Evolutionary Algorithm for State Encoding

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Abstract. This paper presents an encoding technique that is common for many different logic synthesis problems. It enables us to construct a system of Boolean functions, and then to decompose this system into sub-systems in such a way that a dependency of functions, included into each sub-system, on the respective arguments is reduced. For complex applications such type of encoding has a high computational complexity and the paper proposes a novel evolutionary algorithm for the solution of this problem.

1 Introduction

There are many combinatorial tasks that involve encoding algorithms. These tasks appear in particular at various steps in the logic synthesis of digital circuits. One of these tasks is based on such encoding technique that enables us to construct a system of Boolean functions, and then to decompose this system into sub-systems for which we are able to reduce dependency of the functions, included into each sub-system, on the respective arguments [1,2]. Commonly the logic scheme of a finite state machine (FSM) is composed of a combinational circuit and a memory (a set of flip-flops). The combinational circuit implements a system of Boolean functions D₁,...,D_R that depend on variables $x_1,...,x_L,\tau_1,...,\tau_R$. The $x_1,...,x_L$ are external input variables and $\tau_1,...,\tau_R$ bring the code of the state a_{from} from which we have to carry out transition(s). The functions $D_1,...,D_R$ enable the FSM to calculate the code of the next state a_{to} . The lines $\tau_1,...,\tau_R$ are the outputs from the FSM memory and the lines $D_1,...,D_R$ are the inputs to the FSM memory. For example, the FSM can be described as shown in Table 1 (at the beginning let us ignore all symbols enclosed in parenthesis). Here, a_{from} - is an initial state, $K(a_{from})$ and $K(a_{to})$ - are the codes of the states a_{from} and a_{to} , respectively, a_{to} - is the next state, $X(a_{from}, a_{to})$ - is a product of inputs that forces a corresponding transition. We assume that FSM memory is built from D flip-flops. Let us consider various transitions from the same state. We can see that for all

conditional transitions (i.e. for all transitions except from the state a_6), all the subfunctions of $D_1,...,D_3$ that must be activated on transitions from a state, depend on both the state and inputs. We will say that a sub-function is active if it has to be assigned to 1. Since all the sub-functions depend on states and inputs, the relevant Boolean expressions, that are used to calculate the values $D_1,...,D_3$, contain variables from the full set $\{x_1,...,x_L,\tau_1,...,\tau_R\}$, where L - is the number of external inputs (in our example L=5) and R - is the size of the FSM memory (in our example R=3).

Table 1. An example of FSM

a_{from}	K(a _{from})	$X(a_{from}, a_{to})$	a_{to}	$K(a_{to})$	$D(a_{from}, a_{to})$
a_1	000 (000)	x_1x_2	a_1	000 (000)	- (-)
		not_x ₁ not_x ₂	a_2	001 (0 11)	$D_3 (D_2, D_3)$
		$not_x_1x_2$	a_3	010 (0 01)	$D_2(D_3)$
		$x_1 \text{ not} x_2$	a_6	101 (0 10)	$D_{1},D_{3}(D_{2})$
a_2	001 (011)	\mathbf{x}_3	a_3	010 (0 01)	$D_2(D_3)$
		not_x 3	a_5	100 (1 01)	$D_1(D_1,D_3)$
a_3	010 (001)	not_x 1	a_4	011 (1 00)	$D_{2},D_{3}(D_{1})$
		x_1x_2	\mathbf{a}_5	100 (1 01)	$D_1 (D_1, D_3)$
		$x_1 not_x_2$	a_7	110 (1 11)	$D_1,D_2(D_1,$
					D_2, D_3
a_4	011 (100)	not_x 4	a_1	000 (000)	- (-)
		\mathbf{x}_4	a_4	011 (1 00)	$D_{2},D_{3}(D_{1})$
a_5	100 (101)	\mathbf{x}_1	a_1	000 (000)	- (-)
		not_x 1	a_6	101 (010)	$D_1,D_3(D_2)$
a_6	101 (010)	1	a_7	111	D_1, D_2, D_3
a_7	110 (111)	X ₅	a_2	001 (0 11)	$D_3 (D_2, D_3)$
		not_x 5	a_7	110 (1 <i>11</i>)	$D_1,D_2(D_1,$
					D_2, D_3

