Model Contractions on Description Logics*

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Abstract

When using *tableaux algorithms* to reason about *Description Logics* (DLs), new information is inferred from the models considered while trying to achieve knowledge satisfiability. By focusing the *ontology change* problem, we consider an environment where DLs are the logical formalization to express knowledge bases in the web, and the integration of distributed ontologies is developed under new extensions of the *belief revision* theories originally exposed in [1]. Hence, a reinforced theory arises in order to properly apply change operations over models, considering new inferred information and assumed beliefs in each *possible world*. As a result, a new type of *contraction* operator is proposed and its success postulate analyzed.

Keywords: Belief Revision, Description Logics, Tableau Calculi, Ontology Change.

1 Introduction

Our main research interest relays in topics like *Ontology Integration* and *Ontology Merging* [4], for what we have proposed in [7] to use theory change formalizations in order to consistently join terminologies, redefining or reinforcing sub-concepts. But following the reasoning methods exposed for DLs, like satisfiability, solved by tableaux algorithms originally defined in [8], a new area of interest arises. A set of knowledge base extensions is obtained from the models considered during the execution of the DL reasoning service. Here, is imperative to redefine the formalizations of the theory change in order to revise beliefs on each extension and transitively in the knowledge base itself.

A motivating environment in which our proposal seems relevant may be the case of large databases (closed world assumption) managing incomplete information. This means that for some systems, maybe some unnoticed information is inferred from the knowledge base. Moreover, it is possible to have critical information deduced from the base that may take over more undesirable deductions. In order to avoid this kind of scenarios, a database manager could classify those basic beliefs that should not be inferred from the base, and present them as complex queries to the base. In case that some query is verified, he could have the alternative to correct the knowledge that helps to get this

*This article assumes some background knowledge about description logics and belief revision from the reader.

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deduction, *i.e.*, to contract some beliefs in order to avoid problematic deductions. Moreover, this could motivate the writing of some kind of basic rules, possibly exposed as axioms representing some basic knowledge to always be held no matter the upgrade needed to be done to the base.

This is the situation in which we focus our work, anticipating or foretelling undesirable side-effects and / or clashes in large database systems. While some slight details could pass unnoticed by the ordinary people's inspection, the proposed theory could inspect possible worlds by looking some specific assumed beliefs in order to infer new knowledge from the basic knowledge representation. The remainder of this work is disposed as follows, Sect. 2 gives a brief description of tableau-based algorithms and their behavior by achieving satisfiability. Sect. 3 gives a brief overview of kernel contractions in the theory change introducing Sect. 4, which explains the proposed theory where some of the basic definitions for kernel contractions were adapted to deal with model based reasoners. Finally, Sect. 5 concludes making an analysis of the proposal.

2 DLs Reasoning Algorithms

Relevant inference problems usually are reduced to the consistency problem for ABoxes, provided that the DL at hand allows for conjunction and negation. However, for those description languages of DL systems that do not allow for negation, subsumption of concepts can be computed by so-called *structural subsumption algorithms*, *i.e.*, algorithms that compare the syntactic structure of (possibly normalized) concept descriptions.

While usually very efficient, they are only complete for rather simple languages with little expressivity. In particular, DLs with (full) negation and disjunction cannot be handled by structural subsumption algorithms. For such languages, so-called *tableau-based algorithms* have turned out to be very useful.

2.1 Properties for Reasoning

Let first give a very brief description of some important reasoning properties of description logics. Given a terminology $\mathcal{T}$, if there is some interpretation of a concept that satisfies the axioms in $\mathcal{T}$ (a model of $\mathcal{T}$), then the concept denotes a nonempty set for the interpretation, furthermore this concept is known to be *satisfiable* w.r.t. $\mathcal{T}$. Otherwise it is called *unsatisfiable*. Formally,

(Satisfiability) [2] A concept C is *satisfiable* w.r.t. $\mathcal{T}$ if there exists a model $\mathcal{I}$ of $\mathcal{T}$ such that $C^{\mathcal{I}}$ is nonempty. In such a case we say that $\mathcal{I}$ is a *model* of C.

Checking (un)satisfiability of concepts might be considered a key inference given that a number of other important inferences for concepts can be reduced to it. For instance, in order to check whether a domain model is correct, or to optimize concepts, we may want to know whether one concept is more general than another. This is called the *subsumption* problem. A concept C is *subsumed* by a concept D if in every model of $\mathcal{T}$, C is a subset of D.

(Subsumption) [2] A concept C is *subsumed* by a concept D w.r.t. $\mathcal{T}$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for every model $\mathcal{I}$ of $\mathcal{T}$. In such a case we write $C \sqsubseteq_{\mathcal{T}} D$ or $\mathcal{T} \models C \sqsubseteq D$.

A new kind of reasoning algorithms in DLs raised from the approach of considering satisfiability checking as the main inference. These algorithms are known as tableaux and can be understood as a specialized tableau calculi.

2.2 Basics for Tableaux Algorithms

Instead of directly testing subsumption of concept descriptions, these algorithms use negation to reduce subsumption to (un)satisfiability of concept descriptions: $C \sqsubseteq D$ ***iff*** $C \sqcap \neg D$ ***is unsatisfiable.***

We illustrate the underlying ideas by two simple examples taken from [2]. Let A, B be concept names, and let R be a role name. As a first example, assume that we want to know whether $(\exists R.A) \sqcap (\exists R.B)$ is subsumed by $\exists R.(A \sqcap B)$. This means that we must check whether the concept description $C = (\exists R.A) \sqcap (\exists R.B) \sqcap \neg(\exists R.(A \sqcap B))$ is unsatisfiable.

Pushing all negation signs as far as possible into the description yields $C_0 = (\exists R.A) \sqcap (\exists R.B) \sqcap \forall R.(\neg A \sqcup \neg B)$, which is in negation normal form, ***i.e.***, negation occurs only in front of concept names. Then, we try to construct a finite interpretation $\mathcal{I}$ such that $C_0^{\mathcal{I}} \neq \emptyset$. This means that there must exist an individual in $\Delta^{\mathcal{I}}$ that is an element of $C_0^{\mathcal{I}}$. The algorithm just generates such an individual, say b, and imposes the constraint $b \in C_0^{\mathcal{I}}$ on it, this means that b must satisfy all the three interpreted conjunctions that composes C_0.

