Consolidation of Plausiblity Relations in Multi-Agent Systems

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Abstract

Within the context of multi-agent systems, an agent may often find itself in a position where it receives information through informants. These informants are independent agents who have their own interests and, therefore, are not necessarily completely reliable. It is natural for an agent to be more inclined to believe one informant over another, especially if the informant has proven itself reliable over a period of time. This preference is stored in a plausibility relation, a partial order indicating the relative credibility of the agent's informants. Through careless expansion or other means, inconsistencies may be introduced into the plausibility relation. A consolidation operator is proposed and characterized through a set of postulates. Alternative constructions are discussed. A non-prioritized revision operator for plausibility relations, based on consolidation, is also presented.

Keywords: Intelligent agent, Multi-agent systems, Plausibility relation, Belief change, revision, consolidation.

1 Introduction

Within the context of multi-agent systems, an agent may often find itself in a position where it receives information from other agents or, as we will call them throughout this paper, *informants*. These informants are independent agents who have their own interests and, therefore, are not necessarily completely reliable. In fact, on occasion an informant may provide information which is contradictory with that which is provided by another. It would be natural for our agent to be more inclined to believe one informant over another, especially if the informant has proven itself reliable over a period of time.

Situations like these are not infrequent in real-world scenarios. Take for instance a stock-investment agent. Such an agent may receive financial advice from several sources including, but not limited to, human investment advisors and web pages where such financial council is posted. These would fit the role of the previously mentioned informants. It is not hard to imagine the investing agent prefering one informant's advice over another's. Our agent could track the advice of the informants and later on compare it to actual stock market events. This comparison could then be used to somehow raise the relative plausibility of the informants who were correct and lower that of those who were not.

Another real-world scenario could be seen in the case of an agent in need of weather prediction information. There might be several weather-predicting informants available to the agent. As in the previous case, there are for example, a number of web pages providing such services. However, for a given period in time, different informants could predict different conditions. Once again, in such a case the agent would act based on the prediction of the most reliable informant and then use historic information to possibly update these relations.

This paper builds upon the concepts previously introduced by the authors in [SF00]. There, we proposed the organization of the informants into a partial order which compares the plausibility of the relevant informants. Upon noticing contradictory information from two informants, the agent need only check which informant supercedes the other in plausibility according to the partial order (i.e. which one is more *reliable*) and then act upon that informant's information. Through the use of change operators that act on the partial order, this plausibility relation can also be updated to reflect changes in perceived informant plausibility.

As a new concept, we search for a means to restore soundness to generator sets. Such sets may arise as a consequence of careless expansion or perhaps they are obtained from other sources. We propose a consolidation operator, characterized through a set of postulates. We then provide alternative constructions for this operator and, finally, we introduce a non-prioritized revision operator for plausibility relations which bases its definition on the consolidation operator.

This paper is organized as follows. Section 2 presents a summary of the notions presented in [SF00] necessary for the elaboration of the remainder of the paper. It includes our proposal for representation and expansion of plausibility relations for agents relying on informants. Section 3 defines consolidation of plausibility based relations, the alternative operators involved, their characteristic postulates and their construction. Section 4 defines non-prioritized revision of plausibility based relations, based on consolidation as introduced in the previous sections. Finally, section 5 includes this paper's conclusions as well as future work.

For a thorough presentation of the AGM model for belief revision we refer the reader to [AGM85]. Some interesting related work on comparing the credibility of information sources can be found in [Par98, Res76].

2 Representation and revision of plausibility relations

This section contains elements presented in [SF00]. Given that the remaining sections of this paper build upon these concepts, we present the necessary ones here.

2.1 The concept of generator set

Let us assume that we have a universal set of informants, $\mathcal{I}$, and that, of these informants, some are to be considered more reliable than others. This is to say, in any case in which two distinct informants provide an agent with contradictory information the more trustworthy one is to be believed over the other. The agent must, therefore, have a mechanism by means of which the set $\mathcal{I}$ is ordered. To this end we present the following concept.

Definition 2.1: Given a set of informants $\mathcal{I}$ we will call any binary relation $G \subseteq \mathcal{I}^2$ a *generator set* over $\mathcal{I}$. An informant i is *less trustworthy* than an informant j according to G if $(i, j) \in G^*$. □

Graphically, we may represent a generator set as we would a directed graph. We represent the informants in $\mathcal{I}$ as nodes. The tuples in G are then represented as directed arcs: for each tuple $(i, j) \in G$ we add an arc from node i to node j.