Consider all sub-functions of $D_1,...,D_3$ that are generated for proper transitions from a state. For example, sub-functions $D_1^3,...,D_3^3$, that have to be activated in transitions from the state a_3 (later we will also mark such sub-functions with a corresponding superscript) are the following: $D_1^3 = a_3x_1$; $D_2^3 = a_3 \pmod{x_1} \vee \text{not}_{x_2}$; $D_3^3 = a_3 \text{not}_{x_1}$ (these expressions can easily be obtained from Table 1). Note that since there exist 3 transitions from a_3 , they can be distinguished with the aid of just two Boolean variables, such as $D_1^3,...,D_3^3$. As a result, inputs such as x_1 and x_2 can affect (and change) just two variables from the set $\{D_1^3,...,D_3^3\}$ and the remaining variables (in our example one variable from the set $\{D_1^3,...,D_3^3\}$) can be independent of external inputs from the set $X = \{x_1,...,x_L\}$, i.e. they will only depend on the current state (in our example on the state a_3). If the number of different (non coinciding) next states in state transitions from a_m is equal to q_m , then (R-intlog₂ q_m) variables from the subset $D_1^m,...,D_R^m$ can be independent of the input variables from the set X.

Let us suppose now that the states for our example have been coded as shown in parenthesis in Table 1. The values of the sub-functions D_1^r ,..., D_3^r (r=1,2,...,M, M - is

the number of FSM states and for our example M=7) that do not depend on input variables, are marked with bold and italic bold fonts. Note that the bold font has been used for passive values, and italic bold font for active values of the sub-functions.

Now the combinational circuit S of the FSM can be decomposed into two subcircuits in such a way that the first sub-circuit S_{ax} implements all active values of $D_1^1,...,D_3^M$ that are not bold. The functions of S_{ax} depend on both FSM inputs and states. The second sub-circuit S_a implements all active values of $D_1^1,...,D_3^M$ that are bold. The functions of S_a depend only on the states and for our example they are:

$$D_1 = a_3 \vee a_6$$
; $D_2 = a_6 \vee a_7$; $D_3 = a_2 \vee a_6 \vee a_7$;

Such functions are well suited for minimization and are usually very simple. For instance, S_a can be constructed from just four 2-input logic elements of types XOR, AND and OR, which convert values of τ_1, τ_2, τ_3 to values of D_1, D_2, D_3 . On the other hand, such kind of decomposition enables us to essentially simplify the sub-functions of S_{ax} , i.e. the active sub-functions that are not bold in Table 1. Finally, even for our very simple example, the proposed state encoding permits the number of logic elements to be reduced by approximately 20%.

Similar problems appear in a large number of practical applications and we will point out just some of them:

- One-level control circuits based on blocks, such as programmable logic arrays (see [3, p. 182]. It allows reducing essentially the total number of interconnections by eliminating the repeated outputs for different blocks;
- RAM-based implementation of FSMs [2], etc.

The paper presents an evolutionary algorithm that allows the encoding considered above to be achieved. It should be noted that the technique of artificial evolution has been widely used for hardware design [4]. Evolutionary algorithms (EA) are based on a process of "generate-and-test" [5] and this strategy can be applied at different levels. For example, in [6] a genetic algorithm is employed to search for circuits that represent the desired state transition function. Many examples demonstrating EAs that have been successfully employed for hardware design are presented in [7-9]. For some circuits they produced unforeseen results of very high quality (for example, [10]), which have never been obtained by human designers.

The remainder of this paper is organized in four sections. Section 2 presents the detailed description of the proposed evolutionary algorithm. Section 3 discusses feasible variations of the algorithm. Section 4 presents the results of experiments, which clearly demonstrate the advantages of the proposed encoding technique. The conclusion is in section 5.

2 Evolutionary Algorithm

The basic idea of EA was used for the considered problem in the traditional simple way [11]. The algorithm includes the following steps:

- 1. Production of an initial population composed of individuals that represent a set of randomly generated codes for a given number of variables (FSM states).
 - 2. Evaluation of the population and measuring its fitness.

- 3. Variation of the population by applying such operations as reproduction, mutation and crossover. A reproduction operator makes a copy of the individual (with a probability based on its fitness) for inclusion in the next generation of the population. A mutation operator creates new individuals by performing some changes in a single individual, while the crossover operator creates new individuals (offspring) by combining parts of two or more other individuals (parents) [9].
- 4. Performing a selection process, where the most fit individuals survive and form the next generation.

Points 2-4 are repeated until a predefined termination condition is satisfied.

