A Novel MLSD Receiver Architecture for Nonlinear Channels

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Abstract. A new architecture for maximum likelihood sequence detection (MLSD) in nonlinear dispersive channels (NLCs) is presented, and its robustness to inaccurate channel knowledge is analyzed. This architecture is developed by considering a novel orthogonal representation of the NLC, which is exploited to develop a front-end capable of obtaining uncorrelated symbol rate samples, representing a sufficient statistic for information decoding. This front-end is a special form of space-time whitened matched filter (ST-WMF), and the MLSD obtained by using this front-end (ST-WMF-MLSD) requires simple branch metrics due to the signal whitening. The ST-WMF also allows for space-time compression of the equivalent channel, which is exploited for further complexity reduction of the ST-WMF-MLSD. Simulation results show the good trade-off in performance and complexity obtained with the ST-WMF-MLSD, even in the presence of inaccurate channel knowledge.

Keywords: MLSD, nonlinear channel, non-Gaussian noise, optical-fiber communications, whitened matched filter, dimensionality reduction

1 Introduction

Maximum likelihood sequence detection (MLSD) reaches the optimum performance for sequence estimation in communication systems [1]. For linear channels we could highlight Forney's [2] and Ungerboeck's [3] architectures. Forney's receiver consists of a symbol rate sampled whitened matched filter (WMF), followed by a Viterbi detector (VD) with simple euclidean branch metrics. On the other hand, Ungerboeck's receiver consists of a symbol rate sampled matched filter (MF), followed by a VD with suitable branch metrics which take into account the correlation between received samples. Forney's architecture optimally compresses the equivalent dispersive channel energy, which has been exploited in many reduced state schemes (see [4] and references there in). For nonlinear channels (NLCs) the optimal MLSD solution is also known, and many structures have been proposed [5–10]. Most of these schemes consist of a matched

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filter bank (MFB) sampled at the symbol rate, and followed by a VD with suitable branch metrics to take into account the nonlinear intersymbol interference (ISI) and noise correlation. The receiver in [9] consists of a wide-band filter with oversampling to obtain uncorrelated samples representing a sufficient statistic. The structures in [7, 10] use special front-ends to obtain uncorrelated symbol rate samples.

Despite their optimum performance, MLSD receivers may have a prohibitive complexity in highly dispersive NLCs, and suboptimum equalizers may be preferred in many practical applications. Long haul optical intensity modulated (IM) transmissions with direct detection (DD) are extremely low cost communication systems, that have the drawback of being one of the scenarios where the strong nonlinear ISI imposes the use of MLSD receivers to make communication possible. While suboptimum equalizers fail to compensate the nonlinear distortion in channels with a few tens of kilometers, it was observed that the IM/DD link can be compensated with a $\sim$ 3dB loss respect to back-to-back in links up to 1000km [11]. However, the complexity required by known MLSD receivers limits the practical transmission distance to just a few hundreds of kilometers. Thus, reduced complexity MLSD receivers are required in practical applications such as IM/DD optical links, even with actual integration capabilities.

Oversampled MLSD (OS-MLSD) receivers, developed upon the philosophy in [9], are widely used for theoretical and experimental investigations in IM/DD fiber optic transmissions [11, 12]. In this article we present further results obtained with the novel ST-WMF-MLSD receiver presented in [10]. In [10] it is shown how the ST-WMF-MLSD structure presents a smooth degradation of performance when complexity is reduced. The results presented in this article extend those in [10], showing that the ST-WMF-MLSD achieves a good performance even in the presence of imperfect channel knowledge.

This article is structured as follows. Section 2 describes the NLC model, and Section 3 describes the ST-WMF-MLSD architecture. Numerical results are presented in Section 4, while final conclusions are drawn in Section 5.

2 Nonlinear Channel Model

The noisy received signal is given by

$$r(t) = s(t) + z(t), \tag{1}$$

where $s(t)$ is the noise-free signal and $z(t)$ is the noise component, which is assumed to be a white Gaussian process with power spectral density N_0. Component $s(t)$ can be expressed by using its Volterra-series expansion. For example,

in optical IM/DD systems we get[1][2]

$$s(t) = \sum_k a_k f_0(t - kT) + \sum_{m=1}^{N-1} \sum_k a_k a_{k-m} f_m(t - kT), \tag{2}$$

where $f_0(t)$ is the linear kernel, $f_m(t)$, with $m > 0$, is the m-th second-order kernel [13], a_k is the k-th symbol at the input of the nonlinear channel, $1/T$ is the symbol rate, and N is the total number of kernels.