From $b \in (\exists R.A)^{\mathcal{I}}$ we can deduce that there must exist an individual c such that $(b, c) \in R^{\mathcal{I}}$ and $c \in A^{\mathcal{I}}$. Analogously, $b \in (\exists R.B)^{\mathcal{I}}$ implies the existence of an individual d with $(b, d) \in R^{\mathcal{I}}$ and $d \in B^{\mathcal{I}}$. In this situation, one should not assume that $c = d$. Thus:

- *For any existential restriction the algorithm introduces a new individual as role filler, and this individual must satisfy the constraints expressed by the restriction.*

Since b must also satisfy the value restriction $\forall R.(\neg A \sqcup \neg B)$, and c, d were introduced as R-fillers of b, we obtain the additional constraints $c \in (\neg A \sqcup \neg B)^{\mathcal{I}}$ and $d \in (\neg A \sqcup \neg B)^{\mathcal{I}}$. Thus:

- *The algorithm uses value restrictions in interaction with already defined role relationships to impose new constraints on individuals.*

Now c might be such that $c \in (\neg A)^{\mathcal{I}}$ or $c \in (\neg B)^{\mathcal{I}}$. Assume the first possibility leads to an obvious contradiction, so we must choose the second one $c \in (\neg B)^{\mathcal{I}}$. Analogously, we must choose $d \in (\neg A)^{\mathcal{I}}$ in order to satisfy the constraint $d \in (\neg A \sqcup \neg B)^{\mathcal{I}}$ without creating a contradiction to $d \in B^{\mathcal{I}}$. Thus:

- *For disjunctive constraints, the algorithm tries both possibilities in successive attempts. It must backtrack if it reaches an obvious contradiction, i.e., if the same individual must satisfy constraints that are obviously conflicting.*

In the example, we have now satisfied all the constraints without encountering an obvious contradiction. This shows that C_0 is satisfiable, and thus $(\exists R.A) \sqcap (\exists R.B)$ is not subsumed by $\exists R.(A \sqcap B)$. The interpretation generated by the algorithm is $\Delta^{\mathcal{I}} = \{b, c, d\}$; $R^{\mathcal{I}} = \{(b, c), (b, d)\}$; $A^{\mathcal{I}} = \{c\}$ and $B^{\mathcal{I}} = \{d\}$.

In our second example, we now want to know whether $(\exists R.A) \sqcap (\exists R.B) \sqcap \leqslant 1R$ is subsumed by $\exists R.(A \sqcap B)$. The tableau-based satisfiability algorithm first proceeds as above, with the only difference that there is the additional constraint $b \in (\leqslant 1R)^{\mathcal{I}}$. In order to satisfy this constraint, the two R-fillers c, d of b must be identified with each other. Thus:

- *If an at-most number restriction is violated then the algorithm must identify different role fillers.*

The individual $c = d$ must belong to both $A^{\mathcal{I}}$ and $B^{\mathcal{I}}$, which together with $c = d \in (\neg A \sqcup \neg B)^{\mathcal{I}}$ always leads to a clash. Thus, the search for a counterexample to the subsumption relationship fails, and the algorithm concludes that $(\exists R.A) \sqcap (\exists R.B) \sqcap \leqslant 1R \sqsubseteq \exists R.(A \sqcap B)$.

3 Kernel Contractions

A *belief base* is a knowledge state represented by a set of sentences not necessarily closed under logical consequence. Similarly, a *belief set* is a set of sentences closed under logical consequence. In general, a belief set is infinite, being this the main reason of the impossibility to deal with this kind of sets in a computer. Instead, it is possible to characterize the properties that must satisfy each of the change operations on finite representations of a knowledge state.

The classic operations in the theory change [1] are *expansions*, *contractions*, and *revisions*. An *expansion*, noted with "$+$", adds a new belief to the epistemic state, without guaranteeing its consistency after the operation. A *contraction*, noted with "$-$", eliminates a belief α from the epistemic state and, some of those beliefs that make possible its deduction. The sentences to eliminate might represent the *minimal change* on the epistemic state. Finally, a *revision* , noted with "$*$", inserts sentences to the epistemic state, guaranteeing consistency (if it was consistent before the operation). This means that a revision adds a new belief and perhaps it eliminates others in order to avoid inconsistencies.

The *kernel contraction* operator is identically applicable to belief bases and sets. It consist of a contraction operator capable of the selection and elimination of those beliefs in K that contribute to infer α.

Definition 3.1 - Set of Kernels [6]: Let K be a set of sentences and α a sentence. The set $K^{\perp\!\!\perp}\alpha$, called *set of kernels* is the set of sets K' such that (1) $K' \subseteq K$, (2) $K' \vdash \alpha$, and (3) if $K'' \subset K'$ then $K'' \nvdash \alpha$. The set $K^{\perp\!\!\perp}\alpha$ is also called *set of α-kernels* and each one of its elements are called *α-kernel*.

For the success of a contraction operation, we need to eliminate, at least one element of each α-kernel. The elements to be eliminated are selected by an *incision function*.

Definition 3.2 - Incision Function [6]: Let K be a set of sentences and "σ" be an *incision function* for it such that for any sentence α it verifies, (1) $\sigma(K^{\perp\!\!\perp}\alpha) \subseteq \bigcup(K^{\perp\!\!\perp}\alpha)$ and (2) If $K' \in K^{\perp\!\!\perp}\alpha$ and $K' \neq \emptyset$ then $K' \cap \sigma(K^{\perp\!\!\perp}\alpha) \neq \emptyset$.

Definition 3.3 - Kernel Contraction Determined by "σ" [6]: Let K be a set of sentences, α a sentence, and $K^{\perp\!\!\perp}\alpha$ the set of α-kernels of K. Let "σ" be an incision function for K. The operator "$-_\sigma$", called *kernel contraction determined by* "σ", is defined as, $K -_\sigma \alpha = K \backslash \sigma(K^{\perp\!\!\perp}\alpha)$.

