative static results. From an initial situation with credit rationing by loan quality only, the introduction of default risk has an important impact on the loan rate, the deposit rate, and therefore on the spread. Raising portfolio default risk from a probability of $p_l = 0$ to 0.01 raises the spread, from 0.05 to 0.0724 in Table 7. When p_l rises to 0.07, the spread increases to 0.1264. Banks charge the maximum loan rate and the deposit rate must decrease to support an equilibrium. Credit rationing involves both quality and quantity rationing. Loan approvals at cutoff quality level $\theta = 0.92$ fall from 100% ($u = 1$) to 20% as p_l rises. In a general equilibrium setting this result implies disintermediation.

Finally, verification costs also have a first order effect on interest rates and credit rationing.[22] As Claim 5.24 indicates, an increase in bank verification cost reduces the expected return function and therefore increases the loan rate. An increase of 12.5 percent in a bank's verification cost generates a non-equilibrium result for the quality cutoff in this example. On the other hand, an increase in depositors' verification costs increases the default premium. This decreases the bank's expected return and the bank increases the loan rate, as Claim 5.25 indicates. Indeed, an increase of 12.5 percent is not consistent with an equilibrium in this example.

7   Conclusions

This paper analyzes the effect of default risk on banks. We show that the size of the default premium, along with other parameters of the model, affects

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[22]The actual effect of increases in verification costs depend on the initial situation.

which of four possible equilibria occur. (i) Rationing by quality only occurs when the conditions of Proposition 3 are satisfied. Default risk is borne entirely by the loan rate. Neither the extent of credit rationing nor the deposit rate changes, but the loan-deposit rate spread increases due to the default premium. Claim 4.2 shows that the change in the spread is larger than the change in the default premium, a type of multiplier effect. (ii) Credit rationing by loan quantity arises when the bank's expected return for a given quality level is not high enough. This can happen if the default premium is high and/or the distribution of returns is unfavorable. Claim 4.11 shows that under this type of credit rationing, the interest rate on loans is fixed at the maximum return. An increase in the default premium is reflected in an increase in rationing by loan quantity and a decrease in the deposit rate. Therefore, disintermediation results. (iii) Both types of credit rationing can occur when the default premium is sufficiently high. The parametric example showed that small amounts of default risk can induce large interest rate spreads. (iv) For some parameter configurations (e.g., large shocks or monitoring costs), no banking equilibrium exists.

These results are consistent with the stylized facts observed in many developing countries: large interest rate spreads, costly banking crises, and reports of "credit crunches." The model suggests that these problems could be reduced in two ways: First, by reducing the level of default risk. This could be accomplished by better portfolio diversification or insurance opportunities. Second, by improving structural conditions. This could be accomplished by reducing monitoring costs and lowering returns on outside opportunities such as government bonds. However, we believe that it is unlikely that portfolio risk can be eliminated completely, thus the multipliers reported in Table 8 in the Appendix are interesting. They show that even small amounts of default risk can have big effects. This model seems especially appropriate for developing economies where default premium is often an important factor.

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9   Appendix

Williamson (1986) considered the following problem. Entrepreneurs propose loan contracts in a planning period that are analyzed by a lender. A contract is a pair $(L(y, \theta), B_b)$, where $L(y, \theta)$ is the loan repayment from an entrepreneur and B_b is the set of realizations where the entrepreneur is monitored. Given the feasible set of returns $[0, \bar{y}]$, costly state verification occurs on set B_b. No monitoring occurs on the complement $B_b' = [0, \bar{y}] - B_b$.