2.1 Channel Model Orthogonalization

Next we present an alternative representation of the nonlinear signal $s(t)$ [10]. Without loss of generality, we consider here the Volterra-series with a dominant linear kernel $f_0(t)$, and we select the first *pivoting* response as $h_0(t) = f_0(t)$. Let $\mathcal{H}_0$ be the *signal space* spanned by the set $\{h_0(t - kT)\}$ [1]. We assume that signal spaces are Hilbert spaces with inner product defined as $\int_{-\infty}^{\infty} x(t)y^*(t)dt$, where superscript $*$ denotes complex conjugate. From the *projection theorem*, the nonlinear kernels $f_m(t)$ can be uniquely expressed as

$$f_m(t) = \sum_n \lambda_n^{(0,m)} h_0(t - nT) + g_m^{(0)}(t), \tag{3}$$

where $g_m^{(0)}(t)$ is *orthogonal* to the signal space $\mathcal{H}_0$, i.e.,

$$\int_{-\infty}^{\infty} g_m^{(0)}(t) h_0^*(t - jT)dt = 0, \quad m = 1, ..., N-1, \forall j \in \mathcal{Z}, \tag{4}$$

while $\int_{-\infty}^{\infty} |g_m^{(0)}(t)|^2 dt$ is minimum [1], and $\mathcal{Z}$ denotes the set of all integers. We highlight that the first summation in eq. (3) is the *projection* of $f_m(t)$ onto $\mathcal{H}_0$. Define $\mathcal{G}_m^0$ as the signal space spanned by $\{g_m^{(0)}(t - kT)\}$. For $x(t) \in \mathcal{H}_0$ and $y(t) \in \mathcal{G}_m^0$, from (4) note that $\int_{-\infty}^{\infty} x(t)y^*(t)dt = 0$, therefore $x(t)$ and $y(t)$ are orthogonal signals [1]. Replacing (3) in (2) and operating, we can obtain

$$s(t) = s_0(t) + \overline{s}_0(t), \tag{5}$$

where

$$s_0(t) = \sum_k \left[a_k + \sum_{m=1}^{N-1} a_k a_{k-m} \otimes \lambda_k^{(0,m)} \right] h_0(t - kT), \tag{6a}$$

$$\overline{s}_0(t) = \sum_k \sum_{m=1}^{N-1} a_k a_{k-m} g_m^{(0)}(t - kT), \tag{6b}$$

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[1] The DC term of the series expansion is omitted.

[2] Note that (2) only considers kernels up to second-order. This is just to keep the notation as simple as possible, however the ST-WMF-MLSD is not limited in the order of nonlinearity it can handle.

with operator $\otimes$ denoting convolution. Notice that $s_0(t) \in \mathcal{H}_0$ and $\overline{s}_0(t) \in \cup_{m=1}^{N-1}\mathcal{G}_m^0$, therefore signals $s_0(t)$ and $\overline{s}_0(t)$ are orthogonal (see (4) and associated discussion). Without loss of generality, we select the second pivoting response as $h_1(t) = g_1^{(0)}(t)$, then eq. (6b) can be rewritten as

$$\overline{s}_0(t) = \sum_k a_k \left[a_{k-1}h_1(t-kT) + \sum_{m=2}^{N-1} a_{k-m}g_m^{(0)}(t-kT) \right]. \qquad (7)$$

Similarly to (5)-(6), $\overline{s}_0(t)$ can be expressed as $\overline{s}_0(t) = s_1(t) + \overline{s}_1(t)$, where

$$s_1(t) = \sum_k \left[a_k a_{k-1} + \sum_{m=2}^{N-1} a_k a_{k-m} \otimes \lambda_k^{(1,m)} \right] h_1(t-kT), \qquad (8a)$$

$$\overline{s}_1(t) = \sum_k \sum_{m=2}^{N-1} a_k a_{k-m} g_m^{(1)}(t-kT), \qquad (8b)$$

with $\lambda_n^{(1,m)}$ chosen to satisfy

$$\int_{-\infty}^{\infty} g_m^{(1)}(t)h_1^*(t-jT)dt = 0, \text{ for } m = 2, ..., N-1, \forall j \in \mathcal{Z}. \qquad (9)$$