Finally, an operator "$-$" is a kernel contraction operator for K if and only if there exists an incision function "σ" such that $K - \alpha = K -_\sigma \alpha$ for all sentence α.

4 Model Contractions

In formalisms like description logics, the reasoning service is model based, *i.e.*, in order to make deductions the reasoning service looks into every possible world, this means that new deductions are made not only by a classical inference operator "$\vdash$", but also by applying it over every model of the base. Therefore, and considering the motivation exposed in the introductory section of this paper, our interest relays in checking satisfiability for a given possibly complex query α, and considering an affirmative response, breaking thereafter its trueness by applying a base contraction by α .

In this sense, let first analyze the scope of a modeling inference operator "$\models$" such that,

$$\textbf{(Entailment)}\ \Sigma \models \alpha \ \textit{iff}\ \mathcal{M}(\Sigma) \subseteq \mathcal{M}(\{\alpha\})$$

where $\mathcal{M}(\Sigma)$ makes reference to the set of models of a knowledge base Σ, and $\mathcal{M}(\{\alpha\})$ identifies the set of models for a valid sentence α of the language.

Afterwards, a query α to the base Σ, noted as $\Sigma \models^? \alpha$, is solved by using a satisfiability checking process. This is done by generating all possible interpretations that satisfies every sentence in the base Σ, *i.e.*, finding every model for the base Σ. Afterwards, the reasoning process checks for every element in $\mathcal{M}(\Sigma)$, *i.e.*, if every model of the base, let say $\mathcal{M}_i$ is also a model of α. This is exactly the entailment definition.r If this is true, the query is said to be verified and $\Sigma \models \alpha$, *i.e.*, that the base infers α, being `YES` the answer for the query. If the base verifies $\neg\alpha$ then it is said that the query is not verified, being `NO` its answer. The third possibility would be known as an indecision and answered with an `UNKNOWN` if it is not verified α nor $\neg\alpha$.

As seen in Sect. 2.2, new knowledge may be inferred from assumptions made as a consequence of the applied satisfiability process following a tableau-based algorithm. From now on we focuss our attention in checking and adapting the theory change definitions cited in Sect. 3 applied to these extensions.

4.1 Extended Set of Kernels

Following the definitions cited in Sect. 3, a proof for a belief α is given by a set of minimal proofs or α-kernels. But this proofs are enclosed inside the knowledge base Σ itself. By using model based reasoning services, beliefs outside the scope of the base are assumed, exceeding the basic *set of kernels* given in Def. 3.1. Thereafter some definitions of the belief revision model should be redefined in order to be adapted to this new theory. In this sense, let first refer to each model $\mathcal{M}_i$ as an extension of the base Σ from now on noted as Σ_i identified as its i^{th} extension as follows.

Definition 4.1.1 - Base Extension: Let Σ be a knowledge base and $\mathcal{M}(\Sigma) = \{\mathcal{M}_1, ..., \mathcal{M}_n\}$ the set of $\mathcal{M}_i$ models of Σ, where $1 \leq i \leq n$, *i.e.*, $|\mathcal{M}(\Sigma)| = n$. The i^{th} base extension with respect to the model $\mathcal{M}_i$ is identified as Σ_i.

A model $\mathcal{M}_i$ may include several minimal proofs K for a belief α, *i.e.*, $K \subseteq \mathcal{M}_i$, where $\mathcal{M}_i \in \mathcal{M}(\Sigma)$. This also means that every α-proof K is part of a base extension Σ_i and indeed, K is also an element of the set of proofs of that base extension, *i.e.*, $K \in {\Sigma_i}^{\perp\!\!\perp}\alpha$. Moreover, as part of an α-proof K there are *effective* beliefs belonging to the base Σ (this is a subset $K_\Sigma \subseteq K$) and some other *assumed* beliefs that are not part of the base Σ but of some model $\mathcal{M}_i$ of it (equivalently a subset $K_{\mathcal{M}_i} \subseteq K$). Therefore, an α-proof K is such that $K = K_\Sigma \cup K_{\mathcal{M}_i}$. This motivates the following definition by reinforcing the original set of kernels and their components.

Definition 4.1.2 - Extended Set of Kernels: Let Σ be a knowledge base and α be a sentence. The condition $\Sigma \models \alpha$ holds *iff* there exists a set $\Sigma^{\perp\!\!\perp}\alpha$ (namely *extended set of kernels*) of *non-empty* sets ${\Sigma_i}^{\perp\!\!\perp}\alpha$ (called i^{th} *set of extended* α*-kernels*) where each ${\Sigma_i}^{\perp\!\!\perp}\alpha$ is a set of sets K (namely *extended* α*-kernel*) such that the following conditions hold:

(1) $K = K_\Sigma \cup K_{\mathcal{M}_i}$, such that

$i.$ $K_\Sigma \subseteq \Sigma$

$ii.$ $K_{\mathcal{M}_i} \subseteq \mathcal{M}_i$ and $K_{\mathcal{M}_i} \not\subseteq \Sigma$

(2) $K \vdash \alpha$

(3) If $K' \subset K$ then $K' \nvdash \alpha$

The following observation relates an extended set of kernels $\Sigma^{\perp\!\!\perp}\alpha$ and the set of models $\mathcal{M}(\Sigma)$ for a knowledge base Σ in terms of cardinality.