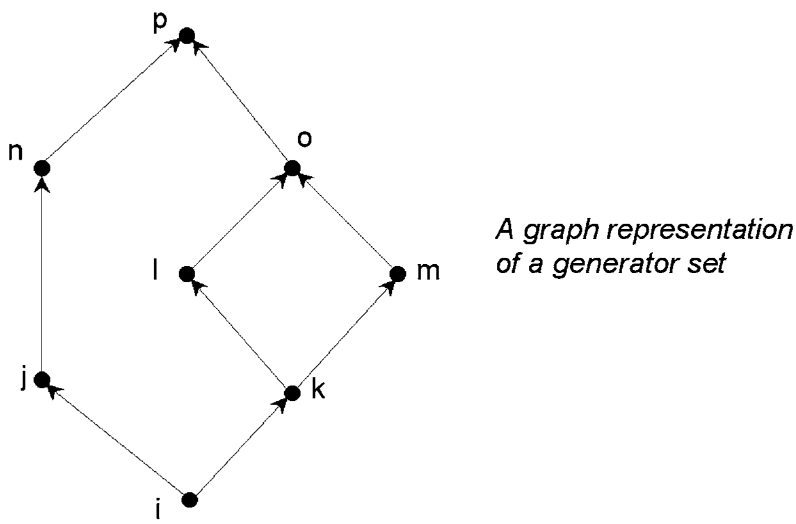

Figure 1: A graph representation of a generator set over a set of informants $\mathcal{I} = \{i, j, k, l, m, n, o, p\}$.

G^* represents the reflexive transitive closure of G. It is desirable for G^* to be a partial order over $\mathcal{I}$, although according to the preceding definition this need not always be the case. We address this matter in the following definition.

Definition 2.2: A generator set $G \subseteq \mathcal{I}^2$ is said to be *sound* if G^* is a partial order over $\mathcal{I}$. □

Example 2.1: For example the generator set $G_1 = \{(i, j), (j, k), (i, l)\}$ is sound. However $G_2 = G_1 \cup \{(k, i)\}$ is *not* sound because $(i, k) \in G_2^*$ and $(k, i) \in G_2^*$. This violates the antisymmetry condition for partial orders. □

Why is it desirable for a generator set to be sound? For a relation to be a partial order it must obey reflexivity, antisymmetry and transitivity. Given a generator set G it is obvious that its reflexive transitive closure, G^*, will obey reflexivity and transitivity. However if antisymmetry is not respected then there is at least one pair of distinct informants, i and j such that both $(i, j) \in G^*$ and $(j, i) \in G^*$. This would mean that both i is less trustworthy than j and that j is less trustworthy than i. Given that these beliefs are contradictory, believing them simultaneously would lead the believing agent to inconsistencies.

Throughout the discussions in the remainder of this paper we will sometimes speak of a tuple as being *entailed* by a generator set. This is no more than a shorthand for saying that the tuple belongs to the reflexive transitive closure of the generator set. We express this formally in the following definition.

Definition 2.3: We will say that a tuple (i, j) is *entailed* by a generator set G if $(i, j) \in G^*$. □

Example 2.2: The tuple (i, l) is among those entailed by the generator set $\{(i, j), (j, k), (k, l)\}$. When we represent a generator set graphically we will show entailed tuples of interest by using a dashed line. □

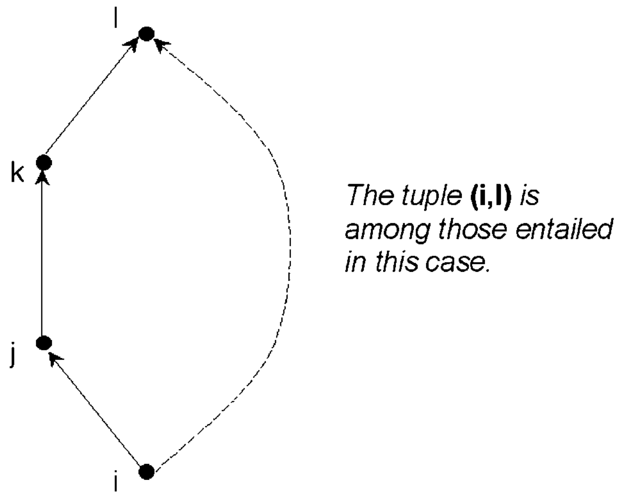

Figure 2: An entailed tuple.