Simple debt is optimal because it minimizes expected monitoring costs. Given realization y, the entrepreneur repays a fixed amount $\bar{L}$ which is not contingent on y, if $y \in B_b'$. If $y \in B_b$, the entrepreneur transfers the entire y to the bank. Incentive compatibility requires a fixed loan repayment $\bar{L} > 0$ in states where no costly state verification occurs. This fixed amount is given by $\bar{L} \cdot \ argmin_{y \in B_b} y$. Williamson showed that the entrepreneur has the incentive to repay $\bar{L}$ when this is feasible because it economizes on deadweight monitoring costs. The entrepreneur keeps the difference, $y - \bar{L}$, as profit. For low realizations $y \in B_b$, the bank monitors, the entrepreneur gets zero, and the bank recovers $y - c_b$. Then $\bar{L}(y, \theta) \cdot \ y, \forall y \in B_b$. Given this condition, $B_b = [0, \bar{L})$, since for any $y \geq \bar{L}$ the entrepreneur prefers to pay $\bar{L}$.

First, Williamson showed that simple debt contract $\bar{L}$ is optimal relative to any other alternative debt contract A because it minimizes expected monitoring costs.[23] Consider two optimal contracts $\bar{L}$ and A in the Figure below. To give the borrower the same expected return, the face value of A must be strictly higher: $\bar{A} > \bar{L}$. Then clearly the expected monitoring costs are less for contract $\bar{L}$ as the Figure illustrates (i.e., the bankruptcy set where costly monitoring occurs $B_b^L \subset B_b^A$).

<div align="center">Insert Figure SDC</div>

Second, Williamson showed that banking (i.e., delegated monitoring) is optimal because it eliminates costly duplicative monitoring. If a bank contracts with m entrepreneurs, then loan demand is mq since q is the scale of each project. In order to satisfy this demand the bank needs $mq - 1$ lenders. The bank receives $L(y, \theta)$ from each entrepreneur and monitors if $L(y, \theta) < \bar{L}$, incurring cost c_b. The bank's total revenue is given by $\pi = q \sum_{j=1}^{m} min(L(y), \bar{L})$. The monitoring cost is given by $C = c_b N(s)$, where $N(s)$ is the number of entrepreneurs that default. As $m \to \infty$, the bank diversifies idiosyncratic risk. By the law of large numbers, the

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[23]In a simple debt contract the lender receives the entire realization when bankruptcy occurs. In an arbitrary debt contracts the borrower may retain some output.

expected revenue for a bank with a loan portfolio of size m is

$$p \lim_{m \to \infty} \frac{1}{mq} \pi = \int_{B_b} \mathrm{L}(y, \theta) dG(y, \theta; s) + \int_{B'_b} \bar{L} dG(y, \theta; s) = \pi(L, \theta)$$

Monitoring cost c_b has a binomial distribution with parameters m and $p = \int_{B_b} dG(y, \theta; s)$. As $m \to \infty$ it follows that[24]

$$p \lim_{m \to \infty} \frac{1}{mq} C = \frac{c_b}{q} \int_{B_b} dG(y, \theta; s)$$

Thus when the bank never fails, i.e., $s = \bar{s}$, the expected return function for a bank that contracts with an infinite number of entrepreneurs is given by (1). Williamson showed that the depositors' expected cost of monitoring the bank goes to zero as portfolio size goes to infinity because the portfolio earns $\bar{L}$ with probability one. The bank can then pay depositors reservation value $\bar{D}$ with certainty and the cost of delegation is nil.

Finally, Williamson showed that the bank's expected return function is concave in loan rate $\bar{L}$, thus it has an interior maximum at some $\bar{L}^*$. This leads to equilibrium credit rationing by loan quantity at $\bar{L}^*$. Even if a rationed borrower offered to pay a loan rate higher than $\bar{L}^*$, the bank would refuse because $\bar{L}^*$ maximizes the bank's expected return. The intuition for this credit rationing by loan quantity is that when failure is costly to the lender, an increase in the loan rate may decrease the bank's expected return because it raises the probability of borrower default.