Observation 4.1.3: If $\Sigma^{\perp\!\!\perp}\alpha$ is an *extended set of kernels* then $|\Sigma^{\perp\!\!\perp}\alpha| = |\mathcal{M}(\Sigma)|$.
Proof:
Supposing by the contrary that $|\Sigma^{\perp\!\!\perp}\alpha| = |\mathcal{M}(\Sigma)|$ does not hold, then two possibilities arise:

(1) If $|\Sigma^{\perp\!\!\perp}\alpha| < |\mathcal{M}(\Sigma)|$ then there exists a model $\mathcal{M}_i \in \mathcal{M}(\Sigma)$ such that there is no $K \subseteq \mathcal{M}_i$ where $K \vdash \alpha$. Therefore, the related i^{th} *set of extended α-kernels* is such that ${\Sigma_i}^{\perp\!\!\perp}\alpha = \emptyset$, but as we know by Def. 4.1.2 this is not plausible, thereafter ${\Sigma_i}^{\perp\!\!\perp}\alpha \notin \Sigma^{\perp\!\!\perp}\alpha$. Besides, by Def. 4.1.2 a condition $\mathcal{M}(\Sigma) \subseteq \mathcal{M}(\{\alpha\})$ holds, therefore $\mathcal{M}_i \in \mathcal{M}(\{\alpha\})$, then necessarily there is a $K \subseteq \mathcal{M}_i$ where $K \vdash \alpha$. *(ABSURD).*

(2) If $|\Sigma^{\perp\!\!\perp}\alpha| > |\mathcal{M}(\Sigma)|$ then there exists a i^{th} *set of extended α-kernels* such that ${\Sigma_i}^{\perp\!\!\perp}\alpha \in \Sigma^{\perp\!\!\perp}\alpha$, with $K \in {\Sigma_i}^{\perp\!\!\perp}\alpha$, where $K \vdash \alpha$. This is verified in a model $\mathcal{M}_i$ in which ${\Sigma_i}^{\perp\!\!\perp}\alpha$ is applied, such that $\mathcal{M}_i \notin \mathcal{M}(\Sigma)$. This implies that $\mathcal{M}_i$ is not a model for Σ, but $\mathcal{M}_i$ is a model for α due to the existence of $K \subseteq \mathcal{M}_i$ where $K \vdash \alpha$, therefore $\mathcal{M}(\Sigma) \nsubseteq \mathcal{M}(\{\alpha\})$. *(ABSURD).* ■

An α-proof K in a model based system may or may not contain assumed beliefs. Note that a knowledge base Σ deducing α, *i.e.*, $\Sigma \models \alpha$ may, in fact, also verify $\Sigma \vdash \alpha$. This means that there is at least one proof that does not need to use any assumed beliefs to achieve its validity. The following definition is given in order to clearly identify the two different kinds of α-proofs in a model based system.

Definition 4.1.4 - Possible (Effective) α-Proof: Let $\Sigma^{\perp\!\!\perp}\alpha$ be an extended set of kernels as specified in Def. 4.1.2 for a given sentence α, and ${\Sigma_i}^{\perp\!\!\perp}\alpha$ the i^{th} set of extended α-kernels K in it contained such that $K = K_\Sigma \cup K_{\mathcal{M}_i}$. Then an α-proof K is referred as a *Possible (*resp. of *Effective) α-Proof* *iff* $K_{\mathcal{M}_i} \subseteq K$ is such that $K_{\mathcal{M}_i} \neq \emptyset$ (resp. of $K_{\mathcal{M}_i} = \emptyset$).

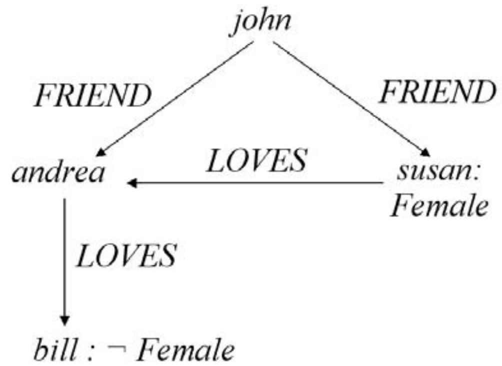
Observation 4.1.5: If K is an effective α-proof in Σ then $\Sigma \vdash \alpha$
Proof:
If K is an effective α-proof then by Def. 4.1.4 it follows that $K_{\mathcal{M}_i} = \emptyset$ for $K_{\mathcal{M}_i} \subseteq K$, hence $K \vdash \alpha$, and since $K \subseteq \Sigma$ and the consequence operator verifies *monotony*[1], it follows $\Sigma \vdash \alpha$. ■

Remark 4.1.6: All considered examples in this paper are reduced in a sense of relevance considering the extended α-proofs. This means that, although it is true that $|\Sigma^{\perp\!\!\perp}\alpha| = |\mathcal{M}(\Sigma)|$, since it is possible to have ${\Sigma_i}^{\perp\!\!\perp}\alpha, {\Sigma_j}^{\perp\!\!\perp}\alpha \in \Sigma^{\perp\!\!\perp}\alpha$ such that ${\Sigma_i}^{\perp\!\!\perp}\alpha \subseteq {\Sigma_j}^{\perp\!\!\perp}\alpha$, those ${\Sigma_i}^{\perp\!\!\perp}\alpha$ self contained by other i^{th} *set of extended α-kernel* are considered not relevant, and finally discarded.

The following example, borrowed from [5], shows the behavior of the new defined theory.

Example 1 : Given a knowledge base Σ as follows,

$$\Sigma = \left\{ \begin{array}{l} FRIEND(john, susan) \\ FRIEND(john, andrea) \\ LOVES(susan, andrea) \\ LOVES(andrea, bill) \\ Female(susan) \\ \neg Female(bill) \end{array} \right\}$$



[1]**(Monotony)** If $\Sigma' \subseteq \Sigma$ then $C_n(\Sigma') \subseteq C_n(\Sigma)$, for a given operator C_n such that $\alpha \in C_n(\Sigma)$ *iff* $\Sigma \vdash \alpha$.

we want to know if $john$ have a $Female$ $FRIEND$ who $LOVES$ a male (*i.e.*, not $Female$) person. This is a query $\Sigma \models^{?} \alpha$ such that α is $\exists FRIEND.(Female \sqcap (\exists LOVES.\neg Female))(john)$. Following the given tableau specifications as a model based reasoning service we have two different models in order to achieve satisfiability of α, furthermore, the extended set of kernels would be such that $\Sigma^{\perp\!\!\perp}\alpha = \{\Sigma_1{}^{\perp\!\!\perp}\alpha, \Sigma_2{}^{\perp\!\!\perp}\alpha\}$, where each $\Sigma_i{}^{\perp\!\!\perp}\alpha$ is directly related to each model $\mathcal{M}_i$. Note that in this example there is only one α-proof K in each extended set of α-kernels.