Sometimes we may find tuples in a generator set which, if removed, would still be entailed by the remaining tuples. In this case we say that the tuple is *redundant* with respect to the generator set. We may also say that the generator set itself is redundant because it contains a redundant tuple. These concepts are introduced by the following definition.

Definition 2.4: Given a tuple (i, j) and a generator set G it is said that (i, j) is *redundant* in G if $(i, j) \in (G \setminus \{(i, j)\})^*$. A generator set is said to be *redundant* if it contains a redundant tuple. Otherwise, the generator set is said to be *non-redundant*. □

Example 2.3: The generator set $G = \{(i, j), (j, k), (i, k)\}$ is redundant because it contains the redundant tuple (i, k). □

2.2 Some interesting properties of Generator Sets

The following are interesting properties associated with generator sets.

• **G1**: A generator set G is sound iff G can be represented by a directed *acyclic* graph.

The fact that G may be represented by a directed graph is trivial. The fact that it must be acyclic arises from the following argument. Let us assume that G contains a cycle of length longer than one. Now let $i, j \in \mathcal{I}$, $i \neq j$ be two vertices of said cycle. Then there exists a path from i to j and from j to i. This would imply that both $(i, j) \in G^*$ and $(j, i) \in G^*$. Since this violates antisymmetry G^* cannot be a partial order and therefore G cannot be sound. By a similar argument the reverse implication may also be proven.

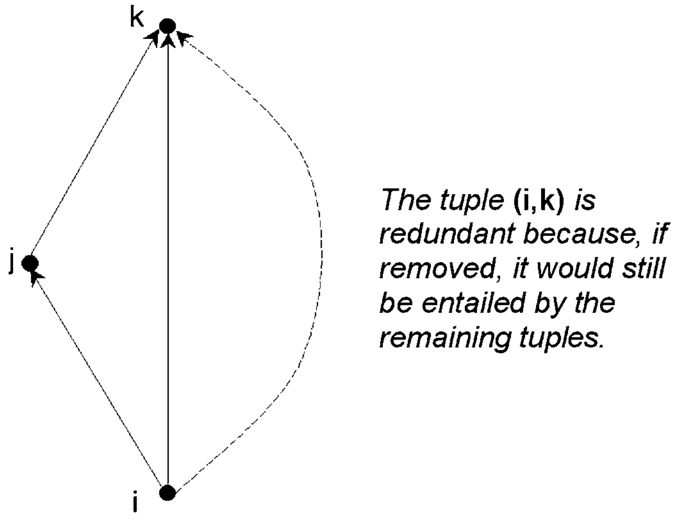

Figure 3: A redundant tuple.

• **G2**: If G is a sound generator set and $(j,i) \notin G^*$ then $G \cup \{(i,j)\}$ is a sound generator set.

Let us assume that G is a sound generator set, $(j,i) \notin G^*$ and that $G \cup \{(i,j)\}$ is *not* sound. If G is sound then it has no cycle. And if $G \cup \{(i,j)\}$ is not sound then it has a cycle and it follows that (i,j) completed it. Therefore, there was a path from j to i in G and hence $(j,i) \in G^*$. This contradicts our initial assumption.

2.3 The expansion operator

Let us assume that an agent learns that, of a pair of informants, one is more reliable than the other. This would warrant the modification of its knowledge accordingly. For this purpose, we define the operator $\oplus : \mathcal{P}(\mathcal{I}^2) \times \mathcal{I}^2 \longrightarrow \mathcal{P}(\mathcal{I}^2)$. This operator adds new tuples to a generator set in order to establish relations between informants. Given a pair of informants and a generator set, this function returns a new generator set in which said agents are now related. According to this new generator set we may say that the first informant is "less reliable" than the second.

In [SF00] we characterize expansion operators through a set of postulates. Then, a construction for an expansion operator is presented and is related to the postulates through a lemma.

• **Success**: $(i,j) \in (G \oplus (i,j))^*$.
• **Inclusion**: $G^* \subseteq (G \oplus (i,j))^*$.
• **Vacuity**: if $(i,j) \in G^*$ then $(G \oplus (i,j))^* = G^*$.
• **Commutativity**: $((G \oplus (k,l)) \oplus (i,j))^* = ((G \oplus (i,j)) \oplus (k,l))^*$.
• **Extensionality**: if $A^* = B^*$ then $(G \oplus A)^* = (G \oplus B)^*$.
• **Conditional Soundness Preservation**: if G is a sound generator set and $(j,i) \notin G^*$ then $G \oplus (i,j)$ is a sound generator set.