9.1 Risky Banking

Equation (1), which must equal $\bar{D}$, and (4), which must be non-negative, specify the banks's expected profit requirements without and with default risk, respectively. The crucial difference is term $\sum_{s=l,h} p_s D(\pi_b(L, \theta; s))$, which is the expected payment from the bank to depositors. Because a risky bank will sometimes default, depositors expect to incur monitoring costs. These expected monitoring costs raise the "effective reservation return" that depositors must receive. We now consider the implications of this.

When there is default risk, as $m \to \infty$ depositor incentive compatibility constraint (3) can be written

$$\sum_{s=l,h} p_s \left[D(\pi_b(L, \theta; s)) + q c_d M(s) \right] \geq \bar{D} \tag{12}$$

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[24]Given that $N(s)$ is a binomial distribution, by the Law of Large Numbers it converges to $m.p$ and m cancels out. With no default risk the bank does not default in the limit.

This equation indicates that depositors must be compensated for expected monitoring costs. As a bank diversifies idiosyncratic risk it obtains $D(\pi_b(L, \theta; s))$ to compensate depositors. But with default risk, the deadweight monitoring cost must be accounted for. For some states $M(s) = 1$, and depositors incur monitoring cost qc_d. If the bank is not risky, then $M(s) = 0$ and (12) simplifies to $\sum_{s=l,h} p_s [D(\pi_b(L, \theta, s))] \geq \bar{D}$.

The key insight is that the bank cannot eliminate default risk, even with an infinite number of projects. Thus rewriting (4), depositors wish to insure that the *bank's* profit is high enough to enable them to recover their expected monitoring costs in bankruptcy states. That is,

$$\sum_{s=l,h} p_s [\int_{B_b} (L(y, \theta) - \frac{c_b}{q}) dG(\cdot) + \int_{B_b'} \bar{L} dG(\cdot)] \geq \sum_{s=l,h} p_s D(\pi_b(L, \theta, s)) \tag{13}$$

The left hand side is the bank's expected return with no default risk. When $M(s) = 0$, (13) reduces to (1). If the bank defaults in state $s = l$, $M(l) = 1$. Then $B_d = [L(y, \theta) : D(\pi_b(L, \theta; l)) < \bar{D}]$, and depositors monitor the bank with probability $p_l \geq 0$. Evaluating depositor incentive constraint (12) gives

$$\sum_{s=l,h} p_s D(\pi_b(L, \theta; s)) \geq \bar{D} + p_l q c_d \tag{14}$$

Given (13), the depositor's incentive constraint can be written

$$\sum_{s=l,h} p_s \left[\int_{B^b} (L(y, \theta) - \frac{c_b}{q}) dG(y, \theta; s) + \int_{B_b'} \bar{L} dG(y, \theta; s) \right] \geq \bar{D} + p_l q c_d \tag{15}$$

The risky bank's expected return must be sufficiently high to compensate a depositor for both the opportunity cost of the reservation project *and* the expected cost of recovering funds from the risky bank when it defaults. Thus, (15) can be written

$$\Pi(L(y, \theta; s), \theta) \geq \bar{D} + p_l q c_d = \psi(\theta) \tag{16}$$

9.2 Proofs

Proof of Proposition 1. Recall (1)

$$\Pi(L(y, \theta), \theta; s) = \int_{B_b} (L(y, \theta) - \frac{c_b}{q}) dG(y, \theta; s) + \int_{B_b'} \bar{L} dG(y, \theta; s)$$

Integrating by parts and solving gives

$$\Pi(L(y,\theta),\theta;s) = [\bar{L} - \frac{c_b}{q}G(\bar{L},\theta;s) - \int_0^{\bar{L}} dG(y,\theta;s)]$$

Let $\bar{L} = x$. Part (a) shows that, as in Williamson, $\pi(x,\theta)$ reaches a maximum for x, given θ. Clearly

$$\pi'(x,\theta;s) = 1 - \frac{c_b}{q}g(x,\theta;s) - G(x,\theta;s) = 0$$

Solving this equation gives $x^* = \eta(\theta)$; $\bar{L}^*(\theta) = \eta(\theta)$ is the optimal loan rate.