$\mathcal{M}_1)$ $Female^{\mathcal{I}} = \{\}, \neg Female^{\mathcal{I}} = \{andrea\}$
$\Sigma_1{}^{\perp\!\!\perp}\alpha = \{\{FRIEND(john, susan), Female(susan), LOVES(susan, andrea), \neg Female(andrea)\}\}$
$K_{\mathcal{M}_1} = \{\neg Female(andrea)\}$

$\mathcal{M}_2)$ $Female^{\mathcal{I}} = \{andrea\}, \neg Female^{\mathcal{I}} = \{\}$
$\Sigma_2{}^{\perp\!\!\perp}\alpha = \{\{FRIEND(john, andrea), Female(andrea), LOVES(andrea, bill), \neg Female(bill)\}\}$
$K_{\mathcal{M}_2} = \{Female(andrea)\}$

□

4.2 Model Selection & Model Incision Function

Let think about an incision function cutting beliefs from a set of kernels in order to achieve a contraction of Σ by α, *i.e.*, we want to get some applied contraction operator " $\ominus$ " based on an incision function "σ" such that the query α turns to fail in the resultant knowledge base, this means that $\Sigma \ominus_{\sigma} \alpha \not\models \alpha$.

In order to achieve such an operation we only need to select one model, *i.e.*, an extension Σ_i, from which to break every α-proof in it such that the erasure is to be done from every K in $\Sigma_i{}^{\perp\!\!\perp}\alpha$ for some i. Moreover, those deleted beliefs from an α-proof should be "*effective*" beliefs, *i.e.*, beliefs from K_{Σ}, not assumed ones. The intuition in this is that no assumed belief could be cut off the knowledge base just because it is not part of the base. No justification can be supported in order to modify the knowledge base by making a specific *possible world* the new epistemic state.

In this sense, let first define a *model selection function* ρ, that following some *preference criterion* among the considered models, it takes the "*most conservative selection*" such that among every considered model, the selected one is the best choice to make a suitable further incision. Formally,

Definition 4.2.1 - Model Selection Function: Let Σ be a knowledge base and $\Sigma^{\perp\!\!\perp}\alpha$ an *extended set of kernels* for a valid sentence α, then a function "ρ" is a *model selection function* determined by some *preference criterion* such that $\rho(\Sigma^{\perp\!\!\perp}\alpha) = \Sigma_i{}^{\perp\!\!\perp}\alpha$, where $\Sigma_i{}^{\perp\!\!\perp}\alpha$ is valid in a model $\mathcal{M}_i$ considered the "*less relevant model*" of Σ.

Inspired in Def. 3.2, we propose a "*model incision function*" determined by a *model selection function* as follows,

Definition 4.2.2 - Model Incision Function: Let Σ be a knowledge base, $\Sigma^{\perp\!\!\perp}\alpha$ an *extended set of kernels* for a valid sentence α, and ρ a *model selection function*, then a function "σ" is defined as a *model incision function* such that it verifies,

(1) $\sigma(\rho(\Sigma^{\perp\!\!\perp}\alpha)) \subseteq \bigcup(\rho(\Sigma^{\perp\!\!\perp}\alpha)) \cap \Sigma$

(2) If $K_{\Sigma} \subseteq K \in \rho(\Sigma^{\perp\!\!\perp}\alpha)$ then $K_{\Sigma} \cap \sigma(\rho(\Sigma^{\perp\!\!\perp}\alpha)) \neq \emptyset$

Example 2 : Following the proposed definition for the *model incision function*, let continue with Ex. 1. As explained before, an incision function cuts beliefs only from one base extension in order to break all proofs in the possible world by it determined. In this sense, suppose a *model selection function* selects the first base extension as the *most suitable model*, such that $\rho(\Sigma^{\perp\!\!\perp}\alpha) = \Sigma_1{}^{\perp\!\!\perp}\alpha$. Therefore, a *model incision function* would be applied to it in order to cut beliefs from every proof in the model $\mathcal{M}_1$. Finally a possibility would be $\sigma(\rho(\Sigma^{\perp\!\!\perp}\alpha)) = \sigma(\Sigma_1{}^{\perp\!\!\perp}\alpha) = \{Female(susan)\}$. [2]□

4.3 Model Contraction Operator

In what follows let define the "*model contraction operator*" by reinforcing the kernel contractions as exposed in Def. 3.3, then a model contraction operator " $\ominus$ " is determined by a *model incision function* "σ" as follows,

Definition 4.3.1 - Model Contraction Determined by σ: Let Σ be a knowledge base, α be a sentence, "σ" be a *model incision function* determined by a *model selection function* ρ for Σ, and $\Sigma^{\perp\!\!\perp}\alpha$ be the extended set of α-kernels of Σ to which the function ρ is applied. The operator " $\ominus_\sigma$", referred as *model contraction determined by* "σ", is defined as,

$$\Sigma \ominus_\sigma \alpha = \Sigma \backslash \sigma(\rho(\Sigma^{\perp\!\!\perp}\alpha))$$

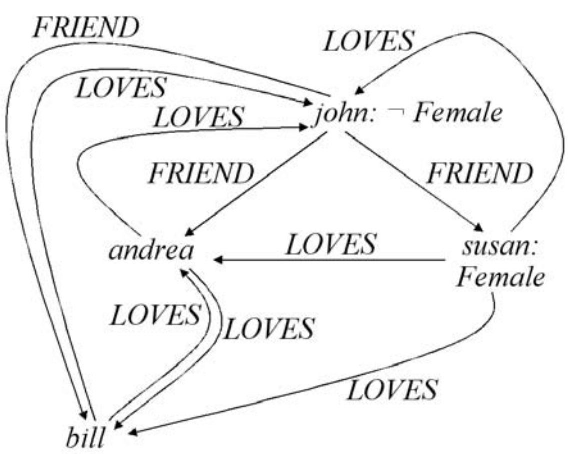
Finally, an operator " $\ominus$ " is a *model contraction* operator for Σ if and only if there exists a *model incision function* "σ" such that $\Sigma \ominus \alpha = \Sigma \ominus_\sigma \alpha$ for all sentence α.