Let us assume that a population π including ν individuals $\pi_1,...,\pi_\nu$ has been randomly generated. In order to evaluate each individual π_t , $t=1,...,\nu$, it is necessary to specify a fitness function. For our problem it is very easy. Let $A=\{a_1,...,a_M\}$ be a set of variables that have to be encoded, M is the number of variables in the set A. The variables in each individual subset $A(a_{from})$ (where $A(a_{from})$ is a set of states to which there exist direct conditional transitions from the state a_{from} , $k_m=|A(a_{from})|>1$, m=1,...,M) have to be encoded in such a way that the number w_m ($w_m=R$ -intlog₂ k_m) of their bits with the same indices have equal values. Here R=intlog₂M is the minimum number of bits in the codes of states assuming binary encoding. Thus, any solution for which the fitness function W is equal to Σw_m ($k_m>1$, m=1,...,M) gives an optimal result (we assume that such result exists, which, in fact, is not true for a general case). Actually we can discover several optimal results and for each of them the function W has the same maximum possible value. Any of these results provides the best solution to the problem so we just have to find out the first of them.

Now the fitness can be estimated very easily. For randomly generated codes π_{ι} we have to calculate the function W_{i} and compare the result with the value W. The less the difference W-W_i the better the fitness for the individual π_{ι} .

The next step produces a variation of the population and can be carried out by applying such operations as reproduction, mutation and crossover. Two kinds of reproduction have been examined and compared. The first one is based on elitist rule [11] where the best solutions in the population are certain to survive to the next generation. This rule has been implemented as follows. For reproduction purposes 10% of individuals with the best fitness have been copied to the next generation of the population. The second kind of reproduction uses the same percentage of individuals, but it is based on proportional selection [11].

The mutation operation runs on one parental individual selected with a probability based on fitness and creates one new offspring individual to be inserted into the new population at the next generation. In order to choose which parents will produce offspring, a fitness proportional selection is employed. Each parent π_i is assigned a weight W_i , calculated at the previous step. The probability of selection for each parent is proportional to its weight. The main idea of the mutation operation will be illustrated by an example of state encoding for FSM with specification presented in [12]. The FSM has 10 states and the following transitions $a_{\text{from}} \Rightarrow A(a_{\text{from}})$: $a_1 \Rightarrow \{a_2, a_3, a_4\}$, $a_2 \Rightarrow \{a_2, a_4, a_5\}$, $a_3 \Rightarrow \{a_6, a_7, a_8, a_9\}$, $a_4 \Rightarrow \{a_5\}$, $a_5 \Rightarrow \{a_3\}$, $a_6 \Rightarrow \{a_5, a_7\}$, $a_7 \Rightarrow \{a_3, a_9\}$, $a_8 \Rightarrow \{a_2, a_{10}\}$, $a_9 \Rightarrow \{a_{10}\}$, $a_{10} \Rightarrow \{a_1\}$. Since M=10 and R=4 for each individual we can chose any 10 from $2^4=16$ possible codes. Suppose that at some step of EA we found the codes for an individual I shown in Table 2 and this individual has to be mutated.

Codes	0000	0001	0010	0011	0100	0101	0110	0111
I	0	0	<u>4</u>	8	0	0	0	3
Codes	1000	1001	1010	1011	1100	1101	1110	1111
I	<u>5</u>	6	2	9	7	0	<u>10</u>	1

Table 2. An individual I that has been selected for mutation operation

For the individual I all the state codes have to be examined and all the weights $w_1^I,...,w_M^I$ that exceed the value 1 have to be calculated. Some (or all for the best result) of these weights correspond to an optimal result. For all weights w_m^I that have an optimal value the respective states a_m have to be selected (see in Table 2 **bold** underlined numbers m of states a_m). For example, we have the following state transitions $a_1 \rightarrow \{K(a_2)=1010, K(a_3)=0111, K(a_4)=0010\}, a_2 \rightarrow \{K(a_2)=1010, K(a_4)=0010\}, K(a_5)=1000\}_{opt}, a_3 \rightarrow \{K(a_6)=1001, K(a_7)=1100, K(a_8)=0011, K(a_9)=1011\}, a_6 \rightarrow \{K(a_5)=1000, K(a_7)=1100\}_{opt},$ etc. Optimal solutions are indicated by subscript "opt" and the respective bits (i.e. bits with coincident indices that have equal values) of the codes are marked with bold font. The mutation operation permits a new child individual to be created and includes the following steps.

Step 1. All the elements that correspond to an optimal solution (see **bold** underlined numbers in Table 2) are included in the new individual (offspring).

Step 2. The codes for the remaining elements will be randomly regenerated in such a way that just free codes (i.e. such codes that have not been already chosen at step 1) can be selected.