The assumption that $1 > \frac{c_b}{q}g(0,\theta;s)$, $\forall\theta$, assures that the profit function reaches an interior maximum for θ. Using the assumption

$$\lim_{x\to 0} \pi'(x,\theta;s) = 1 - \frac{c_b}{q}g(0,\theta;s) - G(0,\theta;s) \geq 0$$

$$\lim_{x\to\bar{y}} \pi'(x,\theta;s) = 1 - \frac{c_b}{q}g(\bar{y},\theta;s) - 1 \cdot \ 0$$

Further,

$$\pi''(x,\theta;s) = -\frac{c_b}{q}g'(x,\theta;s) - g(x,\theta;s)$$

Then $\pi(x,\theta;s)$ reaches a maximum for x as a function of θ, given the assumptions.

To prove part (b), recall that $\bar{L} = \bar{L}^*(\theta)$ is the loan rate that maximizes the expected profit function. Then $\bar{L} = \bar{L}^*(\theta)$ is the value such that

$$\pi'(\bar{L}^*(\theta),\theta;s) = 1 - \frac{c_b}{q}g(\bar{L}^*(\theta),\theta;s) - G(\bar{L}^*(\theta),\theta;s) = 0$$

For $\bar{L}^*(\theta)$ to be a maximum, $\pi''(\bar{L}^*(\theta),\theta;s)$ must be less than zero. This is assured by the assumption $\frac{c_b}{q}g_x(x,\theta;s) + g(x,\theta;s) > 0$. The derivative of $\bar{L}^*(\theta)$ with respect to θ can be calculated using the implicit function theorem

$$\frac{d\bar{L}^*(\theta)}{d\theta} = -\frac{\frac{c_b}{q}g_\theta(\bar{L},\theta;s) + G_\theta(\bar{L},\theta;s)}{\frac{c_b}{q}g_{\bar{L}}(\bar{L},\theta;s) + g(\bar{L},\theta;s)}$$

The assumption that $\bar{L}^*(\theta)$ does not change as θ changes holds as long as $\frac{c_b}{q}g_\theta(\bar{L},\theta;s) +$

$G_\theta(\bar{L}, \theta; s) = 0$. This requires $G_\theta(\bar{L}, \theta; s) = -\frac{c_b}{q} g_\theta(\bar{L}, \theta; s)$. The stochastic dominance assumption implies $G_\theta(\bar{L}, \theta; s) \geq 0$, then $g_\theta(\bar{L}, \theta; s) \cdot$ 0.[25]

Proof of Proposition 2. It follows from differentiation of the expected return function, when $\bar{L} = \bar{L}^*$ and for a given $\bar{D}$, that there is a maximum threshold quality level θ^A.

Suppose that $\bar{L} = \bar{L}^*$, where $\bar{L}^*$ is the value that maximizes the bank's expected revenue for a given θ. Differentiate the expected revenue function with respect to θ, and observe that $G_\theta \geq 0$ by Second Order Stochastic Dominance. Then

$$\pi(\bar{L}^*, \theta; s) = \pi_L(\bar{L}^*, \theta; s) L'(\theta) + \pi_\theta(\bar{L}^*, \theta; s)$$

Since $\bar{L}^*$ maximizes $\Pi(y, \theta)$, the first term is zero. Therefore

$$\pi_\theta(\bar{L}^*, \theta; s) = -\frac{c_b}{q} G_\theta(\bar{L}, \theta; s) - \int_0^{\bar{L}} G_\theta(y, \theta; s) \cdot \ 0$$

Then, the expected revenue function decreases as portfolio quality decreases. For a given $\bar{D}$ banks choose a quality threshold θ^A such that the expected revenue for $\bar{L}^*$ equals the opportunity cost of funds given by $\bar{D}$.