Example 3 : Let conclude with the Ex. 1, and finally apply the contraction operator $\ominus$ to the selection made before in Ex. 2, where $\sigma(\rho(\Sigma^{\perp\!\!\perp}\alpha)) = \{\neg Female(susan)\}$. Then from Def. 4.3.1 follows that, $\Sigma \ominus_\sigma \alpha = \Sigma \backslash \sigma(\rho(\Sigma^{\perp\!\!\perp}\alpha)) = \Sigma \backslash \{Female(susan)\}$. □

Remark 4.3.2: Note that in Ex. 3, the operator " $\ominus$ " successfully achieves the contraction of α such that $\Sigma \ominus_\sigma \alpha \not\models \alpha$. This is directly related to the introductory equivalence defined in Sect. 4 in which a necessary condition, in order to verify $\Sigma \models \alpha$, is that α needs to be modeled by every model in Σ, or equivalently $\mathcal{M}(\Sigma) \subseteq \mathcal{M}(\{\alpha\})$.

Example 4 : Let α be the same query used in previous examples such that $\alpha = \exists FRIEND.(Female \sqcap (\exists LOVES.\neg Female))(john)$, and let Σ be a knowledge base such that,

$$\Sigma = \left\{ \begin{array}{l} FRIEND(john, susan) \\ FRIEND(john, andrea) \\ FRIEND(john, bill) \\ LOVES(susan, andrea) \\ LOVES(andrea, bill) \\ LOVES(andrea, john) \\ LOVES(susan, bill) \\ LOVES(bill, john) \\ LOVES(bill, andrea) \\ LOVES(susan, john) \\ Female(susan) \\ \neg Female(john) \end{array} \right\}$$



The following are the interpretation sets related to each model $\mathcal{M}_i$:

[2]Note that the incision made does not relay on any epistemic condition and has being arbitrarily taken.

$\mathcal{M}_1)$ $Female^{\mathcal{I}} = \{andrea, bill\}, \neg Female^{\mathcal{I}} = \{\}$ $\mathcal{M}_2)$ $Female^{\mathcal{I}} = \{bill\}, \neg Female^{\mathcal{I}} = \{andrea\}$
$\mathcal{M}_3)$ $Female^{\mathcal{I}} = \{andrea\}, \neg Female^{\mathcal{I}} = \{bill\}$ $\mathcal{M}_4)$ $Female^{\mathcal{I}} = \{\}, \neg Female^{\mathcal{I}} = \{andrea, bill\}$

Finally, the extended α-kernels $\Sigma_i{}^{\Perp}\alpha$ in each model $\mathcal{M}_i$, would be:

$$\Sigma_1{}^{\Perp}\alpha$$

$$\left\{\begin{array}{l} FRIEND(john, susan), Female(susan), \\ LOVES(susan, john), \neg Female(john) \end{array}\right\}$$
$$\left\{\begin{array}{l} FRIEND(john, andrea), Female(andrea), \\ LOVES(andrea, john), \neg Female(john) \end{array}\right\}$$
$$\left\{\begin{array}{l} FRIEND(john, bill), Female(bill), \\ LOVES(bill, john), \neg Female(john) \end{array}\right\}$$

$$\Sigma_2{}^{\Perp}\alpha$$

$$\left\{\begin{array}{l} FRIEND(john, susan), Female(susan), \\ LOVES(susan, john), \neg Female(john) \end{array}\right\}$$
$$\left\{\begin{array}{l} FRIEND(john, bill), Female(bill), \\ LOVES(bill, john), \neg Female(john) \end{array}\right\}$$
$$\left\{\begin{array}{l} FRIEND(john, bill), Female(bill), \\ LOVES(bill, andrea), \neg Female(andrea) \end{array}\right\}$$
$$\left\{\begin{array}{l} FRIEND(john, susan), Female(susan), \\ LOVES(susan, andrea), \neg Female(andrea) \end{array}\right\}$$

$$\Sigma_3{}^{\Perp}\alpha$$

$$\left\{\begin{array}{l} FRIEND(john, susan), Female(susan), \\ LOVES(susan, john), \neg Female(john) \end{array}\right\}$$
$$\left\{\begin{array}{l} FRIEND(john, susan), Female(susan), \\ LOVES(susan, bill), \neg Female(bill) \end{array}\right\}$$
$$\left\{\begin{array}{l} FRIEND(john, andrea), Female(andrea), \\ LOVES(andrea, bill), \neg Female(bill) \end{array}\right\}$$
$$\left\{\begin{array}{l} FRIEND(john, andrea), Female(andrea), \\ LOVES(andrea, john), \neg Female(john) \end{array}\right\}$$

$$\Sigma_4{}^{\Perp}\alpha$$

$$\left\{\begin{array}{l} FRIEND(john, susan), Female(susan), \\ LOVES(susan, john), \neg Female(john) \end{array}\right\}$$
$$\left\{\begin{array}{l} FRIEND(john, susan), Female(susan), \\ LOVES(susan, andrea), \neg Female(andrea) \end{array}\right\}$$
$$\left\{\begin{array}{l} FRIEND(john, susan), Female(susan), \\ LOVES(susan, bill), \neg Female(bill) \end{array}\right\}$$

A *model selection function* would select one of the considered base extensions in order to cut beliefs from each α-proof in it contained by applying a *model incision function* "σ". In such a case, let consider $\rho(\Sigma^{\Perp}\alpha) = \Sigma_4{}^{\Perp}\alpha$ as the selected i^{th} set of extended α-kernels, then the *model incision function determined by "ρ"* would be $\sigma(\rho(\Sigma^{\Perp}\alpha)) = \sigma(\Sigma_4{}^{\Perp}\alpha) = \{Female(susan)\}$. [3] Finally, the application of a *model contraction* would be $\Sigma \ominus_{\sigma} \alpha = \Sigma \backslash \{Female(susan)\}$. □

Remark 4.3.3: In the Ex. 4 it is shown that if the base deduces by itself α, *i.e.*, that there exists an *effective α-proof*, therefore the respective proof would be part of every set of extended kernels.