Thus, note that $\overline{s}_1(t)$ is orthogonal to the signal spaces $\mathcal{H}_0$ and $\mathcal{H}_1$, spanned by $\{h_0(t-kT)\}$ and $\{h_1(t-kT)\}$, respectively. Repeating the processing on eqs. (4)-(6) and (7)-(9), and generalizing, we can get

$$s(t) = \sum_{n=0}^{N-1} s_n(t) = \sum_{n=0}^{N-1} \sum_k b_k^{(n)} h_n(t-kT), \qquad (10)$$

where $h_n(t)$ is the response of the n-th channel *path*, and $b_k^{(n)}$ is given by

$$b_k^{(n)} = \begin{cases} a_k + \sum_{m=1}^{N-1} a_k a_{k-m} \otimes \lambda_k^{(0,m)} & \text{if } n = 0 \\ a_k a_{k-n} + \sum_{m=n+1}^{N-1} a_k a_{k-m} \otimes \lambda_k^{(n,m)} & \text{if } 0 < n < N-1 \\ a_k a_{k-N+1} & \text{if } n = N-1, \end{cases} \qquad (11)$$

with

$$\int_{-\infty}^{\infty} h_m(t)h_n^*(t-jT)dt = 0 \qquad m \neq n, \forall j \in \mathcal{Z}. \qquad (12)$$

From (10) and (12) note that

$$\int_{-\infty}^{\infty} s_m(t)s_n^*(t-jT)dt = 0, \qquad \forall \, j \in \mathcal{Z} \text{ with } m \neq n. \qquad (13)$$

Equation (13) is a sort of *extended* orthogonality condition[3], that will be an important property for the development of the ST-WMF-MLSD in Section 3.

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[3] The *classical* orthogonality condition corresponds to the particular case of (13) with $j = 0$.

Note that at the i-th orthogonalization step, the energy of the signal component orthogonal to the subspace $\cup_{l=0}^{i-1}\mathcal{H}_l$ is minimized (see (3)-(4)). Then, a proper selection of the pivoting response at each step will maximize the signal energy in the first paths.

3   ST-WMF-MLSD for Nonlinear Channels

The MLSD receiver chooses the sequence $\{a_k\}$ that minimizes the metric

$$J = \int_{-\infty}^{\infty} |r(t) - s(t)|^2\, dt. \tag{14}$$

Using (9) and (10) in (14), it can be shown that all the information in the received signal $r(t)$, needed to estimate the information sequence $\{a_k\}$, is contained in the sequence vector $\bar{\mathbf{r}}_k = [\bar{r}_k^{(0)}, \bar{r}_k^{(1)}, \ldots, \bar{r}_k^{(N-1)}]^T$, where

$$\bar{r}_k^{(n)} = r(t) \otimes h_n^*(-t)|_{t=kT}, \quad n = 0, 1, \ldots, N-1 \tag{15}$$

are the symbol rate sampled outputs of a bank of filters matched to the path responses $\{h_n(t)\}_{n=0}^{N-1}$, as shown in Fig. 1 (see the first block denoted as S-WMF).

Let $\bar{r}_k^{(n)} = \bar{s}_k^{(n)} + \bar{z}_k^{(n)}$ be the decomposition of the n-th MF sampled output into its signal and noise components. It follows from (13) that $E[\bar{z}_l^{(n)}\bar{z}_{l-k}^{(m)*}] = N_0\rho_k^{(n)}\delta_{n-m}$, with $\rho_k^{(n)} = \int_{-\infty}^{\infty} h_n(t)h_n^*(t-kT)dt$, and δ_i denotes the Kronecker's delta. Then, the MFB in (15) produces a vector process $\bar{\mathbf{r}}_k$ with spatially uncorrelated components (independent because of the Gaussian assumption). We call this filter bank and symbol rate sampler as the space-whitening MF (S-WMF), as shown in the first block of Fig. 1. Note the S-WMF in Fig. 1 is implemented with P filters instead of N. The parameter choice $P < N$ can be used to reduce complexity by spatial truncation, which introduces negligible degradation as discussed in Section 2.1, and we shall verify it in Section 4 by numerical simulations in IM/DD optical transmissions.