When $\bar{L} < \bar{L}^*$, for a given $\bar{D}$ and a fixed quality threshold θ, the bank chooses an interest rate on loans $\bar{L}$ such that expected revenue equals the opportunity cost given by $\bar{D}$. Any attempt to increase revenue would induce more depositors to become banks, which would drive expected revenue down.

Table 8: The Multipliers

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[25]The numerical example in Section 6 uses mean preserving changes in the variance as a measure of the quality of the distribution of project returns, θ. In the example there is no change in $\bar{L}^*(\theta)$ as θ changes. Other parameters can generate changes in either direction. Jaffe and Stigtliz (1990) analyze a similar problem and note that as the expected return function shifts down, the optimal loan rate can increase, decrease or stay the same. If the success probability of a risky project is reduced by the same proportion as the reduction in the success probability of the safe project, then the optimal loan rate does not change. If the risky project's success probability is reduced by more than proportionally compared with the safe project, then the loan rate will increase.

	Quantity Multiplier	Ex.	Quality Multiplier	Ex.
$\frac{du}{d\rho}$	$-\frac{\alpha w(1-\delta)}{\bar{x}(1-\alpha)qH(\theta)} < 0$	13.11	0	0
$\frac{dD}{d\rho}$	-1	-1	0	0
$\frac{dL}{d\rho}$	$-$	$-$	$\frac{1}{\pi_{\bar{L}}} > 0$	12.69
$\frac{du}{dq}$	$-\left[\frac{u}{q} + \frac{\alpha w(1-\delta)}{\bar{x}(1-\alpha)qH(\theta)}\right] < 0$	-15.61	$-$	$-$
$\frac{dD}{dq}$	$-\frac{\rho}{q} < 0$	-0.008	$\frac{(1-\alpha)H(\theta)\bar{x}}{\alpha w(1-\delta)} > 0$	0.038
$\frac{dL}{dq}$	$-$	$-$	$\frac{1}{\pi_{\bar{L}}}\left(\frac{dD}{dq} + \frac{\rho}{q}\right)$	0.38
$\frac{du}{d\bar{x}}$	$-\frac{\alpha w D(1-\delta)}{\bar{x}(1-\alpha)qH(\theta)} < 0$	-0.80	$-$	$-$
$\frac{dD}{d\bar{x}}$	0	0	$\frac{D}{\bar{x}} > 0$	0.97
$\frac{dL}{d\bar{x}}$	$-$	$-$	$\frac{D}{\pi_{\bar{L}}\bar{x}} > 0$	12.3
$\frac{d\theta}{dc_b}$	0	0	0	0
$\frac{du}{dc_b}$	$\frac{\pi_{c_b}\alpha w(1-\delta)}{\bar{x}(1-\alpha)qH(\theta)} < 0$	0.738	$-$	$-$
$\frac{dD}{dc_b}$	$\pi_{c_b} < 0$	-0.056	0	0
$\frac{dL}{dc_b}$	$-$	$-$	$-\frac{\pi_{c_b}}{\pi_{\bar{L}}} > 0$	0.714
$\frac{du}{dc_d}$	$-\frac{p_l q\alpha w(1-\delta)}{\bar{x}(1-\alpha)qH(\theta)} < 0$	-0.062	$-$	$-$
$\frac{dD}{dc_d}$	$-p_l q < 0$	-0.02	0	0
$\frac{dL}{dc_d}$	$-$	$-$	$\frac{p_l q}{\pi_{\bar{L}}} > 0$	0.254
$\frac{dspread}{dp_l}$	$-$	2.9	$-$	20

9.3 Log Normal Distribution

Assume. The distribution of project returns is log normal.

- expected value $\bar{y} = e^{\mu + \frac{\sigma^2}{2}}$

- variance $Var[y] = e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2}$

An increase in the variance, increases the mean return. We now show that the model is robust to this specification of the return distribution.[26] All parameters are the same, except the mean return is $\bar{y} = 0.35$ (to make results comparable). For $\theta = 0.92$,[27] the bank's expected return is maximized with a lower loan rate: $\bar{L} = 0.126$.