Crossover is the most complicated operation of the considered EA. The main idea of this operation will also be illustrated by the same example of FSM. Suppose that at some step of EA we have found the codes for two individuals I1 and I2 shown in Table 3 and these individuals were chosen to be parents for creating a new individual that is a child. For all weights $\mathbf{w}^{11}_{m}(\mathbf{w}^{12}_{s})$ that have an optimal value the respective states \mathbf{a}_{m} (\mathbf{a}_{s}) have to be selected (see **bold** underlined numbers m of states \mathbf{a}_{m} for the first individual I1 and *italic* underlined numbers s of states \mathbf{a}_{s} for the second individual I2). The parents are chosen on the base of proportional selection [11].

Codes Ι1 I2 Codes <u>10</u> Ι1 Ι2

Table 3. The results of encoding for two individuals

The crossover operation permits a new child individual to be created and includes the following steps.

Step 1. The first solution (see Table 4) is formed from the selected elements of the first individual I1 (see **bold** underlined numbers in Table 3).

- Step 2. Permitted selected elements from the second individual I2 (see *italic* underlined numbers in Table 3) are added to the first solution (i.e. to the child). An element is allowed for step 2 if:
 - a) It was not included into the child during the first step; and
 - b) It does not have a code that has already been used during the first step.

Table 5 shows the result of step 2 for our example.

- Step 3. All the remaining permitted elements from the first and the second individuals are added to the child. An element is allowed for step 3 if it has not yet been included in the child and:
 - a) It has the same code for both individuals I1 and I2; or
- b) It is included in the second individual I2 and the respective code of the first individual I1 was not used for the states; or
- c) It is included in the first individual I1 and the respective code of the second individual I2 was not used for the states;

Table 6 shows the result of step 3 for our example.

Table 4. The result of step 1

Codes	0000	0001	0010	0011	0100	0101	0110	0111
Child			4					
Codes	1000	1001	1010	1011	1100	1101	1110	1111
Child	5		2		7		10	

Table 5. The result of step 2

Codes	0000	0001	0010	0011	0100	0101	0110	0111
Child			4					
Codes	1000	1001	1010	1011	1100	1101	1110	1111
Child	5		2	3	7		10	

Table 6. The result of step 3

Codes	0000	0001	0010	0011	0100	0101	0110	0111
Child	8		4			1		
Codes	1000	1001	1010	1011	1100	1101	1110	1111
Child	5	6	2	3	7		10	

Step 4. All the remaining states that have not been assigned yet are recorded in free boxes for codes from left to right.

Table 7 presents the final result of the crossover operation.

Individuals I1, I2 and the child can be evaluated as follows: $W_{I1}=W_{I2}=11$, $W_{child}=13$ (i.e. the child is better than any of the parents I1 and I2) and the optimal weight W=15.

There are two termination conditions for the considered EA: obtaining an optimal solution or exceeding a specified time limit.

Codes Child Codes Child

Table 7. The result of step 4 that gives the final result of the crossover operation

3 Variations of the Evolutionary Algorithm

Note that for many practical applications it is allowed that a state has more than one code. If the FSM circuit is constructed from RAM blocks then using multiple codes does not make the circuit more complicated [2]. Moreover applying this technique enables us to improve the results of encoding. It should be noted that for some practical problems an optimal solution, that only permits each state to be assigned a single unique code, cannot be obtained. For example, such solution cannot be found for the following set of state transitions: $a_1 \Rightarrow \{a_1, a_2\}, a_2 \Rightarrow \{a_2, a_3\}, a_3 \Rightarrow \{a_4, a_5\}, a_4 \Rightarrow \{a_1, a_3\}, a_5 \Rightarrow \{a_1\}$. However, if more than one code is permitted for the states we can find an optimal solution, which is: $K(a_1)=000, K(a_2)=001, K(a_3)=100$ and $101, K(a_4)=011, K(a_5)=111$. The EA can be modified slightly in order to produce the proper solution. Indeed if an optimal result cannot be found within a predefined time interval we can allow using more than one code for states. Thus, the algorithm is relatively flexible when it comes to future improvements and modifications.

4 Experimental Results

The results of the proposed EA were estimated for more than 100 digital circuits that required the considered above encoding technique within the respective process of synthesis. Fig. 1 shows these results for 25 FSMs. We considered block-based decomposition of FSMs [3], where $R=intlog_2M$ and R_{av} is an average number of outputs for the blocks. So the considered technique makes possible the number of outputs required for each block to be decreased on average by 1.8.

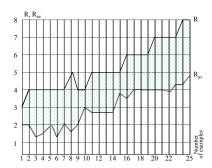


Fig. 1. The results of experiments.