Let $S_n(z) = M_n(z)M_n^*(1/z^*)$ be the folded spectral factorization of $S_n(z)$, with $S_n(z)$ and $M_n(z)$ being the Z-transforms of the sequence $\rho_k^{(n)}$ and the minimum-phase response $m_k^{(n)}$, respectively (see eq. (5.80) in [1] for more details). Then, it is straightforward to show that by filtering $\bar{\mathbf{r}}_k$ with a bank of scalar whitening filters (WF) $\{M_n^{-*}(1/z^*)\}_{n=0}^{N-1}$, results in the process

$$\tilde{\mathbf{r}}_k = \left[\tilde{r}_k^{(0)}\ \tilde{r}_k^{(1)} \ldots \tilde{r}_k^{(N-1)}\right]^T, \tag{16}$$

that besides being spatially independent is also time independent. Let $\mathbf{w}$ be the vector with the noise components of $\tilde{\mathbf{r}}_k$, then its power spectral density results $S_{\mathbf{W}} = N_0\mathbf{I}$, where $\mathbf{I}$ is the identity matrix. Then, the MLSD results in the minimization of the simple euclidean metric $J = \|\tilde{\mathbf{r}}_k - \tilde{\mathbf{s}}_k\|^2$, where $\tilde{\mathbf{s}}_k$ is the signal component of $\tilde{\mathbf{r}}_k$, and is given by

$$\tilde{\mathbf{s}}_k = \left[\tilde{s}_k^{(0)}\ \tilde{s}_k^{(1)} \ldots \tilde{s}_k^{(N-1)}\right]^T = \mathbf{M}_k \otimes \mathbf{b}_k, \tag{17}$$
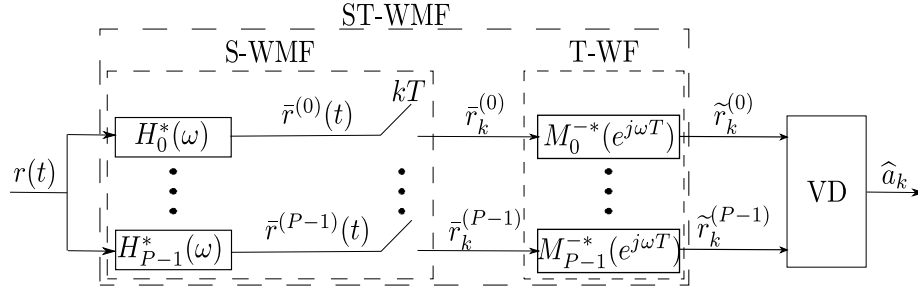
Fig. 1. Block diagram of the P-dimensional ST-WMF-MLSD receiver.

while $\mathbf{M}_k$ represents the $N \times N$ diagonal matrix

$$\text{diag}\{\mathbf{M}_k\} = \left[m_k^{(0)} \, m_k^{(1)} \ldots m_k^{(N-1)} \right], \tag{18}$$

and

$$\{\mathbf{b}_k\} = \left[b_k^{(0)} \, b_k^{(1)} \ldots b_k^{(N-1)} \right]^T. \tag{19}$$

Figure 1 shows the implementation of the time-whitening filter (T-WF) $\mathbf{M}_k$. The concatenation of the S-WMF and the T-WF results in the ST-WMF, also indicated in Fig 1. The MLSD based on the ST-WMF (ST-WMF-MLSD) then minimizes $J = \|\tilde{\mathbf{r}}_k - \tilde{\mathbf{s}}_k\|^2$, with a traditional VD with simple P-*dimensional* branch metrics. Note that $P < N$, as suggested in Fig. 1, can be used to reduce complexity thanks to the energy compression obtained with the novel orthogonal representation proposed in Section 2.1. As the scalar components of the response $\mathbf{M}_k$ are minimum-phase, its energy is compressed in time [1]. These properties can be exploited to reduce the number of states in the VD and the complexity of the branch metrics.