Credit Rationing by Loan Quality: $u = 1$

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[26]Boyd and Smith (1997), (1998) use a uniform distribution in numerical examples.

[27]The index is based on the variance in the numerical calculations: $\rho = 0.675$ for $Var[y] = 0.07$.

Table 9: Effect of Default Risk: $c_d = 0.6$, $q = 1.5$

Variable	No Default	Default Risk
$\bar{L}$	0.106	0.112
$\bar{D}$	0.076	0.076
$Spread$	0.03	0.036
θ	0.92	0.92
u	1	1
p_l	0	0.001
ρ	0	0.009

Quality and Quantity Rationing: $u < 1$

Table 10: Increase in Default Risk: $c_d = 0.6$, $q = 1.5$

p_l	0	0.01	0.02	0.03	0.05	0.06	0.07
$\bar{L}$	0.123	0.126	0.126	0.126	0.126	0.126	0.126
$\bar{D}$	0.0777	0.0687	0.0597	0.0507	0.0327	0.0237	0.0147
$Spread$	0.0453	0.0573	0.0663	0.0753	0.0933	0.1023	0.1113
θ	0.92	0.92	0.92	0.92	0.92	0.92	0.92
u	1	0.90	0.78	0.66	0.42	0.31	0.19
ρ	0	0.009	0.018	0.027	0.045	0.054	0.063

Π[θ,]

D+ρ   B

D   A   Π[θ^B,]

0.06

0.04   Π[θ^A,]

0.02

0.05   0.1   L*   0.2   L
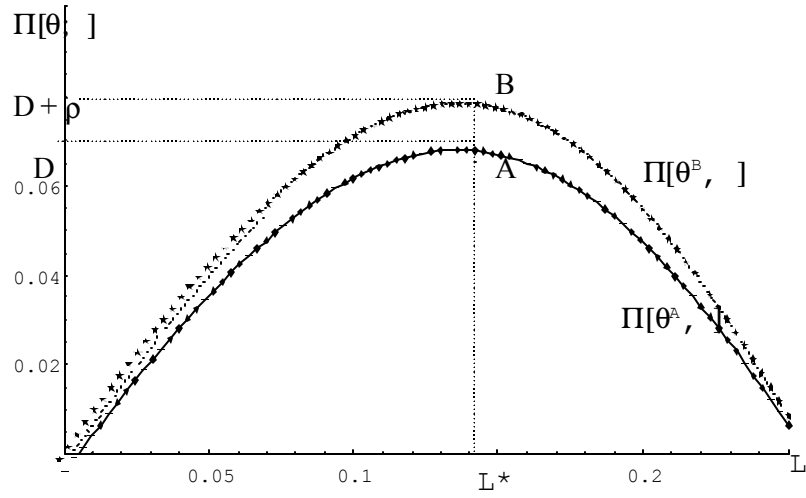
FIGURE 1: Bank Expected Return Function