The EA has been analyzed in several contexts. Firstly, we evaluated primary genetic operations that are reproduction (based on elitist rule and proportional selection criteria), mutation and crossover. The considered options A, B, C and D are listed below:

A: the crossover operation was carried out in order to form 90% of population for the next generation and 10% of population for the next generation was chosen with a probability based on fitness (i.e. based on proportional selection);

B: firstly the crossover operation was carried out in order to form 100% of population, secondly the mutation operation based on proportional selection was performed for 10% of individuals of the new generation and finally 10% of individuals in the next generation were replaced with 10% of randomly generated individuals;

C: the mutation operation based on proportional selection was carried out in order to form 100% of population for the next generation and 10% of individuals in the next generation were replaced with 10% of randomly generated individuals;

D: the crossover operation was carried out in order to form 90% of population for the next generation and 10% of population for the next generation was chosen based on elitist rule.

Fig. 2 shows how the execution time for all four options depends on the number of individuals in population. This dependency was considered for an FSM with 15 states (M=15) and with at maximum 4 transitions from each state. The experiments were performed on PentiumIII/800MHz/256MB. Fig. 3 shows how the number of required generations for all four options depends on the number of individuals in population.

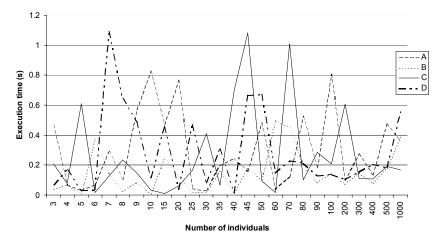


Fig. 2. Dependency of execution time on the number of individuals in population

Secondly, we examined practical applications that could benefit from the considered encoding technique. This enabled us to estimate some parameters, such as the expectable size of codes. Table 8 presents examples of control circuits used in assembly lines for manufacturing purposes. The number of individuals in population

was chosen to be 15. Here NG is the number of generations, W is the optimal weight, W_e is the obtained weight, ET is the execution time, $G_{min} = max (intlog_2|A(a_m)|)$,

Thirdly, we performed a set of experiments for randomly generated examples with different initial data (such as the number of individuals in populations) and variable requirements (such as using one code for each state of FSM or employing more than one code for some states). Table 9 shows the best results for options A, B, C, D obtained for arbitrary selected examples (the option that gave the best result is indicated in the first column in parentheses). For the examples aex7 and aex8 just the option D was used and we received We<W because the value W cannot be obtained when using just one code for each state. If we allow to employ more than one code for some states then the result with W_e =W can be easily found (ET = 41.12 s for aex7 and ET = 9.38 s for aex8).

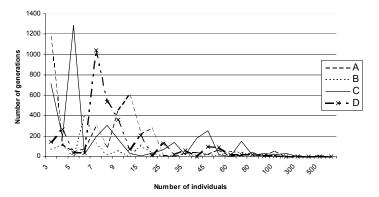


Fig. 3. Dependency of the number of generations on the number of individuals in population

ET (s) Example NG W.

Table 8. The results of experiments for practical examples

Example	NG	W	W _e	ET (s)	M	G_{\min}
Ex1	134	32	32	0.424	20	2
Ex2	63	36	36	0.261	29	2
Ex3	99	7	7	0.202	12	4
Ex4	111	30	30	0.255	15	2

Table 9. Experiments with arbitrary selected FSMs

Example	NG	W	W _e	ET (s)	M	G_{\min}
aex1 (B)	15	21	21	0.052	21	2
aex2 (B,D)	4	25	25	0.017	28	2
aex3 (D)	146	15	15	0.216	10	2
aex4 (B)	23682	34	34	61.767	15	3
aex5 (B)	490	31	31	3.897	47	2
aex6 (B)	12854	42	42	49.201	16	3
aex7	5534	88	87	1000	52	2
aex8	1024	38	37	1000	49	3

5 Conclusion

In the previous discussion we have presented the evolutionary algorithm for state encoding that allows Boolean functions to be decomposed in such a way that the dependency of sub-functions obtained as a result of the decomposition on the arguments can be reduced. The algorithm has been analyzed in several contexts. Firstly, we evaluated the primary genetic operations that are reproduction, mutation and crossover. Secondly, we examined practical applications that require the considered encoding technique. This enabled us to estimate some parameters, such as the expectable size of codes. Thirdly, we performed a set of experiments with different initial data (such as the number of individuals in populations) and variable requirements (such as using one code for each state of finite state machine or employing more than one code for some states). The examples in the paper and the results of experiments with a C++ program that implements the proposed evolutionary algorithm have shown that the considered approach is very effective.

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