4 ST-WMF-MLSD in IM/DD Optical Systems

Next we analyze the proposed ST-WMF-MLSD receiver in transmissions over IM/DD fiber-optic systems with on-off keying (OOK) modulation. We focus on two key aspects of ST-WMF-MLSD: its performance (in comparison with current solutions based on OS-MLSD), and its ability to reduce complexity (e.g., number of states of VD). Complexity reduction is possible owing to (i) the minimum-phase property of the equivalent channel response provided by ST-WMF, and (ii) the spatial compression property. The latter gives rise to reduction of the ST-WMF dimension, P. This is achieved by using the most significant P paths of the nonlinear channel in (10).

Figure 2 depicts the optical system under consideration. The transmitter modulates the intensity of the transmitted signal using NRZ OOK modulation. Data rate is $1/T = 10$Gb/s, and the transmitted pulse has an unchirped Gaussian envelope $e^{-t^2/2T_0^2}$ with $T_0 = 36$ps. The standard single mode fiber (SMF)
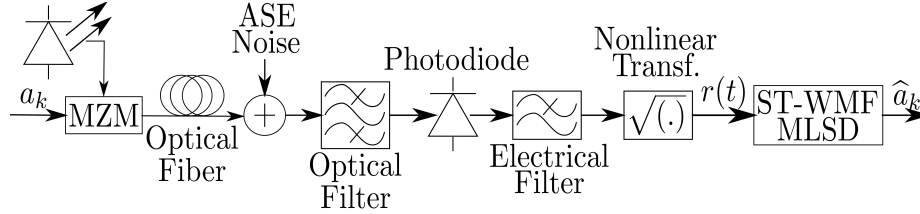
Fig. 2. IM/DD fiber-optic system with ST-WMF-MLSD receiver.

introduces CD (17 ps/(nm-km)), as well as attenuation. Optical amplifiers deployed along the fiber introduce amplified spontaneous emission (ASE) noise in the signal, which is modeled as AWGN in the optical domain. The received optical signal is filtered by a Lorentzian optical filter (15GHz), and then converted to a current with a PIN diode or avalanche photodetector. The resulting photocurrent is filtered by a fourth-pole Butterworth electrical filter (10GHz). The noise component after the electrical filtering is non-Gaussian and signal-dependent [14]. Therefore, the electrical signal is first processed by a memory-less nonlinear transformation. It has been found that after a square root transformation, the noise can be assumed Gaussian and signal-independent [12, 15]. Furthermore, channel nonlinearities can also be reduced by using the square root transformation [16], which also improves the space compression used to reduce the receiver dimension (i.e., most of the channel energy is concentrated on the linear kernel). The split-step Fourier method is used to compute the propagation of optical signals through the fiber. Oversampled linear and nonlinear kernels are extracted from the electrical signal after the square root transformation. The oversampling factor is $T/T_s = 16$. Then, we compute $h_n(kT_s)$ and $\lambda_k^{(n,m)}$ according to (6a) and (8a), and the symbol rate channel response matrix $\mathbf{M}_k$ can be easily obtained from (18). Since the noise after the square root transformation is approximately Gaussian and signal-independent [15], the theory proposed in [17] is used to evaluate the bit error probability. All the kernels of the nonlinear channel are used to compute the error probability, independently of the receiver dimension, P.

4.1 Performance with Perfect Channel Knowledge

Figure 3-A shows the penalty of the optical signal-to-noise ratio (OSNR) at BER= 10^{-3}, as a function of the fiber length, L. We present results for two *unconstrained* complexity receivers (i.e., without reduction of the number of states of the VD): ST-WMF-MLSD with $P = 2$, and OS-MLSD with 2 samples/bit (note that the branch metric *dimensions* of both VDs are the same). From Fig. 3-A we observe that both receivers have essentially the same performance, with a negligible loss with ST-WMF-MLSD, caused by using $P < N$.