Volume

u<1, θ

ED {

Case 2

L_1D

Case 1

u=1, θ

L_2D

L

0.05   0.1   0.15   0.2   0.25

$\Pi[\theta;.]$

D+ρ

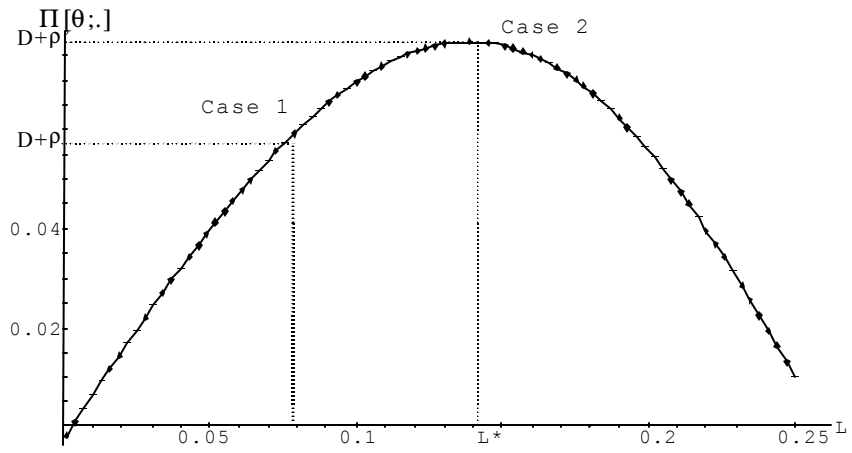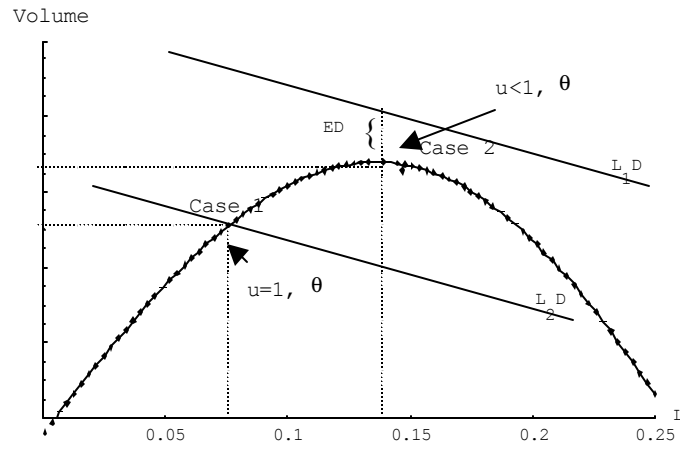Case 2

D+ρ

Case 1
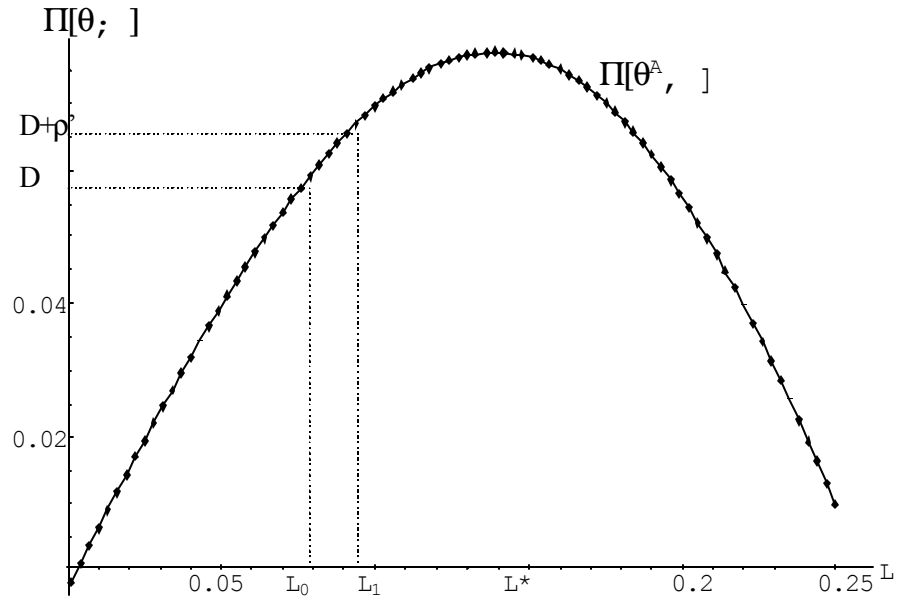
0.04

0.02

L

0.05   0.1   L*   0.2   0.25

FIGURE 2: Credit Rationing Cases

FIGURE 3: Case 1 Credit Rationing