4.4 Anti-Shielding Model Contraction

The proposal of a contraction operation not verifying the success postulate is discussed in several works, in order to analyze the success of a model contraction, let first propose the success and inclusion postulates, inspired in the AGM postulates for contractions originally defined in [1].

(Success) If $\not\vdash \alpha$ then $\Sigma \ominus \alpha \not\models \alpha$
(Inclusion) $\Sigma \ominus \alpha \subseteq \Sigma$

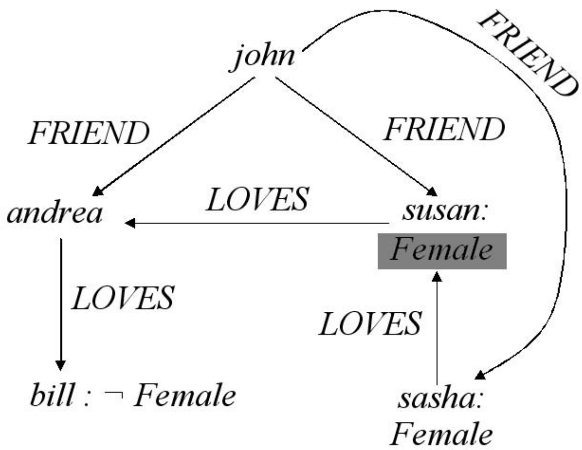
Although the success and inclusion postulates have being verified in previous examples, this is not always possible. Moreover, while inclusion is always verified by a model contraction as exposed before, success does not. In a model contraction as previously specified, some information in a knowledge base may generate the non-satisfiability of the *success postulate*. This information is

[3] As seen before, we adopt an incision which minimizes the quantity of beliefs to be cut off.

being referred as *shielding set* [4] since it represents an epistemic state supporting (in "*background*") an inference of α even after its model contraction. This is clearly shown by the following example:

Example 5 : Let Σ be a knowledge base as exposed below, and α be the same query used in Ex. 1 such that $\alpha = \exists FRIEND.(Female \sqcap (\exists LOVES.\neg Female))(john)$.

$$\Sigma = \left\{ \begin{array}{l} FRIEND(john, susan) \\ FRIEND(john, andrea) \\ FRIEND(john, sasha) \\ LOVES(susan, andrea) \\ LOVES(andrea, bill) \\ LOVES(sasha, susan) \\ Female(susan) \\ \neg Female(bill) \\ Female(sasha) \end{array} \right\}$$



Therefore, the extended α-kernels $\Sigma_i{}^{\perp\!\!\perp}\alpha$ would be:

$\mathcal{M}_1$) Model $\mathcal{M}_1$ is such that $\neg Female^{\mathcal{I}} = \{andrea\}, Female^{\mathcal{I}} = \emptyset$

$$\Sigma_1{}^{\perp\!\!\perp}\alpha = \left\{ \left\{ \begin{array}{l} FRIEND(john, susan), Female(susan), \\ LOVES(susan, andrea), \neg Female(andrea) \end{array} \right\} \right\}$$

$\mathcal{M}_2$) Model $\mathcal{M}_2$ is such that $\neg Female^{\mathcal{I}} = \emptyset, Female^{\mathcal{I}} = \{andrea\}$

$$\Sigma_2{}^{\perp\!\!\perp}\alpha = \left\{ \left\{ \begin{array}{l} FRIEND(john, andrea), Female(andrea), \\ LOVES(andrea, bill), \neg Female(bill) \end{array} \right\} \right\}$$

A *model selection function* ρ may solve the selection as $\rho(\Sigma^{\perp\!\!\perp}\alpha) = \Sigma_1{}^{\perp\!\!\perp}\alpha$, then the *model incision function "σ" determined by "ρ"* would be $\sigma(\rho(\Sigma^{\perp\!\!\perp}\alpha)) = \sigma(\Sigma_1{}^{\perp\!\!\perp}\alpha) = \{Female(susan)\}$, and therefore, the application of a *model contraction* would result as $\Sigma \ominus_\sigma \alpha = \Sigma \backslash \{Female(susan)\}$.

Note that the success postulate does not hold due to the existence of a *shielding set* H:

$$H = \{FRIEND(john, sasha), Female(sasha), LOVES(sasha, susan)\}$$

where $H \cup \{\neg Female(susan)\} \vdash \alpha$, such that $\neg Female(susan)$ is, after the application of the model contraction, a new assumed belief that helps to verify the query α, and as seen before, its opposite $Female(susan)$ also helps to verify the same query α. So by the application of the contraction as before, we have just generated a new possible world, and afterwards the original query applied to the resultant knowledge base $\Sigma \ominus \alpha \models \alpha$ does still hold. □

Note that although the success postulate does not hold, there is a "weak" version always verified:

(Weak-Success) If $\not\vdash \alpha$ then $\Sigma \ominus \alpha \not\vdash \alpha$

Observation 4.4.1: A *model contraction* operator " $\ominus$ " determined by a *model incision function* "σ" does verify *weak-success*.
Proof:

[4]The notion of *shielding* was borrowed from [3], where a contraction operator not verifying success was proposed.