Definition 2.5: Given a pair of informants $i, j \in \mathcal{I}$ and generator set $G \subseteq \mathcal{I}^2$, we define the *expansion* of G by (i,j) as $G \oplus (i,j) = G \cup \{(i,j)\}$. □

Lemma 2.1: Let $\oplus$ be an expansion operator as defined in Definition 2.5. Then $\oplus$ satisfies *success*, *inclusion*, *vacuity*, *commutation*, and *extensionality*. □

Assume we have a pair of informants i and j and a generator set G, and the agent now has received information saying that i is less reliable than j. Such information could be provided by a set denoting a minimal path from i to j. It is possible for the expansion operator to be generalized to allow path as inputs as per the following definitions.

Definition 2.6: Given a pair of informants $i_1, i_n \in \mathcal{I}$, we define a *path* between them as an ordered set of tuples $path(i_1, i_n)$ of the form $\{(i_1, i_2), (i_2, i_3), \ldots, (i_{n-1}, i_n)\}$. □

Definition 2.7: Given a pair of informants $i, j \in \mathcal{I}$ and generator set $G \subseteq \mathcal{I}^2$, we define the *generalized expansion* of G by (i, j) as $G \oplus path(i, j) = G \cup path(i, j)$. □

In [SF00] operators for contraction and revision are also presented. However, for what follows, only expansion need be introduced.

3 Restoring soundness to generator sets

Situations may arise in which a given generator set is no longer sound. For example, through expansions, cycles may have been introduced. It would be interesting to have an operator that, when applied to such a set, produces a new, sound generator set based on the original. For this purpose we will define the *consolidation operator* $! : \mathcal{P}(\mathcal{I}^2) \longrightarrow \mathcal{P}(\mathcal{I}^2)$. In the following subsection we characterize this operator through postulates.

3.1 Postulates for consolidation operators

The following postulates have been modified from Hansson's works [Han96] to be applied in our framework.

- **Soundness**: $G!$ is sound.
  Soundness states that the consolidated generator set does not contain cycles.

- **Inclusion**: $G! \subseteq G$.
  This postulate states that in the process of consolidation, we can only remove pairs from the generator set.

- **Vacuity**: If G is sound then $G! = G$.
  Vacuity states that no changes occur when G is sound. Changes are only needed in the case where it is not.

- **Relevance**: If $(i, j) \in (G \setminus G!)$ then there is some G' such that $G! \subseteq G' \subseteq G$, G' is sound but $G' \oplus (i, j)$ is not sound.
  Relevance states that, if some pair (i, j) is excluded when a generator set is consolidated, it must have contributed to making the original generator set cyclic.

- **Core retainment**: If $(i, j) \in (G \setminus G!)$ then there is some G' such that $G' \subseteq G$, G' is sound but $G' \oplus (i, j)$ is not sound.
  This postulate is a weaker version of the above postulate.

3.2 Construction of consolidation operator

The following definitions are needed to define different consolidation operators.

Definition 3.1: Given a generator set $G \subseteq \mathfrak{I}^2$ we say that $i \in \mathfrak{I}$ is a *vertex* of G if for some k in $\mathfrak{I}$ it holds that $(i,k) \in G$ or $(k,i) \in G$. $\square$

Definition 3.2: Given a generator set $G \subseteq \mathfrak{I}^2$ we define a *minimal cyclic path* to be a subset H of G such that:

1. For every pair of vertices i, j in H then $(i,j) \in H^*$ and $(j,i) \in H^*$.
2. If $H' \subset H$ then H' is acyclic.

$\square$

Definition 3.3: Given a generator set $G \subseteq \mathfrak{I}^2$ we define the *set of minimal cyclic paths* as:

$$\mathbf{mcp}(G) = \{H \subseteq G : H \text{ is a minimal cyclic path of } G\}$$

$\square$

Definition 3.4: Given a generator set $G \subseteq \mathfrak{I}^2$ we define a *maximal acyclic generator set* to be a subset H of G such that:

1. For every pair of vertices i, j in H such that $(i,j) \in H^*$ then $(j,i) \notin H^*$.
2. If $H \subset H'$ then H' contains a cycle.

$\square$

Definition 3.5: Given a generator set $G \subseteq \mathfrak{I}^2$ we define the *set of maximal acyclic generator sets* as:

$$\mathbf{mags}(G) = \{H \subseteq G : H \text{ is a maximal acyclic generator set } G\}$$

$\square$

Example 3.1: Let $G = \{(i,j),(i,k),(j,l),(k,l),(l,i),(l,n),(m,n)\}$ be a generator set (see Figure 4). Then we have that:

$\mathbf{mcp}(G) = \{\{(i,j),(j,l),(l,i)\},\{(i,k),(k,l),(l,i)\}\}$.