Figure 3-B depicts the OSNR penalty at BER= 10^{-3} versus the number of states of the VD for $L = 700$ km. We also present results for ST-WMF-MLSD with $P = 1$ (i.e., only one filter in the bank). For OS-MLSD with 2 samples/bit,
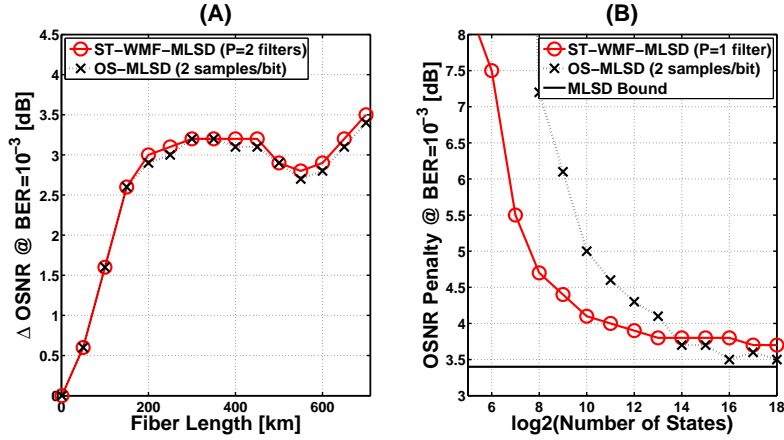
Fig. 3. (A): OSNR penalty at $BER = 10^{-3}$ versus fiber length with unconstrained complexity receivers. (B): OSNR penalty at $BER = 10^{-3}$ versus number of states of the VD with $L = 700$ km.

where the reduction of states is achieved by truncation and optimization of the sampling phase in order to minimize BER (8 uniformly distributed phases in the interval $T/2$ were tested). From Fig. 3-B, we verify that the number of states of the VD at a penalty of 4.6 dB can be reduced from 2048 to 256 with ST-WMF-MLSD and $P = 1$. Notice that this performance is achieved by using a VD with one sample per bit. Furthermore, we emphasize that these benefits widely outperform the extra complexity required by the linear filter and the channel estimator.

4.2   Performance with Imperfect Channel Knowledge

The performance evaluation of the proposed ST-WMF-MLSD architecture in Fig. 3 has been achieved assuming a perfect knowledge of the channel $(\{f_n(t)\}_{n=0}^{N-1})$. Then we present a simple analysis about the impact of the channel estimation inaccuracy on the performance of the ST-WMF-MLSD architecture in transmissions over IM/DD optical systems. This study shows that the performance degradation in ST-WMF-MLSD receivers caused by an imperfect channel estimation is low (~ 0.2) dB, and similar to that achieved by oversampled OS-MLSD receivers in the $L = 700$ km fiber link used in the example of Fig. 3-B.

The estimation of the oversampled linear and nonlinear kernels is required to implement both MLSD-based receivers. Let $R = T/T_s$ be the oversampling factor. Based on the polyphase filter representation of the oversampled channel response, the received samples can be expressed as

$$r_n^{(i)} = \sum_k a_k f_0^{(i)}[n-k] + \sum_k \sum_{m=1}^{N-1} a_k a_{k-m} f_m^{(i)}[n-k] + z_n^{(i)}, \qquad (20)$$

where $r_n^{(i)} = r(nT + iT_s)$, $z_n^{(i)} = z(nT + iT_s)$, and $f_m^{(i)}[n] = f_m(nT + iT_s)$ with $i = 0, ..., R - 1$. Notice that there are R different symbol rate sequences $r_n^{(i)}$.

Since symbols a_k are assumed zero-mean and i.i.d. real random variables with $\sigma_a^2 = E\left\{|a_k|^2\right\}$, we can verify that

$$
\begin{aligned}
f_0^{(i)}[k] &= \frac{1}{\sigma_a^2} E\{r_n^{(i)} a_{n-k}\}, \\
f_m^{(i)}[k] &= \frac{1}{\sigma_a^4} E\{r_n^{(i)} a_{n-k} a_{n-k-m}\},
\end{aligned} \qquad i = 0, ..., R - 1. \qquad (21)
$$

From (21), a simple estimator of the oversampled linear and nonlinear kernels can be implemented with an averaging filter as follows[4]:

$$
\begin{aligned}
\hat{f}_0^{(i)}[k] &= \frac{1}{\sigma_a^2 L_A} \sum_{n=n_0}^{n_0+L_A-1} r_n^{(i)} \hat{a}_{n-k}, \\
\hat{f}_m^{(i)}[k] &= \frac{1}{\sigma_a^4 L_A} \sum_{n=n_0}^{n_0+L_A-1} r_n^{(i)} \hat{a}_{n-k} \hat{a}_{n-k-m},
\end{aligned} \qquad (22)
$$