Two possibilities arise relaying on the existence of effective α-proofs in Σ. Suppose there is no effective α-proof then from Obs. 4.3.3, $\Sigma \nvdash \alpha$ holds, and since the classic consequence "$\vdash$" verifies *monotony* and " $\ominus$ " verifies *inclusion* it follows that $\Sigma \ominus \alpha \nvdash \alpha$. For the other case in which we suppose that there exists an effective α-proof K in Σ, such that $K \in \rho(\Sigma^{\perp\!\!\perp}\alpha)$, it follows that $K_{\mathcal{M}} = \emptyset$ for $K_{\mathcal{M}} \subseteq K$ and therefore $K = K_{\Sigma}$. Hence, from Def. 4.2.2 an incision function "σ" verifies $K_{\Sigma} \cap \sigma(\rho(\Sigma^{\perp\!\!\perp}\alpha)) \neq \emptyset$, and since $\Sigma \ominus \alpha = \Sigma \backslash \sigma(\rho(\Sigma^{\perp\!\!\perp}\alpha))$ it follows that $K_{\Sigma} \nsubseteq \Sigma \ominus \alpha$. Finally $\Sigma \ominus \alpha$ has no effective α-proof and therefore $\Sigma \ominus \alpha \nvdash \alpha$. ■

The latter example motivates the proposal of a new postulate in order to avoid any *shielding set* in the resultant knowledge base $\Sigma \ominus \alpha$.

(Anti-Shielding) If $\beta \in \Sigma$ and $\beta \notin \Sigma \ominus \alpha$ then $H \cup \{\neg\beta\} \not\models \alpha$ for any $H \subseteq \Sigma \ominus \alpha$

Definition 4.4.2 - Anti-Shielding Model Contraction: Let " $\ominus$ " be a model contraction operator satisfying the *anti-shielding*, then it is referred as *anti-shielding model contraction operator*.

The latter postulate may not only be a property satisfied by some model contractions, this means that by considering the anti-shielding postulate at the time the incision function is being applied, we could always achieve an anti-shielding model contraction operator " $\ominus$ ". A model contraction satisfying anti-shielding is always a desirable operator due to the following observation:

Observation 4.4.3: An *anti-shielding model contraction* operator " $\ominus$ " does verify *success*.
Proof:

If " $\ominus$ " verifies *anti-shielding* it follows that for any $\beta \in \Sigma$ and $\beta \notin \Sigma \ominus \alpha$ then $H \cup \{\neg\beta\} \not\models \alpha$ for any $H \subseteq \Sigma \ominus \alpha$. This proof is shown by supposing to the contrary that *success* is not verified (*i.e.*, $\Sigma \ominus \alpha \models \alpha$).

By Obs. 4.4.1 we know that any model contraction operator does always verify *weak-success* (*i.e.*, $\Sigma \ominus \alpha \nvdash \alpha$), and hence no effective α-proof exists in $\Sigma \ominus \alpha$. This means that there are only possible α-proofs in $\Sigma \ominus \alpha$, thus, we have at least two models $\mathcal{M}_i$ and $\mathcal{M}_j$ with assumed beliefs $\varphi \in \mathcal{M}_i$ and $\neg\varphi \in \mathcal{M}_j$. Therefore, there exist at least two possible α-proofs $K \in ((\Sigma \ominus \alpha)_i{}^{\perp\!\!\perp}\alpha)$ and $K' \in ((\Sigma \ominus \alpha)_j{}^{\perp\!\!\perp}\alpha)$ such that $K = K_{(\Sigma \ominus \alpha)} \cup K_{\mathcal{M}_i}$ and $K' = K'_{(\Sigma \ominus \alpha)} \cup K'_{\mathcal{M}_j}$, where $\varphi \in K_{\mathcal{M}_i}$ and $\neg\varphi \in K'_{\mathcal{M}_j}$.

This situation is always captured by a model contraction operator, this means that φ was not an assumed belief in Σ (*i.e.*, $\varphi \in \Sigma$), and it has being cut off such that $\varphi \notin \Sigma \ominus \alpha$. Moreover, since $K'_{(\Sigma \ominus \alpha)} \cup K'_{\mathcal{M}_j} \vdash \alpha$ and $\neg\varphi \in K'_{\mathcal{M}_j}$, we have that $K'_{(\Sigma \ominus \alpha)} \cup \{\neg\varphi\} \models \alpha$. Note that assuming $\beta = \varphi$ and $H = K'_{(\Sigma \ominus \alpha)}$ we achieve the absurd $H \cup \{\neg\beta\} \models \alpha$, contradicting *anti-shielding*.

Finally, *success* is verified (*i.e.*, $\Sigma \ominus \alpha \not\models \alpha$). ■

5 Conclusions & Future Work

Some of the theory change classic definitions and postulates exposed in [1] have being generalized in [4] in order to match extra-classic logics like DLs. Considering DLs reasoning services like tableau-based algorithms to solve satisfiability, not only set us up in a more direct theory formalization, but also allow us to work purely description languages with no need to translate beliefs to fragments of first order logic, as we have done before in [7]. Tableaux algorithms are nowadays probably the most important reasoning algorithms used in the area. A distinctive feature of this reasoning service is the way it reasons about incomplete information, inferring new beliefs by combining assumed ones (from the generated models) and effective knowledge (from the base). All this happens while proving clauses' satisfiability, and indeed, while contracting a sentence from the base. By this, we have a completely different way of reasoning about knowledge due to a multiple generation of base extensions. This all is what motivates the definition of a model contraction and its several components.

Basically, a model contraction is a kind of kernel contraction reinforced in a way such that possible worlds are considered when proving knowledge satisfiability. In this sense, the set of deductions made by this type of reasoning service exceeds the traditional deductions made in a knowledge base. Therefore, a set of kernels in a model contraction (namely extended set of kernels) should consider assumed information, *i.e.*, those beliefs that are part of some possible worlds only. Moreover, an incision function should cut beliefs off in a given model in order to break a proof, for this matter a selection function is defined in order to decide which model is preferable to be incised. Finally, a contraction eliminates those incised beliefs from the knowledge base thus achieving the elimination of the proof in some possible world. Hence, the sentence at issue is no longer verified.

A first approach to some model contraction postulates is given, concentrating this investigation mostly on the success postulate. Here a new type of information sets is recognized as *shielding*, which is a kind of *"protected proof"* for a sentence α standing in *"background"*. Thereafter, a special postulate is proposed in order to deal with this tradeoff, hence achieving success by the model contraction.

As said before, kernel contractions seem to be a special case of model contractions where every required proof is an effective-α proof. In this sense, a deeper investigation is being taking over in order to give a formal characterization of model contractions by means of kernels contractions, relating postulates and axiomatic representations.

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