$\mathbf{mags}(G) = \{G_1, G_2, G_3, G_4, G_5\}$ where:

$$\begin{aligned}
G_1 &= \{(i,j),(i,k),(j,l),(k,l),(l,n),(m,n)\},\\
G_2 &= \{(j,l),(k,l),(l,i),(l,n),(m,n)\},\\
G_3 &= \{(i,j),(i,k),(l,i),(l,n),(m,n)\},\\
G_4 &= \{(i,k),(j,l),(l,i),(l,n),(m,n)\},\\
G_5 &= \{(i,j),(k,l),(l,i),(l,n),(m,n)\}.
\end{aligned}$$

$\square$

We shall define two possible constructions for the consolidation operator.

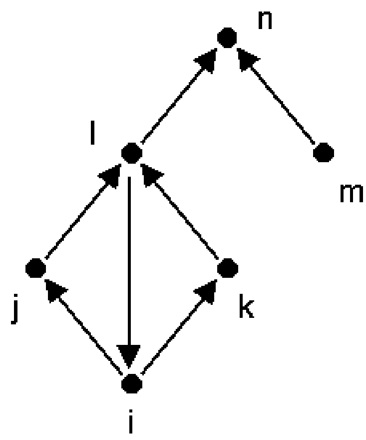

Figure 4: A graph representation of G.

- Kernel Consolidations: this type of operation is based on eliminating the "least prefered" arcs of each *minimal cyclic path*.
- Partial Meet Consolidations: this type of operation is based on intersecting the "most prefered" *maximal acyclic generator sets*

Now, their remains the matter of defining this selection mechanism which allows us to prefer one arc over another. We can define two preference criteria:

1. **Qualitative**: in which we rely on additional information that tells us which arcs of every *minimal cyclic path* are the worst or, in the case of *maximal acyclic generator sets*, which of these are the best.
2. **Quantitative**: in which we seek to minimize or maximize the *number* of arcs that remain in the *minimal acyclic path* or the *maximal acyclic generator sets* respectively.

In the case of a qualitative criterion, we must define the corresponding selection functions and present the necessary algorithms making use of said functions. In the quantitative case, we must be more detailed in the algorithm's definition, but we need not have any additional information concerning arc value.

3.3 Kernel and Partial Meet Consolidation

We will define kernel and partial meet consolidation operators. Let us define the necessary selection mechanisms for each of these.

Definition 3.6: Let G be a generator set. We say that σ is an *incision function* for G if and only if it holds that:

1. $\sigma(\mathbf{mcp}(G)) \subseteq \cup(\mathbf{mcp}(G))$.
2. If $H \in \mathbf{mcp}(G)$ then $\sigma(\mathbf{mcp}(G)) \cap H \neq \emptyset$.

$\square$

Observation 3.1: If G does not contain cycles then $\mathbf{mcp}(G) = \{\emptyset\}$. $\square$

Definition 3.7: Let G be a generator set and σ be an incision function for G. The *the kernel consolidation of G based on* σ is defined as $G!_\sigma = G \setminus \sigma(\mathbf{mcp}(G))$. $\square$

Example 3.2: Let G and $\mathbf{mcp}(G)$ be the sets presented in the Example 3.1. Some possible results of kernel consolidation $G!_\sigma$ are:

1. $\{(j, l), (k, l), (l, i), (l, n), (m, n)\}$.
2. $\{(i, k), (j, l), (l, i), (l, n), (m, n)\}$.
3. $\{(i, j), (i, k), (j, l), (k, l), (l, n), (m, n)\}$.
4. $\{(l, i), (l, n), (m, n)\}$.
5. $\{(k, l), (l, n), (m, n)\}$.