where L_A is the length of the averaging filter, n_0 is an arbitrary time index, and $\hat{a}_k$ is the detected symbol. The accuracy of the channel estimation given by (22) depends on the precision of the decisions $\hat{a}_k$, the length of the averaging filter L_A, and the channel noise power. We consider that decisions provided by the forward error correction (FEC) decoder are available, therefore the effect of decision errors can be neglected (i.e., $\hat{a}_k = a_k$). We highlight that this assumption is still valid if pre-FEC decisions are used as a result of the *low* bit error rates experienced in this link (e.g., $\sim 10^{-3}$). From the above, we conclude that the goodness of the estimates (22) shall mainly depend on the filter length L_A and the channel noise power.

The precision of (22) improves as the value of L_A increases. On the other hand, the maximum value of L_A shall be imposed by the speed of temporal variations of the fiber optic channel. As a result of its dependence on stress and vibrations, as well as on random changes in the state of polarization of the laser, PMD is nonstationary. Fluctuations with a time scale of a hundreds of microseconds have been considered in previous works (e.g., [18]). Therefore, the response time of channel estimation algorithms for PMD mitigation must be less than 1 ms (in practice a response time less than 100 μs is required [15]). This imposes, e.g., that the bandwidth of the averaging filter ($\sim 1/(4L_A T)$) should be $\gtrsim$ 20 KHz in order to efficiently track the channel variation.

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[4] In order to improve the channel tracking capability, an efficient implementation of the channel estimator with the well-known LMS algorithm may be preferred. Nevertheless, the objective is to shed light on the impact of imperfect knowledge of the fiber dispersion on the orthogonalized Volterra model. Therefore, practical aspects of the receiver architecture (e.g., buffers, number of taps of the WF, finite precision arithmetic effects, etc.) are not considered.
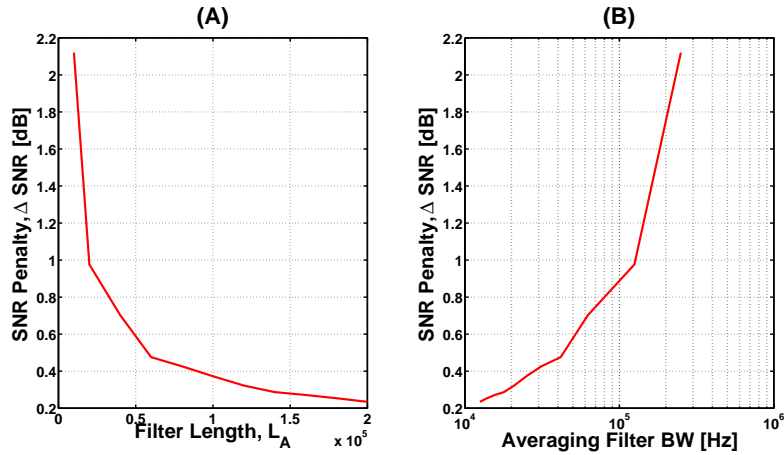
Fig. 4. (A) Penalty of SNR caused by imperfect channel estimation versus length of the averaging filter, and (B) the bandwidth of the averaging filter.

The received signal *seen* by an OS-MLSD receiver in the presence of imperfect knowledge of the channel dispersion can be expressed as

$$\tilde{r}_n^{(i)} = \sum_k a_{n-k}\left(f_0^{(i)}[k] + \sum_{m=1}^{N-1} a_{n-k-m} f_m^{(i)}[k]\right) + z_n^{(i)} + \hat{z}_n^{(i)} = s_n^{(i)} + z_n^{(i)} + \hat{z}_n^{(i)} \quad (23)$$

where $s_n^{(i)} = s(nT + iT_s)$, $\hat{z}_n^{(i)} = \hat{s}(nT + iT_s) - s(nT + iT_s)$ is the *estimation error* component, while $\hat{s}(nT + iT_s)$ is the synthesized signal obtained from (22).