$\square$

Definition 3.8: Let G be a generator set. We say that γ is an *selection function* for G if and only if holds that $\emptyset \subset \gamma(\mathbf{mags}(G)) \subseteq \mathbf{mags}(G)$. $\square$

Observation 3.2: For all generator set G holds that $\mathbf{mags}(G) \neq \emptyset$. $\square$

Definition 3.9: Let G be a generator set and γ be an selection function for G. The *the partial meet consolidation of G based on* γ is defined as $G!_\gamma = \cap\gamma(\mathbf{mags}(G))$. $\square$

Example 3.3: Let G and $\mathbf{mags}(G)$ be the sets presented in the Example 3.1. Some possible results of partial meet consolidation $G!_\gamma$ are G_1, G_2, G_3, G_4, G_5, $(G_1 \cap G_2)$, $(G1 \cap G_2 \cap G_3)$, $(G_1 \cap G_2 \cap G_3 \cap G_5)$ or $(G_1 \cap G_2 \cap G_3 \cap G_4 \cap G_5)$. $\square$

Proposition 3.1: Let ! be a kernel consolidation operator. Then ! satisfies *soundness*, *inclusion*, *vacuity* and *core retainment*. $\square$

Proposition 3.2: Let ! be a partial meet consolidation operator. Then ! satisfies *soundness*, *inclusion*, *vacuity* and *relevance*. $\square$

Corolary 3.1: If ! is a partial meet consolidation operator then ! is a kernel consolidation operator. $\square$

4 Non-prioritized revision operator

Suppose that an agent with a generator set G suspects an informant i may be less reliable than another j. The addition of this new pair (i, j) into G may introduce a cycle. For this purpose we define the non-prioritized revision operator $\circ : \mathcal{P}(\mathcal{I}^2) \times \mathcal{P}(\mathcal{I}^2) \longrightarrow \mathcal{P}(\mathcal{I}^2)$. The basic task of the $\circ$ operator is, given a generator set G and a path from an informant i to another j to produce a new generator set, noted $G \circ \mathit{path}(i, j)$, in which the relation (i, j) could be entailed or not. The decision will depend on whether, in this particular case, preference is given to the pre-existing or to the newly acquired knowledge.

4.1 Postulates for non-prioritized revision operator

We will present different postulates to be satisfied by a non-prioritized revision operator.

• **Soundness**: $(G \circ path(i,j))^*$ is sound.

It would be convenient if the generator set were also modified, when necessary, so that the revised generator set is sound.

• **Inclusion**: $(G \circ path(i,j))^* \subseteq (G \oplus path(i,j))^*$.

Here, we say that a non-prioritized revision is included in a generalized expansion.

• **Congruence**: If $G \oplus path(i_1,j_1) = G \oplus path(i_2,j_2)$ then $G \circ path(i_1,j_1) = G \circ path(i_2,j_2)$.

Here, we say that if the expanded sets are equal then the respective non-prioritized revised sets are equal too.

• **Relative Success**: $(G \circ path(i,j)) = G$ or $(i,j) \in (G \circ path(i,j))^*$.

Relative Success states that (i,j) is included in the revised generator set or there are no changes in it.

• **Disjunctive Success**: $(i,j) \in (G \circ path(i,j))^*$ or $(j,i) \in (G \circ path(i,j))^*$.

Here we say that (i,j) or (j,i) is included in the revised generator set.

• **Path Irrelevance**: If $(G \cup path(i_1,j_1))^* = (G \cup path(i_2,j_2))^*$ then $(G \circ path(i_1,j_1))^* = (G \circ path(i_2,j_2))^*$.

4.2 Construction of non-prioritized operator revision

We define the non-prioritized revision operator using two steps:

1. Add the all the pairs in the path connecting i and j.
2. Consolidate the resulting set.

Notice that there is an intermediate state in which the generator set may be inconsistent.

Definition 4.1: Let G be a generator set, $path(i,j)$ be a path connecting i and j, and ! be a consolidation operator. The *non-prioritized revision* of G by $path(i,j)$, noted by $G \circ path(i,j)$, is defined as:

$$G \circ path(i,j) = (G \oplus path(i,j))!$$

□

Proposition 4.1: Let $\circ$ be a non-prioritized revision operator constructed as described above. Then $\circ$ satisfies *soundness*, *inclusion* and *congruence*. □

Note that $\circ$, in general, does not satisfy relative success, disjunctive success nor path irrelevance.

5 Conclusions and Future Work

We have presented a means for consolidating plausibility relations among informants in the context of a multi-agent system. Using the operator provided an agent can resolve any and all inconsistencies to be found in said relation. This operator was characterized through postulates and alternative constructions were presented and discussed.

Based on the new consolidation operator, an operator for non-prioritized revision was introduced. This operator allows for an agent to tentatively add to its plausibility relation. The new relation will contain a sound mix of the new and previously existing information.

Clearly, what follows is to devise ways of handling the perception of changing plausibilities in real sources of information. Such is the case of the weather forecasting systems and predictors of stock market behavior mentioned in this paper's introduction. From these examples, and others, we will seek to understand the complexities of dynamic updating in decision making and advising systems.

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