Figure 4-A shows a first approximation of the SNR penalty caused by the imperfect channel estimation as a function of L_A, obtained from computer simulations. We consider $1/T = 10$ GHz, $R = 16$, and the fiber link with $L = 700$ km, as used in Fig. 3-B. The approximate SNR penalty caused by an imperfect channel estimation is then computed as

$$\Delta SNR = \frac{\sigma_z^2 + \sigma_{\hat{z}}^2}{\sigma_z^2}, \qquad (24)$$

where σ_z^2 is the channel noise power required to achieve a $BER = 10^{-3}$ with an unconstrained complexity OS-MLSD receiver, and $\sigma_{\hat{z}}^2$ is the variance of the estimation error component[5]. Notice that the penalty is ~ 0.4 dB for $L_A = 10^5$. This value of L_A represents a BW of $\sim 1/(4L_A T) = 25$ KHz (see Fig. 4-B). Assuming that the estimation error is white Gaussian noise, with power $\sigma_{\hat{z}}^2$, from Fig. 4-A we infer that the SNR penalty caused by an imperfect channel knowledge in OS-MLSD receivers with $L_A = 10^5$ should be $\lesssim \Delta SNR \sim 0.4$ dB.

On the other hand, Fig. 5 depicts the SNR penalty at $BER = 10^{-3}$ versus the number of states of the VD with $L = 700$ km. In this case the performance is estimated considering the missmatched ST-WMF and VD receivers, which are

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[5] The mean penalty of 20 runs with different seeds of the random number generator is presented.
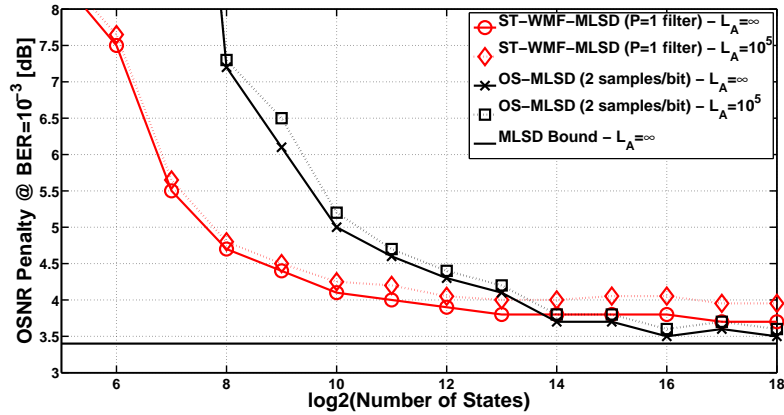
Fig. 5. OSNR penalty at $BER = 10^{-3}$ versus number of states of the VD for $L = 700$ km.

designed according to $\{\hat{f}_n(t)\}_{n=0}^{N-1}$, instead of $\{f_n(t)\}_{n=0}^{N-1}$. The error probability is computed as described in [17]. We present results with perfect knowledge of the fiber dispersion (denoted as $L_A = \infty$), and for imperfect channel estimation with $L_A = 10^5$. We see that the mean penalty caused by inaccuracies of channel estimation agrees with that expected from Fig. 4 with $L_A = 10^5$ (i.e., ~ 0.14 dB $< \Delta SNR \sim 0.4$ dB). Furthermore, we observe that the impact of imperfect channel knowledge on the performance is similar in both MLSD receivers (i.e., ~ 0.14 dB and 0.18 dB for OS and ST-WMF, respectively). This result can be understood from the fact that the filters $h_0^*(-t)$ and $m_k^{(0)}$ are computed from the samples of the estimated linear kernel $\hat{f}_0(t)$. Taking into account that the energy of the linear component is significantly higher than the nonlinear kernels [16] due to the spatial compression property, an accurate estimation of $h_0(t)$ can be achieved for the channel considered. Then, we infer that the energy loss of the signal component at the output of $\hat{m}_0(t)$ will be small. Therefore, and based on eq. (23), notice that the performance of OS-MLSD and ST-WMF-MLSD with $P = 1$ should degrade in a similar way.

5 Conclusions

We have introduced and analyzed a novel MLSD structure for nonlinear dispersive channels, the ST-WMF-MLSD. A procedure to maximize the spatial compression of ST-WMF-MLSD has been also proposed, and its benefits have been demonstrated by computer simulations. Performance degradation caused by imperfect channel knowledge has been analyzed. Simulation results have shown the excellent robustness of ST-WMF-MLSD to imperfect channel knowledge. These results make the ST-WMF-MLSD an architecture to be considered for high speed and high performance applications over dispersive NLCs.

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